Rate-Dependent Modelling of the Meso-Mechanics of Z-Pins Bridging Mixed Mode Delaminations

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Abstract

In this paper, a rate dependent model is developed to simulate the bridging performance of z-pins subject to Mode I and Mode II loading. Z-pins are modelled as beams embedded in a Winkler elastic foundation with friction. The latter is described via Ruina's state dependent friction model in order to incorporate the rate effect. The Ruina's model parameters were calibrated against a Mode I and Mode II test carried out at a pullout speed of 5m/s and 1m/s respectively. The calibrated model was then validated against a mode I and mode II test conducted at 12m/s and 1.5m/s respectively. The predicted force versus opening-displacement trend is in excellent agreement with the experimental one. The error of predicted apparent fracture toughness is within 5% of the experimental value.

1. Introduction

1.1. Motivation

Carbon Fibre Reinforce Composites (CFRP) are widely used in primary aerospace structures. However, the mechanical behaviour of ply interfaces is dominated by polymeric matrix, which is relatively weak (Ref. [1]). Thus Laminated CFRP structures are susceptible to delamination, particularly as a consequence of impact loading. Through thickness reinforcement (TTR) enhances the delamination resistance properties of laminated CFRP composites.

Z-pinning is a form of TTR that consists in the orthogonal insertion of either fibrous or metallic rods into CFRP laminates (Ref. [2]). The use of z-pins has been shown to yield a 20 fold increase in Mode I apparent interlaminar fracture toughness of pre-preg based laminates (Ref. [3-6]). In Mode II, z-pin leads to a 4 fold increase in apparent fracture toughness when compared to laminates without TTR (Ref. [7]). The use of z-pins in low-speed impact scenarios reduced delaminated area by upto 64%; this leads to a 4 fold increase in compression-after-impact strength (CAI) (Ref. [8]).

Nonetheless, the literature addressing the performance of z-pinned composites under high-velocity impacts is quite scarce.

1.2. Background

The analysis of laminated composites with TTR is usually carried out via a multi-scale modelling approach, which generally involves two different meso-mechanical levels **(**Ref. [9]**)**. Meso-level 1 refers to the characterization and prediction of the mechanical response of single z-pin. The bridging behaviour of an array of TTR (multiple z-pins) will be denoted as meso-level 2. The transition from meso-level 1 to meso-level 2 requires the introduction of a suitable homogenization scheme, in order to represent the behaviour of multiple interacting z-pins in a "smeared" fashion over a bridged interface. In finite element analysis (FEA), the latter can be accomplished via the introduction of a bridging-dependent cohesive zone represented by zero-thickness interface elements. This paper is focussed on meso-level 1, i.e. "single" z-pin, modelling.

The bridging performance of z-pins have been predicted by utilizing "high fidelity" simulation strategies, based on 2D and 3D Finite Element Analysis (FEA). Meo et al (Ref. [10]) addressed the simulation of mode I pullout tests. Cui et al. presented a more advance 2D plane-strain model in (Ref. [11]) which investigated the z-pin bridging performance under mixed-mode loading. In this approach, cohesive elements were used to model the debonding between the TTR rod and the embedding laminate, as well as the internal splitting of the z-pin. A full 3D model was introduced by Zhang et. al (Ref. [12]) in order to simulate the z-pin pull-out and progressive failure while accounting for the the effects of thermal residual stresses due to post-cure cool down.

Although the FEA approach provide a comprehensive description of the complex behaviour of z-pins, the associated computational cost is too high for concurrent runs at meso-level 2 and, most importantly, at the macro-scale. Therefore, "reduced order" approaches proposed based on a 1D idealization of the TTR elements have been proposed (Ref. [13-19]). Cox (Ref. [13]) presented a 1D model to describe the traction-displacement response of a TTR element subject to pure mode II, where the interaction between the z-pin and the laminate is described as that of a rigid punch ploughing through a perfectly plastic medium. Such an assumption might only be valid for a unidirectional laminate, where the z-pin can plough through the resin eyelet that surrounds it. Cox model was further extended in (Ref. [14,15]) to mixed-mode and mode I scenarios. However, a perfectly plastic response of the TTR in shear was assumed, wheareas fibrous z-pins typically exhibit brittle failure. Cox also recognised the key role that the friction enhacement due to "snubbing" has on the z-pin response. Plain and Tong (Ref. [20]) proposed a model for describing the bridging performance of metallic z-pins, where the TTR was assumed to behave as an Euler-Bernoulli beam embedded in elastic foundation. A similar approach was adopted by Bianchi and Zhang (Ref.[18]) to model single z-pin behaviour in pure mode II. Allegri et al. (Ref. [19]) proposed a mixed-mode model of z-pin bridging performance based on geometrically nonlinear beam theory coupled with Weibull's strength theory to predict the TTR failure.

These reduced-order approaches are orders of magnitude more computationally efficient than meso-level 1 FEA, while providing a reasonable level of accuracy in terms of bridging performance prediction. Nonethless, all the aforementioned reduced order models have been introduced for quasi-static loading.

1.3. Paper Overview

This paper focusses on the introduction of a meso-level 1 model describing the rate-dependent response of single z-pins when bridging mode I and II interlaminar cracks. Our approach is based on describing z-pins as a Timoshenko peridynamic beams embedded in an elastic foundation and experiencing rate dependent firctional forces. The aim is to develop and validate a reduced-order model which is suitable for application to high-rate scenarios.

2. Model Formulation

2.1. Problem Statement

A z-pin of length L is orthogonally inserted into a composite laminate. The composite laminate is split into two sub-laminates by a delamination. The interlaminar fracture plane intersects the z-pin at a known depth, cutting the TTR rod into two segments (i.e. "lower" and "upper") having lengths L^- and L^+, respectively. This is illustrated in Figure 1. An insertion asymmetry parameter α is introduced as:

$$\alpha = \frac{L^-}{L^- + L^+} \tag{1}$$

Therefore, for a mid-plane delamination, one has α=1/2. The z-pin counteracts the opening and sliding displacement by exerting bridging forces on the delamination surfaces. The z-pin is subject to two main sets of distributed forces: 1) friction in the axial direction; 2) a lateral elastic foundation force in the transverse direction. The experimental evidence suggests that, the z-pin experiences completely pullout in mode I, whereas in mode II the TTR rod fails at a characteristic sliding displacement.

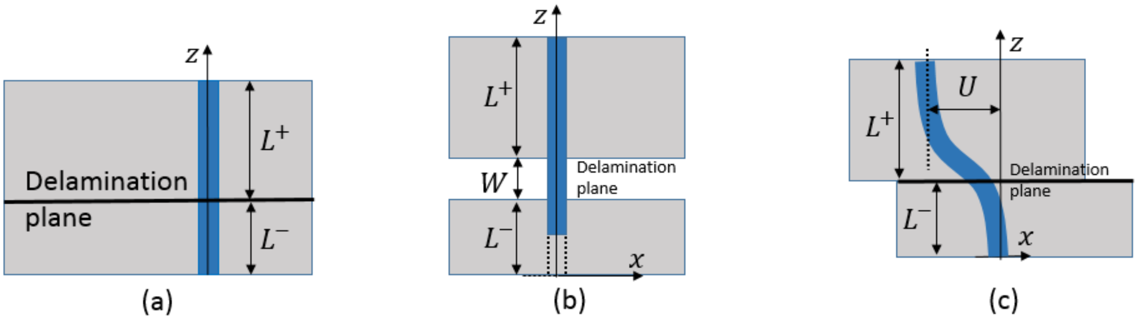

Figure 1: Bridging Kinematics of z-pins. (a) Reference mode, (b) pure mode I, and (c) pure Mode II.

2.2. Model Formulation

Taking into account external, inertial and damping forces, the dynamic equilibrium equations for a Timoshenko peridynamic beam experiencing moderate rotations are:

$$\begin{aligned}\rho A\ddot{w} &= c_a \int_{H_z} S_w dz' - c_s \int_{H_z} S_\gamma dz' + p - q\theta \\ \rho A\ddot{u} &= c_s \int_{H_z} S_\gamma dz' + c_a \int_{H_z} S_w dz' + q + p\theta \\ \rho A\ddot{\theta} &= c_b \int_{H_z} S_\theta + \frac{1}{2} c_s \int_{H_z} S_\gamma (z'-z) dz' + \frac{1}{2} c_a \int_{H_z} S_w (\theta' - \theta)(z'-z) dz'\end{aligned} \tag{2}$$

$$Where \quad S_w = \frac{w'-w}{|z'-z|}, \quad S_\gamma = [\tfrac{u'-u}{|z'-z|} - \tfrac{1}{2}(\theta' + \theta)(z'-z)], \quad S_\theta = \frac{\theta'-\theta}{|z'-z|}$$

Where, w, u and θ referes to the axial extension, lateral defelection and cross section rotation respectively. z is the coordinate of a peridynamic particle along the beam. A is the cross sectional area of the z-pin and ρ its density. The coefficients b_w, b_u and b_θ represent the axial, lateral and rotational damping. Finally, c_a, c_s and c_b are the peridynamic axial, shear and bending coefficient.

The lateral force q is modelled using a simple Winkler foundation formulation, as shown in equation (3) below, where, k_x is the laminate punch resistance and u_{rel} is the relative lateral displacement between the z-pin and the laminate. In the cuurent problem definition, since the lower laminate is assumed fixed, then the relative displacement in the lower and upper laminates is given respectively by: ${u_{rel}}^{-} = u$ and ${u_{rel}}^{+} = u + U$.

$$q = -k_x u_{rel} \tag{3}$$

The axial force p acting along z-pin axis is described using the Ruina's state-based friction model (Ref. [21]), i.e.

$$\begin{cases} \tau = \sigma(\mu_0 + \theta + A ln\left(\frac{V}{V_c}\right) \\ \frac{d\theta}{dt} = -\frac{V}{l}\left(\theta + B ln\left(\frac{V}{V_c}\right)\right) \end{cases} \tag{4}$$

Where, τ is friction stress; σ is the normal stress; μ_0 is the static friciton coefficient; θ is the state; V is the slip velocity; V_c is a critical cut-off velocity, in this case set as 0.01mm/s which is the velocity used to conduct quasi static testing (Ref. [6]); l is a characteristic length scale; A and B are constants to be determine.

2.3. Numerical Solution

In order to solve the model equation, an explicit centered-difference scheme was employed for the time variable. The integrals in Eqs. (3) were calculated using the trapezoidal rule. In order to ensure stability, the time step was set to the time required for axial waves (i.e. the fastest ones) to travel across half the particle horizon length.

The sliding friction is obtained by solving Eq. (4) using a centred difference scheme for the state variable. Eq. (4) is solved concurrently with Eqs. (3).

3. Model Calibration

The model proposed has been implemented in Matlab for a z-pin with geometrical and mechanical properties presented in Table *1*. Considering the model and the data in Table *1*, there are 6 remaining parameters that need to be estimated. These parameters include the laminate punch resistance (k_x), the static coefficient of friction (μ_0), and the constants *A*, *B* and l for the state-dependent friction model

A genetic algorithm (GA) is used to identify the remaining 6 unknowns. The cost function to be minimised is defined as shown in Eq. (5). In Eq. (5), ε_I^2 and ε_{II}^2 are the relative error at each data point on the force against displacement graph for tests conducted at 5 m/s in mode I and at 1.5m/s in mode II. These tests are described in detail in (Ref. [22]).

$$C = \sqrt{\varepsilon_I^2 + \varepsilon_{II}^2} \tag{5}$$

Table 1. Z-pin Parameters.

$\boldsymbol{E}$(GPa)	$\boldsymbol{\rho}$(kg/mm^3)	L(mm)	D(mm)	$\boldsymbol{\alpha}$
115	1.6e-06	8	0.28	0.5

4. Discussion and Validation

Using the calibrated model parameters, the z-pin bridging response was obtained for Mode I at two different pullout speeds, namely 5m/s and 12m/s. The predicted performance is plotted alongside the experimental results from Ref. [23] in Figure 2. Similarly, the predicted response in mode II at a loading rate of 1.5m/s is shown in Figure 3. It is worth noting that no failure model has been implemented in the current model formulation for mode II. This explains why for relatively large displacements the experimental results show a drop in force, while predicted trend continues to increase. The numerical results for both mode I and mode II are in excellent agreement with the experimental data, proving the effectiveness of the model. In addition, the energy dissipated by the z-pin during briding was calculated for both the experimental tests and the simulated load-versus-displacement curves. The difference between the values of dissipated energies is around 5% for all the cases considered. This proves that the calibrated model provides an excellent prediction capability.

Furthermore, an important consideration is the computational cost of the model. The model runs in under a minute, while a detailed dynamic finite element model of a z-pinned coupon of similar size requires a runtime of the order of days.

The model proposed here can be employed as part of a multi-scale approach, where the solution procedure can be implemented into a user subroutine that provides bridging tractions to a commercial finite element package. In this case the FE package can perform structural analysis on z-pinned laminates at much reduced computational time, while ensuring the fidelity of the simulations.

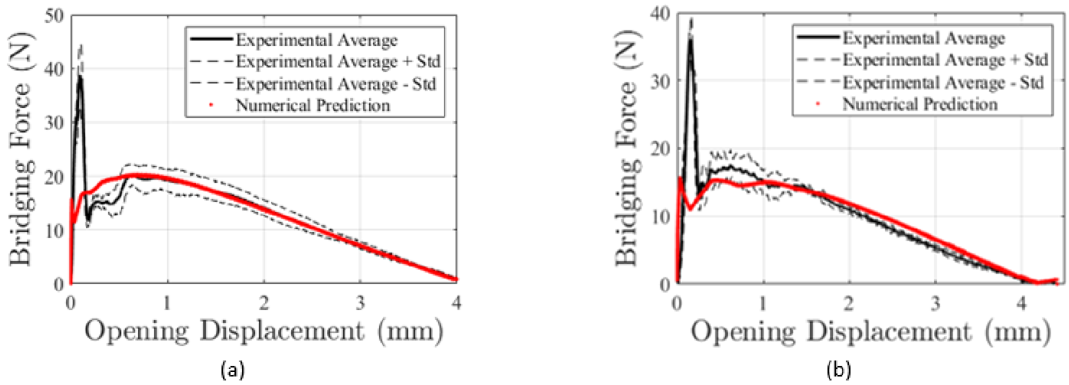

Figure 2. The predicted and experimental reposne under mode I loading at (a) 5m/s and (b) 12m/s.

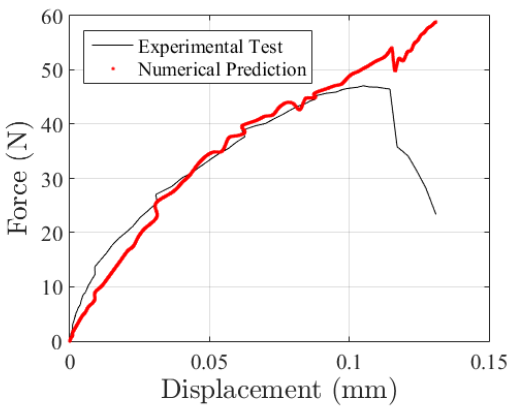

Figure 3. The predicted and experimental response under Mode II loading at 1.5m/s.

5. Conclusions

A rate-dependent model has been proposed to predict the crack bridging performance of z-pins subject to mode I and mode II loading. The governing equations were derived from first principles using peridynamic formulation. A state-based friction model was employed to account for the effect of rate on friction. The model parameters were obtained by calibrating the model against experimental data in literature for a pullout rate of 5m/s. The calibrated parameters were then used to test the model predictive capability at a different rate of 12m/s and at a different loading mode (mode II).

The model provides an excellent predictive capability for the bridging tractions as functions of the opening/sliding displacements. The difference between the predicted and the actual fracture toughness enhancement values was within 5%.

The main advantage of this model is the very low computational cost when compared to meso-level 1 FEA. The proposed model runs in the order of minute, compared to the days required by meso-level1 FE simulation.

Future work will include implementing a failure model for predicting the z-pin failure in mode II dominated scenarios.

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