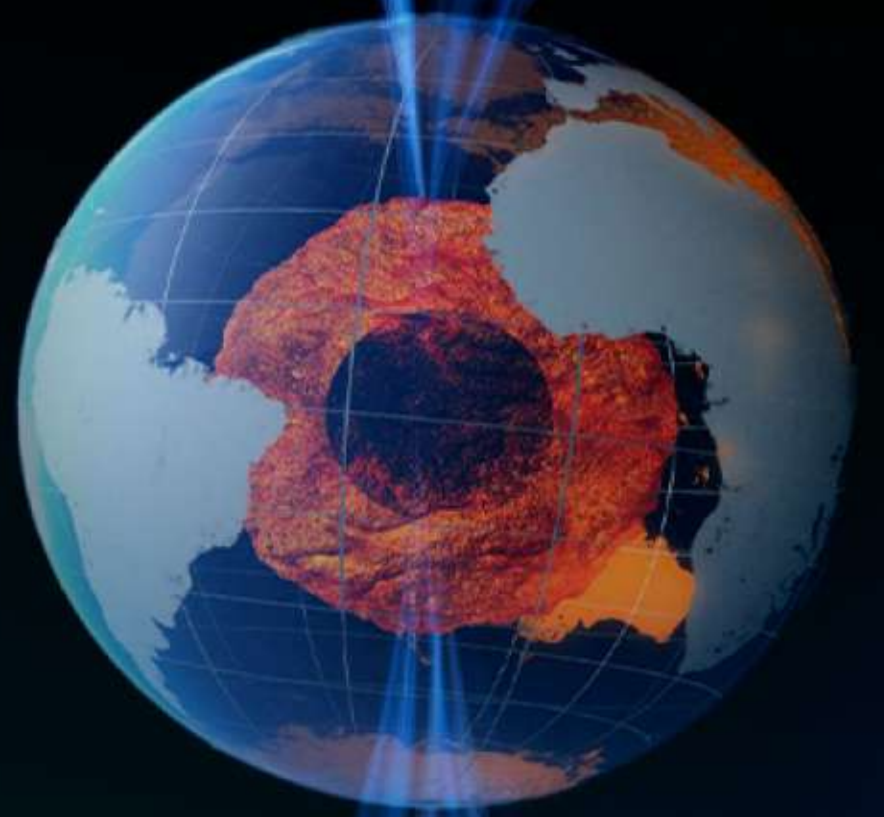

Radial shear in the core surface flow from Swarm data

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Swarm 10 Year Anniversary & Science Conference 2024

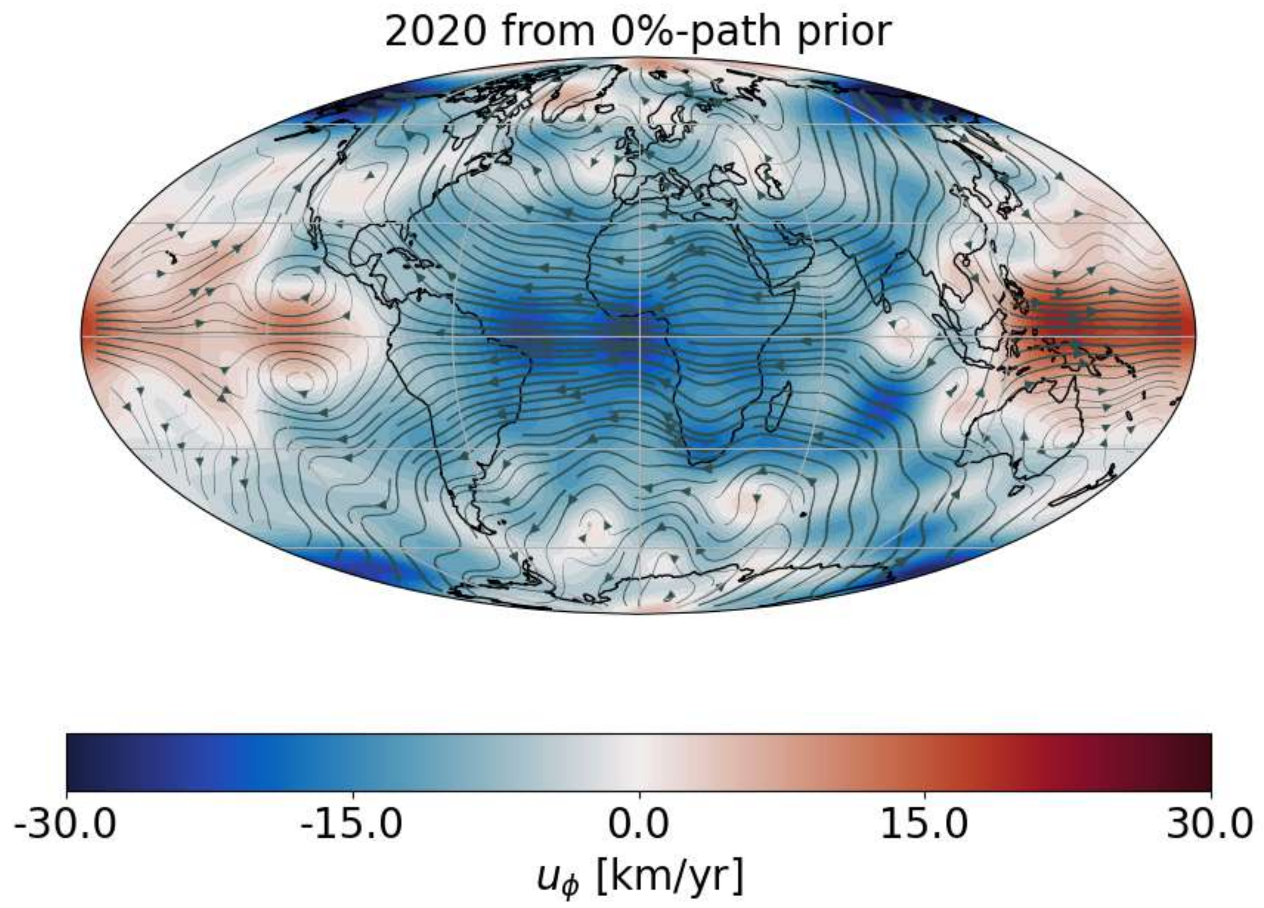
Acknowledgements

- Research in the framework of the ESA consortium Swarm + 4D Deep Earth: Core; from 09/2019 to 09/2024
- Based on the PhD thesis of Ilya Firsov, supported by the ERC Synergy project GRACEFUL (V Dehant, M Mandea, A Cazenave)

Beyond mapping flows at the surface of the core

- Ingredients: magnetic field model + statistics from geodynamo simulations, for example covariance matrix for the flow coefficients
- Limitation: geomagnetic time series too short to build statistics → crucial role of the geodynamo simulations
- Stress-free simulations ($\partial(\boldsymbol{u}/r)/\partial r = 0$ at the core surface $r = r_c$): enable to resolve the boundary layer attached to the core-mantle interface and thus to attain low viscosity and short time-scale; flow directly extracted as the flow at $r = r_c$
- Tool: radial component of the induction equation at $r = r_c$, $\dfrac{\partial B_r}{\partial t} = -\nabla_H \cdot (\boldsymbol{u} B_r)$
- Better codes (*e.g.* XShells from N. Schaeffer) and computers: it becomes feasible to obtain statistics and synthetic data from no-slip boundary simulations ($\boldsymbol{u} = 0$ at $r = r_c$) with 'extreme' values of the parameters
- Description of the boundary layer: prerequisite to use the three components of the induction equation at the core-mantle boundary (CMB)
- First step: estimation of $\boldsymbol{u}$ at $r = r_c$, magnetic field model used at this stage and at this stage only
- Second step: imposing that $\partial \boldsymbol{B}/\partial t$ matchs with a vector field $-\nabla\Phi$ deriving from a potential Φ at $r = r_c$ gives a relationship between the radial shear in the flow $\boldsymbol{\delta} = r\partial\boldsymbol{u}/\partial r$ and the flow $\boldsymbol{u}$

Flow at the core surface for 2020 CHAOS-7 model (Finlay et al., 2020) Stress-free prior (Aubert et al., 2013)



Superposition of a steady anticyclonic planetary-scale eccentric gyre and a growing Eastward flow under the Pacific Ocean (see Ropp & Lesur, 2023)

Horizontal components of the induction equation

Neglecting electrical currents at the core surface ($r = r_c$):

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

In the presence of a conducting layer at the bottom of the mantle:

$$\boldsymbol{B} = \boldsymbol{B}_\delta \,|_{r=r_c} - \nabla \Phi, \text{ at } r = r_c$$

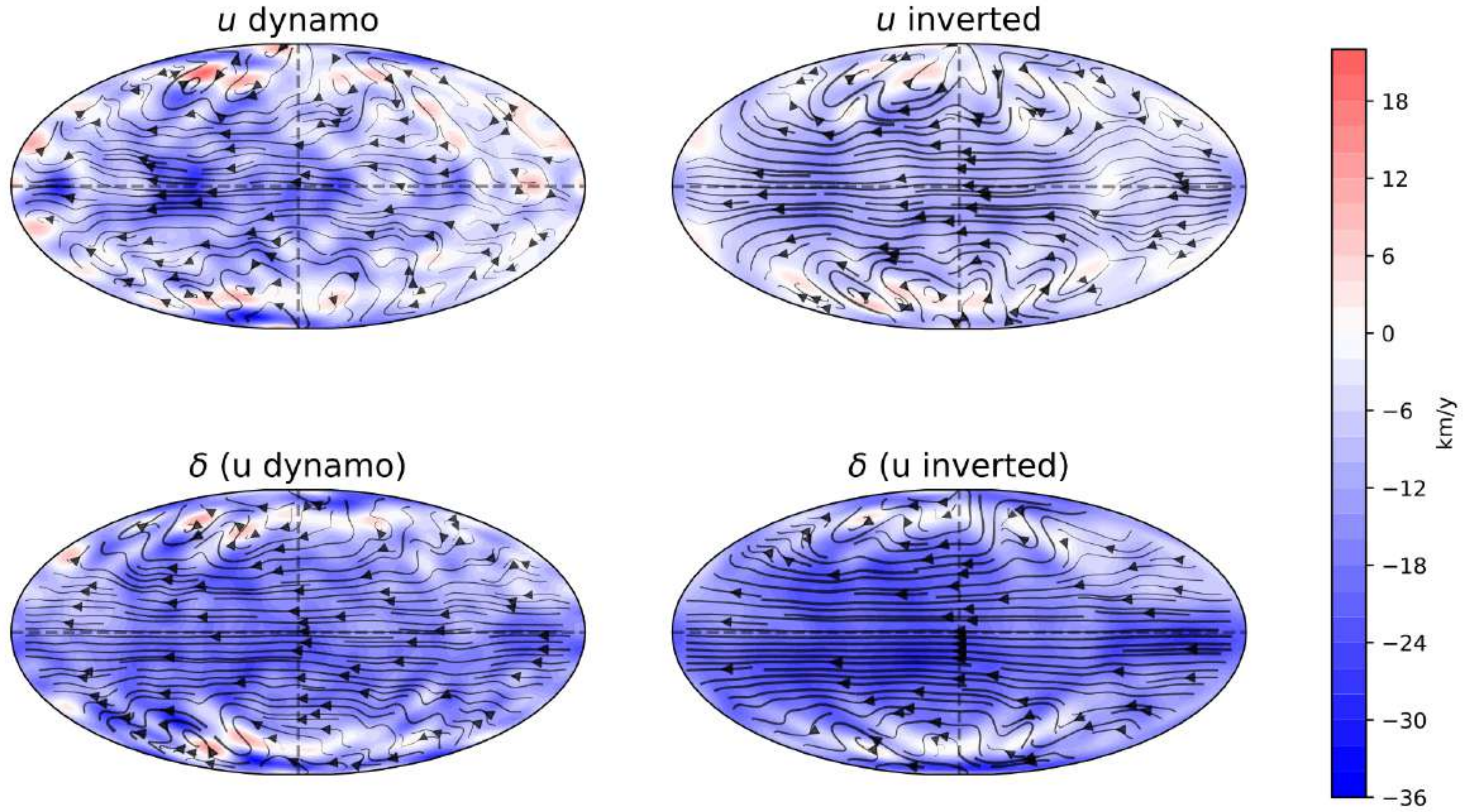
and $\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \dfrac{\partial \boldsymbol{B}_\delta}{\partial t}$ matches a potential field

This boils down to

$$A(B_r)\boldsymbol{u} + B(B_r)\left(\boldsymbol{\delta} + \tau_G \frac{\partial \boldsymbol{u}}{\partial t}\right) = 0 \text{ with } \boldsymbol{\delta} = r\frac{\partial \boldsymbol{u}}{\partial r} \text{ and } \tau_G = r_c \mu_0 \sigma_m d_m$$

Term dependent on the mantle electrical conductivity important at high frequency

If an independent relationship between $\boldsymbol{u}$ and $\boldsymbol{\delta}$ is available, we can estimate τ_G and thus the conductance $\Sigma = \sigma_m d_m$, where d_m is the thickness of the conducting layer at the bottom of the mantle



Radial shear in the flow at $r = r_c$,

stress-free dynamo simulation ($\boldsymbol{\delta} = \boldsymbol{u}$ at $r = r_c$)

$\tau_G \simeq 0$, correlation $c = 0.76$

Firsov, Jault, Gillet, Aubert, Mandea, GJI, 2023

Motivations

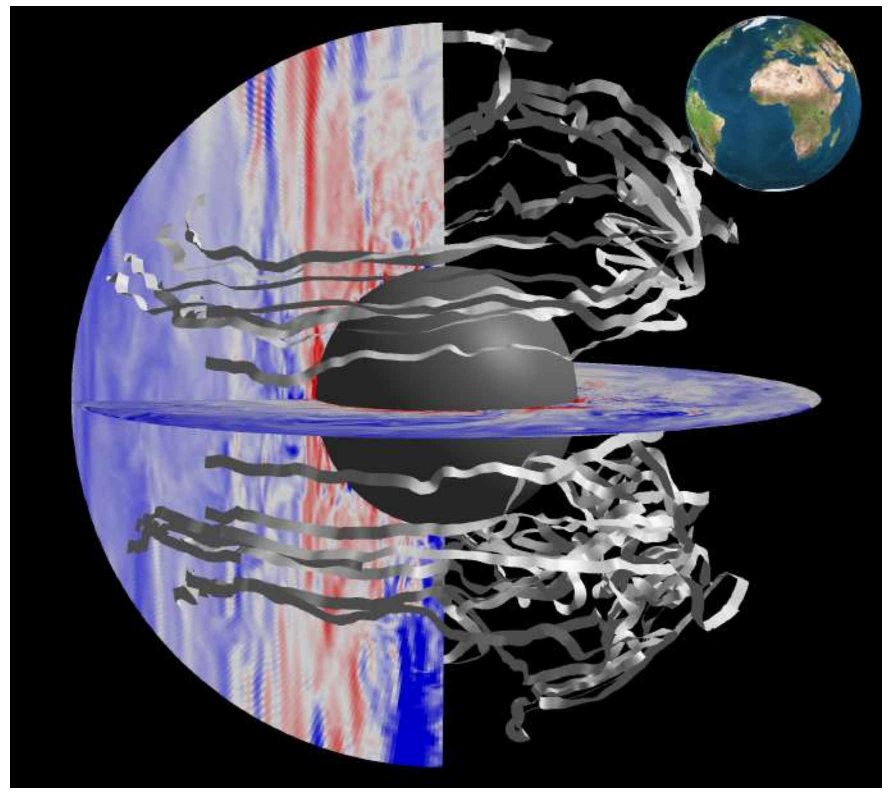
- Testing physical models about the flow in the Earth's core: invariance of the flow in the direction parallel to the axis of rotation (quasi-geostrophy) vs radial stratification

- Improving models of the flow at the Earth's core surface

- Gaining insight on the physics of the core: is the relative geometry of the motions and of the ambiant magnetic field really controlled by the boundary condition on the magnetic field changes at $r = r_c$?

- Probing the electrical conductivity of the lowermost mantle

Radial shear in the flow $\boldsymbol{\delta}$ at the top of the core below the boundary layer from the geomagnetic model Cov-Obs-x2 (based on Swarm data)



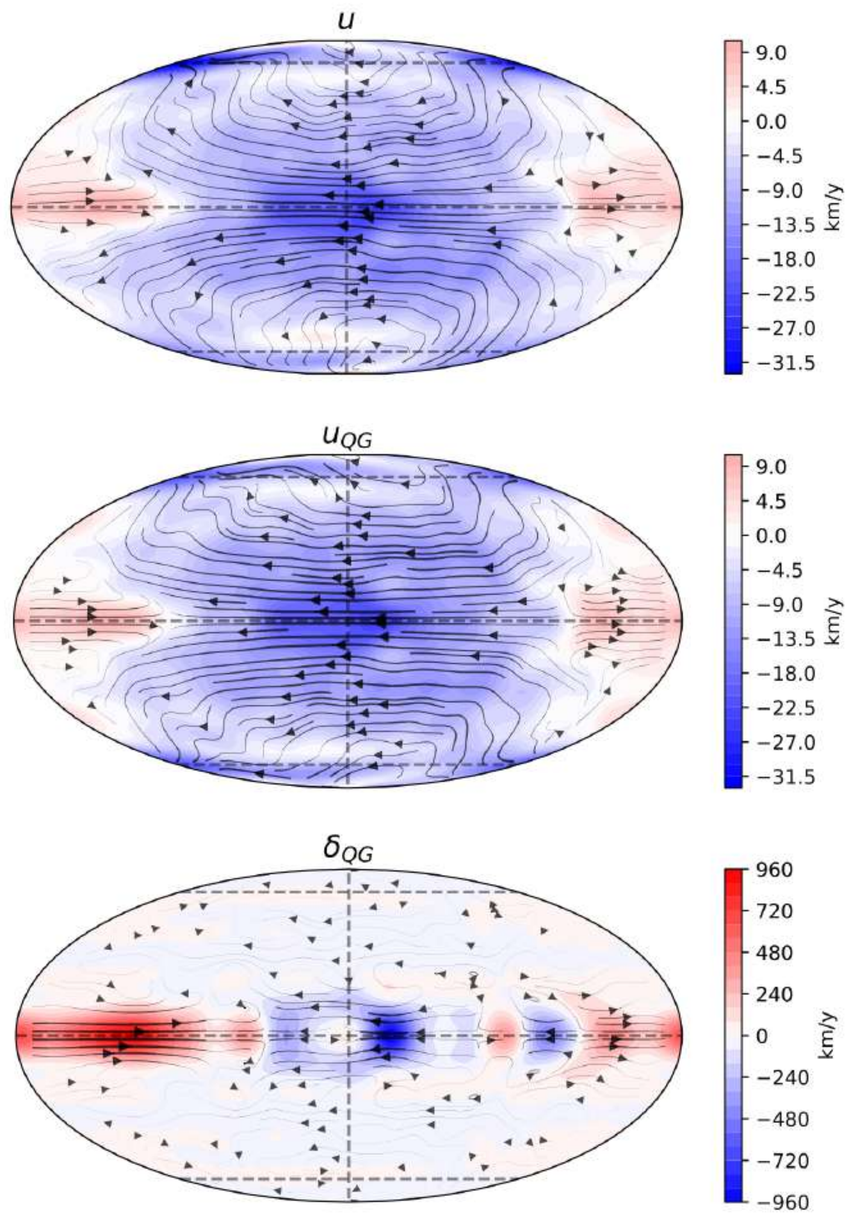
Numerical simulation of Aubert & Gillet (2021), from Finlay et al. (2023)

Quasi-geostrophy:

- u_s, u_ϕ invariant along the rotation axis (i.e. z-invariant)
- $u_z \propto z$
- $\boldsymbol{u}$ at the surface $\rightarrow$ $\boldsymbol{u}$ everywhere, and in particular $\boldsymbol{\delta} = r\partial\boldsymbol{u}/\partial r$ below the core-mantle boundary

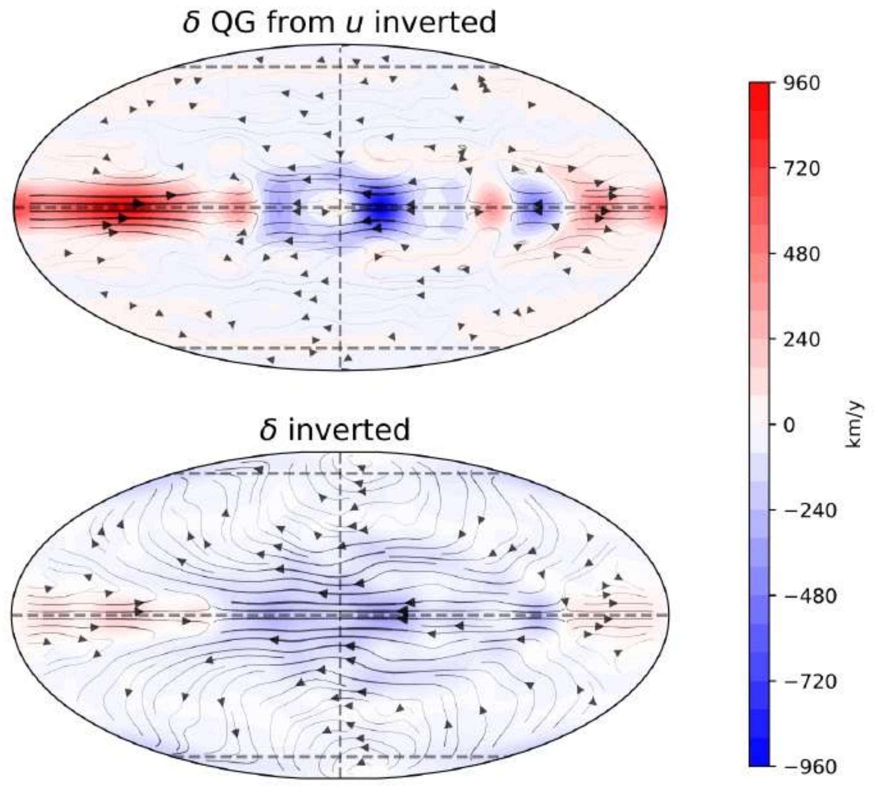
Derivation of the shear $\boldsymbol{\delta}$ from the flow $\boldsymbol{u}$ assuming quasi-geostrophy

Inverted flow Cov-Obs-x2 for 2018



$$\delta \sim 30\,u$$

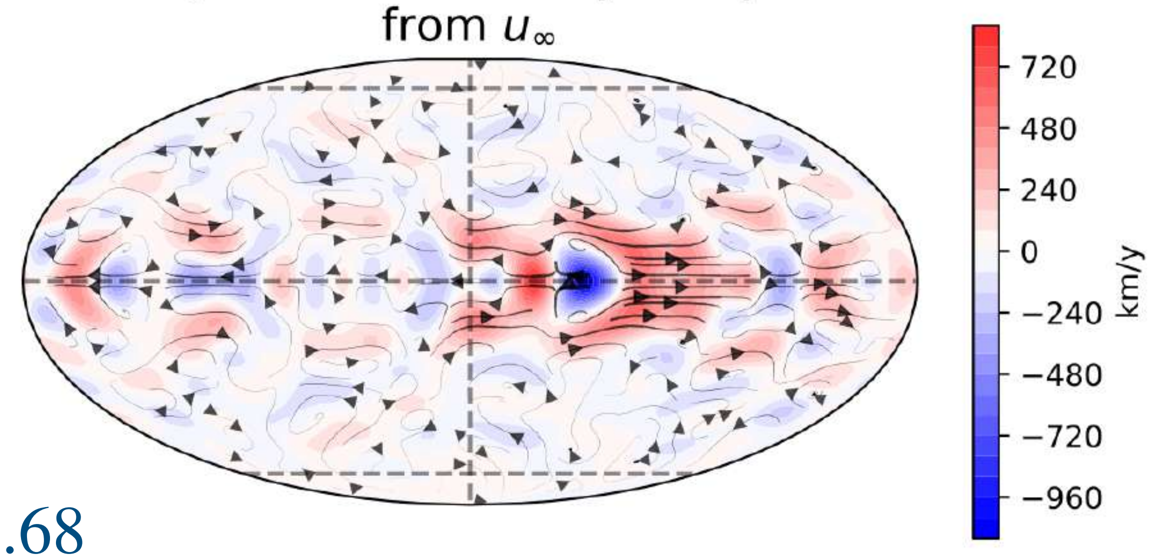
Agreement (mainly next to the equator) between the two estimates for the shear; correlation $c = 0.79$



Radial shear in the flow in no-slip geodynamo simulations

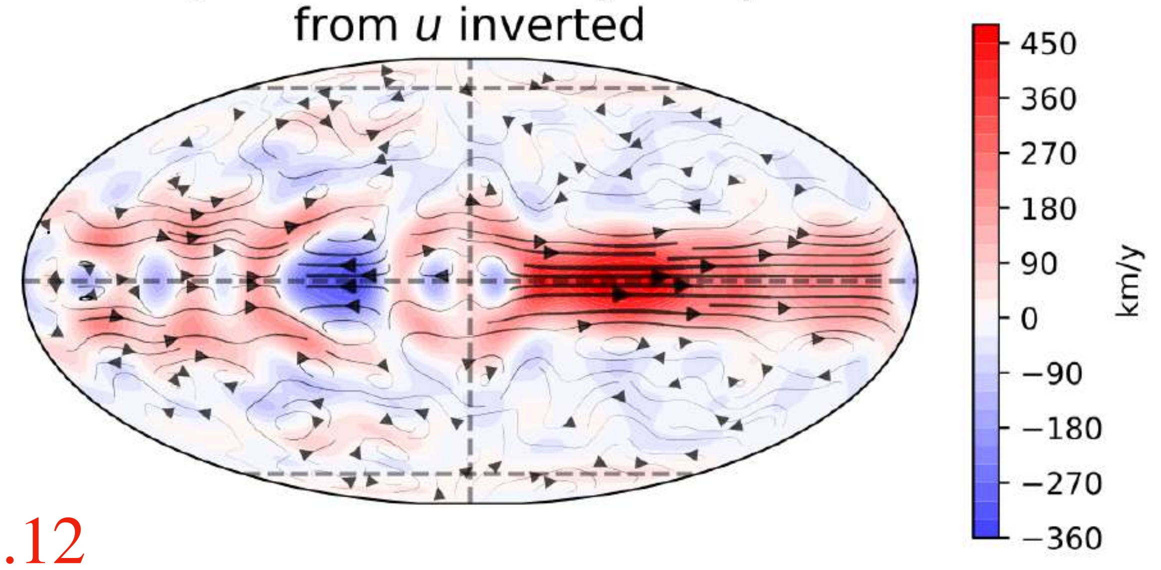
$$r = r_c - 18\, d_E$$

δ QG at 18 Ekman layer depth from u_∞



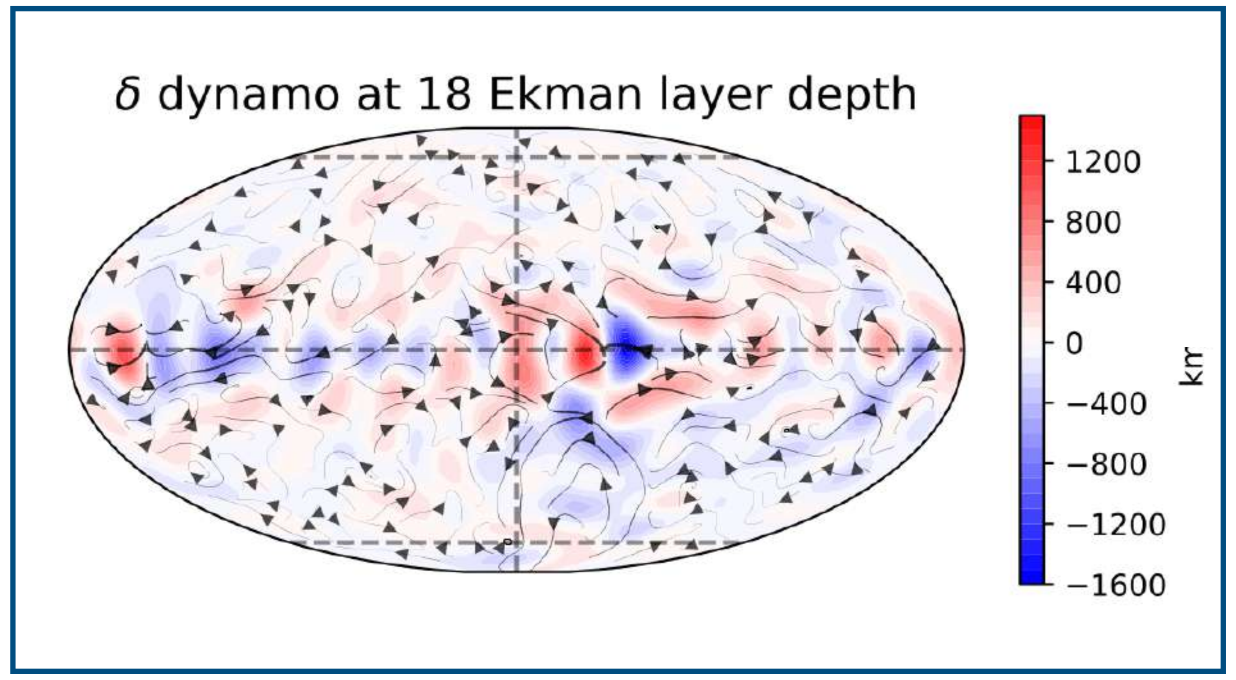
$c = 0.68$

δ QG at 18 Ekman layer depth from u inverted



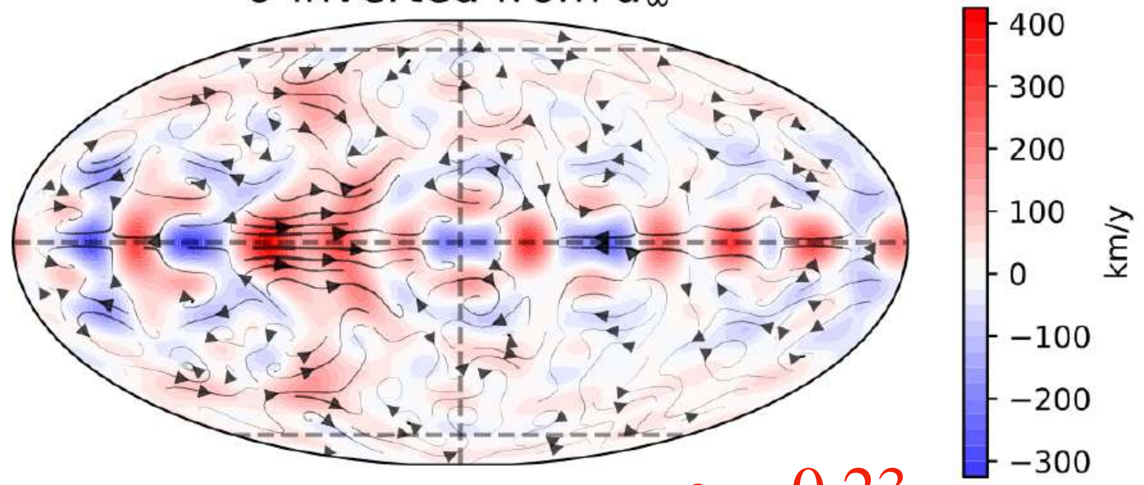
$c = 0.12$

δ dynamo at 18 Ekman layer depth



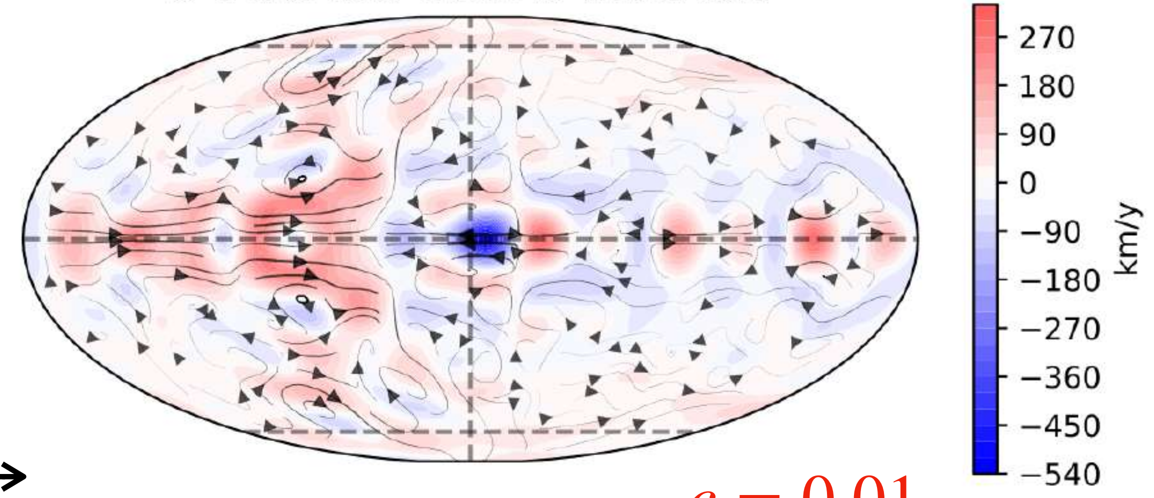
Reference

δ inverted from u_∞



$c = 0.23$

δ inverted from u inverted



$c = 0.01$

$c = 0.4$

$$d_E = \sqrt{\frac{\nu}{\Omega}} = E^{1/2} D$$

$$E = \frac{\nu}{\Omega D^2}$$

$$D = r_c - r_i$$

$$P_m = \frac{\nu}{\eta}$$

Simulation S1:

$$E = 10^{-6}$$

$$R_m \simeq 500$$

$$\sqrt{P_m} = 0.45$$

Radial shear reasonably well predicted from the QG hypothesis

No correlation between the shear estimated as in the geophysical application and the actual shear in the simulation

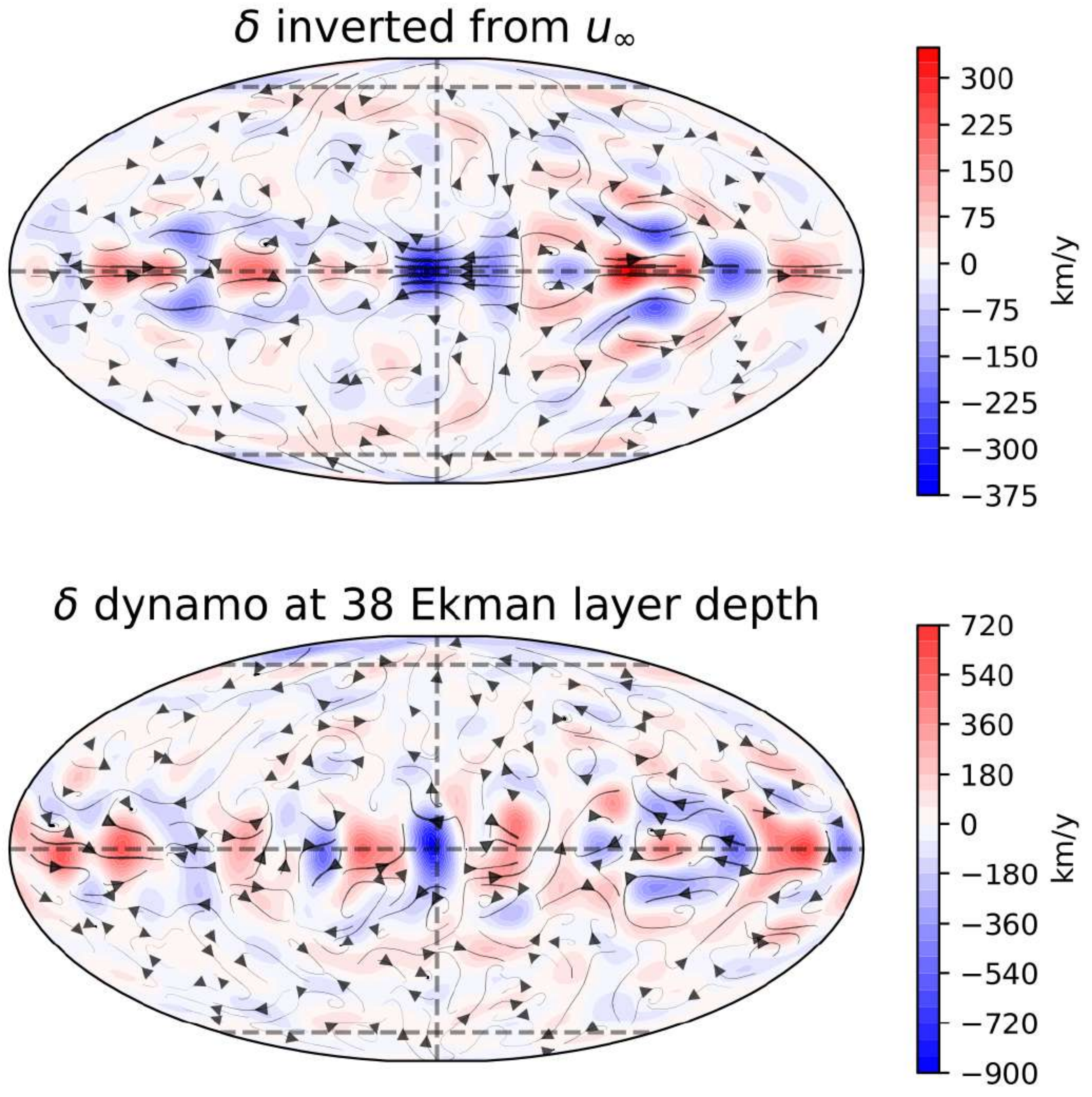
$\boldsymbol{u}_\infty$ asymptotic limit of the flow in the Ekman layer (see the poster ‘Flow at the top of the free stream in geodynamo calculations’)

Schaeffer, Jault, Nataf & Fournier, GJI, 2017

Depth at which the shear is best estimated

Correlation between the shear estimated from $\boldsymbol{u}_\infty$ and the actual shear in the geodynamo simulation (S1)

Depth below the CMB	25 %	median	75 %
6 d_E	0.14	0.18	0.24
18 d_E	0.24	0.28	0.34
38 d_E	0.3	0.35	0.4


δ inverted from u_∞

δ dynamo at 38 Ekman layer depth

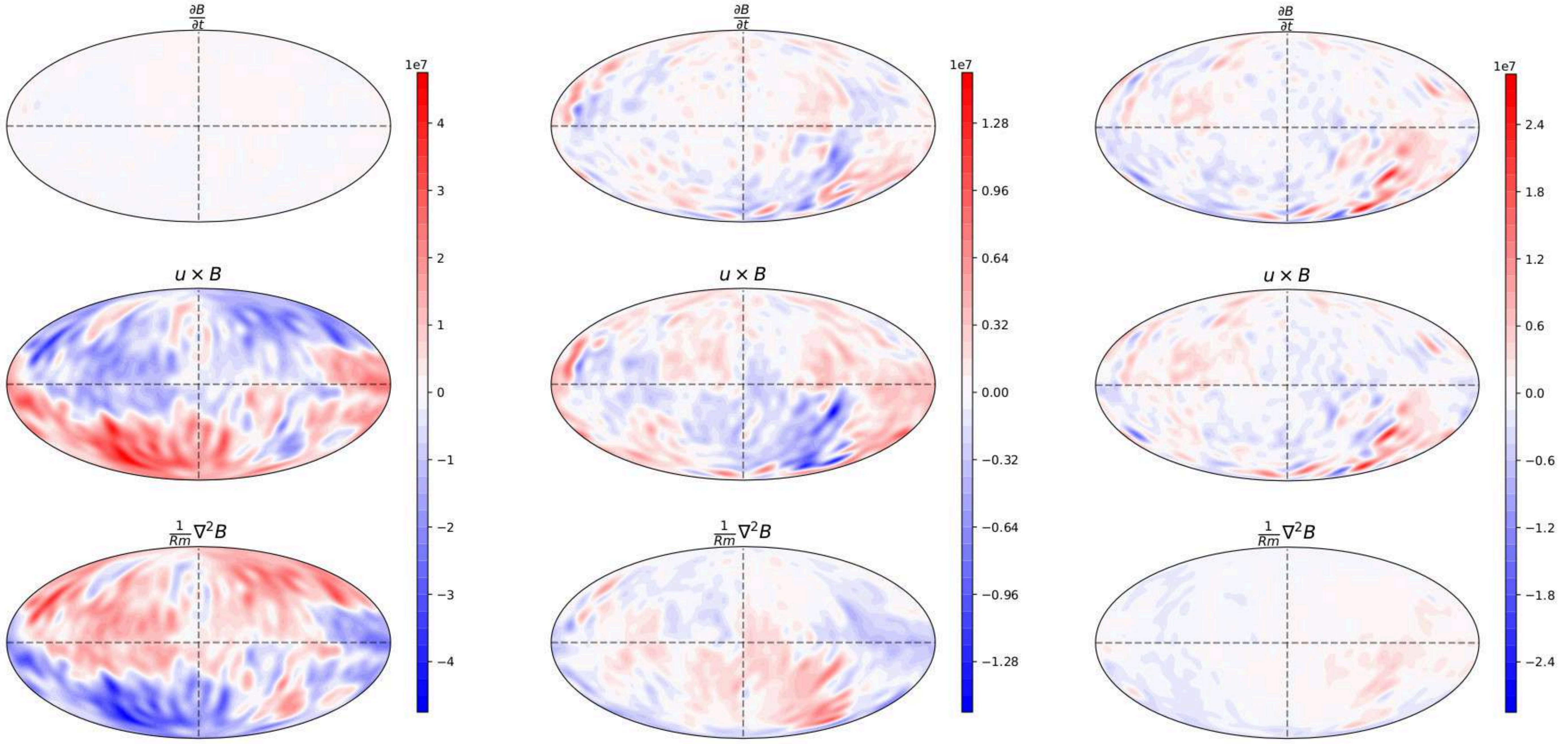
$c = 0.4$

Shear from synthetic data, first lessons

- Radial shear in the flow the strongest next to the equator, as expected for quasi-geostrophic (QG) flows
- Radial shear reasonably estimated from the QG hypothesis
- First difficulty: estimation of the core surface flow; some success using $\boldsymbol{u}_\infty$ but not the estimated flow (to date) → need to improve the estimation of the flow
- Location for which the shear is best estimated: not just below the viscous boundary layer where magnetic diffusion remains important; illustration for $r = 0.975 r_c$ whereas the Ekman depth d_E is $6.5\,10^{-4}\,r_c$ only

Diffusion below the core surface

Simulation S1 of Schaeffer et al. (2017), toroidal coefficient



$r = r_c - 2.5\, d_E$

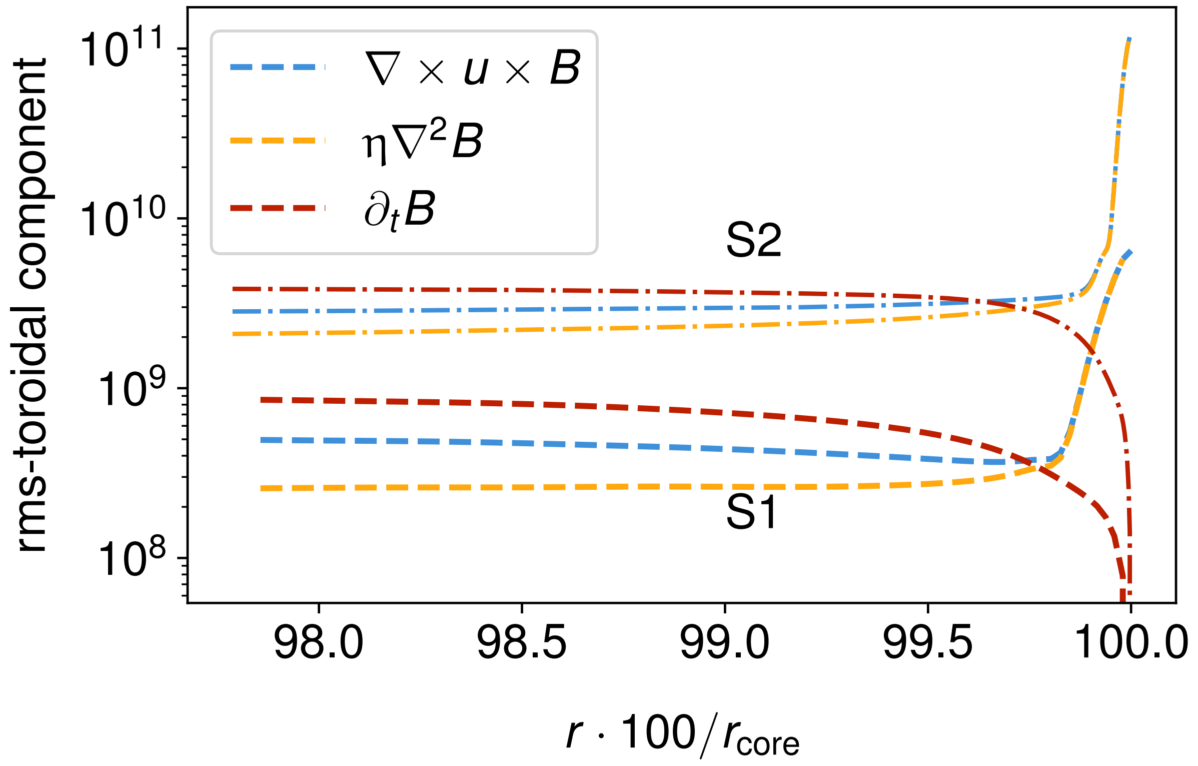
Inside the Ekman layer

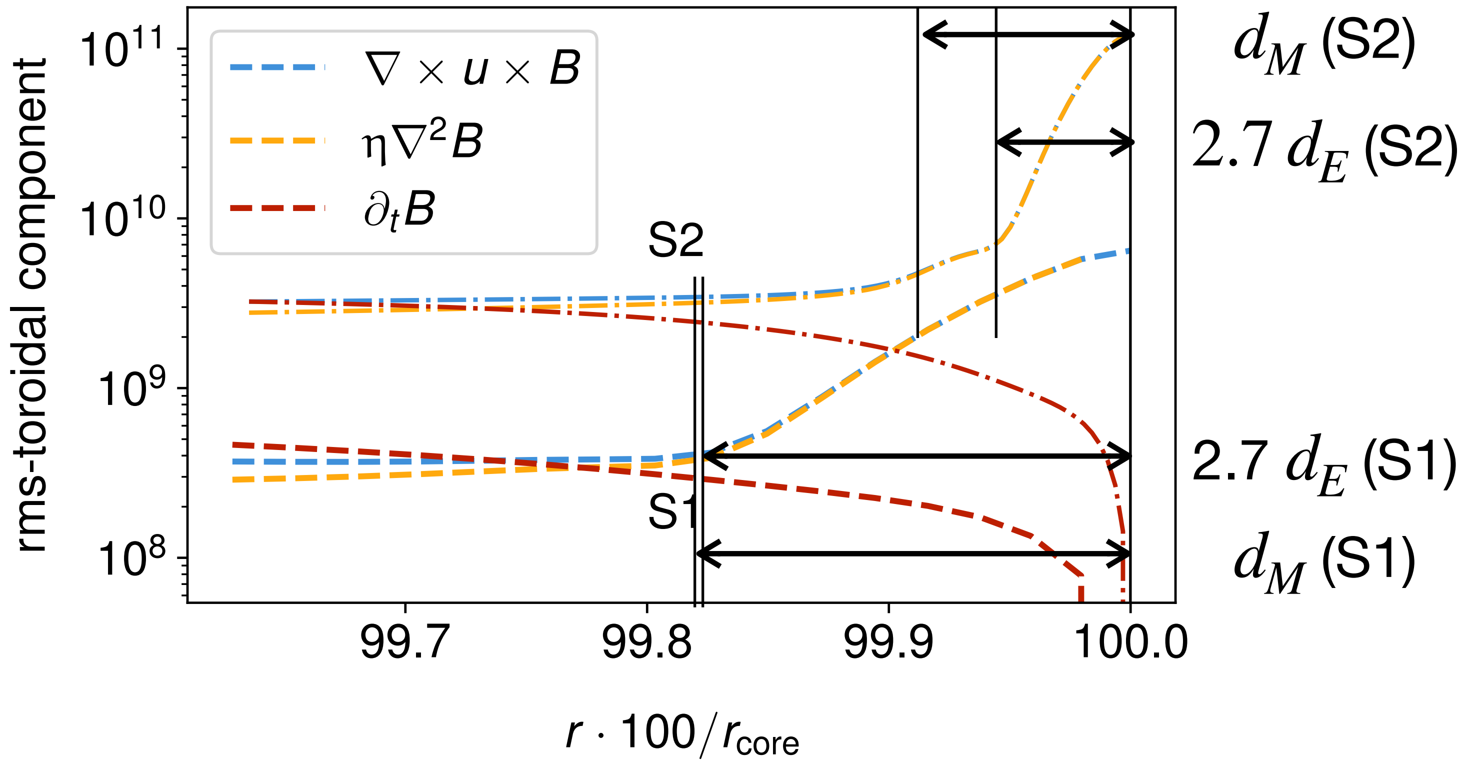
$r = r_c - 18.5\, d_E$

$r = r_c - 58.5\, d_E$

In the bulk

Comparison between
the simulations S1 and S2:

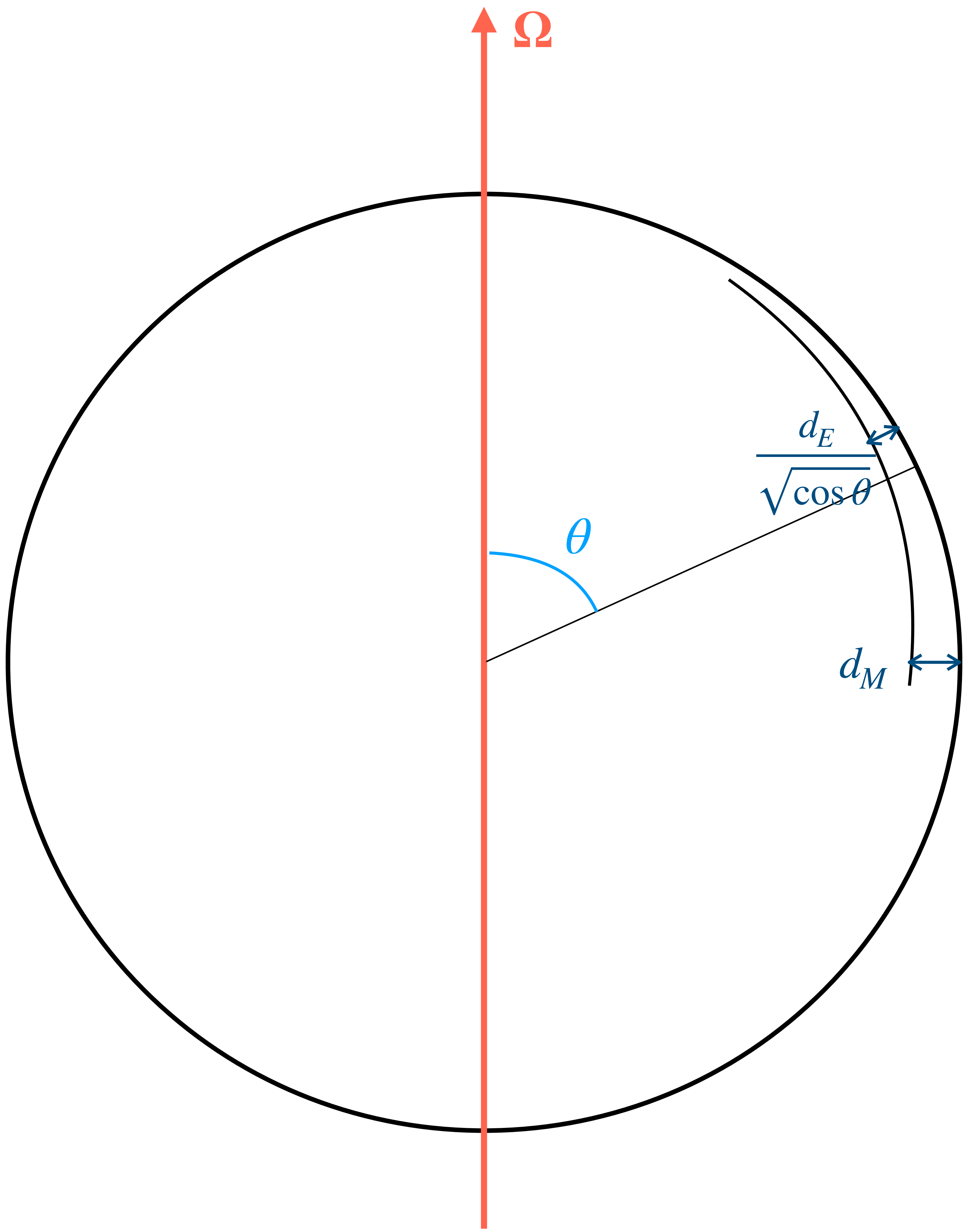




Parameters for S2: $E = 10^{-7}$

$R_m \simeq 500 \quad \sqrt{P_m} = 0.32$

Sketch of the viscous boundary layer



Thickness of the Ekman layer: $\frac{d_E}{\sqrt{\cos\theta}}$

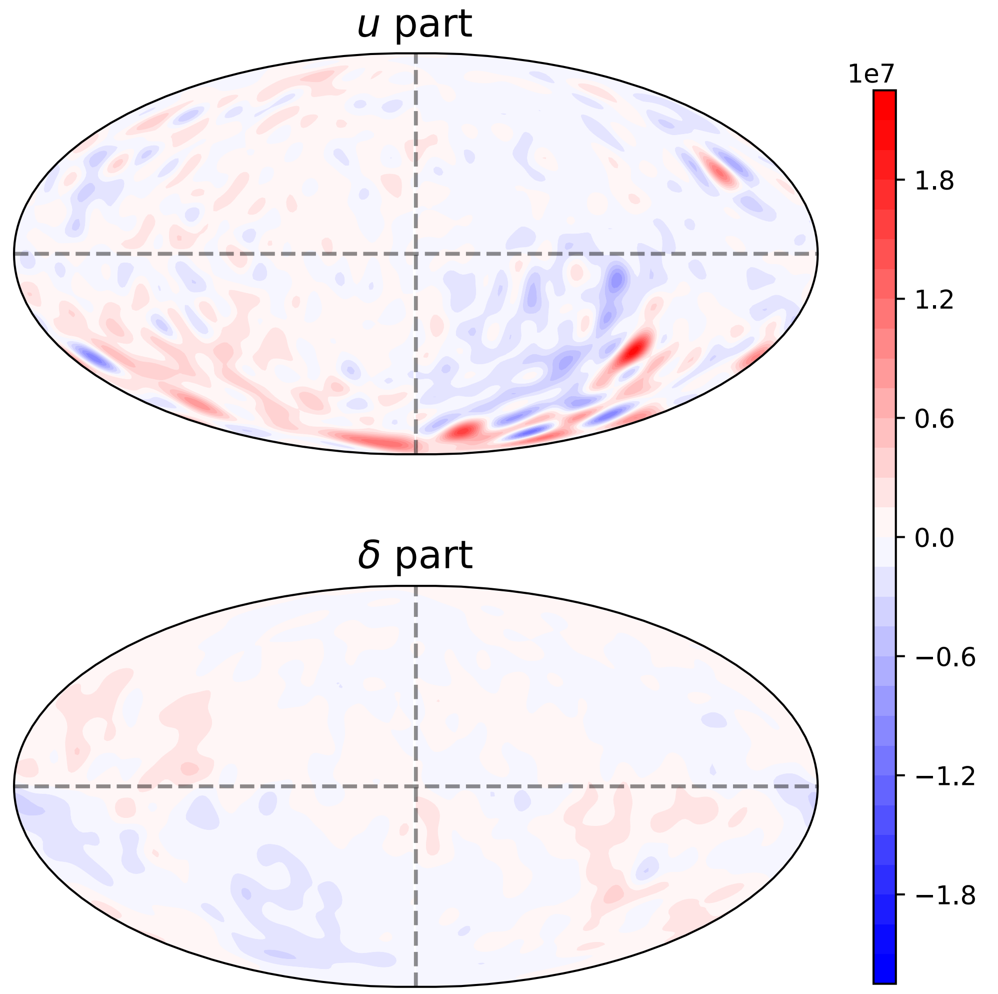
replaced at the Equator by an Hartmann

boundary layer of depth $d_M = \frac{\sqrt{\rho\mu\nu\eta}}{B|_{r=r_c}}$, which

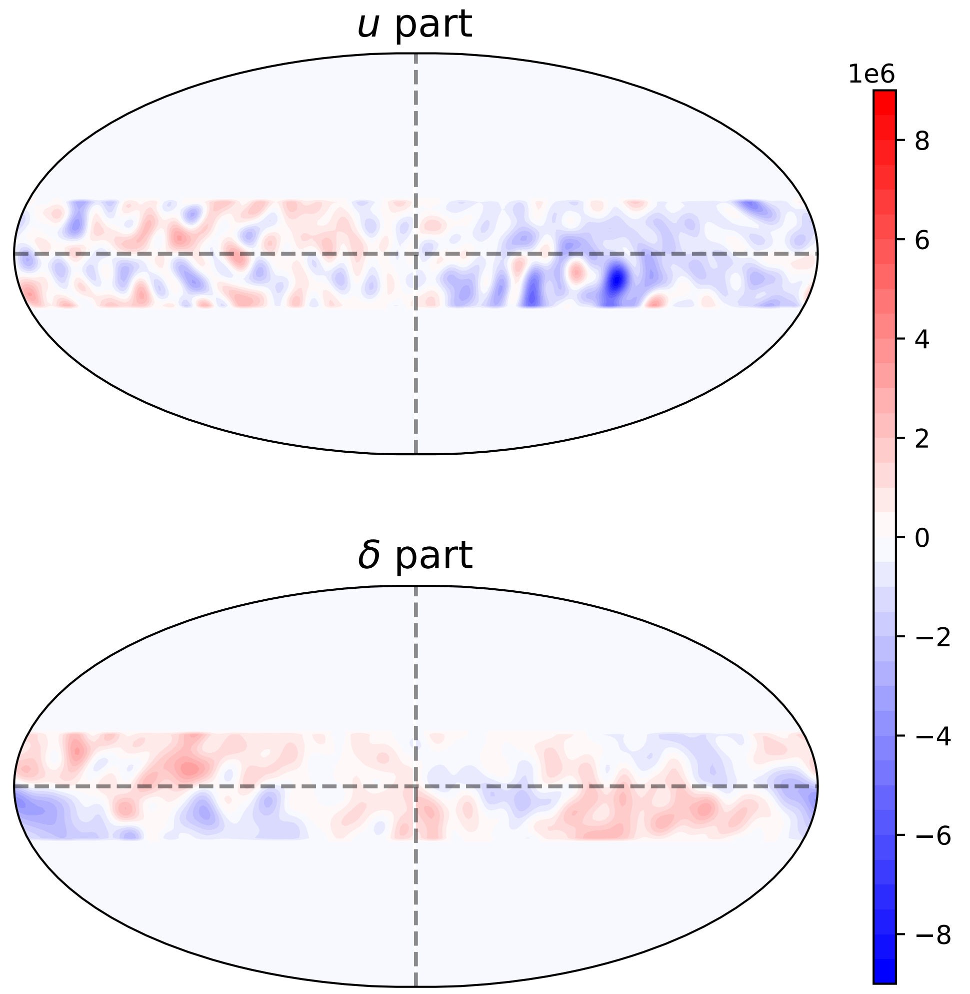
is independent of the rotation

Test of the forward problem: toroidal part of the induction term $\nabla \times (\boldsymbol{u} \times \boldsymbol{B})$

- Simulation S1
- $r = r_c - 58.5\, d_E$
- Test of the forward problem $A\boldsymbol{u} + B\boldsymbol{\delta} = 0$
- Starting hypothesis, anti-correlation $c = -1$
- diagnostics slightly better next to the equator



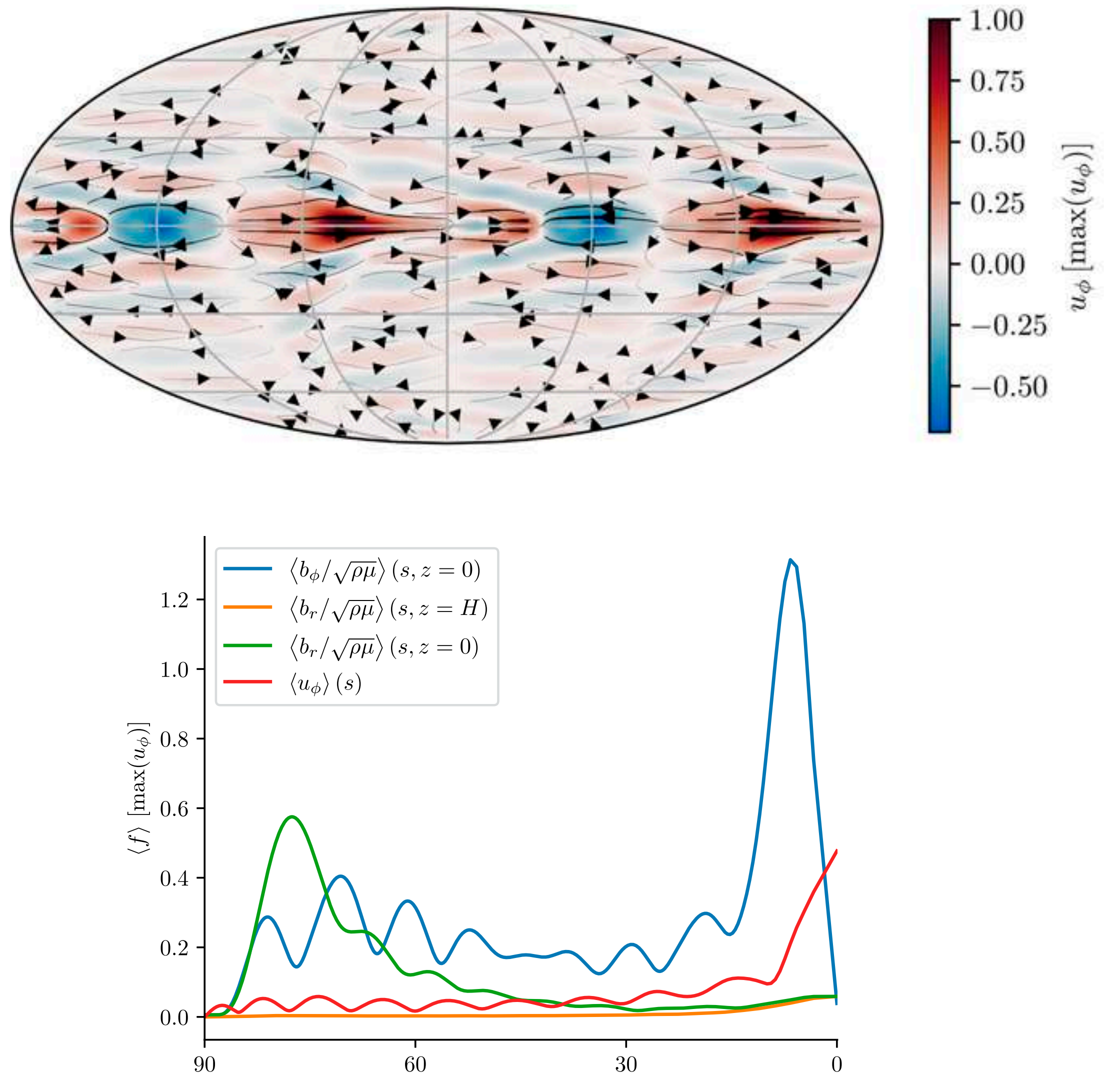
$c = -0.21$



$c = -0.29$

Perspectives

- No-slip geodynamo simulations for more extreme parameters, *e.g.* $E = 10^{-7}$ *(as in S2),* $R_m = 2000$ (instead of 500 for S1 and S2) and smaller magnetic Prandtl number P_m to better model the physics next to the Equator
- To reproduce the geophysical case: $\sqrt{P_m} \ll 1$
- Search for the time-scales and length-scales for which the forward model gives the best results
- Extraction of wave-like motions in the equatorial region which present small length-scales in the cylindrical radial direction
- Taking $\boldsymbol{\delta} \sim 30\,\boldsymbol{u}$, the term that depends on the the conductance Σ becomes significant for motions with period T about 5 yrs when the conductance $\Sigma \sim 10^8$ S; linear dependence of the perceptible Σ on T
- Investigation of geodynamo models with electrically conducting mantle
- Key to estimate the mantle conductivity σ_m from below: increasing the time resolution of core surface flow models as permitted by the availability of satellite data (Swarm and Macau Science satellites)



Numerical calculation of a Magneto-Coriolis mode of period about 7 years (Gerick et al, 2021; Gillet et al., 2022)