Asymmetric Huber distribution for geomagnetic data and application to jerks

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INTRODUCTION: THE HUBER DISTRIBUTION AND RESIDUALS OF GEOMAGNETIC DATA

Space weather, the ionosphere, tidal and induced fields add a complicated set of noise to observations of the core field, meaning that residuals in geomagnetic data have heavier tails than would be expected from a Gaussian distribution. The common method of accounting for this is to assume that the noise follows a Huber distribution. Residuals of magnitude $\leq \delta\sigma$ fit a Gaussian curve with standard deviation σ, but larger residuals fit an exponential function. The parameter δ is most commonly set to 1.5. An example of the non-Gaussian distribution of geomagnetic noise is shown in Figure 1.

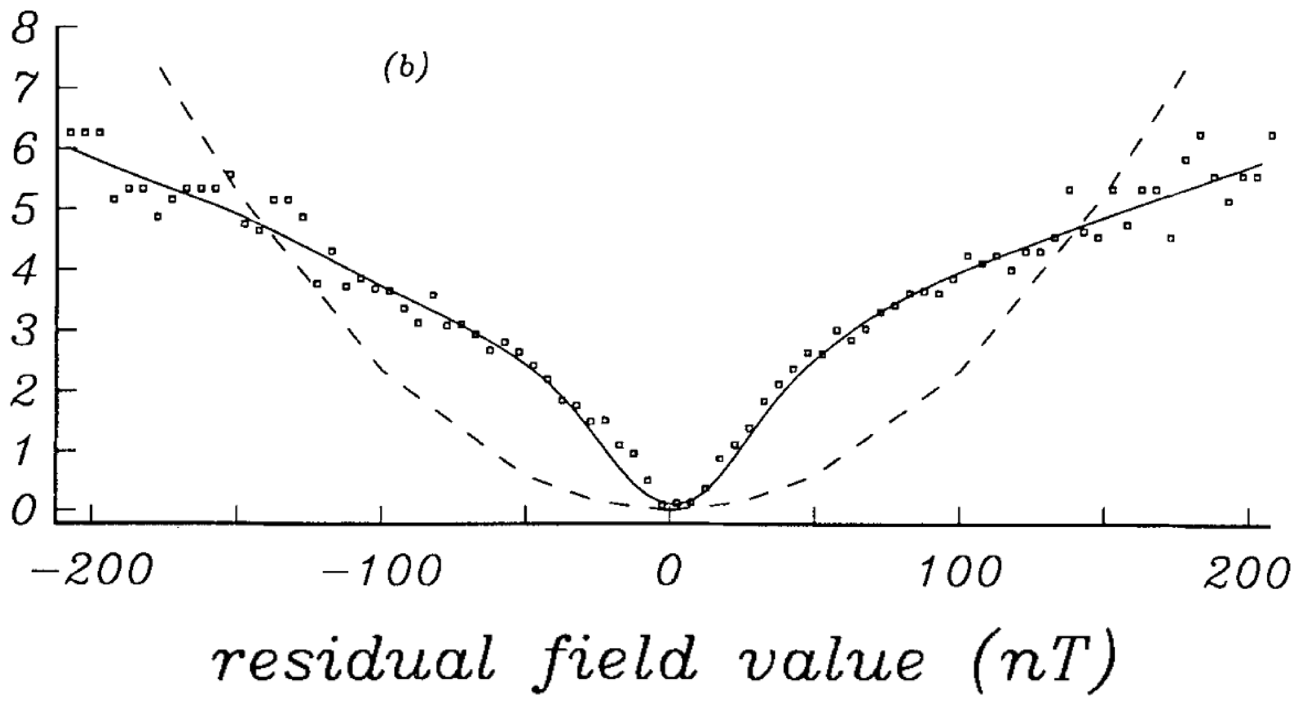

Figure 1: Figure adapted from Constable (1988). Negative log-likelihood of Maximum likelihood estimator (solid) and Gaussian distribution (dashed) against binned residuals from Z (downward) component of the magnetic field at Yellowknife (YKC).

SKEWED DISTRIBUTIONS AND THE AURORAL OVAL

At auroral latitudes in particular, the noise distribution relative to the any slowly-varying magnetic field model shows a strong skew for many observatories. Local conditions also affect this skew, meaning there is considerable variation even between observatories at similar latitudes. Complicated models of conductivity combined with details of each space weather event may be an approach that could help decompose the magnetic field's background and space-weather-induced components, but comes at a significant cost in data processing.

We account for the asymmetry more simply by fitting each observatory's data to an asymmetric Huber distribution: the parameter δ is allowed to be different in each direction, and the two values δ_- and δ_+ are derived from a maximum likelihood calculation. Some examples of the outcome of this are shown in Figure 2.

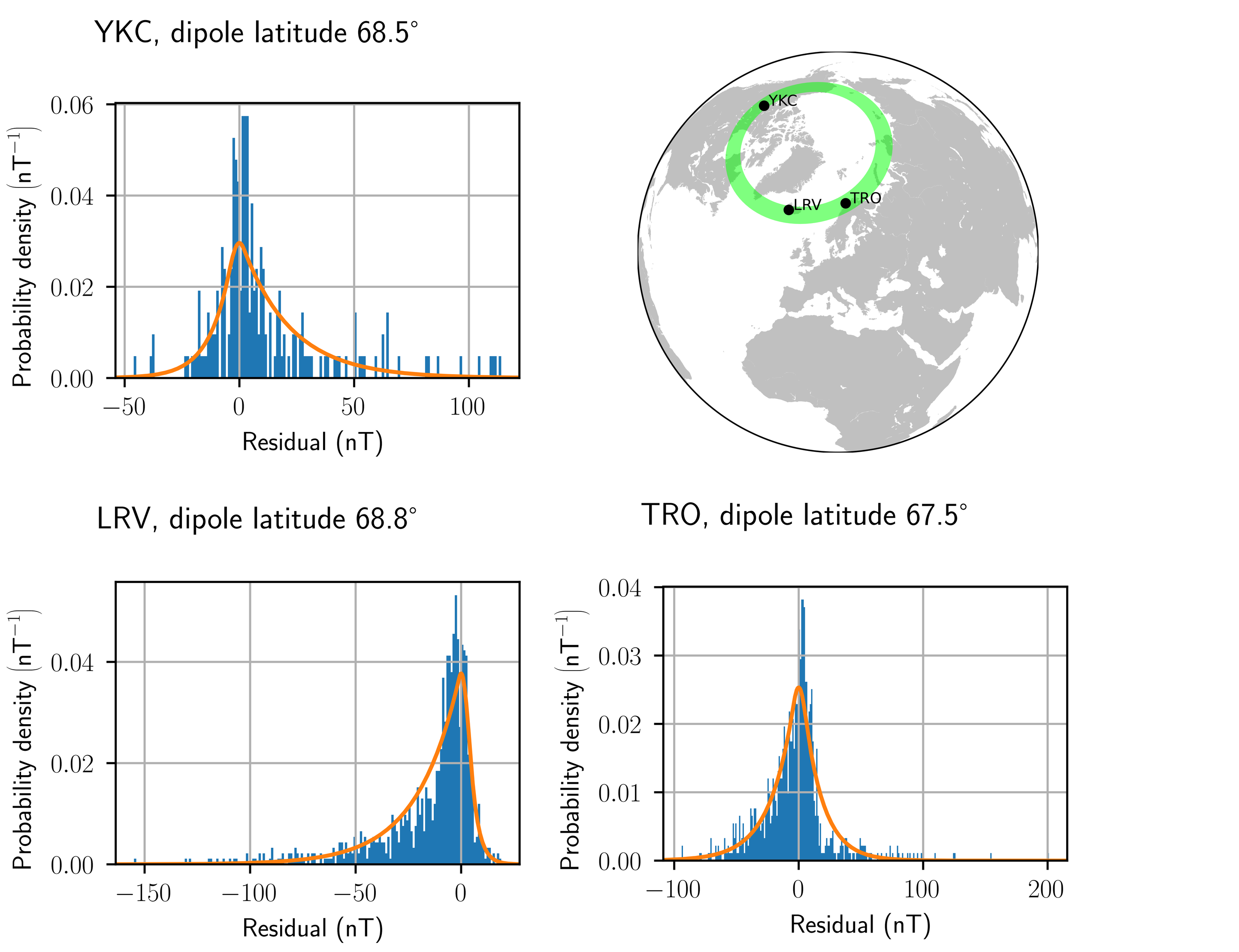

Figure 2: Histograms of noise in the Z (downward) direction for three observatories in the northern auroral zone. For each observatory, night-time hourly mean data filtered for quiet space weather conditions was taken for the period 2020-01-01 to present, and fitted to a maximum-likelihood estimator asymmetric Huber distribution, with time dependence via a constant secular acceleration (second time derivative of the local magnetic field vector) and an annual sinusoidal component. Residuals are relative to the mode of the resulting distribution, shown in orange. Top right: location map with approximate quiet night-time auroral location shown in green (dipole latitude 65–70°).

APPLICATION: FINDING SMALL AND RECENT JERKS

A *jerk* in the magnetic field is a sudden change in the secular acceleration. These are difficult to predict and are the source of many errors in magnetic field forecasts. The earliest signs of a jerk may first be noticed in the preparation of quasi-definitive data for a single observatory (e.g. Torta et al. (2015)), at which time there is often little other data to compare to. This initially rules out using correlations between observatories to filter noise, an otherwise useful technique in the study of jerks (Li et al., 2023). The large external fields seen in higher-latitude data also makes some jerks more difficult to study directly, requiring the use of global magnetic field models (Tozzi et al., 2009).

Through the use of the asymmetric Huber distribution we are able to find clear evidence for a jerk in early 2023 or late 2022 at a wide variety of observatories considered individually, as shown in the global map in Figure 3. This includes observatories at auroral latitudes such as Abisko (ABK), shown in further detail in Figure 4. Here the asymmetric fit (in dark grey) is a much better fit to the densest cluster of data points than the standard Huber fit (in light grey). This allows a clearer detection of a recent jerk, though the short span of data after the jerk likely causes the magnitude of the jerk to be exaggerated. In a similar fashion, the use of an asymmetric Huber distribution may assist in the detection of small jerks whose significance in data is weaker.

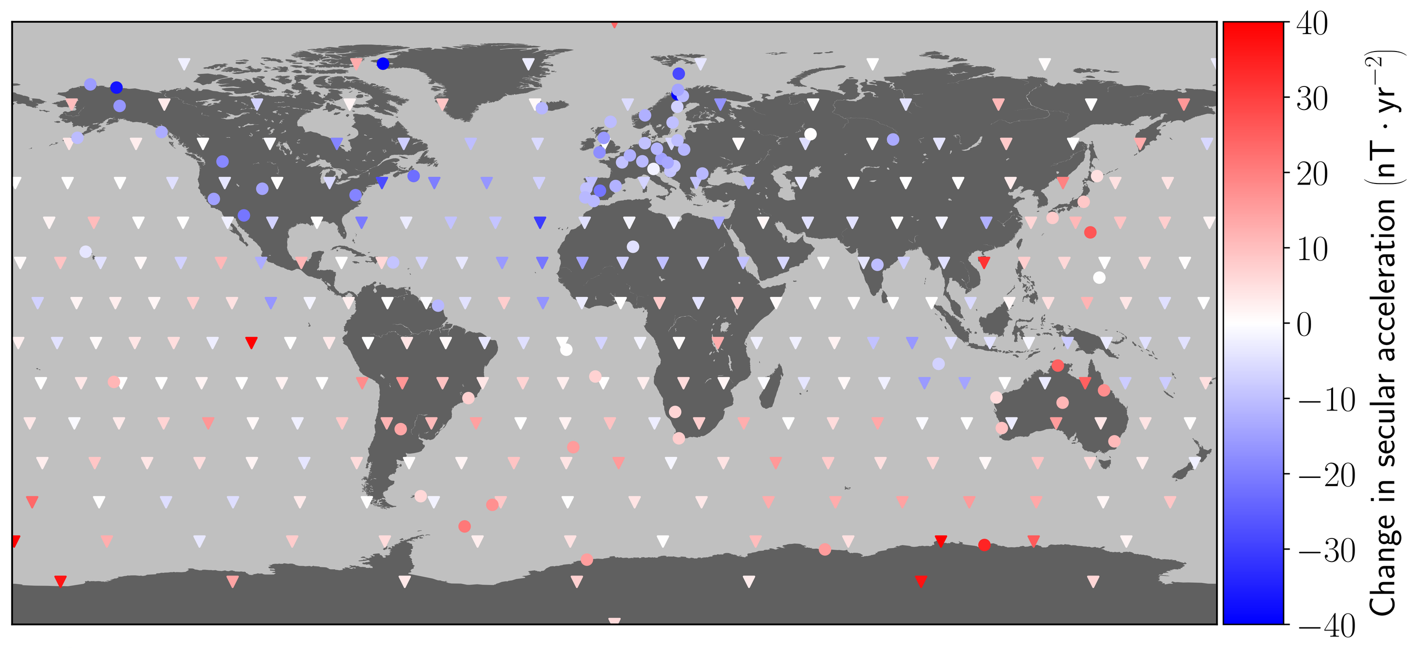

Figure 3: Global map of geomagnetic observatories (circles) coloured by magnitude and sign of jerk observed in the Z (downward) direction. Data was used in the form of hourly means beginning 2020-01-01, under quiet space weather conditions at night. Only observatories with at least two years of data ending on or after 2023-07-01 are shown. For comparison are the Geomagnetic Virtual Observatories (triangles), which use Swarm data collected into an observatory-like format and are monthly means.

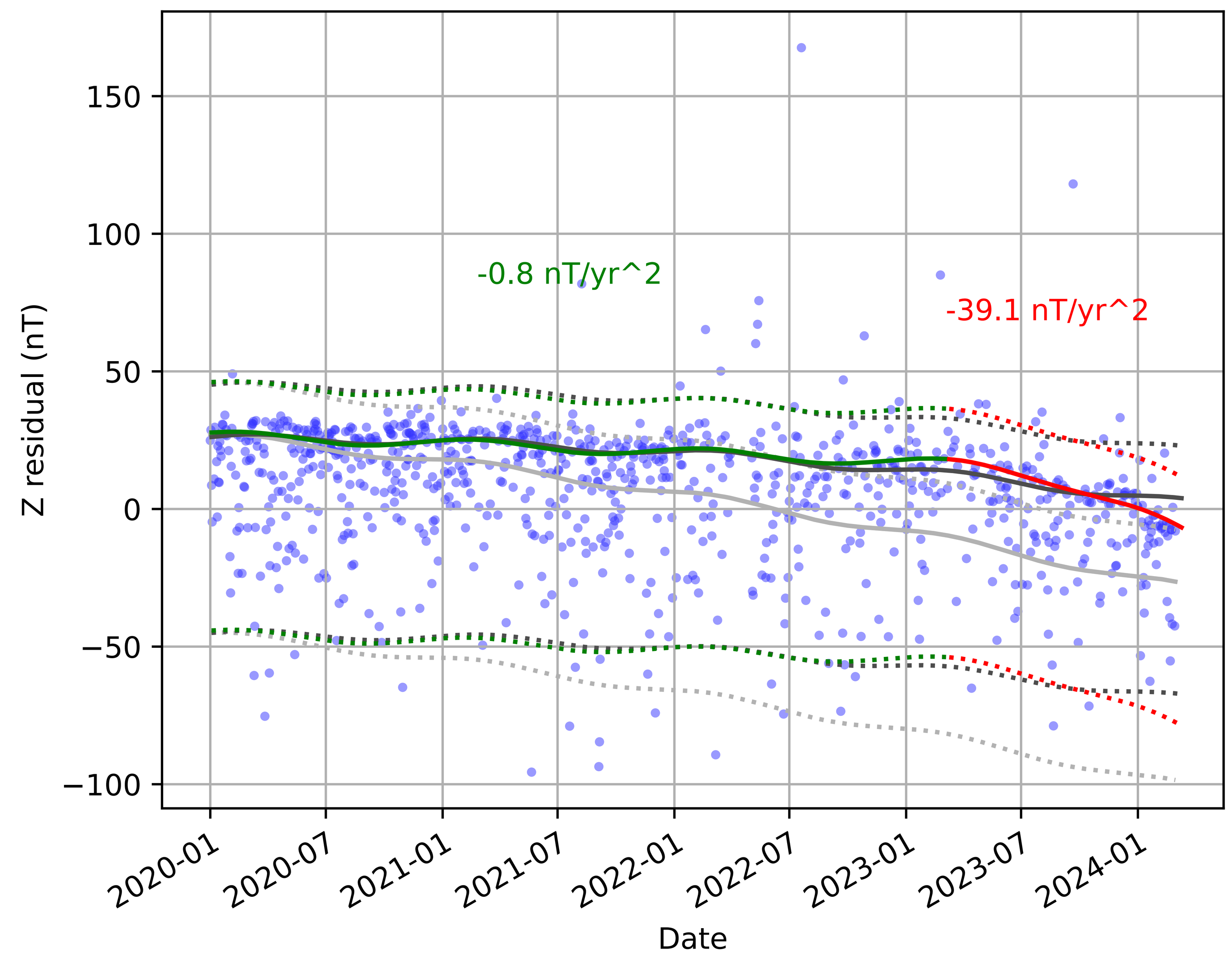

Figure 4: Scatter plot of observations at Abisko observatory (dipole latitude: 66.3°) in the Z (downward) direction, with the International Geomagnetic Reference Field subtracted. To this data, maximum-likelihood curves were fitted, assuming constant or piecewise constant secular acceleration with an annual sinusoidal component added. In light grey: a fit assuming residuals have a standard Huber distribution. In dark grey: the mode of a fit to an asymmetric Huber distribution. Red and Green use an asymmetric Huber distribution but allow a jump in secular acceleration. Dotted lines show a 95% confidence interval for each curve.

CONCLUSIONS

The asymmetric Huber distribution provides a flexible yet simple way of observing trends in the core field that is robust to skewed noise distributions as found in the auroral zone. Given many Scandinavian observatories are among the fastest to publish quasi-definitive magnetic field data, the use of this distribution can reduce latency in the detection of geomagnetic jerks, particularly in northern Europe. By finding a better fit to the magnetic field found in the quietest space weather conditions, improvements to global magnetic field models are also possible.

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