

## EQUIVALENT MECHANICAL PROPERTIES OF FOAM REINFORCED IN THICKNESS

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### Abstract

The reinforcements in the thickness of the sandwich structures represent one of the technological solutions that improve the mechanical properties of this type of structures. We are talking about 3 D sandwich structures.

In this work, the equivalent properties of reinforced sandwich structures in the thickness are evaluated. The finite element analysis homogenization is used to calculate the equivalent mechanical properties of foam reinforced by circular glass fiber reinforcements at  $\pm 45^\circ$ .

This study shows the interest of reinforcing sandwich structures in the thickness. All the results show that the improvement of the mechanical properties is significant, especially the properties in normal and shear in the out-plane direction.

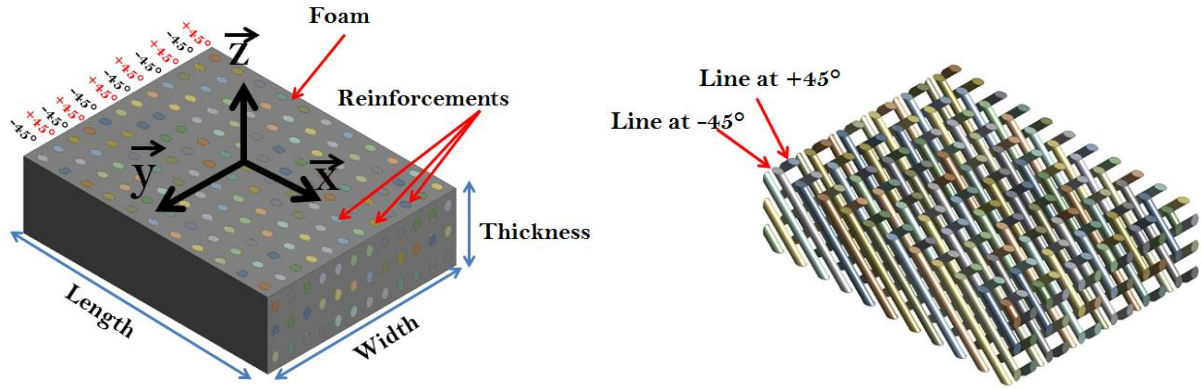
### 1. Introduction

The industrial players in different sectors are expected to operate structures with higher performance materials with low densities and competitive production costs. The use of multifunctional products, such as composite structures, has become an increasingly important concern. Sandwich structures are increasingly used in various industrial sectors such as aerospace, transportation, energy, construction, and sports. The combinations of different components provide complementary properties. That leads to obtain a structure able to meet a very demanding specification.

Traditionally, sandwich structures are made of two components, the skins and the core. In this case we are talking about 2D sandwich structures. The 2D sandwich materials have good mechanical properties especially in flexion. But, their performances are limited mainly under the out-plane shear behavior. Hence, introduce reinforcements in the out-plane direction of sandwich structures seems an appropriate solution to improve out-plane mechanical properties. In this case we are talking about 3D sandwich structures.

Several studies have shown the interest of reinforcements in out-plane direction of sandwich structures. All works reveal that the improvement of the mechanical properties is significant, especially the normal and out-plane shear properties [1, 2 and 3]. In this work, homogenization by

finite element analyzes is used to evaluate the equivalent mechanical properties of a foam reinforced in the thickness. Circular fiberglass reinforcements at  $\pm 45^\circ$  are considered Figure 1.



**Figure 1.** Representation of reinforced foam in the thickness.

## 2. Homogenization by finite element analysis

### 2.1. Concept

The material is considered orthotropic and subsequently it is described only by 9 constants. The relation between stress and strain for such material is written by the generalized Hooke law as follows:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{xx}} & -\frac{\nu_{xy}}{E_{xx}} & -\frac{\nu_{xz}}{E_{xx}} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_{xx}} & \frac{1}{E_{yy}} & -\frac{\nu_{xz}}{E_{yy}} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_{xx}} & -\frac{\nu_{xz}}{E_{yy}} & \frac{1}{E_{zz}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} \quad (1)$$

The 9 coefficients defining the elastic mechanical properties are:

- $E_{xx}$ : Elastic modulus in X direction
- $E_{yy}$ : Elastic modulus in Y direction
- $E_{zz}$ : Elastic modulus in Z direction
- $G_{xy}$ : Shear modulus in (X,Y) plane
- $G_{xz}$ : Shear modulus in (X,Z) plane
- $G_{yz}$ : Shear modulus in (Y,Z) plane
- $\nu_{xy}$ : Poisson's ratio representing the ration between X strain and Y strain
- $\nu_{xz}$ : Poisson's ratio representing the ratio between X strain and Z strain
- $\nu_{yz}$ : Poisson's ratio representing the ration between Y strain and Z strain

The work consists in determining the equivalent elastic mechanical properties of reinforced foam. Analytical models are often used to determine the mechanical properties of composites from the properties of constituents [4, 5, 6, and 7]. However there are significant differences between the models. The result depends on the considered geometric model of the composite. Hence, the experimental determination of the elastic mechanical properties of a heterogeneous structure is also a very complicated task, especially for out-plane shear modulus.

To predict the homogenized mechanical properties of the heterogeneous structures, it is necessary to take into account the physical and mechanical properties of the constituents. Finite element analysis is an appropriate method to simulate several configurations representing different physical parameters of the material.

Generally, homogenization consists in calculating the mechanical properties from the mechanical response of a representative elementary volume (REV) of the heterogeneous material. Hence, several cases of loading are necessary to estimate the different elastic properties of the material.

## 2.2 Modeling

### 2.2.1 Boundary conditions

The boundary conditions used to determine the homogenized properties of a heterogeneous material are defined in this section. For an orthotropic material, the nine elastic properties are determined from six loading cases. In fact, the representative elementary volume is submitted to appropriate boundary conditions to modelling the different loading cases.

To ensure the periodicity of stress and strain fields, periodic boundary conditions are applied to the REV. The different loading cases are applied to the REV in the form of a displacement which generates a deformation on the REV. The reaction forces and the generated displacements are used to calculate the homogenized mechanical properties of the representative elementary volume. Normal load are applied on the REV to determine the properties in the main directions. Axial loading, transversal loading and out-plane loading are performed.

To represent these three test cases, the boundary conditions applied on the REV are presented below:

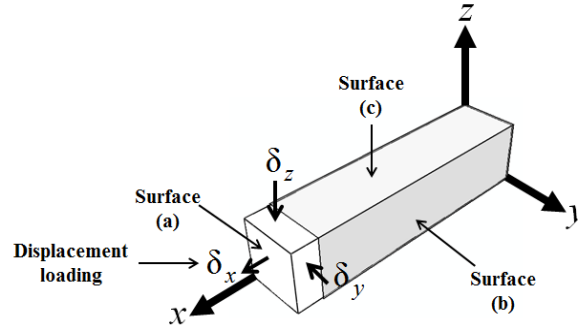
$$\begin{cases} u(0, y, z) = 0 \\ u(a, y, z) = \delta_x \\ v(x, 0, z) = 0 \\ v(x, b, z) = \delta_y \\ w(x, y, 0) = 0 \\ w(x, y, c) = \delta_z \end{cases} \quad (2)$$

The three loading cases are applied independently. For example, the axial loading is modeled by a constant displacement  $\delta_x$  acting on the surface (a) in the x direction. While the surfaces (b) and (c) undergo a constant displacement respectively in the y and z directions.

In the figure below (Figure 2) a representation of the axial test case on the representative elementary volume is shown. The elastic properties determined by this loading case are calculated with the following formulas.

$$E_{xx} = \frac{2.a.R_x}{b.c\delta_x} ; \nu_{xy} = -\frac{a.\delta_y}{b.\delta_x} ; \nu_{xz} = -\frac{a.\delta_z}{b.\delta_x} \quad (3)$$

$R_x$  is the reaction force on the surface (a).  $\delta_y$  and  $\delta_z$  are respectively the displacements undergone by the surfaces (b) and (c). With the same way transversal loading and out-plane loading are modeled.



**Figure2.** Representation of the axial test case on the representative elementary volume.

To determine the three shear modulus, tangent displacements on the surfaces of the REV are applied. For example, longitudinal shear loading is modeled by a tangent displacement applied to the surface ( $y = b$ ) in the  $x$  direction. The boundary conditions applied on the REV are:

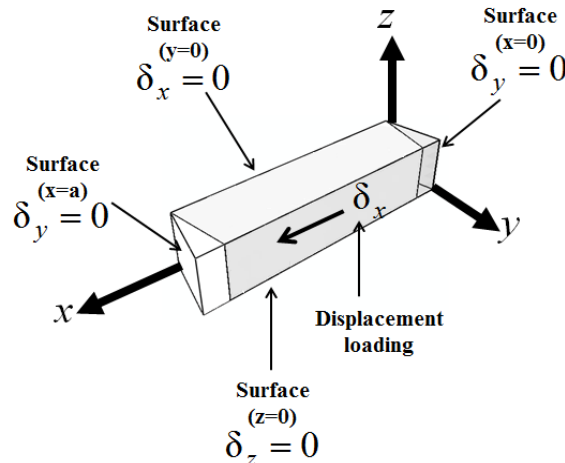
$$\begin{cases} u(x, b, z) = \delta_x \\ u(x, 0, z) = 0 \\ v(0, y, z) = 0 \\ v(a, b, z) = 0 \\ w(x, y, 0) = 0 \end{cases} \quad (4)$$

In the figure below (Figure 3) a schematic representation of the longitudinal shear test on a representative elementary volume is shown.

The longitudinal shear modulus is calculated by the following equation:

$$G_{xy} = \frac{2.b.R_x}{a.c.\delta_x} \quad (5)$$

$R_x$  represents the reaction force on the surface ( $y = b$ ). With the same way the transverse and out-plane shear modulus are calculated.



**Figure3.** Representation of the longitudinal shear loading test case on the representative elementary volume.

### 2.2.2 Representative elementary volume

The finite element analysis is done with ANSYS-Workbench software. The representative elementary volume used for numerical homogenization is shown in the figure below (Figure 5). Two lines of reinforcements crossed at  $\pm 45^\circ$  immersed in a foam is modeled. A convergence study was performed, showing that the results begin to converge at 450 mm of VER's length. In the Figure 4, the evolution of  $E_{xx}$  and  $G_{xz}$  depending on the length of the representative elementary volume is shown. We specify that all the modulus do not evolve according to the width.

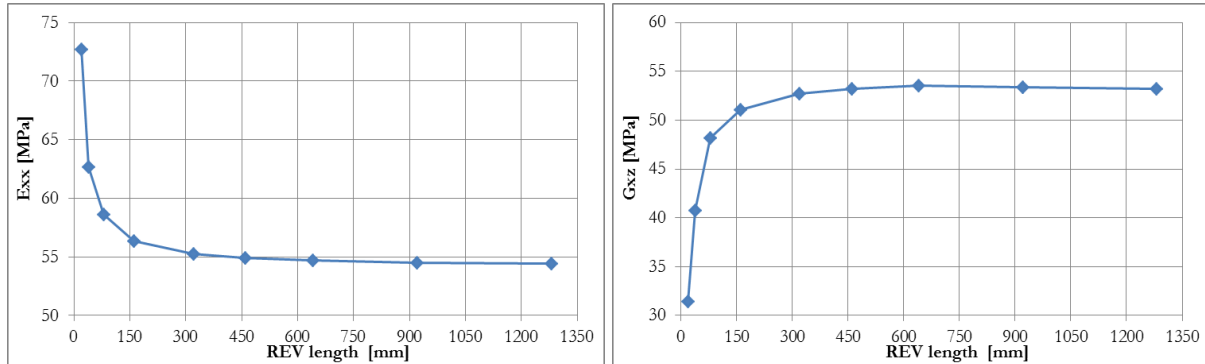


Figure 4.  $E_{xx}$  and  $G_{xz}$  depending in the REV length.

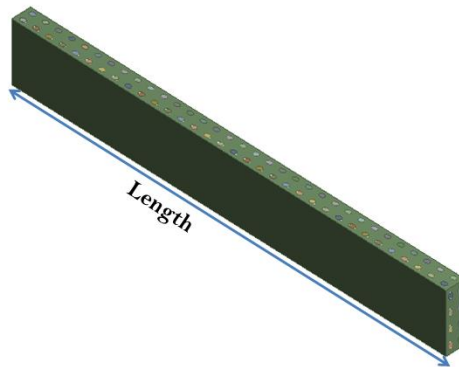


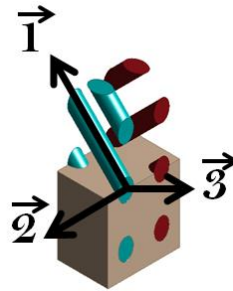
Figure 5. Representation elementary volume.

The mechanical properties of the foam and the reinforcements at different fiber volume fraction are presented in the table below. Three fiber volume fractions are considered.  $V_F$  is the fiber content in the reinforcements. The mechanical properties of the reinforcements are defined considering the same numerical simulation concept. The same boundary conditions were applied on Representative elementary volume representing a glass fiber included in the epoxy resin.

In Figure 6 a representative elementary volume pattern (shown in Figure 5) is presented. This pattern shows the local coordinate system of the reinforcement at  $\pm 45^\circ$ . In this local coordinate the mechanical properties of the reinforcements (Table 1) are defined.

Table 1. Mechanical properties of the foam and the reinforcements at different fiber volume fraction

	$V_F$	$E_{11}$ (MPa)	$E_{22}$ (MPa)	$E_{33}$ (MPa)	$N_{12}$	$N_{23}$	$N_{13}$	$G_{12}$ (MPa)	$G_{23}$ (MPa)	$G_{13}$ (MPa)
Reinforcements	5 %	8356	5641	5641	0,34	0,41	0,34	2015	1985	2015
	30 %	25134	9363	9363	0,30	0,41	0,30	3146	2747	3146
	50 %	38557	15119	15119	0,28	0,32	0,28	4814	3744	4814
Foam	-	28	28	28	0.07	0.07	0.07	13	13	13



**Figure 6.** Local coordinate system for reinforcements.

### 3. Results

#### 3.1. Equivalent properties depending on the fiber content in the reinforcements and reinforcements content in the foam

In this part the equivalent mechanical properties depending on the fiber content in the reinforcements and the reinforcements content in the foam are evaluated (Table 2).  $V_R$  and  $V_F$  are respectively reinforcements content in the foam and fiber content in the reinforcements. The main conclusions are:

- Evolution of  $E_{xx}$ ,  $E_{zz}$  and  $G_{xz}$  with respect to the fiber content in the reinforcements and reinforcements content in the foam.
- Insensitivity of  $E_{yy}$ ,  $G_{xy}$ ,  $G_{yz}$  and Poisson's ratio to the fiber content in the reinforcements
- Increasing of the fiber content in the reinforcements brings more rigidity to the structure with high reinforcements content in the foam.

**Table 2.** Equivalent properties depending on the fiber content in the reinforcements and reinforcements content in the foam

$V_R$	$V_F$	$E_{xx}$ (MPa)	$E_{yy}$ (MPa)	$E_{zz}$ (MPa)	$\nu_{xy}$	$\nu_{yz}$	$\nu_{xz}$	$G_{xy}$ (MPa)	$G_{yz}$ (MPa)	$G_{xz}$ (MPa)
14 %	5 %	55	36	65	0,05	0,02	0,4	16	17	53
	30 %	59	36	75	0,05	0,02	0,4	17	17	63
	50 %	61	36	85	0,05	0,01	0,4	17	17	67
38 %	5 %	104	55	125	0,04	0,01	0,45	24	25	150
	30 %	117	56	149	0,04	0,01	0,45	25	25	195
	50 %	128	56	173	0,04	0,01	0,45	25	25	215

#### 3.2. Equivalent properties at different thickness of the foam

The mechanical properties with three thicknesses of foam are estimated (Table 3). The results show that out-plane shear properties change with the thickness of the core. Reinforcement at higher thicknesses is more important. Between 60 mm and 80 mm we have about 19% more stiffness in out-plane shear modulu, between 60 mm and 120 mm we have around 50% more stiffness in out-plane shear modulu. The other properties remain almost constant.

**Table 3.** Equivalent properties at different thickness of the foam

Thickness (mm)	$E_{xx}$ (MPa)	$E_{yy}$ (MPa)	$E_{zz}$ (MPa)	$\nu_{xy}$	$\nu_{yz}$	$\nu_{xz}$	$G_{xy}$ (MPa)	$G_{yz}$ (MPa)	$G_{xz}$ (MPa)
60	55	36	65	0,05	0,02	0,4	16	17	53
80	56	37	64	0,05	0,02	0,4	17	17	53
120	56	36	66	0,05	0,01	0,4	17	17	80

### 3.3. Equivalent properties at different diameter of reinforcements

Mechanical properties at three diameter of reinforcements are estimated (Table 4). The results show that the out-plane shear properties progress with the diameter of the reinforcements. Reinforcements with a small diameter provide more rigidity. With 5.6 mm in diameter of reinforcements the out-plane shear modulus increase by 23% compared to reinforcements with 8 mm in diameter. Thus, with 4 mm in diameter of reinforcements the out-plane shear modulus increased by 90% compared to reinforcements 8 mm in diameters.

**Table 4.** Equivalent properties at different thickness of the foam

Diameter (mm)	$E_{xx}$ (MPa)	$E_{yy}$ (MPa)	$E_{zz}$ (MPa)	$\nu_{xy}$	$\nu_{yz}$	$\nu_{xz}$	$G_{xy}$ (MPa)	$G_{yz}$ (MPa)	$G_{xz}$ (MPa)
8	54	35	63	0,06	0,02	0,4	16	17	46
5.6	55	36	65	0,05	0,02	0,4	17	17	57
4	55	37	66	0,05	0,01	0,4	17	17	87

### 4. Conclusion

In this work the homogenization by finite element analyzes was used to evaluate the equivalent mechanical properties of reinforced foam in the thickness. Cross reinforcement at  $\pm 45^\circ$  fiberglass with circular sections are considered. A parametric study is carried out showing the interest of reinforcement in the thickness of sandwich structures. We retain the following results:

- Evolution of  $E_{xx}$ ,  $E_{zz}$  and  $G_{xz}$  with respect to the fiber content in the reinforcements and reinforcements content in the foam.
- Insensitivity of  $E_{yy}$ ,  $G_{xy}$ ,  $G_{yz}$  and Poisson's ratio to the fiber content in the reinforcements
- Increasing of the fiber content in the reinforcements brings more rigidity to the structure with high reinforcements content in the foam.
- Reinforcements at higher thicknesses are more important.
- Small diameter of the reinforcements provide more out-plane stiffness.

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