Spectral Decomposition of the Mistral Wind

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Main scientific question: What is the spectral decomposition of the Mistral and its impact on air-sea fluxes?

Background

The Mistral is a northwesterly flow through the Rhône Valley, that brings dry and cool air out over the Gulf of Lion. Typically, it occurs with a sister, westerly flow through the Aude Valley, called the Tramontane, which combines with the Mistral over the sea (the "Mistral" will refer to both). They're both driven by the presence of a cyclone in the the Genoa region (see Fig. 1), but events have been caused by cyclones as far east as the Adriatic.

The Mistral leads to large surface cooling over the Gulf of Lion, which, when combined with the seasonal change in atmospheric surface forcing, can cause large vertical mixing in the gulf called deep convection. Deep convection then aids the general overturning the Mediterranean Sea (Waldmann et al. 2018).

In prior work, the Mistral has been determined to have a high frequency component and a low frequency component (Keller et al. 2022), however the spectral properties of the Mistral haven't been fully investigated until this work.

Methodology

The Fourier transform was utilized to explain the spectral composition of the Mistral. It is defined as:

$$\mathcal{F}\{f(x)\} = \hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-j\omega x}\,dx$$

And, due to its integral nature, is a linear operator:

$$\mathcal{F}\{f(x) + g(x) + h(x)\} = \hat{f}(\omega) + \hat{g}(\omega) + \hat{h}(\omega)$$

Which will be important in our analysis.

Hourly ERA5 reanalysis data from 1940 to 2023 was used to build an index of Mistral events (Hersbach et al. 2020). These events were determined with thresholds for the wind speed (greater than 2 m/s), wind direction (northerly flow between -135 and -45 degrees) at two locations (Montélimar and the Gulf of Lion), and the presence of a Genoa low (a complete mean sea level minimum contour) touching the white bounding box (see in Fig. 1; Givon et al. 2021, Keller et al. 2022). Additional processing was done to group these indices into events (where indices within 2 days of each other were grouped into an event and events less than 1 day in duration were discarded).

To understand the spectral composition of the Mistral, Gaussian curves were then fitted to the wind speed taken from the Gulf of Lion point using the event information determined in the step before. The curve fitting was only applied to the wind speed profile contained within the individual Mistral events, resulting in a fitted curve per event (see Fig. 3). The spectral information was then extracted from the fitted Gaussian curves with the Fourier transform. The Gaussian function and its Fourier transform are shown in Fig. 2 (functions g(t) and ĝ(ω), respectively).]

To see the impact of the spectral representation of the Mistral on air-sea fluxes, we note the following formulas for latent and sensible heat, respectively:

$$Q_E = \rho \Lambda C_E \Delta q |\Delta u|$$
$$Q_H = \rho c_p C_H \Delta\theta |\Delta u|$$

Where ρ is the density of the air, Λ is the latent heat of evaporation constant, c_p is the specific heat of enthalpy constant, and C_E and C_H are coefficients of latent and sensible heat, respectively (Large and Yeager 2004, 2008). Δq, Δθ, and Δu are the specific humidity, temperature, and wind speed and ocean current differences between the air and sea surface, respectively.

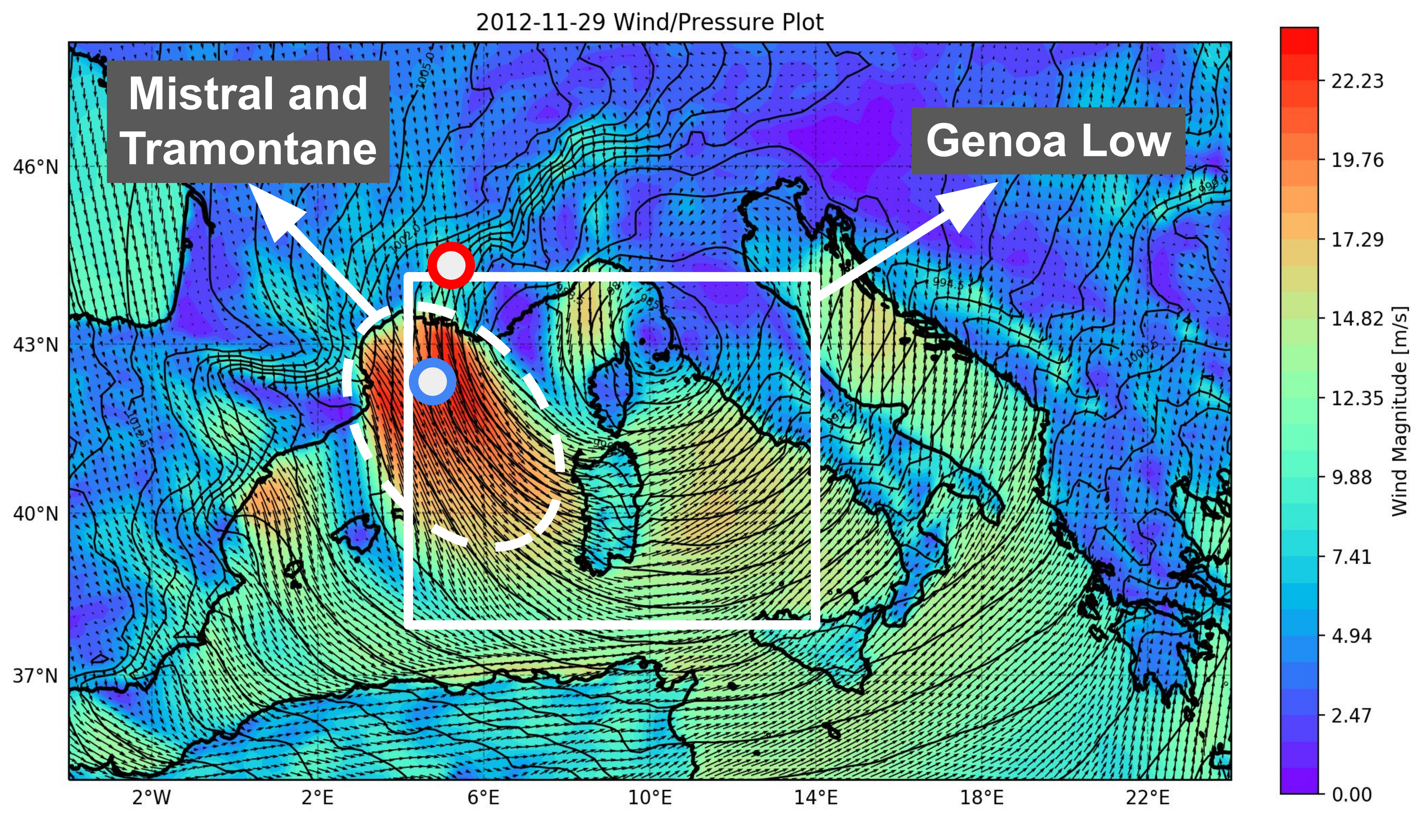


Fig. 1: Wind speed magnitude and sea level pressure contour plots (not labeled) displaying the structure of the Mistral, Tramontane (main "heart" heart of the winds shown by the dashed ellipse), and Genoa Low. If a cyclone mask touches the bounding box (38 to 44 N; 4 to 14 E) it is considered a Genoa Low. Montélimar is marked by the red outlined dot (44.56 N, 4.75 E) and the Gulf of Lion representative wind location is marked by the blue outlined dot (42.67 N, 4.44 E).

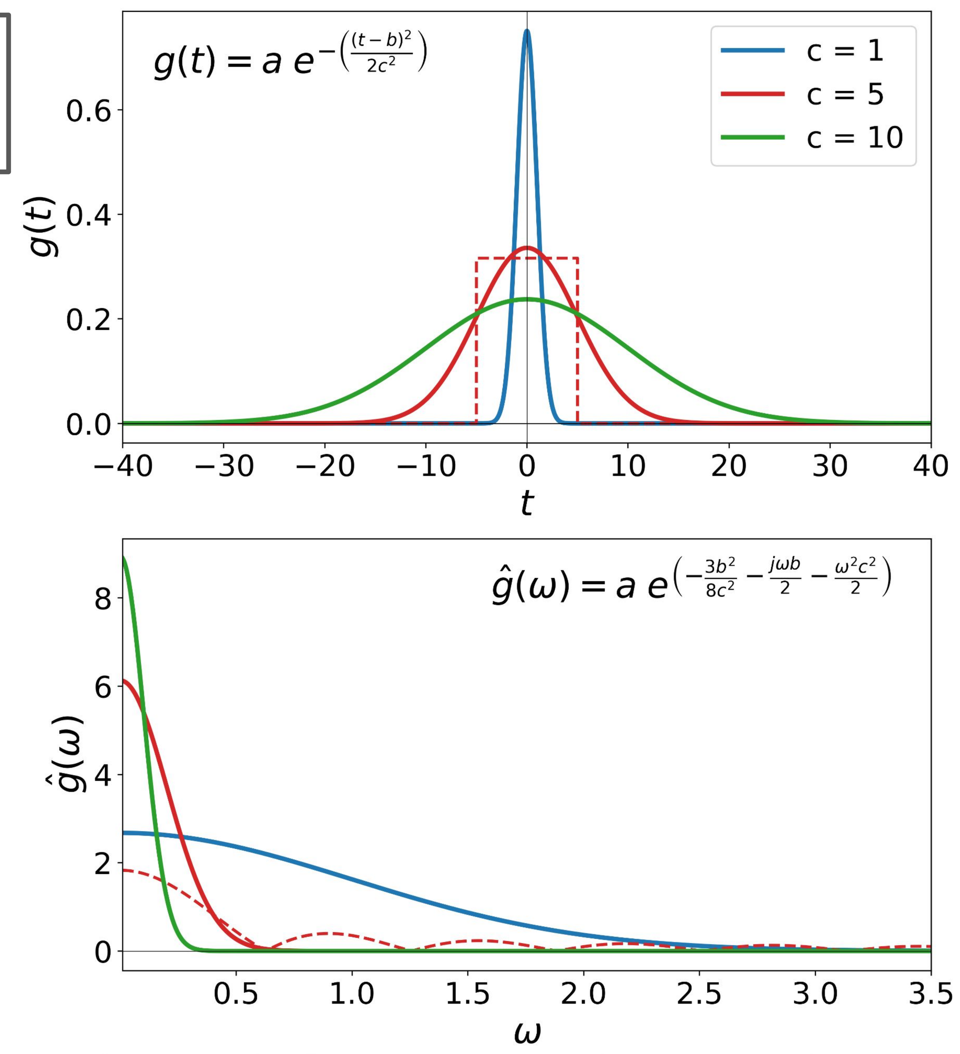
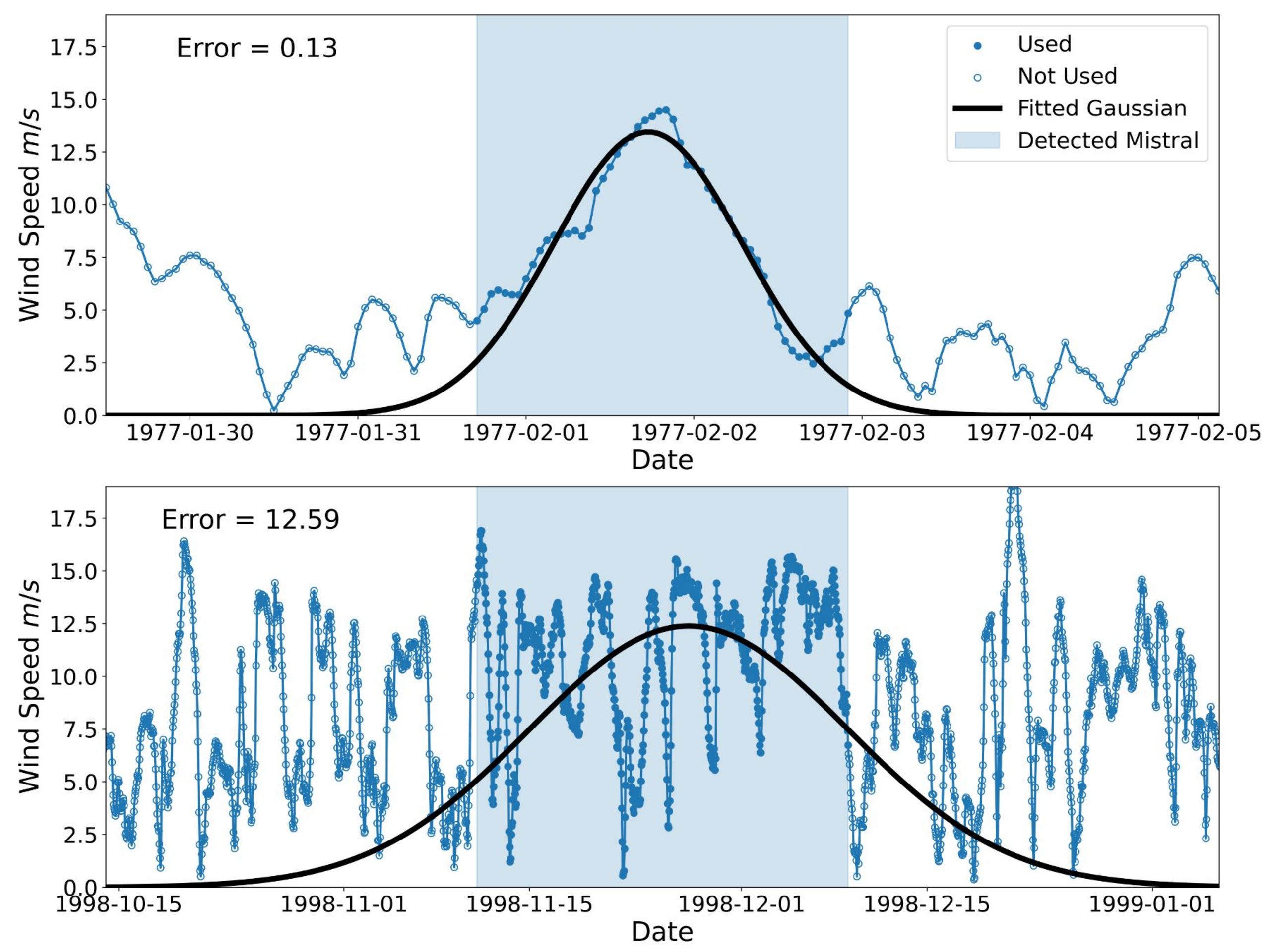


Fig. 2: Example Gaussian curves (each normalised by the signal energy). Top plot shows the time domain of the curves; Bottom plot shows the frequency domain. a, b, and c are all parameters of the Gaussian and its transform, a = amplitude, b = time intercept, and c = width. A square pulse (dashed lines) with c = 5 (it's respective width) is shown too (equations not shown).



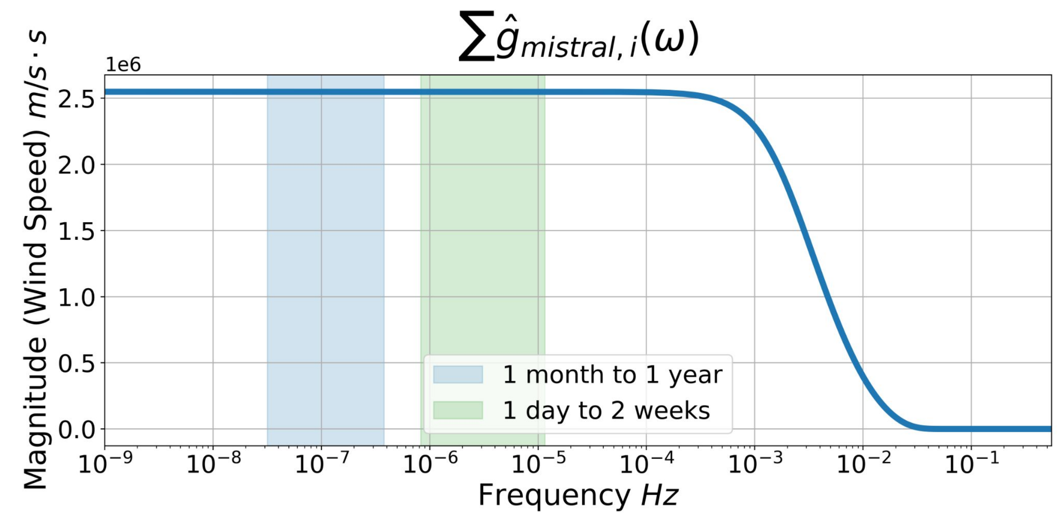


Fig. 4: Composite Mistral spectrum. All 2410 curve fitted events are summed and presented.

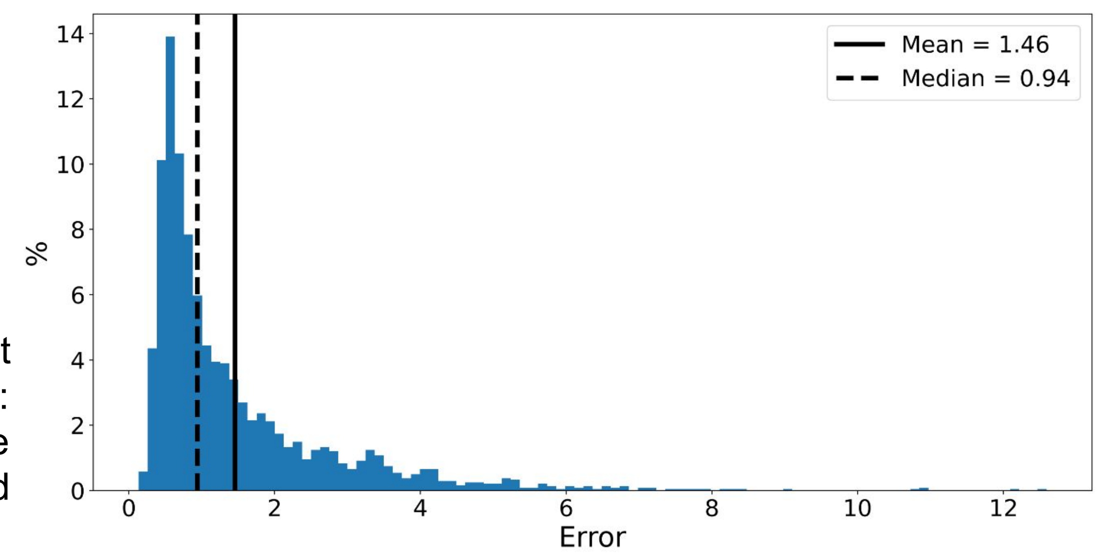


Fig. 3: Best and worst case of Gaussian curve fitting with the Mistral event data (fitted to wind speed and constrained by event duration). Above top: best case. Above bottom: worst case. Right: error distribution for the different cases. For above, the filled dots were used to fit the curve, unfilled were excluded.

Results and Discussion

Our discussion starts with the use of Gaussian curves to represent the Mistral wind. In Keller et al. 2022, the Mistral was assumed to be a square pulse function. This is easy to fit to wind data, as it has a simple definition in time domain, but is not as clean to analyze in the frequency domain (the Fourier transform is a sinc function; dashed curves in Fig. 2). On the other hand, Gaussian curves are fairly simple to fit in the time domain *and analyze in the frequency domain*, as its Fourier transform is another Gaussian. This has a very interesting consequence on how the Mistral behaves in the frequency domain, as the pulse has power with a broad representation in low, medium, and high frequencies. Unsurprisingly, shorter in duration pulses have more high frequency components and vice versa for longer in duration pulses (see the different curves with c, the pulse width, in Fig. 2). We can then use the linear properties of the Fourier transform to rebuild the composite spectrum of all Mistrals to see which frequencies this phenomenon primarily acts on.

2410 Mistral events were detected with our methodology in the ERA5 reanalysis data. Figure 3 shows the best and worst case of Gaussian curve fitting, as well as the errors for the rest of the cases (the error was calculated by taking the magnitude of the standard deviation vector for the fitment of the three Gaussian parameters: a, b, and c). Generally speaking, the Gaussian appears to match the pulse-like behavior of the Mistral quite well, as the large majority of the events have a low fitting error (below a mean of 1.46), with even the worst case scenario capturing the overall structure of the event.

We find that the composite spectrum of all Mistral events ($\Sigma\hat{g}_{mistral,i}(\omega)$ in Fig. 4) covers a very broad spectrum, with the energy only trailing off starting at 1000 Hz and higher (note the log formatting of the x-axis in Fig. 4). The Mistral clearly impacts the monthly to yearly frequency range similarly to the daily to bi-weekly range (with the former necessarily larger), demonstrating its broad spectrum impact.

Consequently, we can determine the Mistral's impact on air sea fluxes, which are predominantly in the latent and sensible heat fluxes (Givon et al. 2021, Keller et al. 2022). If we assume ρ, Λ, c_p, C_E, C_H, Δq, and Δθ to all be constant (the latter two assumptions are ***not*** true but can be assumed so for first order approximation) and if we assume Δu ≈ u(t), the wind speed, (as the ocean current is typically an order or two magnitudes slower; cm/s for ocean current vs m/s for wind speed), then we can directly apply our spectrum to analyze the heat fluxes:

$$Q_i = \int_{-\infty}^{\infty} \phi_i u(t) e^{-j\omega t}\,dt = \phi_i \int_{-\infty}^{\infty} u(t) e^{-j\omega t}\,dt = \phi_i \hat{u}(\omega)$$

Where φ_i is the combination of the constants for the i-th flux (E or H). Therefore, as our Mistral spectrum is given by the transformation of the wind speed (just curve fitted), we can conclude that the latent and sensible heat flux spectra will have the same shape, just with a shifted magnitude due to the multiplication of the constants, φ_i.