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PROBABILISTIC FORECASTING OF RENEWABLE ENERGY PRODUCTION AND ELECTRICITY LOAD USING HOURLY GAUSSIAN COPULAS ENSEMBLE FORECAST (GCH-EN)

Summary

This paper introduces a multivariate probabilistic model called GCH-En (Gaussian Hourly Copula - Ensemble) designed to jointly forecast photovoltaic (PV) generation, electricity demand, and wind speed over short-term horizons ranging from 10 minutes to 12 hours ahead. The proposed approach relies on constructing distinct Gaussian copulas for each combination of time of day and meteorological season. We propose a multivariate approach to explicitly capture the cross-dependence between the target variables. Marginal distributions are estimated non-parametrically, and probabilistic forecasts are generated through conditional sampling based on a Cholesky decomposition of the copula. The model is evaluated using historical data and compared against both a multivariate benchmark model (MuPEn) and standard univariate models, including XGBoost and Quantile Random Forests. In terms of nCRPS, the GCH-En yields highly competitive marginal forecasts compared to univariate models. Moreover, its fundamental advantage lies in modelling joint distributions. As evidenced by the Energy and Variogram scores, GCH-En distinctly outperforms all benchmarks in capturing complex multivariate dependencies, particularly for lead times beyond two hours. Ultimately, these results demonstrate that explicitly modelling time-dependent structural dependencies substantially enhances the reliability of joint scenarios in practice.

Keywords: probabilistic forecasting, Gaussian copula, renewable energy, ensemble forecast, multivariate dependence, CRPS, Energy-Score, Variogram-Score

Field of application for the findings: Power system management, renewable energy integration, smart grids

Addressed audience: Researchers and practitioners in solar energy forecasting, grid operators

1. Introduction

As deterministic forecasts are no longer sufficient, probabilistic approaches, which allow the estimation of the inherent uncertainty associated with a prediction, are increasingly used in the Variable Renewable Energy (VRE) community (Gneiting and Raftery, 2007). However, most of these models remain univariate (predicting each variable separately) rather than multivariate (modelling the joint dynamics of multiple variables simultaneously). Treating interconnected variables independently ignores their shared meteorological drivers, which often leads to unrealistic joint scenarios in operational settings. Multivariate copulas address this limitation by explicitly modelling cross-dependencies (Schefzik et al., 2013). In this paper, we propose the GCH-En (Gaussian Copula Hourly – Ensemble) model, extending this framework with time-of-day and seasonal stratification to capture these complex joint dynamics more accurately.

2. Methodology

The GCH-En model is designed to capture the joint distribution of present observations and future values across multiple forecast horizons, while accounting for daily and seasonal patterns in dependency structures.

2.1 Joint data construction and seasonal stratification

Let $\mathbf{x}_t \in \mathbb{R}^{d_x}$ denote the vector of observed variables at time t, composed of the target variables (PV production, electricity demand, and wind speed) and exogenous meteorological predictors, specifically surface temperature and relative humidity. Let $\mathbf{y}_t = [\mathbf{x}_{t+h_1}, \ldots, \mathbf{x}_{t+h_K}] \in \mathbb{R}^{d_y}$ represent the concatenation of future target values over

K forecast horizons. We assign the joint vector $\mathbf{w}_t = [\mathbf{x}_t, \mathbf{y}_t] \in \mathbb{R}^d$ (with $d = d_x + d_y$) to a specific subset indexed by a temporal key combining the time of the day (HH:MM) and the season.

This stratification produces a distinct subset of joint vectors for each (hour, season) combination, ensuring that the learned dependency structures are specific to the prevailing diurnal and seasonal patterns.

2.2 Non-parametric marginal estimation

Within each subset, the empirical CDF $\hat{F}_j$ of each variable j is computed from the N measurements of the training dataset:

$$\hat{F}_j(v) = \frac{1}{N+1} \sum_{i=1}^{N} \mathbf{1}\{w_{i,j} \leq v\} \tag{1}$$

2.3 Gaussian space transformation and conditional sampling

Each marginal is transformed into the Gaussian space using the probit transform:

$$z_{i,j} = \Phi^{-1}\big(\hat{F}_j(w_{i,j})\big), \quad \forall i, j \tag{2}$$

The dependence structure is then represented by a Gaussian copula characterised by its correlation matrix, partitioned into observed variables ($\mathbf{z}_x$) and target variables ($\mathbf{z}_y$), following standard copula-based approaches (Schefzik et al., 2013).

At forecasting time, a new observation $\mathbf{x}_{\text{obs}}$ is mapped to the Gaussian space. The conditional distribution of the target variables is a multivariate Gaussian with parameters $\mu_{y|x}$ and $\Sigma_{y|x}$ (Bishop, 2006). An ensemble of size m is generated through:

$$\mathbf{z}_y^{(k)} = \mu_{y|x} + \mathbf{L}\,\varepsilon^{(k)}, \quad \varepsilon^{(k)} \sim \mathscr{N}(\mathbf{0}, \mathbf{I}_{d_y}), \quad k = 1, \ldots, m \tag{3}$$

Where $\mathbf{L}$ is obtained from the Cholesky decomposition of $\Sigma_{y|x}$. The simulated values are then mapped back to the original space using the inverse marginal transformations.

2.4 Data and evaluation

The model is evaluated using data from the Terre-Sainte campus in La Réunion, which provides a representative case of a highly variable tropical environment, including high-resolution measurements of both power grid dynamics and local weather conditions. The dataset has a temporal resolution of 10 minutes, with one year used for training and another for testing.

Forecast performance is assessed using the Continuous Ranked Probability Score (CRPS) and its decomposition into reliability and resolution (Gneiting and Raftery, 2007; Lauret et al., 2019). To evaluate the multivariate dependencies, the Energy-Score (ES) (Gneiting et al., 2008) and Variogram-Score (VS) (Scheuerer and Hamill, 2015) are also computed. The CRPS is normalised by the mean observed values (nCRPS) to facilitate comparison, whereas for the calculation of the ES and VS, each variable is scaled using min-max normalisation.

We compare GCH-En against:

- MuPEn (Multivariate benchmark), proposed by Van Der Meer (2021)
- Gradient Boosting Decision Trees (XGBoost)
- Quantile Random Forest (QRF)

3. Results

GCH-En consistently outperforms the multivariate benchmark (MuPEn) across all metrics. For marginal performance (nCRPS), it rivals univariate models (QRF, XGBoost), matching or surpassing them at longer horizons (e.g., wind speed), despite XGBoost's slight edge at 10 minutes forecasts.

Crucially, GCH-En excels at modelling joint distributions (ES, VS). By explicitly capturing cross-variable and temporal dependencies, it distinctly dominates all benchmarks for lead times beyond 2 hours, where univariate models rapidly degrade. Furthermore, unlike these black-box machine learning algorithms, GCH-En acts as a transparent white-box model, offering fully interpretable dependency structures and thus providing the essential explainability required for operational decision-making by grid operators.

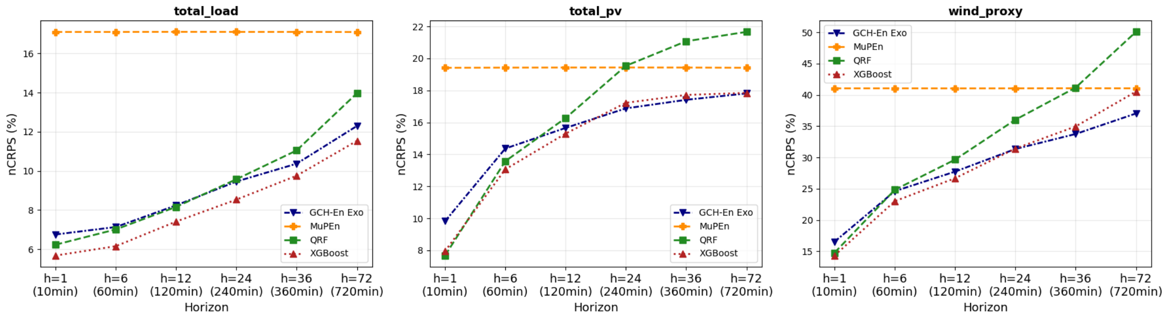

Fig. 1: Comparison of probabilistic models — nCRPS (% of the mean value) for total load, PV production, and wind speed across all forecast horizons.

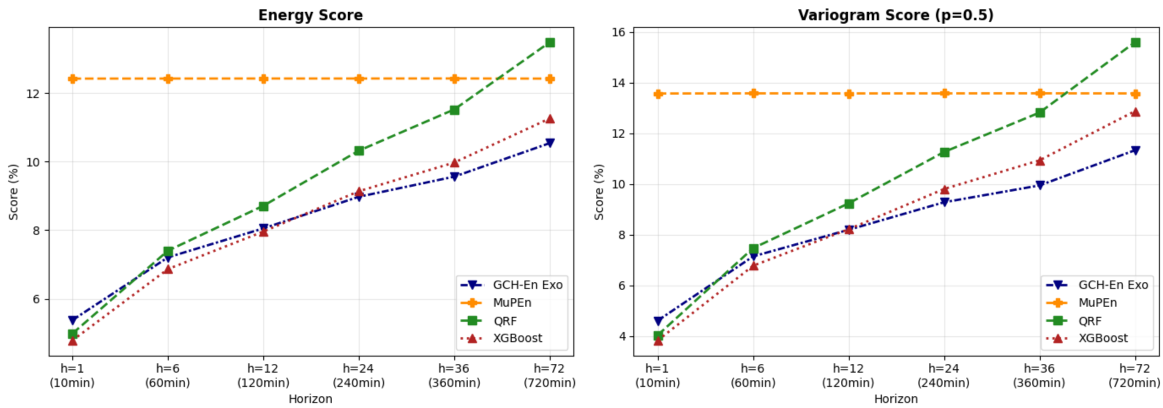

Fig. 2: Comparison of probabilistic models — Energy-Score (left) and Variogram-Score (right) for all horizons.

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