EXPERIMENTAL AND COMPUTATIONAL STUDY OF MULTIDIRECTIONAL GLASS/EPOXY LAMINATES SUBJECTED TO MULTIAXIAL LOADING

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Abstract

A nonlinear constitutive model based on non-associative plasticity for unidirectional (UD) composites subjected to multiaxial in-plane loading is proposed. The model can be calibrated using a relatively simple biaxial experiment based on a novel Modified Arcan Fixture (MAF) and Digital Image Correlation (DIC). The 'plasticity' model accounts for the cumulated nonlinear effects arising from plasticity in the resin, micro cracking and geometric nonlinearity. The UD composite is assumed to be transversely isotropic with negligible nonlinearity in the fibre direction. A Drucker-Prager type yield function is used to account for the nonlinear, pressure dependent response in transverse and shear loading and a non-associative flow rule is proposed. Model predictions show good agreement with nonlinear experimental transverse and shear stress-strain curves. Thus, the model can be used to analyse the nonlinear stress/strain states in a multidirectional laminate made from UD composites.

1. Introduction

Multidirectional composite laminates are widely used across many industrial sectors due to their ability to be tailored/optimised with respect to their mechanical properties to meet design requirements. Often unidirectional (UD) fibre reinforced polymer composites (e.g. glass/epoxy or carbon/epoxy) are used in multidirectional laminates due to their high relative stiffness and strength [1]. When multidirectional laminates are subjected to even just a simple uniaxial state of stress, the individual UD plies are subjected to a multiaxial state of stress. UD composites are anisotropic and exhibit nonlinear and pressure sensitive behaviour when subjected to multiaxial stress states especially in shear and transverse compression [2,3]. To predict ply-by-ply stress and strain states in a multidirectional laminate accurately, it is therefore crucial to account for the nonlinear, pressure sensitive constitutive response of the UD plies. The nonlinear constitutive behaviour is due to a combination of physical effects, including plasticity, micro-cracking and geometric nonlinearities (e.g. fibre rotation) [4]. The modelling framework of plasticity can be used conveniently to account cumulatively for these effects [4-8].

Often UD composites are assumed to be transversely isotropic [9,10] with linear elastic fibres and the nonlinear transverse and shear responses accounted for with a Drucker-Prager type yield function [11]. Simple 'plasticity' models [4-6] assume associative flow, where the direction of the 'plastic' strain tensor is the gradient of the yield function, f. More complex models [7,8], use non-associative flow where the 'plastic' strain tensor is related to the gradient of a plastic potential function g which is different from *f*. All 'plasticity' models require reliable multiaxial data for model calibration which for composites is not readily available and can only be obtained from challenging experiments [7,12].

The overarching aim of the research described in the paper is to develop a material model that accounts for the nonlinear, pressure sensitive behaviour of UD composites, which can be calibrated using relatively simple tests. A novel procedure based on a Modified Arcan Fixture (MAF [13]) and Digital Image Correlation (DIC) is proposed which allows the characterisation of materials in the full combined transverse tension/compression and shear stress domain using a single test fixture and a single specimen geometry. The compression/shear domain, which cannot be tested using the conventional Arcan rig [14], is crucial because this is where the nonlinear, pressure sensitive behaviour of UD composites is most significant [2,3]. The proposed model was calibrated for RP-528 glass/epoxy [15] using the MAF and strain data obtained from DIC.

2. Biaxial MAF Rig and Experimental Set-up

A front view of the Modified Arcan Fixture (MAF) is shown in (Fig. 1a), where the biaxial stress state is defined by the loading angle α, which can be adjusted from pure tension over combined tension/shear, pure shear, combined compression/shear to pure compression. The test specimen takes the form of a 'butterfly specimen' as shown in Fig. 1b, where the fibres of the unidirectional laminate (1- direction) are aligned in parallel to the waisted gauge section. The butterfly geometry was chosen to promote controlled failure at the waisted gauge section. Digital image correlation (DIC) is used on both sides of the specimens to measure the transverse and shear stress strain fields.

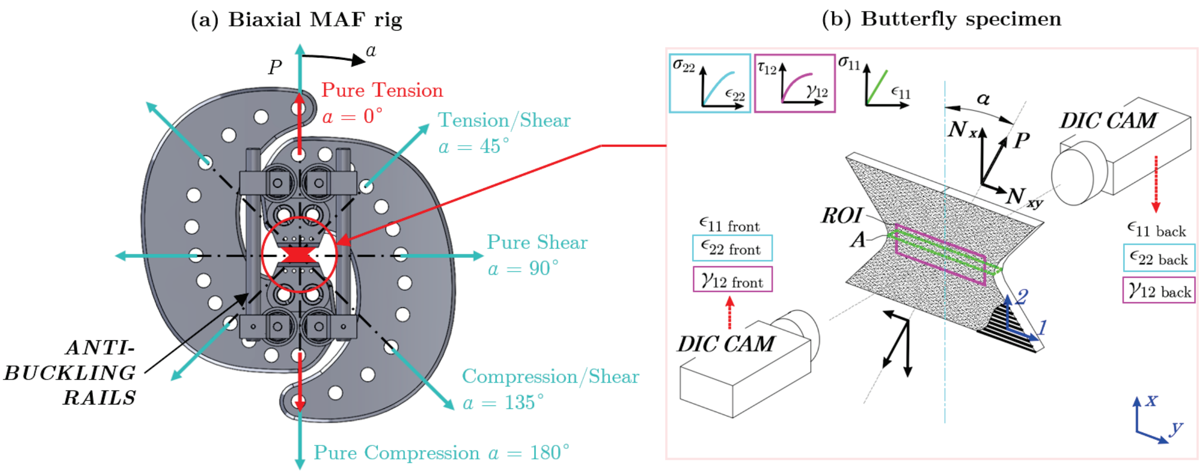

Figure 1. (a) The biaxial MAF rig and (b) the butterfly specimen and the front and back DIC system set-up measuring the nonlinear transverse and shear strains.

The averaged biaxial stress state in the gauge section, comprising of transverse normal stress $\sigma_{22\ \mathrm{AVG}}$ and shear stress $\tau_{12\ \mathrm{AVG}}$, is given by α, the applied force P and the gauge area A, defined in (Fig. 1), as follows:

$$\sigma_{22\ \mathrm{AVG}} = \frac{Nx}{A} = \frac{P}{A}\cos(\alpha) \tag{1}$$

$$\tau_{12\ \mathrm{AVG}} = \frac{Nxy}{A}\frac{P}{A}\sin(\alpha) \tag{2}$$

The strains are averaged at the gauge section over the region of interest (ROI) on both sides of the specimen as shown in (Fig. 1b). The transverse $\epsilon_{22\ \mathrm{AVG}}$ and shear $\gamma_{12\ \mathrm{AVG}}$ strains associated to the stresses in (Eq. 1-2) to construct the stress-strain curves were then averaged on the front and backside as:

$$\epsilon_{22\,\mathrm{AVG}} = 1/2\,(\epsilon_{22\,\mathrm{front}} + \epsilon_{22\,\mathrm{back}}) \tag{3}$$

$$\gamma_{12\,\mathrm{AVG}} = 1/2\,(\gamma_{12\,\mathrm{front}} + \gamma_{12\,\mathrm{back}}) \tag{4}$$

3. Proposed non-associative 'plasticity' model

The 'plasticity' model is based on [4-6] but uses a non-associative flow rule instead, which avoids the prediction of non-physical 'plastic' strains. The total strain is assumed to be composed of an elastic and 'plastic' component [16]:

$$\epsilon = \epsilon^e + \epsilon^p \tag{5}$$

where '*e*' denotes elastic and '*p*' plastic. A Drucker-Prager type [11] yield function f is proposed for plane stress:

$$f(\sigma_{ij}) = \sqrt{H\sigma_{22}{}^2 + \tau_{12}{}^2} + J\sigma_{22} \tag{6}$$

where i,j = 1, 2, 3 and H and J are yield coefficients. The effective 'plastic' yield stress is defined as:

$$\bar{\sigma} = f(\sigma_{ij}) \tag{7}$$

A non-associative plastic flow potential g (*i.e.* plastic flow potential $g \neq$ yield function f) is proposed and defined similarly to the yield function in (Eq. 6), but dropping the pressure sensitive term, *i.e.*:

$$\mathrm{g}(\sigma_{ij}) = \sqrt{H\sigma_{22}{}^2 + \tau_{12}{}^2} \tag{8}$$

The 'plastic' strain increment $d\epsilon_{ij}^p$ is defined as the gradient of the flow potential g scaled by the 'plastic' multiplier $d\lambda$:

$$d\epsilon_{ij}^p = \frac{\partial g(\sigma_{ij})}{\partial \sigma_{ij}} d\lambda \tag{9}$$

Using the definition for the effective stress $\bar{\sigma}$ in (Eq. 7) and the equivalence of incremental 'plastic' work per unit volume [6]:

$$dW_p = \sigma_{ij} d\epsilon_{ij}^p = \bar{\sigma} d\bar{\epsilon}_p \tag{10}$$

where $d\bar{\epsilon}_p$ is the increment of the effective 'plastic' strain, and repeating indices imply Einstein summation, the 'plastic' multiplier $d\lambda$ can be found by substituting the 'plastic' flow rule in Eq. (9) into Eq. (10):

$$\sigma_{ij} \frac{\partial g(\sigma_{ij})}{\partial \sigma_{ij}} d\lambda = \bar{\sigma} d\bar{\epsilon}_p \Rightarrow g(\sigma_{ij}) d\lambda = f(\sigma_{ij}) d\bar{\epsilon}_p \Rightarrow d\lambda \frac{f(\sigma_{ij})}{g(\sigma_{ij})} d\bar{\epsilon}_p \tag{11}$$

Substituting the plastic multiplier in (Eq. 11) into the flow rule in (Eq. 9), the 'plastic' strain increment is obtained:

$$d\epsilon_{ij}^p = \frac{\partial g(\sigma_{ij})}{\partial \sigma_{ij}} d\lambda = \frac{\partial g(\sigma_{ij})}{\partial \sigma_{ij}} \frac{f(\sigma_{ij})}{g(\sigma_{ij})} d\bar{\epsilon}^p \tag{12}$$

The effective 'plastic' stress-strain curve (or hardening curve) relating (Eq. 7) and (Eq. 12) can be approximated by [17]:

$$\overline{\epsilon}^p = (\frac{\overline{\sigma}}{K})^{1/m} \tag{13}$$

In summary, two yield (H, J) and two hardening coefficients (K, m) have to be calibrated in a biaxial experiment. Therefore, a cost function is defined as the vector of the differences between all the effective 'plastic' strains extracted from the experimental stress-strain curves at all tested loading angles α according to (Eqs. 1-12) and the effective 'plastic' strains given by the hardening rule in (Eq. 13). The optimised 'plastic' coefficients are found by minimizing the least-squares error of the cost function. In other words, the model is calibrated when the effective-plastic stress-strain curves as a function of H and J collapse into the hardening curve described by Eq. (6). For RP-528 glass/epoxy the optimised coefficients are $H = 0.19$, $J = 0.069$, $K = 105.343$ and $m = 0.174$.

4. Preliminary Results

Using the calibrated analytical model (Eqs. 5-13), the transverse and shear stress-strain curves under compression/shear loading were predicted and compared against experimental data in (Fig. 2). For brevity, only the compressive domain is shown where the most significant nonlinear effects are observed. Overall, the analytical model predictions and the experimental data show good agreement.

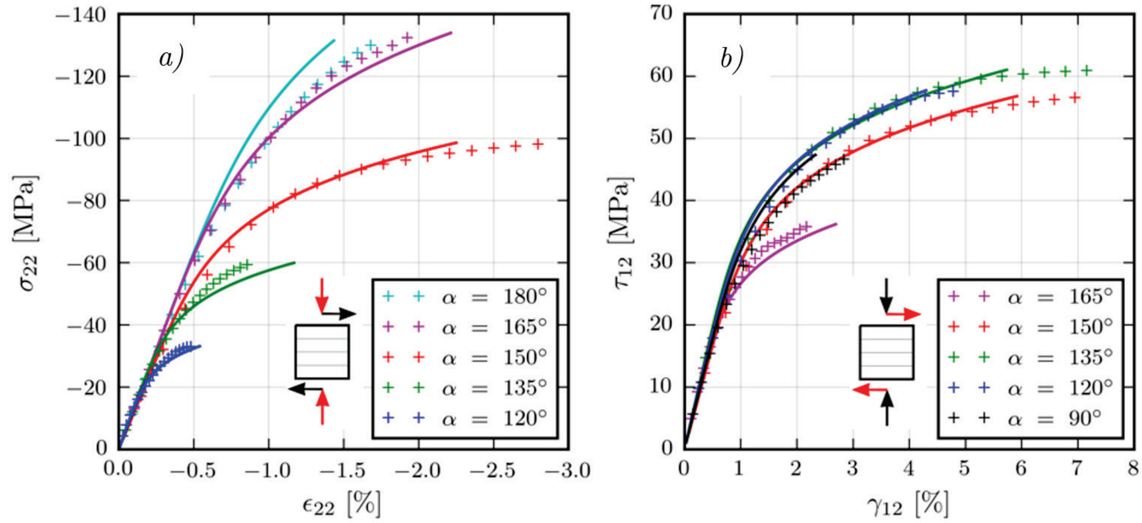

Figure 2. Non-associative model predictions (solid line) in comparison with experimental transverse (a) and shear (b) stress-strain curves (+ markers) under compression/shear loading.

5. Conclusions

A simple nonlinear constitutive model based on non-associative 'plasticity' has been developed for UD composites subjected to multiaxial in-plane loading. It is demonstrated that the model parameters (H, J, K, m) can be calibrated using biaxial experimental data obtained from the MAF/DIC procedure. Comparison of model predictions to the experimental stress-strain curves have shown that the proposed non-associative 'plasticity' model can predict the nonlinear pressure dependent constitutive behaviour of UD composites with good accuracy. The material model can therefore be implemented as a user defined material subroutine in a commercial FE software, which can potentially be used to investigate the nonlinear multiaxial stress states in a multidirectional laminate of a critical composite component or structure.

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References

[1] I. M. Daniel. *Engineering mechanics of composite materials*. Oxford University Press, Oxford, 2005.

[2] E. S. Shin and K. D. Pae. Effects of hydrostatic-pressure on the torsional shear behavior of graphite epoxy composites. *J. Compos. Mater.*, 26:462–485, 1992.

[3] P. J. Hine, R. A. Duckett, A. S. Kaddour, M. J. Hinton, and G. M. Wells. The effect of hydrostatic pressure on the mechanical properties of glass fibre/epoxy unidirectional composites. *Compos. Part A Appl. Sci. Manuf.*, 36:279–289, 2005.

[4] T. Yokozeki, S. Ogihara, S. Yoshida, and T. Ogasawara. Simple constitutive model for nonlinear response of fiber-reinforced composites with loading-directional dependence. *Compos. Sci. Technol.*, 67:111–118, 2007.

[5] C. T. Sun and J. L. Chen: A simple flow rule for characterizing nonlinear behavior of fiber composites. *J. Compos. Mater.*, 23:1009–1020, 2002.

[6] K. W. Gan, M. R. Wisnom, S. R. Hallett, and G. Allegri. A simple plasticity model for predicting transverse composite response and failure. *Proceedings of the 19th international conference on composite materials ICCM-19*, Montréal, Canada 2013.

[7] M. Vogler, R. Rolfes, and P. P. Camanho. Modelling the inelastic deformation and fracture of polymer composites-Part I: Plasticity model. *Mech. Mater.*, 59:50–64, 2013.

[8] G. M. Vyas, S. T. Pinho, and P. Robinson. Constitutive modelling of fibre-reinforced composites with unidirectional plies using a plasticity-based approach. *Compos. Sci. Technol.*, 71:1068–1074, 2011.

[9] J. P. Boehler and A. Sawaczuk. Application of representation theorems to describe yielding of transversely isotropic solids. *Mech. Res. Comun.*, 3:277–283, 1976.

[10] A. J. M. Spencer. Plasticity theory for fibre-reinforced composites. *J. Eng. Math.*, 26:107–118, 1992.

[11] D. C. Drucker and W. Prager. Soil mechanics and plastic analysis or limit design. *Q. Appl. Math.*, 10:157–165, 1952.

[12] M.J. Hinton, A.S. Kaddour, P.D. Soden. A comparison of the predictive capabilities of current failure theories for composite laminates, judged against experimental evidence. *Compos. Sci. Technol.*, 62:1725-1797, 2002.

[13] K. W. Gan, T. Laux, S. T. Taher, J. M. Dulieu-Barton, and O. T. Thomsen. A novel fixture for determining the tension/compression-shear failure envelope of multidirectional composite laminates. *Compos. Struct.*, 184:662–673, 2018.

[14] M. Arcan, Z. Hashin, A. Voloshin. A method to produce uniform plane-stress states with applications to fibre-reinforced composites, *Experimental Mechanics*, 18:141-146, 1977.

[15] PRF Composites: Product data prepreg systems RP-528. [Online]. Available: http://www.prfcomposites.com/. [Accessed: 20-Sep-2016].

[16] E.A. de Souza Neto, D. Peric, D.R.J. Owen. *Computational methods for plasticity – Theory and Applications*. John Wiley & Sons Ltd, Chichester, West Sussex, 2008

[17] W. Ramberg and W. Osgood. Description of stress-strain curves by three parameters, *NACA Technical Note No. 903*, 1943.