THE APPLICATION OF A REDUCED VOLUME METHOD FOR THE SIMULATION OF THE CHARACTERISATION OF A CARBON FIBRE PRESSURE VESSEL

Martinus P. Widjaja[1], Sébastien Joannès[2], Anthony Bunsell[2], Georg Mair[1], and Alain Thionnet[3]

[1]BAM Federal Institute for Materials Research and Testing, Berlin, 12203, Germany
Email: martinus-putra.widjaja@bam.de
Email: georg.mair@bam.de
[2]MINES ParisTech, PSL - Research University, MAT - Centre des Matériaux, CNRS UMR 7633 BP 87, 91003 Evry cedex, France
Email: sebastien.joannes@mines-paristech.fr
Email: anthonybunsell@gmail.com
[3]Université de Bourgogne, Mirande, Dpt. IEM, BP 47870, Dijon, France
Email: alain.thionnet@mines-paristech.fr

Keywords: composite pressure vessel, fibre-break, integral range, representative volume element

Abstract
The characteristics of advanced composite materials make them ideally suited for use in pressure vessels for storing gas as fuel for ground transport vehicles. It has been found that the failure process starts with randomly distributed fibre breaks and as the loading continues, they coalesce into clusters of fibre breaks which lead to failure [4]. However, improvements are needed to reduce computational times when performing full-scale simulations. The reduced volume method is therefore applied to the stochastic fibre break model related to the concept of an integral range. This method allows the calculation for a certain volume of a laminate that statistically represents the same physical properties as a full-scale pressure vessel. A convergence study with multiple configurations has been done and shows that the assignment of fibre failure strength values at each integration points is not the same if the related configurations were rotated. The integral method has been successfully applied to the 3D case and by using only 23 elements for 1 simulation, 95% of confidence level can be achieved. Another important remark is that the result from the model is highly sensitive with the Weibull characteristics value that was used to produce the fibre rupture values.

1. Introduction

The markets for composite materials have widened greatly to cover many different structures. Thanks to the orthotropic material behaviour, a composite structure can be designed accordingly to withstand the loads expected. In the automotive industry, CNG and hydrogen technologies have been developed as the substitutes for petroleum-based fuels. The main goal is to reduce the CO2 emissions, the challenge, however, arises from quantifying the safety standards.

According to ISO standards, two or three composite cylinders should be tested experimentally to evaluate their strengths. Composite materials have particular degradation kinetics which are markedly different from other materials. Evaluating the reliability of such material requires a comprehensive understanding of how this material behaves under specific loads and environmental conditions. Currently, the available standards are not sufficient to clearly evaluate the strength properties of a composite structure. BAM, therefore has developed an analytical tool that analyses the result from an experiment using a statistical-probabilistic approach. This approach shall give the assumed statistical property of the burst pressure population [9].

Blassiau et al. have found that the failure process starts with randomly distributed fibre breaks and as the loading continues, they will coalesce together into a cluster of fibre breaks, sooner or later those clusters will cause a total failure [1-3]. On the other hand, BAM has found an interesting finding from aged pressure vessels that showed an altered reliability when tested with slower than usual pressurisation rates (slow burst test). As the linear viscous behaviour is used to describe the matrix inside the fibre-break model, this could help to explain the phenomenon occurred during the residual strength testing.

The fibre break model (FBM) is using the fibre rupture values based on Weibull distribution as the input parameter. A sensitivity study then required to investigate the failure strength predicted by the model. Defining the minimum number of the representative volume element (RVE) contained in a simple structure which will not significantly change its failure strength, for a given structural loading, compared with a similar structure with a higher number of RVE is the main objective of this part of the study. A method called integral range then will be used to give a confidence level from the simulated results. The approach used in this paper needs to be justified so that the study on a real size pressure vessel can be performed, however, this will become the part of the next study. It is also worth to be mentioned that a relation between the results from the convergence study and the probabilistic approach made by BAM must be made. Though it has not yet been studied, a hint for the scatter of the failure strength would become an added value.

2. Integral Range Method

The failure strength values estimated from the model may fluctuate depending on the number of realisations as these values are randomly assigned at each integration point. All the information in this section is based on the study from [5-8]. According to [8], the convergence mode of the ergodic stationary random function or simply put, the fluctuations of the estimated failure strength is not very restrictive, it is applicable to increasing sequences of balls or parallelepipeds.

Integral range method, therefore, was used to calculate the variance $D_z^2(V)$ of its average value $Z(V)$ over the volume V. Equation 1 is valid for a large specimen where $V \gg A_3$ [5]. $D_z^2(V)$ is the point variance and A_3 is the integral range of the random function, of which in this case is the random assignment of the fibre failure strength value.

$$D_z^2(V) = D_z^2 \frac{A_3}{V} \tag{1}$$

$$D_z^2(V) = D_z^2 \left(\frac{A_3}{V}\right)^{\gamma} \tag{2}$$

When $A_3 \neq 0$, an integer number N can be found, which leads to equation 4,

$$\frac{V}{A_3} \approx N \tag{3}$$

$$D_z^2(V) \approx \frac{D_z^2}{N} \tag{4}$$

Equation 4 represents the variance of a mean of N independent realisations as if the domain V had been divided into N independent domains of the same size A_3 [8]. The scale of the phenomenon then can be interpreted in A_3, which in this case is the phenomenon of the fibre breaks and V is the scale of observation, i.e. pressure vessels or parallelepipeds.

The term $D_z^2 A_3^\gamma$ in equation 2 can be defined as K, as explained in [5]. K and γ can then be numerically defined by fitting a power function to the log-log equation below,

$$\log D_z^2(V) = \log K - \gamma \log V \tag{5}$$

The number of realisations is represented by n and the sampling error of the estimated property can be calculated as

$$\epsilon_{abs} = \frac{2\, D_z\, (V)}{\sqrt{n}} \qquad\qquad \epsilon_{rel} = \frac{\epsilon_{abs}}{\overline{Z}} \rightarrow \epsilon_{rel}^2 = \frac{4\, D_z^2 A_3^\gamma}{\overline{Z}^2\; n\; V^\gamma} \tag{6}$$

By rearranging the equation 6 above, the representative reduced volume is defined,

$$V_{RVE} = \left(\frac{4\, K}{\overline{Z}^2\; n\;\; \epsilon_{rel}^2} \right)^{1/\gamma} \tag{7}$$

3. Methodology

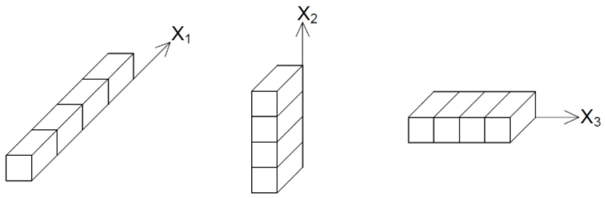
The FBM uses the macroscopic stress state value from usual FEM calculation to study the fibre breaks in a microscopic scale, this was implemented in a simplified FE^2 multiscale approach. At each integration point on FEM calculation, it requires 5 fibre failure values to change the state of the RVE from C32 (undamaged state) until C1 (final damage state). These values were based on a two parameters Weibull function to describe the statistical characteristic of fibre failure and have been produced by a Monte-Carlo process.

The structure studied, denoted as S, is a material system in movement in the physical space, modelled with an affine space with 3 dimensions ε^3, for which the spatial reference frame is R, assumed to be Galilean, the origin of R is O and its cartesian orthonormal basis is b = $(\overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_3})$. The structures studied has a parallelepiped shape, that will be referred as specimens from now on. It is limited with section plan S_{+a} and S_{-a}, that is located respectively according to the equation $x_1 = +a$ and $x_1 = -a$, section plan S_{+b} and S_{-b}, located respectively according to the equation $x_2 = +b$ and $x_2 = -b$, section plan S_{+c} and S_{-c}, located respectively according to the equation $x_3 = +c$ and $x_3 = -c$. The length L, width l, and the thickness e of the specimens is defined as $L = 2a$, $l = 2b$, $e = 2c$. The boundary condition of the simulation will be as follows:

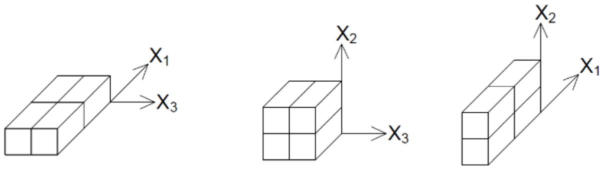
- A density of surface force $\overrightarrow{F}_{-a}\,(M,t) = -F(t)\overrightarrow{x_1}$ is applied on section S_{-a};
- A density of surface force $\overrightarrow{F}_{+a}\,(M,t) = +F(t)\overrightarrow{x_1}$ is applied on section S_{+a};
- The other surfaces are free of forces;
- The applied load $F(t)$ is a monotonic increasing function of time.

The cases are distinguished by its stacking direction (1D, 2D, and 3D). The 1D – case is stacked in x_1, x_2 and x_3 direction while for the 2D – case in (x_1,x_2), (x_1,x_3) and (x_2,x_3) directions. To compare between 1D and 2D case, the number of the RVE is limited from 4 to 100. The 3D – case therefore shall have the number of RVE from 8 to 1000 due to its cuboid nature. The study then continues to implement the integral range method to discover the minimum required simulation with acceptable deviation of failure strength.

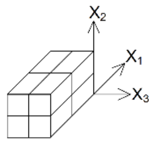
Each specimen from three specimens set will be subjected to the same monotonic increasing load and use the same behaviour. Multiple N calculations then will be performed and it is distinguished by the fact that the fibre break values required for these N calculations are obtained by N Monte-Carlo process with the same Weibull function. These N calculations therefore shall give the N fibre break values (in MPa) that will be used to compute its average and standard deviation from each specimen.



a. 1D Assemblies Case



b. 2D Assemblies Case



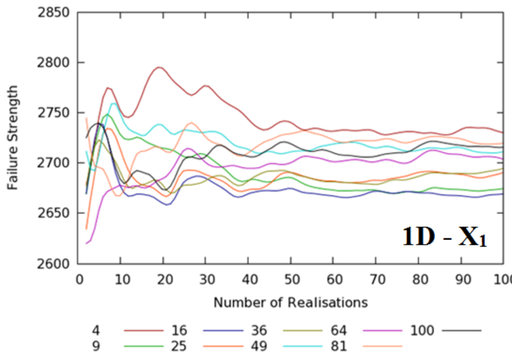
c. 3D Assemblies Case

Figure 1. Stacking direction of multiple specimens

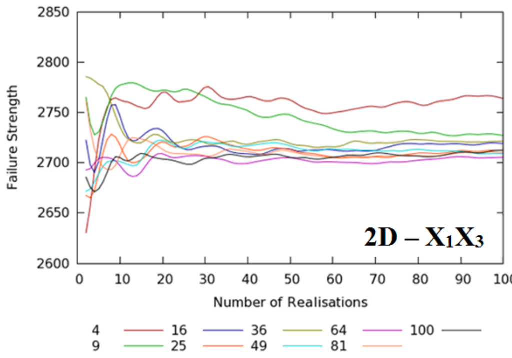
4. Results

4.1. Study on Number of Realisations

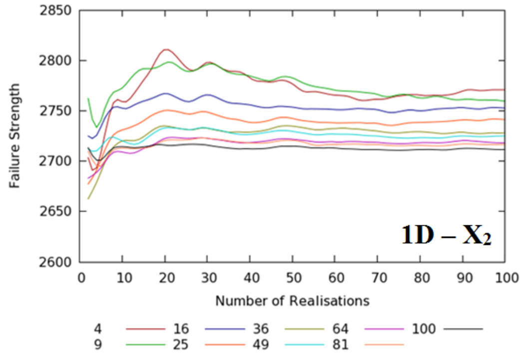
Figure 2 explains the relation between the number of simulations and the average failure strength value on 2 different cases. Based on figure 1, the results between 1D – X_2 and 1D – X_3 cases should be the same, as well as between the 2D – X_1X_3 and 2D – X_1X_2 cases. However, a slight difference on the 1D – case was noticed and the more apparent difference appears in the 2D **-** case. It is believed that such difference occurs due to the distribution of the fibre failure strength at each gauss points which is implemented by Monte-Carlo simulation. It should be noted that the result from the 1D – X_1 specimen does not give any good results as expected. This is because that the fibre break only occurs along the fibre direction which is the same direction with the loading. Consequently, when a fibre breaks, the computation of the stress state becomes inaccurate and leads to unrelated results with increasing number of elements.



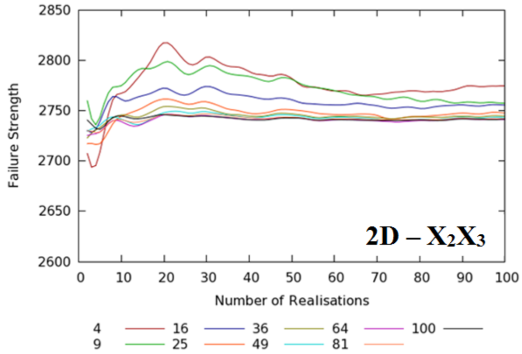
a. 1D – X_1 Case



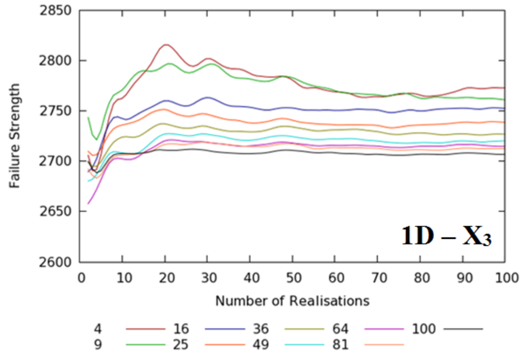
b. 2D – X_1X_3 Case



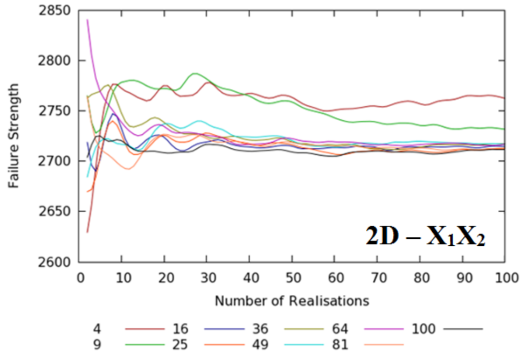
c. 1D – X_2 Case



d. 2D – X_2X_3 Case



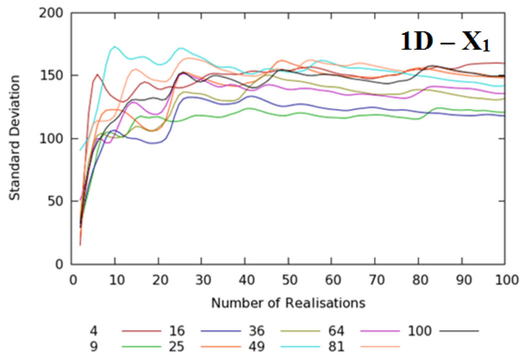
e. 1D – X_3 Case



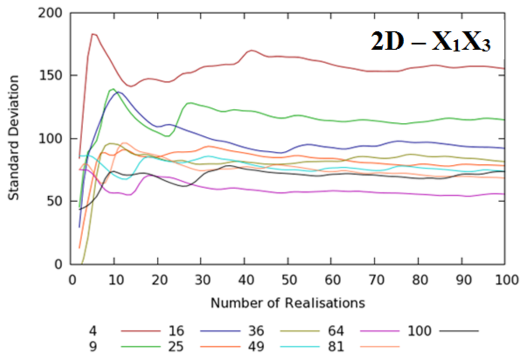
f. 2D – X_1X_2 Case

Figure 2. Failure strength (FS) on 1D and 2D cases

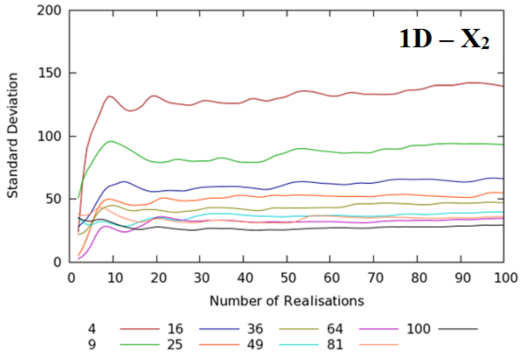
Currently, the model uses 5 fibre failure strength value assigned at each Gauss points (1 RVE). Thus, for one element that has 8 Gauss points, 40 values are required. If we assume that the predicted failure strength converges by using 125 elements, it means (40 x 125 = 5000) value would be used. On the other hand, the total fibres inside 1 RVE are 32, and if each fibre had one fibre failure strength, then 256 (32 x 8) value would be assigned to 1 elements. As a result, it took only 20 elements (5000/256) to reach the converged result. This is an important remark to improve the fibre break model, as this might vastly affect the computation time.



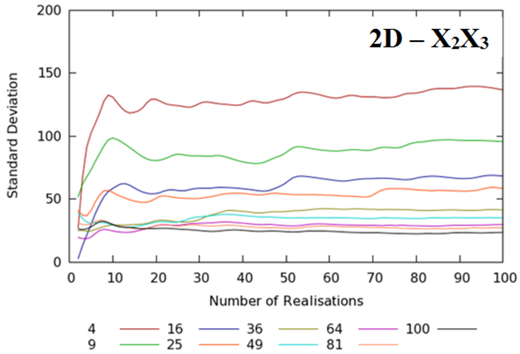
a. 1D – X_1 Case



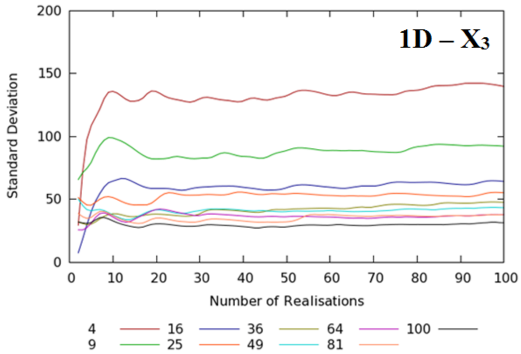
b. 2D – X_1X_3 Case



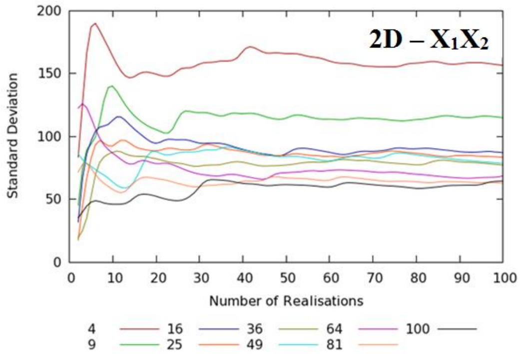
c. 1D – X_2 Case



d. 2D – X_2X_3 Case



e. 1D – X_3 Case

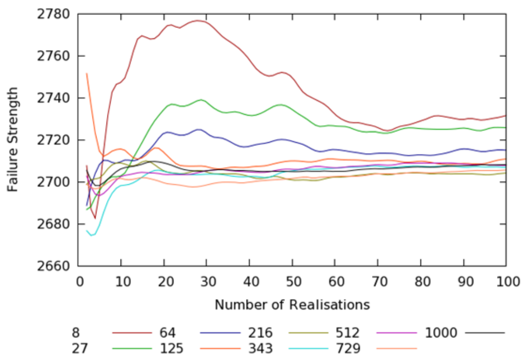


f. 2D – X_1X_2 Case

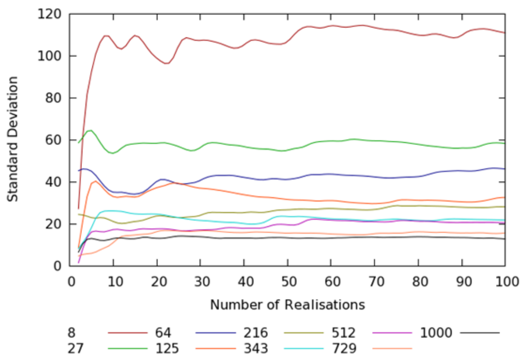
Figure 3. Standard deviation (SD) on 1D and 2D cases

4.2. Study on Number of Elements

The results from 1D – X_2, 1D – X_3, and 2D – X_2X_3 show that the average failure strength value seems converged into one value as the number of elements increases. The convergence on the 2D – X_2X_3 specimen is more noticeable as predicted because of its higher dimension (figure 2d). Moreover, figure 4a shows the results with relatively small deviation after using more than 100 elements. This confirms that the usage of such element size on the fibre break model gives more reliable result after using more than 100 elements. It is also clearly depicted in figure 5a, that even using a smaller number of elements on 3D specimen already gives a closer result to the converge one with smaller standard deviation as well. After certain extent, the average failure strength value unsteadily converges with smaller standard deviation value as shown in figure 5b.

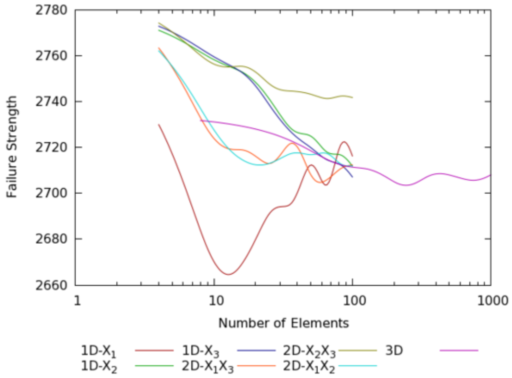


a. Effect on failure strength

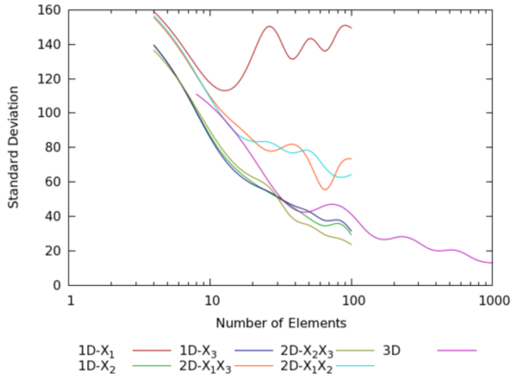


b. Effect on standard deviation

Figure 4. FS and SD in 3D case



a. Effect on failure strength



b. Effect on standard deviation in all cases

Figure 5. Comparison between all cases

4.3. Integral Range Implementation

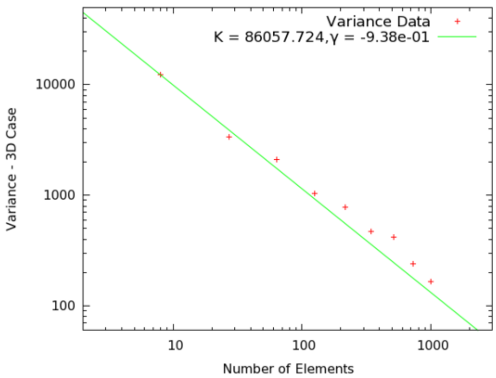


Figure 6. Log-graph fitting

Figure 6 is a fitting result based on equation 5 that has been calculated from the 3D – case results (figure 4). These variables then will be used to calculate the representative volume which refers to equation 7 (table 1). A higher number of an element is needed to really find the converged solution, but also the computation will become more and more ineffective. Nevertheless, the integral range method gives a result with a certain confidence level, for instance, it is calculated by 100% - 0.10% = 99.9% where 705 elements were used and 100 realisations/computations has been performed. It became much faster by performing only 1 computation with the same number of elements, however, the confidence level is reduced to 100% - 1.00% = 99%.

Table 1. Result from integral range method

n Realisations	Intended Relative Error			
	5.00%	1.00%	0.5%	0.10%
	V_{RVE} (Representative Volume)			
100	1	5	23	705
50	1	11	48	1476
5	4	127	556	17184
1	23	705	3090	95567

5. Conclusion

The integral range approach has successfully been used for the current study. A slight difference results have been noticed in 1D – case and 2D – case due to the random assignment of fibre rupture values at each node (integration points) in the geometry model. Based on the 3D – case results that show an asymptotic behaviour for the standard deviation value, it is predicted that after a certain extent, a required number of element will be found, hence more simulation shall be performed with higher number elements (over 1000 elements) to ensure the findings. Nevertheless, the 95% confidence level result is selected as this is the common confidence value to be used in statistical analysis. To be noted that these predicted failure strength is highly sensitive with the input data of fibre failure strength coming from the Weibull distribution. In addition, the relation between the confidence level of the proposed method and the probabilistic approach developed by BAM will also be studied on the next stage.

Acknowledgments

The research leading to these results has been done within the framework of the FiBreMoD project and has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 722626.

References

[1] S. Blassiau, A. Thionnet, and A. Bunsell. Micromechanisms of loads transfer in a unidirectional carbon fibre reinforced epoxy composite due to fibre failures: Part 1. micromechanisms and 3d analysis of load transfer (elastic case). *International Journal of Composite Structures*, 74:303-318, 2006.

[2] S. Blassiau, A. Thionnet, and A. Bunsell. Micromechanisms of loads transfer in a unidirectional carbon fibre reinforced epoxy composite due to fibre failures: Part 2. influence of viscoelastic and plastic matrices on the mechanisms of load transfer. *International Journal of Composite Structures*, 74:319-331, 2006.

[3] S. Blassiau, A. Thionnet, and A. Bunsell. Micromechanisms of loads transfer in a unidirectional carbon fibre reinforced epoxy composite due to fibre failures: Part 3. multiscale reconstruction of composite behaviour. *International Journal of Composite Structures*, 83:312-323, 2008.

[4] H. Chou, A. Bunsell, G. Mair, and A. Thionnet. Effect of the loading rate on ultimate strength of composites. application: Pressure vessel slow burst test. *International Journal of Composite Structures*, 104:144-153, 2013.

[5] J. Dirrenberger, S. Forest, and D. Jeulin. Towards gigantic rve sizes for 3d stochastic fibrous networks. *International Journal of Solids and Structures*, 51:359-376, 2014.

[6] P. Easwaran, F. Hahn, M. J. Lehmann, C. Redenbach, and K. Schladitz. Representative domain size study on simulated 3d fibre systems. *Proceedings of Filtech Exhibitions, Cologne, Germany* 2016.

[7] D. Jeulin, T. Kanit, and S. Forest. *Representative Volume Element: A Statistical Point of View*. Kluwer Academic Publishers, Printed in the Netherlands, 2004.

[8] C. Lantuejoul. 1991. Ergodicity and integral range. *Journal of Microscopy*, 161:387-403, 1991.

[9] G. W. Mair. *Safety Assessment of Composite Cylinders for Gas Storage by Statistical Methods*. Springer, 2017.