Swarm 10 Year Anniversary & Science Conference 2024

Beyond steady state:
Solving the induction equation in the ionosphere

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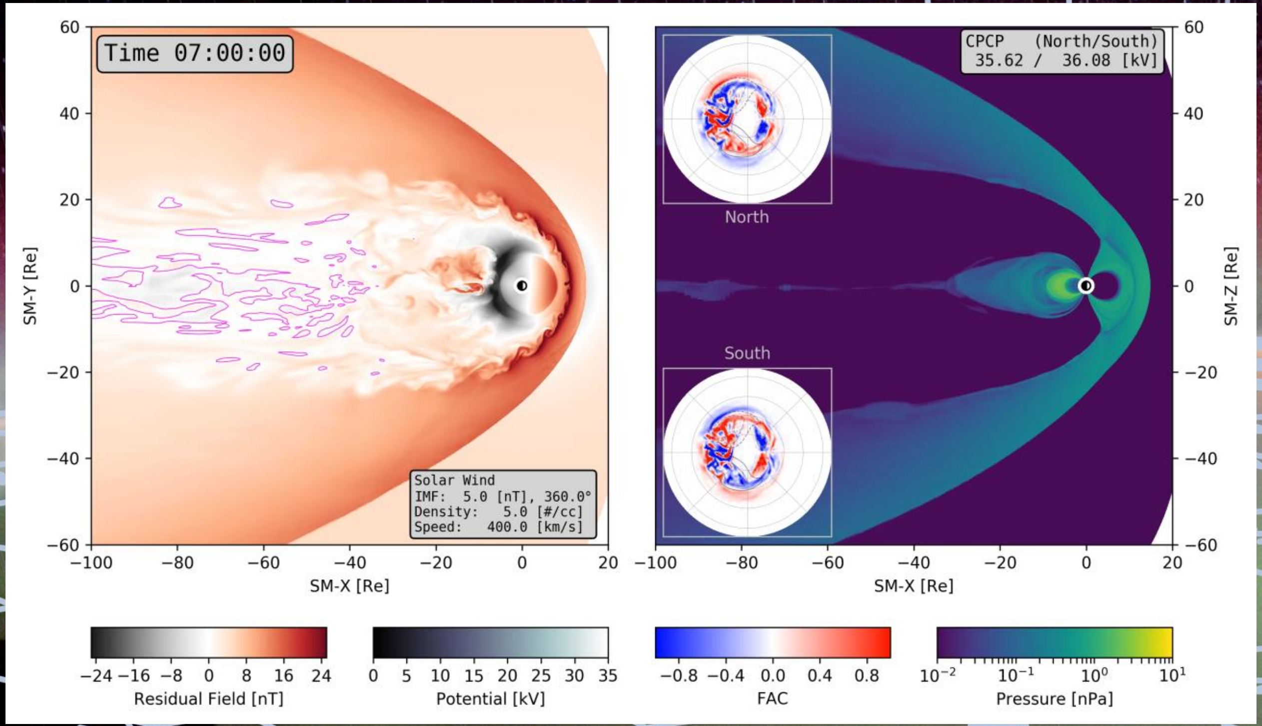
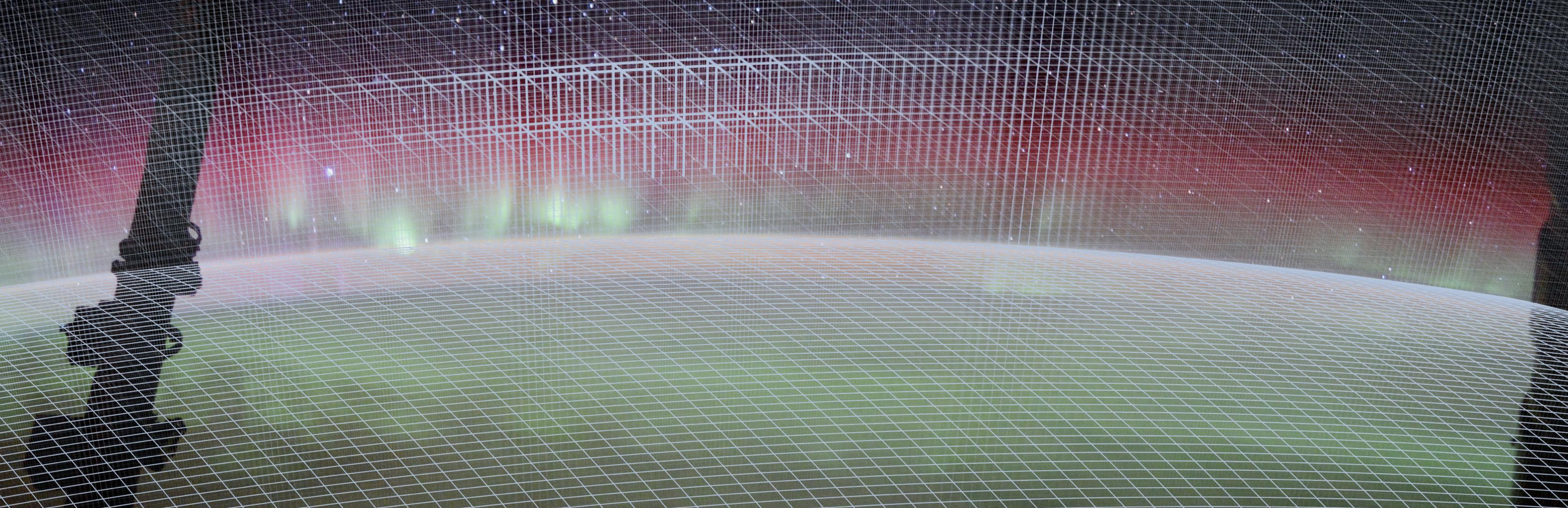
Conventional magnetosphere-ionosphere coupling

- Conventional M-I coupling represents the ionosphere as an electric circuit
- The circuit analogy is only valid in steady state and neglects induction. That is, $\frac{\partial B}{\partial t} = 0$

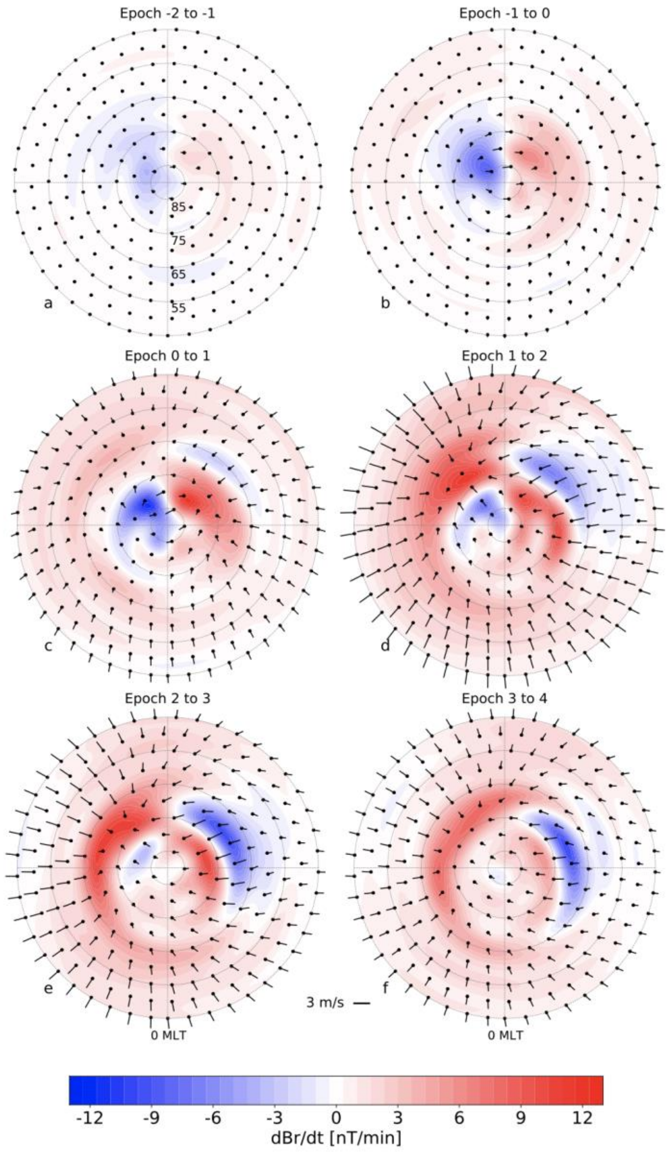


A fluid perspective – inductive MI coupling

- In space plasmas, **B** and **v** are primary variables and **E** and **j** are derived. The ionosphere is no exception (Vasyliunas 01, 05, 12, ..., Parker 96, 07, ...)
- To understand magnetic field variations, we can not neglect induction



Is it detectable?



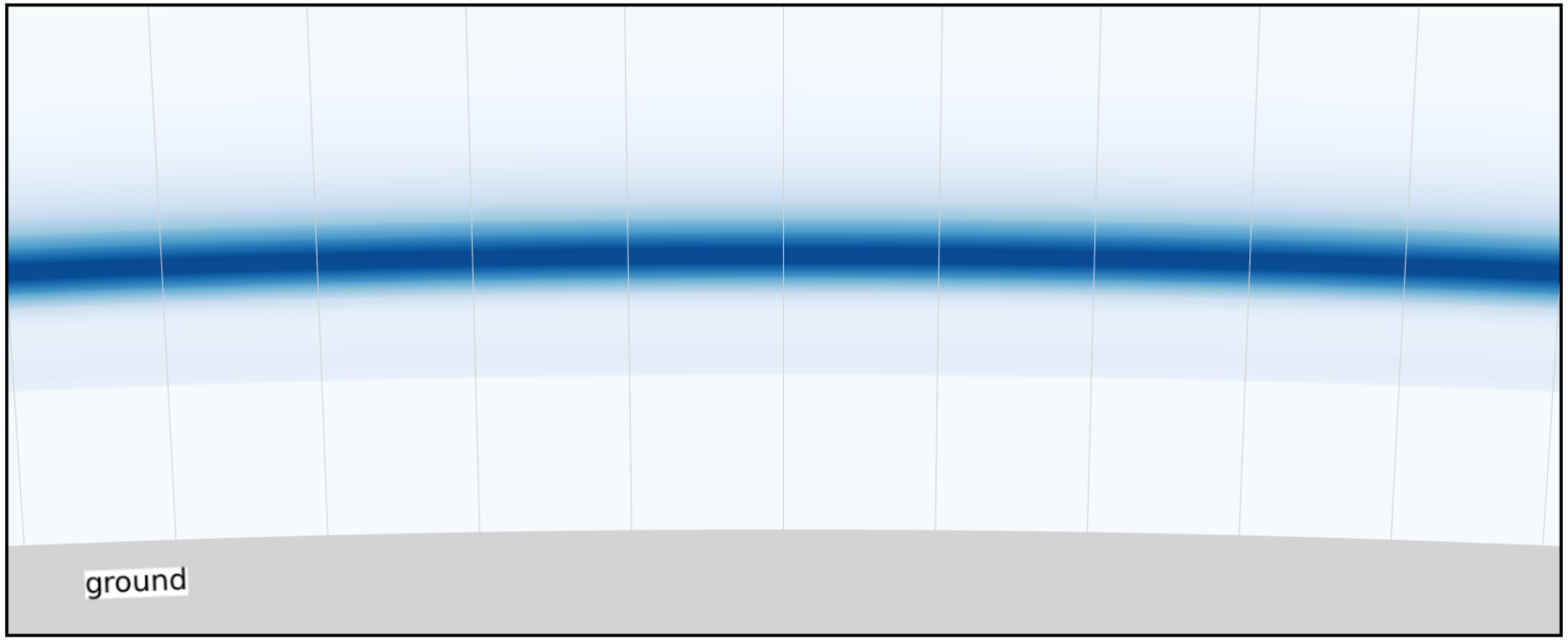
- Observations of $\partial \boldsymbol{B}/\partial t$ on ground can be used to estimate $\nabla \times \boldsymbol{E}$ in the ionosphere – analogous to what is done with main field models and the core-mantle boundary
- *Madelaire et al.* (GRL, accepted), calculate $\nabla \times \boldsymbol{E}$ and the corresponding plasma flow based on average magnetic field variations during a solar wind pressure increases
- The velocities are small, but conceptually very important – this is *why* magnetic field changes

A 2D dynamic model

Goal:
Simulate dynamic ionospheric response to magnetic field implied by FACs

Input:
Magnetic field in space associated with FACs
Conductances

Output:
B_r, E, J_S



Example Pedersen conductivity from EISCAT (to scale!)

A 2D dynamic model

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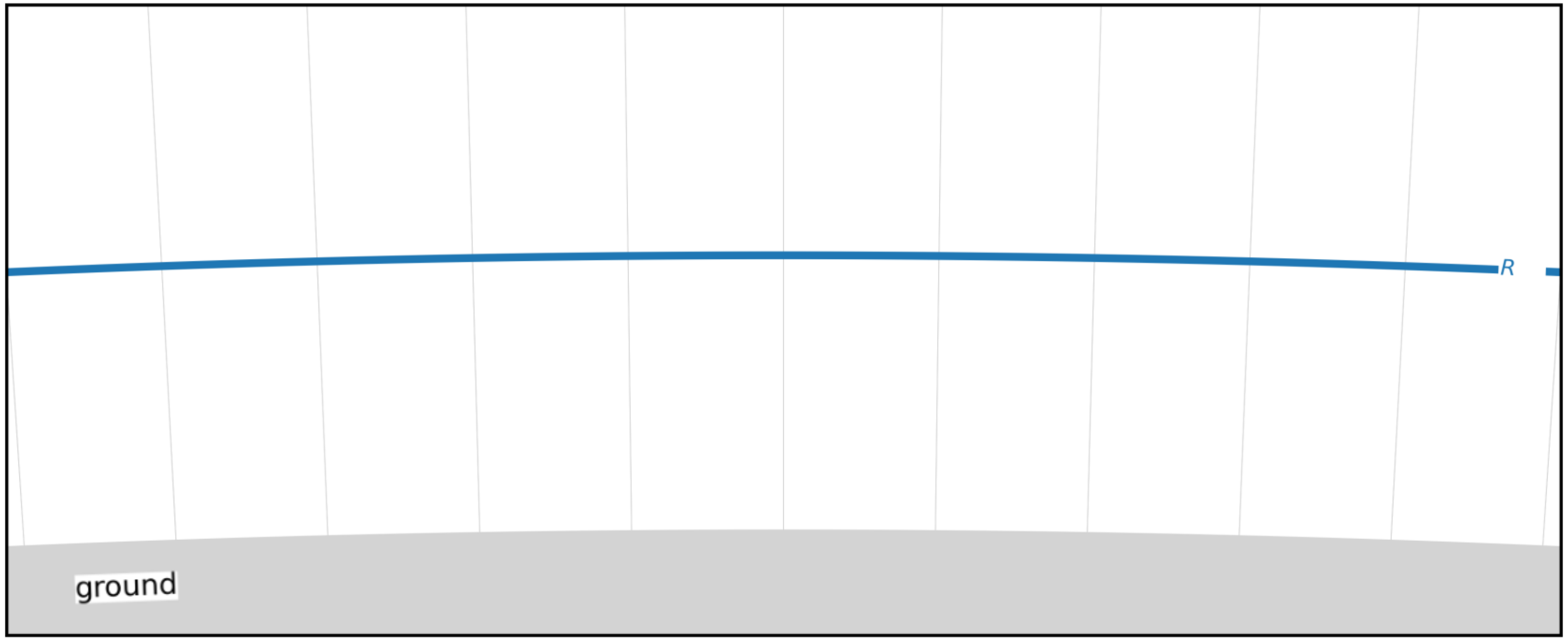

Input:
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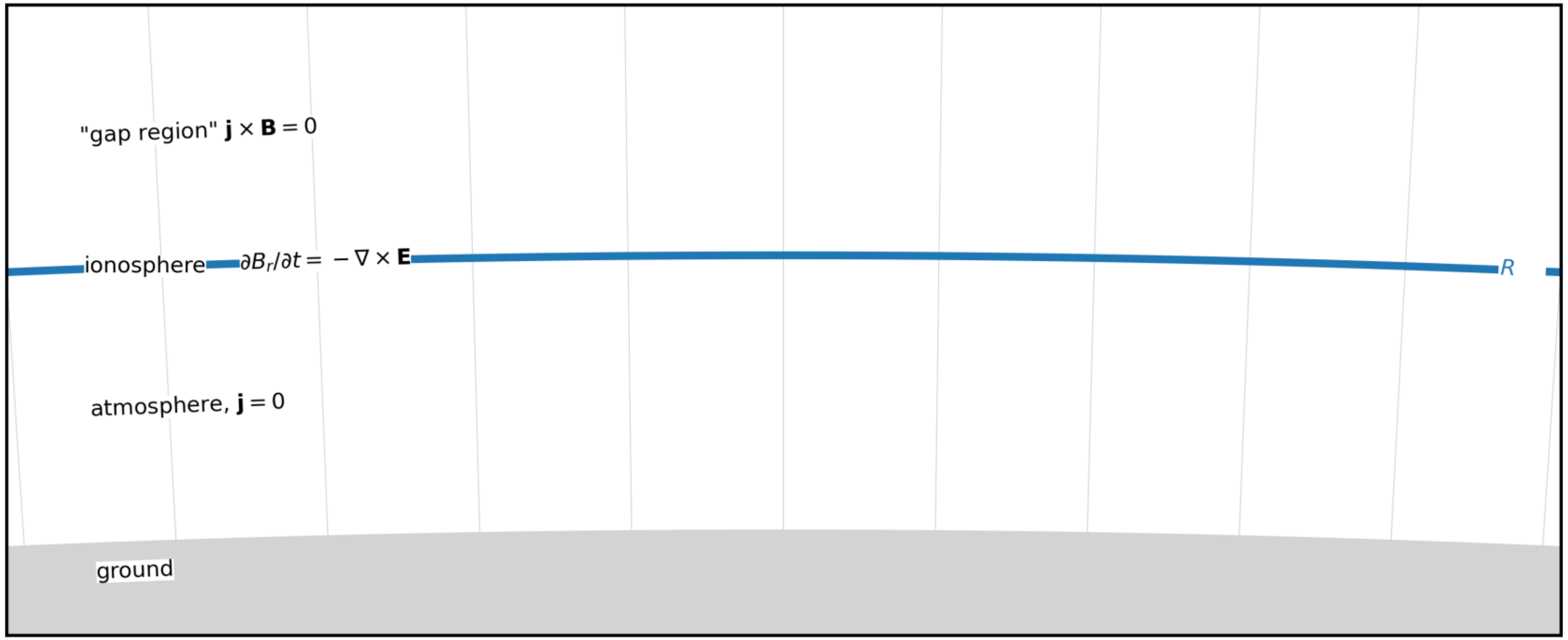
A 2D dynamic model

The electric field can be derived from the momentum equations for ions and electrons. It is known as the **Generalized Ohm's law**

$$\mathbf{E} = \frac{\sigma_P \hat{\mathbf{b}} \times (\mathbf{j} \times \hat{\mathbf{b}}) + \sigma_H \mathbf{j} \times \hat{\mathbf{b}}}{\sigma_P^2 + \sigma_H^2} - \mathbf{u} \times \mathbf{B}_0$$

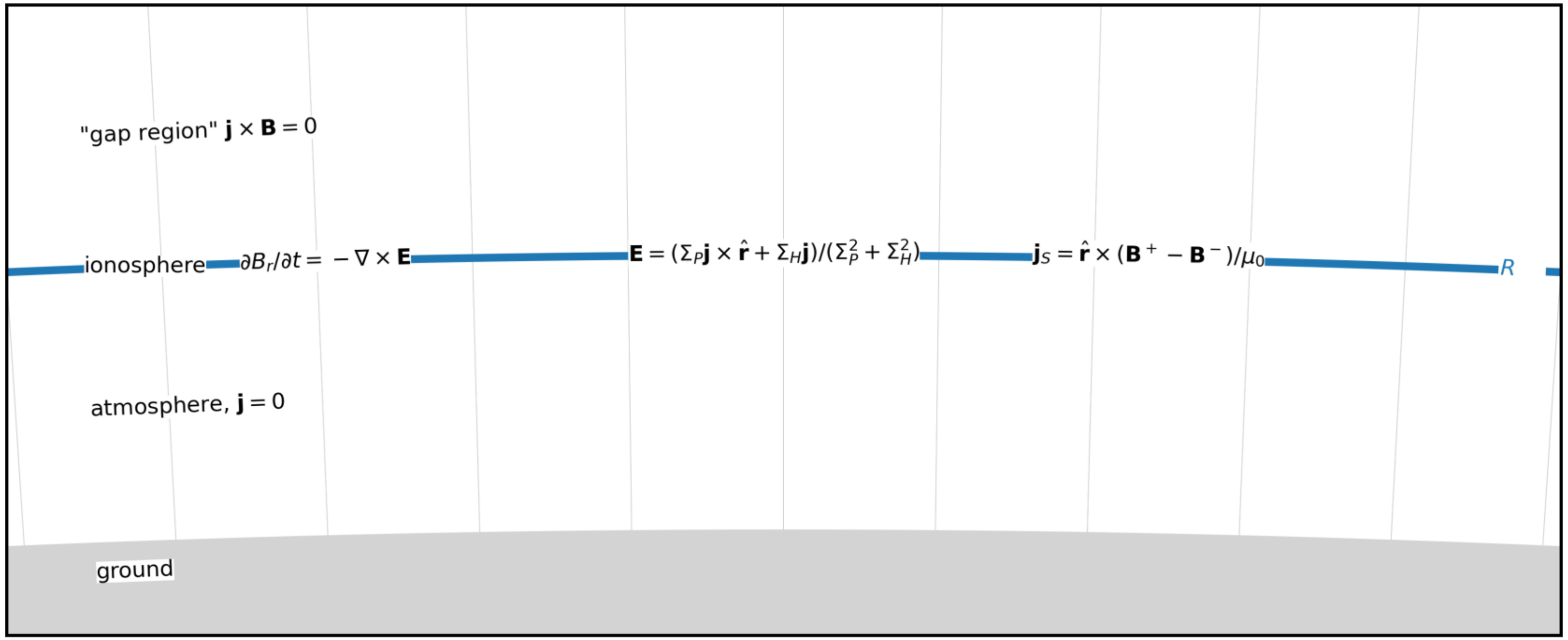
See, e.g., Leake et al. 2014, SSR



A 2D dynamic model

Since the ionosphere is a delta function, it is easy to integrate in altitude (we ignore neutral wind):

$$\mathbf{E}_S = \hat{\mathbf{r}} \times \left(\frac{\Sigma_P \hat{\mathbf{b}} \times (\mathbf{J}_S \times \hat{\mathbf{b}}) + \Sigma_H \mathbf{J}_S \times \hat{\mathbf{b}}}{\Sigma_P^2 + \Sigma_H^2} \times \hat{\mathbf{r}} \right)$$

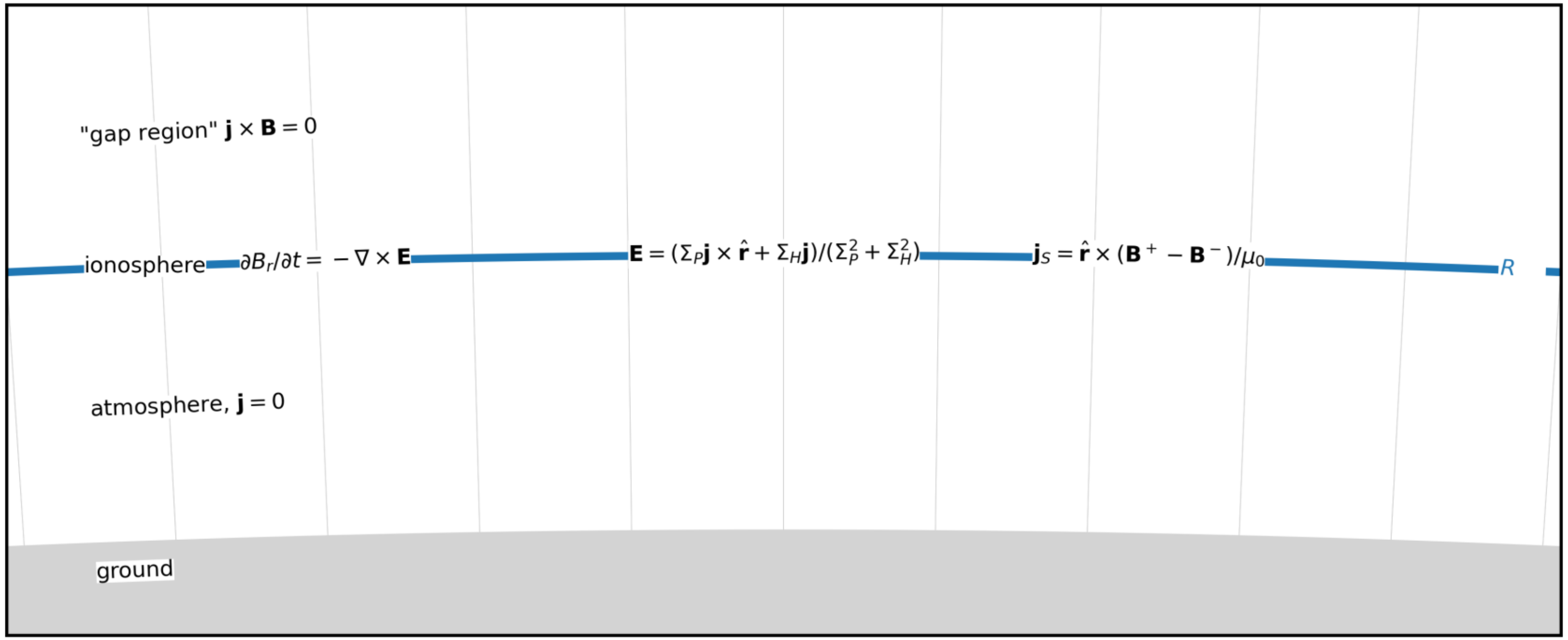


A 2D dynamic model

Faraday's law becomes:

$$\frac{\partial B_r}{\partial t} = -\nabla\times\left[\eta_P \begin{pmatrix} b_\phi^2+b_r^2 & -b_\theta b_\phi \\ -b_\theta b_\phi & b_\theta^2+b_r^2\end{pmatrix}\begin{pmatrix}J_\theta\\ J_\phi\end{pmatrix} + \eta_H \begin{pmatrix}0 & b_r\\ -b_r & 0\end{pmatrix}\begin{pmatrix}J_\theta\\ J_\phi\end{pmatrix}\right]$$

$$\begin{pmatrix}J_\theta\\ J_\phi\end{pmatrix} = \frac{1}{\mu_0}\begin{pmatrix}B_\phi^- - B_\phi^+\\ B_\theta^+ - B_\theta^-\end{pmatrix} \qquad \eta_{P,H} = \frac{\Sigma_{P,H}}{\Sigma_P^2+\Sigma_H^2}$$



Input:
≈Magnetic field in space associated with FACs
Conductances

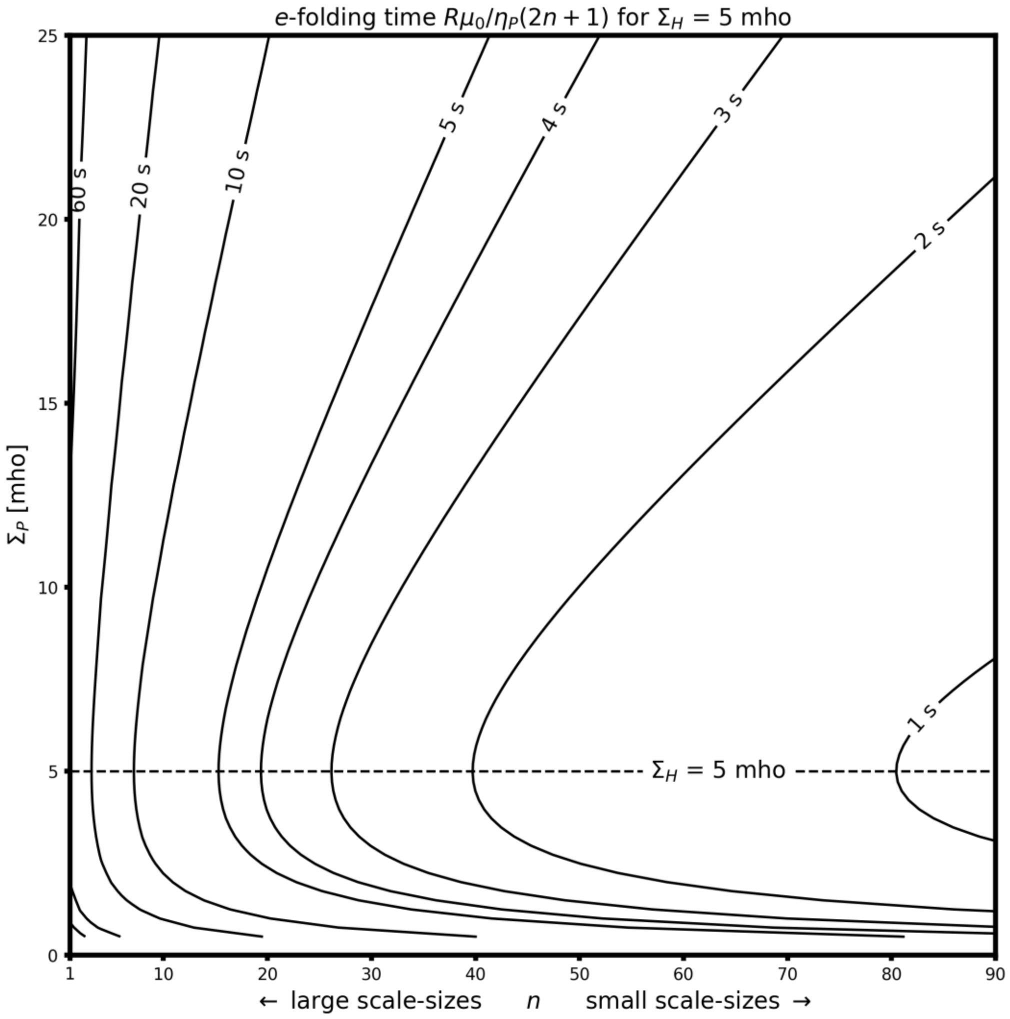
Output:
B_r, E, J_S

An analytical solution

- Assuming *uniform conductance* and *radial field lines,* we find an analytical solution

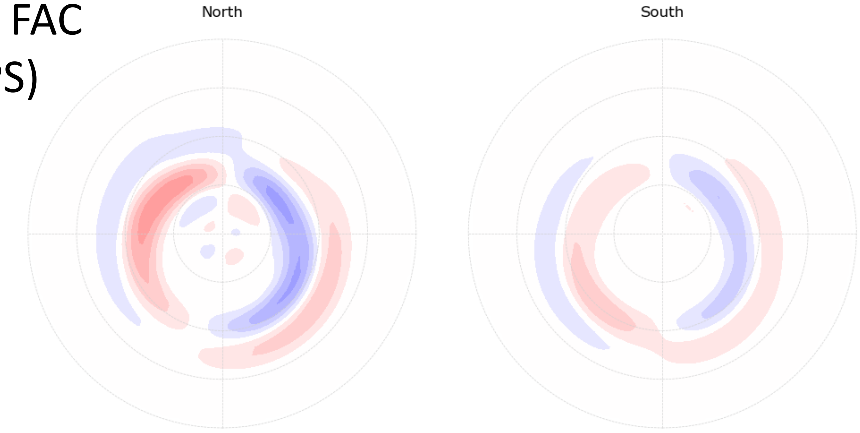
- B_r (and E) change as $e^{-\frac{\Sigma_P(2n+1)}{R\mu_0(\Sigma_P^2+\Sigma_H^2)}t}$
-

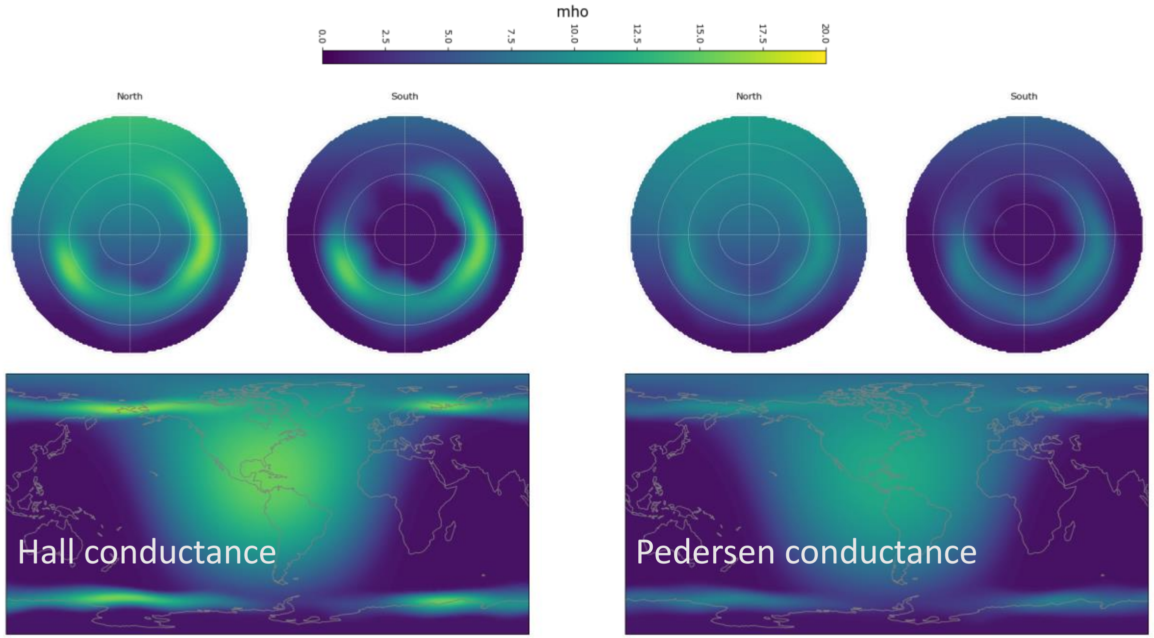
  We see that:
  - Large scales change more slowly
  - Changes are faster when $\Sigma_P \approx \Sigma_H$
  - This is not negligible with 1Hz data

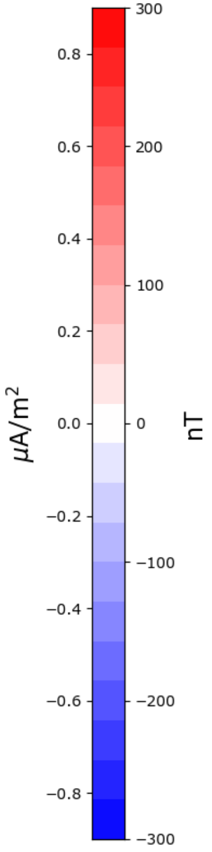


Global dynamic response with (semi)-realistic conductance and dipole magnetic field

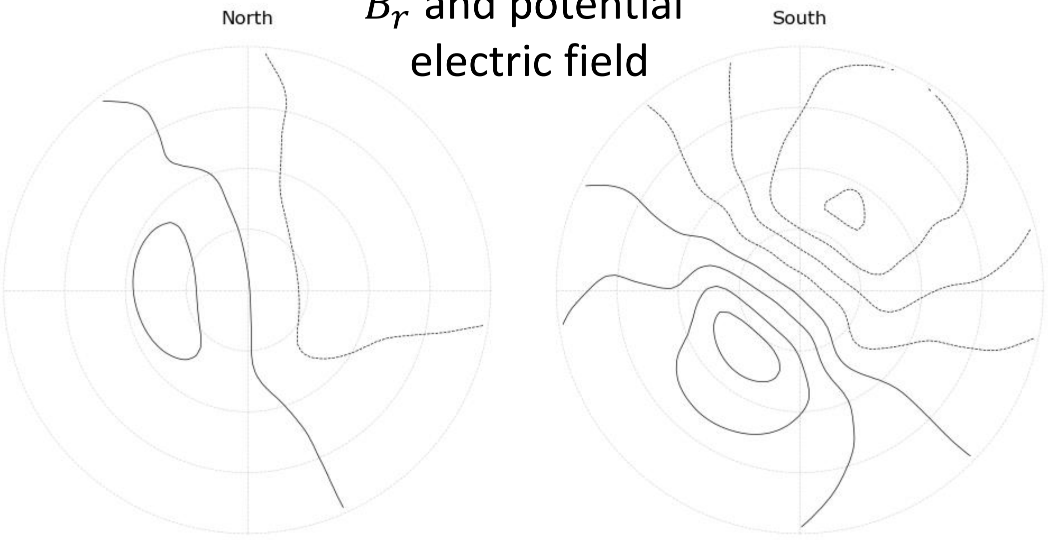
Input FAC (AMPS)

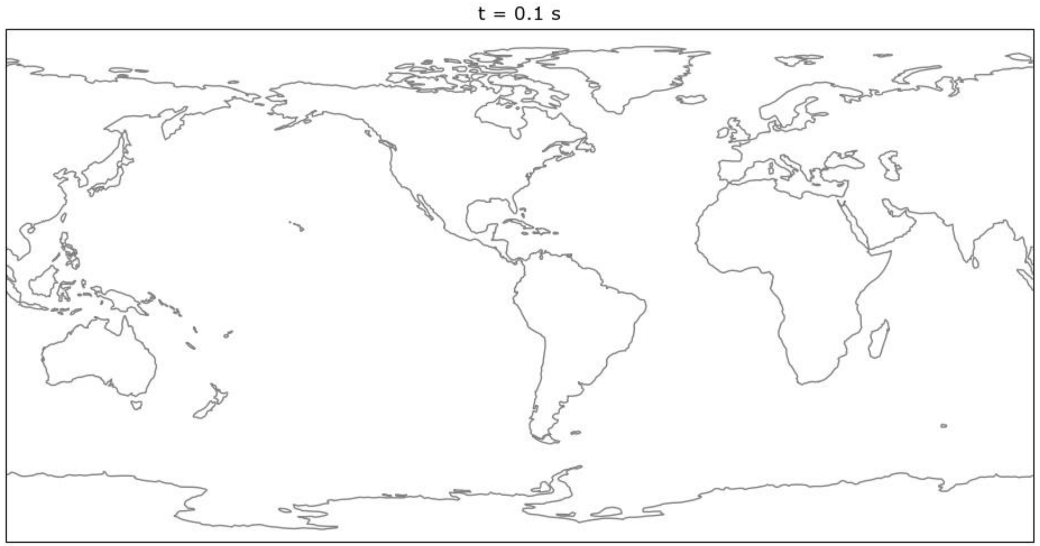




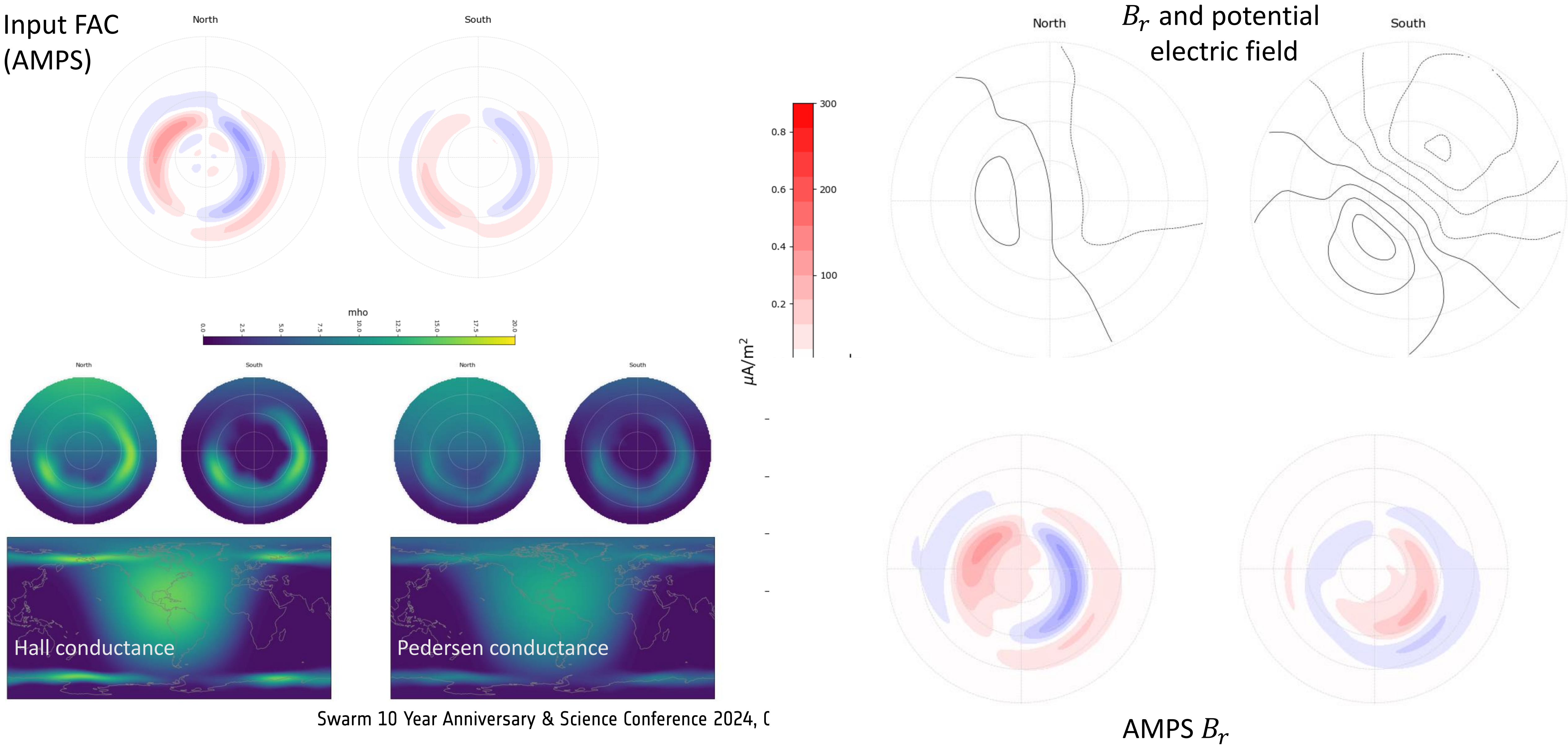


B_r and potential electric field



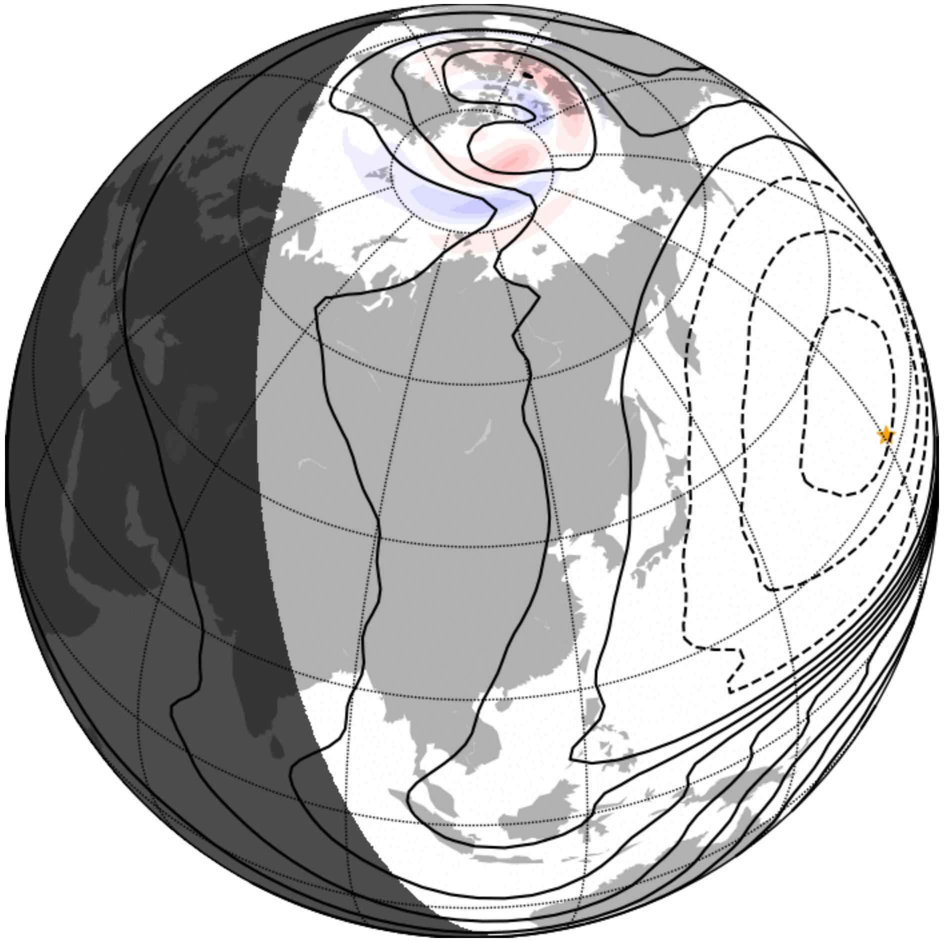


Global dynamic response with (semi)-realistic conductance and dipole magnetic field



The AMPS and SWIPE models

- Global average magnetic field disturbances and associated currents, derived with magnetometer data from CHAMP and Swarm.
  - New: Most recent update uses all data until late 2023
  - Web interface: https://birkeland.uib.no/data/amps/
  - Python code https://github.com/klaundal/pyamps
- See also SWIPE: A recently developed model of electric field, Swarm Hi-C, and quantities derived from it and AMPS (assuming steady state)
  - https://github.com/Dartspacephysiker/pyswipe
  - Poster by Spencer Hatch
  - Hatch, S. M., Vanhamäki, H., Laundal, K. M., Reistad, J. P., Burchill, J., Lomidze, L., Knudsen, D., Madelaire, M., and Tesfaw, H.: Does high-latitude ionospheric electrodynamics exhibit hemispheric mirror symmetry?, EGUsphere [preprint], https://doi.org/10.5194/egusphere-2023-2920, 2023.



Conclusions

- Faraday's law should not be neglected in the ionosphere...
  - If interested in ~0.1 Hz variations (depending on scale size)
  - **Or** if interested in the process that *causes* magnetic field disturbances
- 2D simulation is a promising alternative to current MI coupling
- **Todo:**
  - Account for the poloidal part of the magnetic field of FACs
  - Evolve fluid equations (density, momentum, energy)
  - Couple with magnetospheric MHD simulation
  - Properly handle low latitudes
    - Winds
    - Current continuity between hemispheres
  - A full 3D treatment of the dynamics