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Spintronics II:

Hall effects and spin torques in ferromagnets

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[@ChrisMarrows](#)





- Hall Effects in Ferromagnets
 - Ordinary Hall Effect (OHE)
 - Spin Hall Effect (SHE)
 - Anomalous Hall Effect (AHE)
 - Planar Hall Effect (PHE)
 - Quantum Spin Hall Effect (QSHE)
 - Topological Hall Effect (THE)
- Current-Driven Torques
 - Spin-Transfer Torques in Nanopillars
 - Spin-Transfer Torques at Domain Walls
 - Spin-Orbit Torques in ferromagnet/heavy metal bilayers



Hall Effects in Ferromagnets

Schoolchild Version

$$V = IR$$

Vector Version

$$\vec{E} = \rho \vec{J}$$

Now the resistivity ρ is a tensor. Assume magnetic field along z-axis.

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \times \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

If $\rho_{xy} = -\rho_{yx} \neq 0$ then \vec{E} and \vec{J} are no longer parallel and there is a Hall effect. The current must still flow along the conductor, but there is now some transverse component to the electric field.

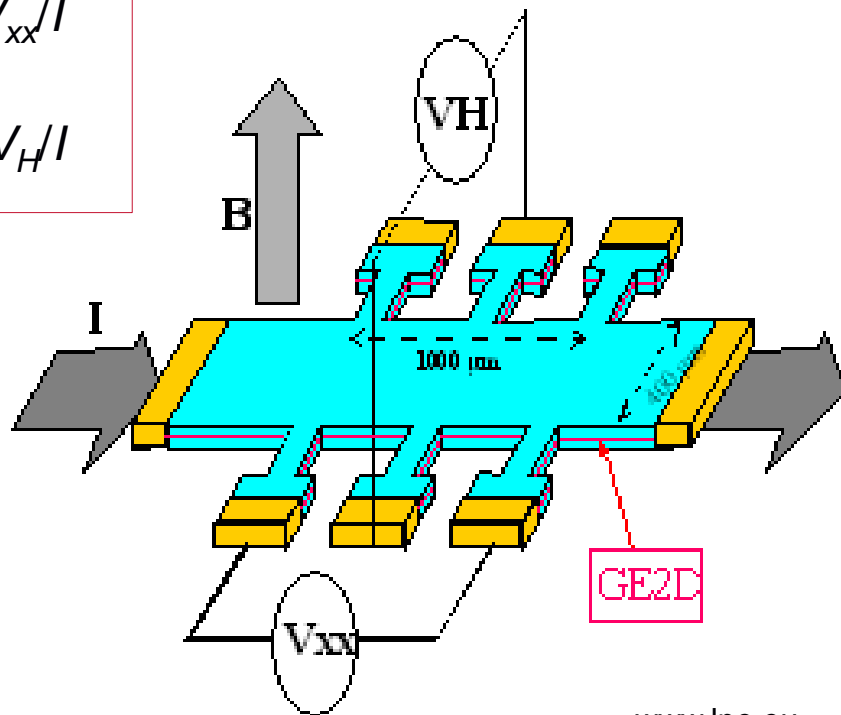
Hall bar



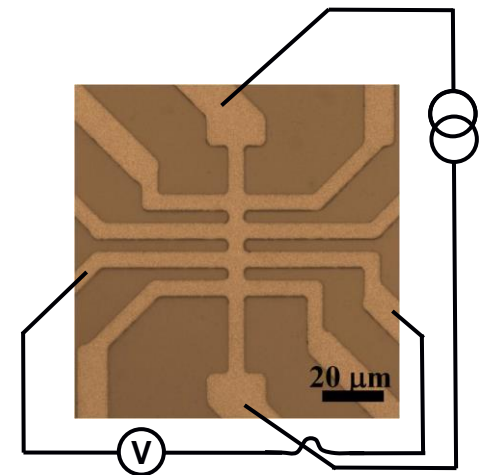
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Cut sample into a neat strip with voltage contacts down the sides

$$R_{xx} = V_{xx}/I$$
$$R_{xy} = V_H/I$$



www.lne.eu



Hall bar in FeCoSi
patterned by
photolithography,
hard mask, ion
milling.

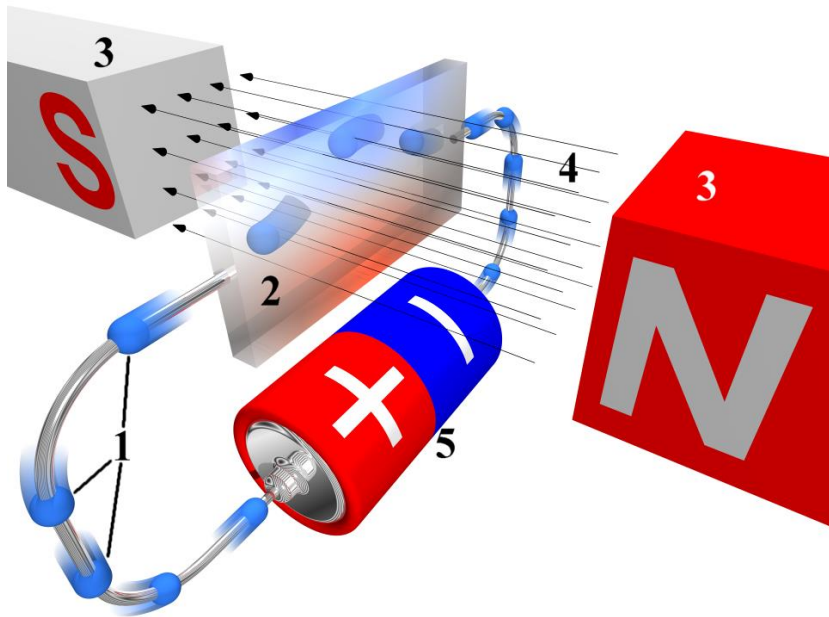
Use van der Pauw method for irregular sample shapes

Ordinary Hall Effect



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Lorentz deflection of electrons – discovered by Edwin Hall in 1879.



Wikimedia Commons

In the OHE

$$\rho_{xy} = R_0 B$$

R_0 is the ordinary Hall coefficient, where

$$R_0 = -\frac{1}{ne}$$

This makes the Hall effect a very useful way to measure carrier type (electron or hole) and density n .

Spin Hall Effect



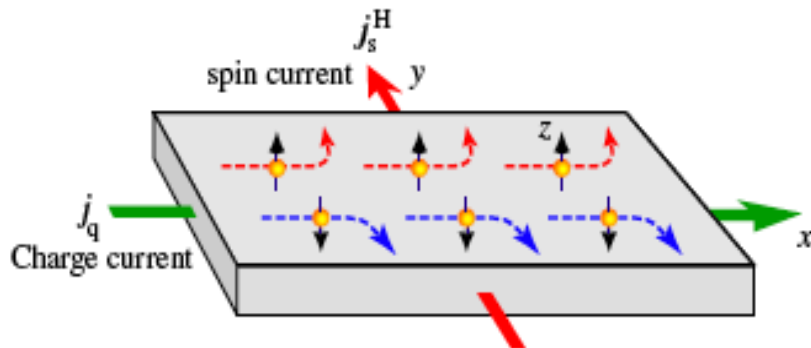
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Predicted in 1971 by D'yakonov and Perel, observed 30 years later by Wunderlich et al. and Kato et al.

In the direct SHE, a charge current density \vec{J} drives a spin current \vec{J}_S that is orthogonal to both \vec{J} and the spin direction $\vec{\sigma}$.

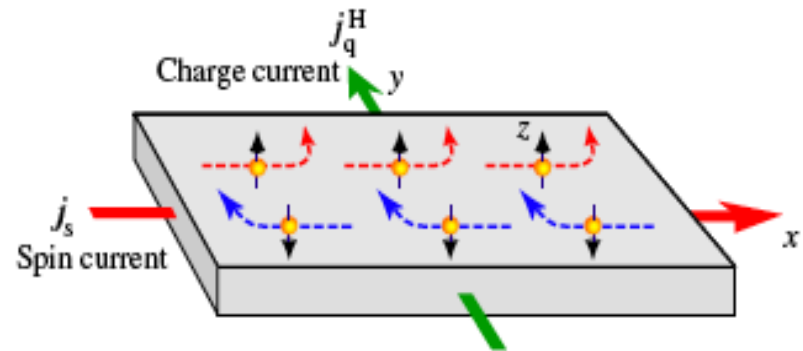
Direct SHE

$$\vec{J}_S = \alpha_{\text{SHE}} \vec{J} \times \vec{\sigma}$$



Inverse SHE

$$\vec{J} = \alpha_{\text{SHE}} \vec{\sigma} \times \vec{J}_S$$



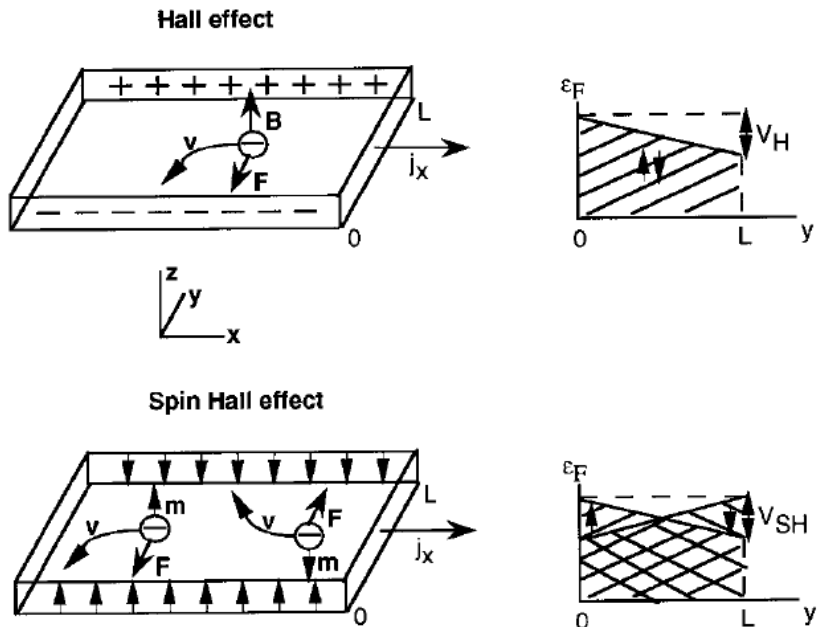
Spin Hall angle $\alpha_{\text{SHE}} = \frac{J_S}{J}$

Spin Hall Effect

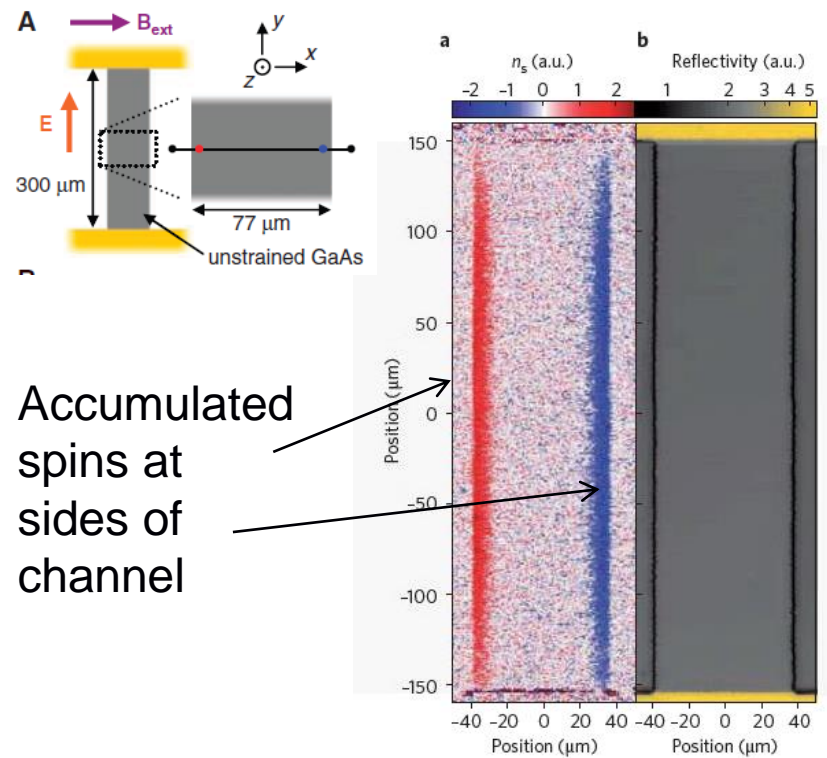


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In SHE, spin-orbit interactions scatter spins to opposite sides of 2D slab.



Magneto-optical detection in GaAs



D'Yakonov and Perel, Physics Letters A (1971)
 Hirsch, Phys. Rev. Lett (1996)

Kato et al., Science (2004).

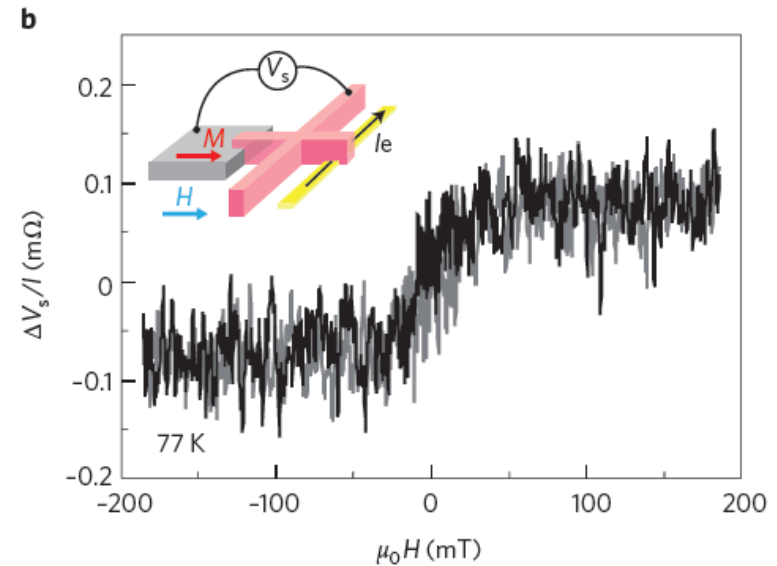
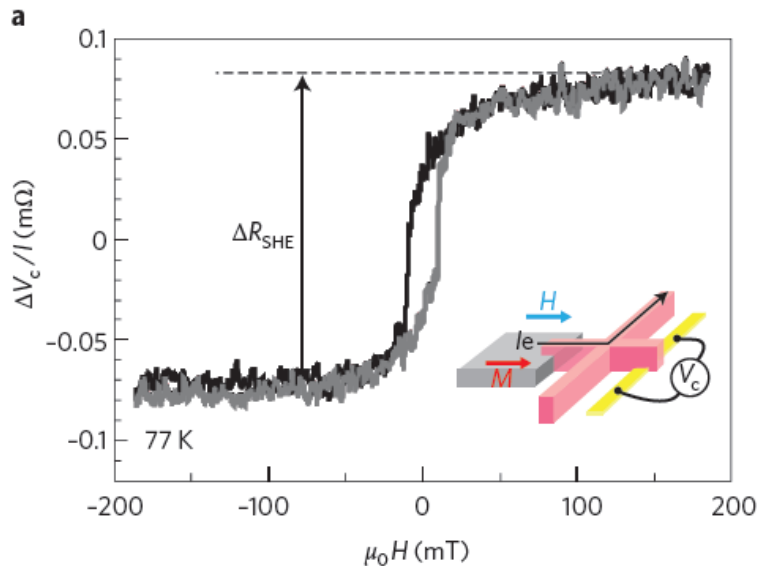
Invertible Spin Hall Effect



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Inverse spin Hall effect
Spin current => charge current

Spin Hall effect
Charge current => spin current



Grey = Permalloy; Pink = Copper; Yellow = Platinum

Kimura et al., Phys. Rev. Lett. (2007).

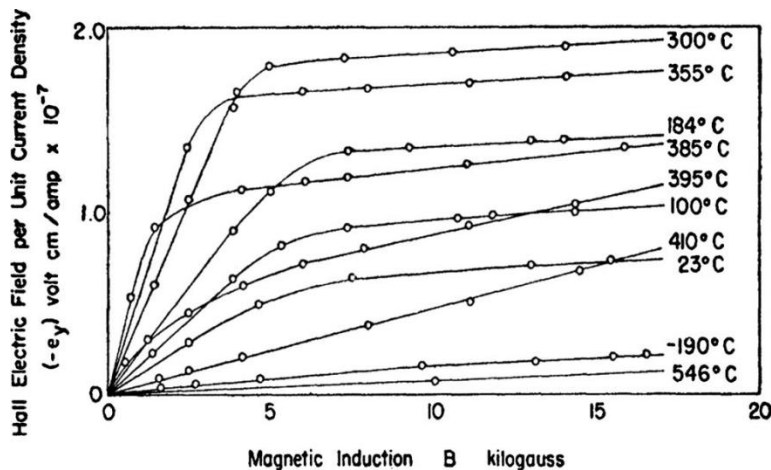
Anomalous Hall Effect



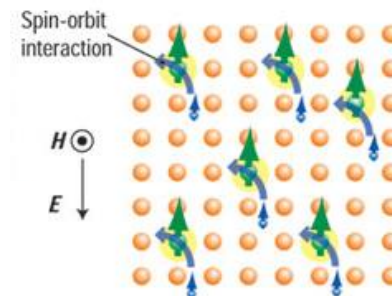
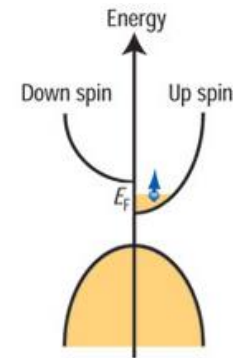
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Sometimes called Extraordinary Hall Effect (to distinguish from ordinary). Discovered by Hall in 1881. Proportional to magnetisation M , and hence useful for nanomagnetometry.

$$\rho_{xy} = R_0 B + R_S M$$



The Hall effect in Ni. Data from Smith, 1910. From Pugh and Rostoker, 1953.



Toyosaki et al., Nature Materials 3, 221 (2004)

Best understood as SHE in the presence of spin-polarised carriers.

Anomalous & Spin Hall Effects: Mechanisms



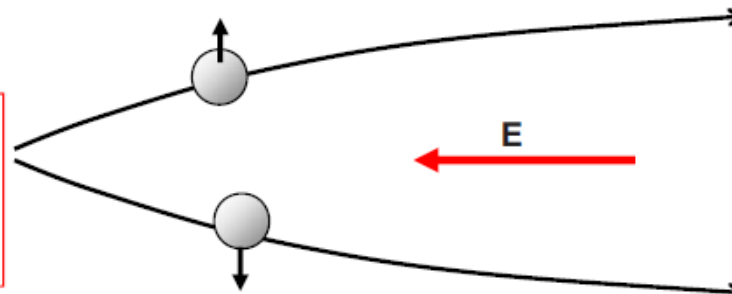
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a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

$$\frac{d\langle \vec{r} \rangle}{dt} = \frac{\partial E}{\hbar \partial \vec{k}} + \frac{e}{\hbar} \vec{E} \times \vec{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature



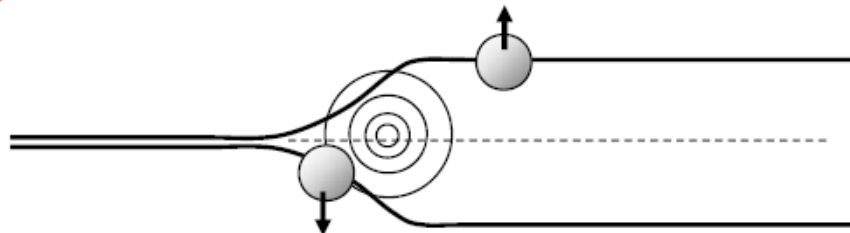
k-space Berry phase curvature in perfect crystals

$$\rho_{xy} \propto \rho_{xx}^2$$

(Karplus & Luttinger 1954;)

b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.



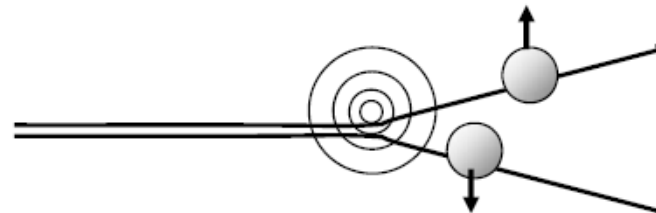
Spin-orbit scattering from impurities

$$\rho_{xy} \propto \rho_{xx}$$

(Smit 1955, 1958)

c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



Spin-orbit scattering from impurities – independent of impurity concentration!

$$\rho_{xy} \propto \rho_{xx}^2$$

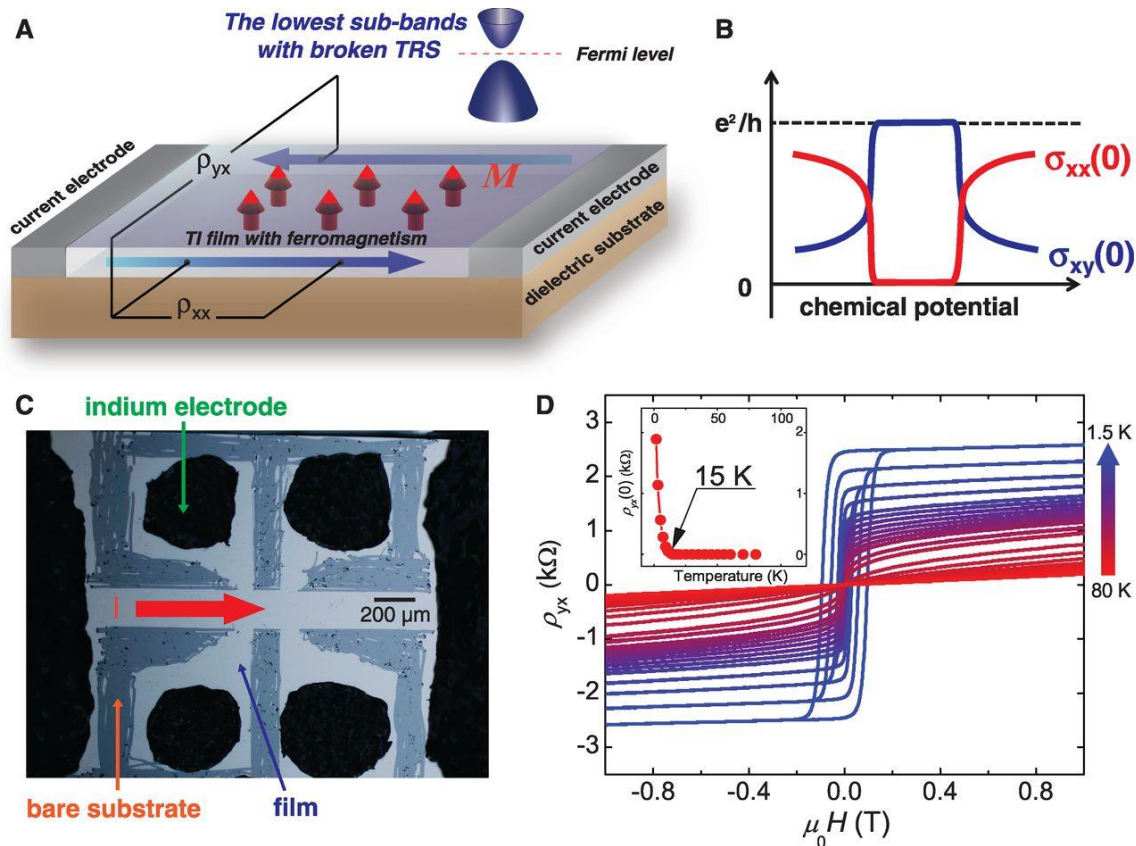
(Berger 1970)



Quantum anomalous Hall effect

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Observed in ferromagnetic topological insulators



Chang et al., Science **340**, 167 (2013)

Planar Hall Effect



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Actually a manifestation of the anisotropic magnetoresistance.

Recall that for AMR $\rho_{||} \neq \rho_{\perp}$.

For a single domain magnet with in-plane angle ϕ

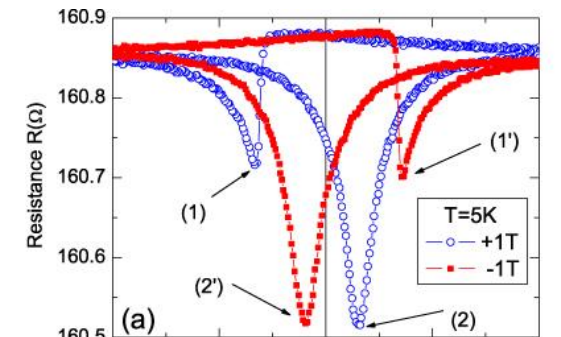
$$E_x = J\rho_{\perp} + J(\rho_{||} - \rho_{\perp}) \cos^2 \phi$$

$$E_y = J(\rho_{||} - \rho_{\perp}) \sin \phi \cos \phi$$

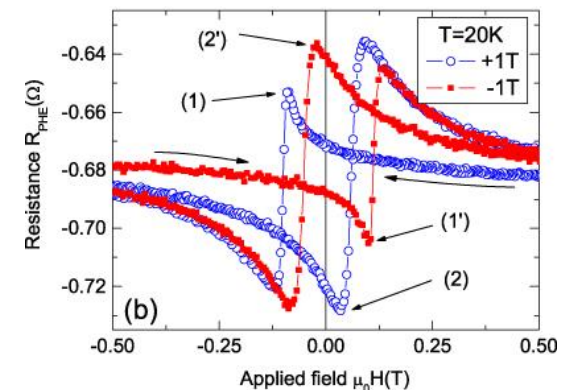
In-plane magnetisation can be manipulated with an in-plane field, giving changes in E_y that are picked up as Hall voltages.

Can be mistaken for true Hall effects.

exchange biased Co/YMnO₃ bilayer



ρ_{xx}



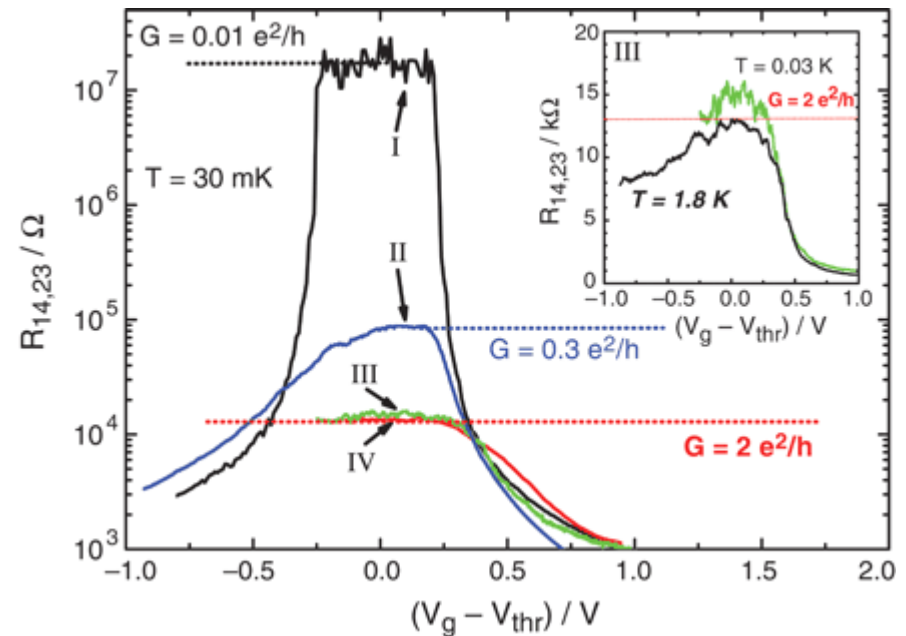
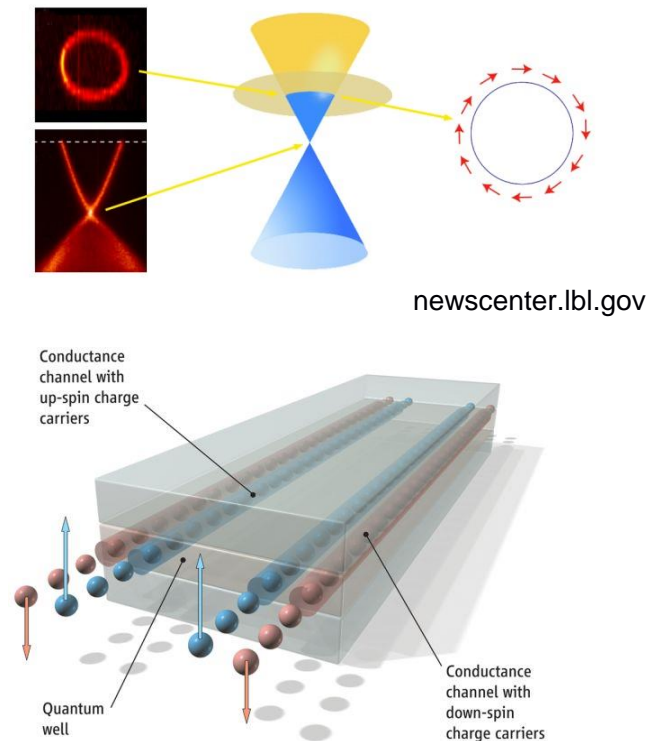
ρ_{xy}

Quantum Spin Hall Effect



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Found in 2-dimensional topological insulators, e.g. HgTe quantum wells.
Edge channels where spin is locked to momentum through the Rashba SOI.



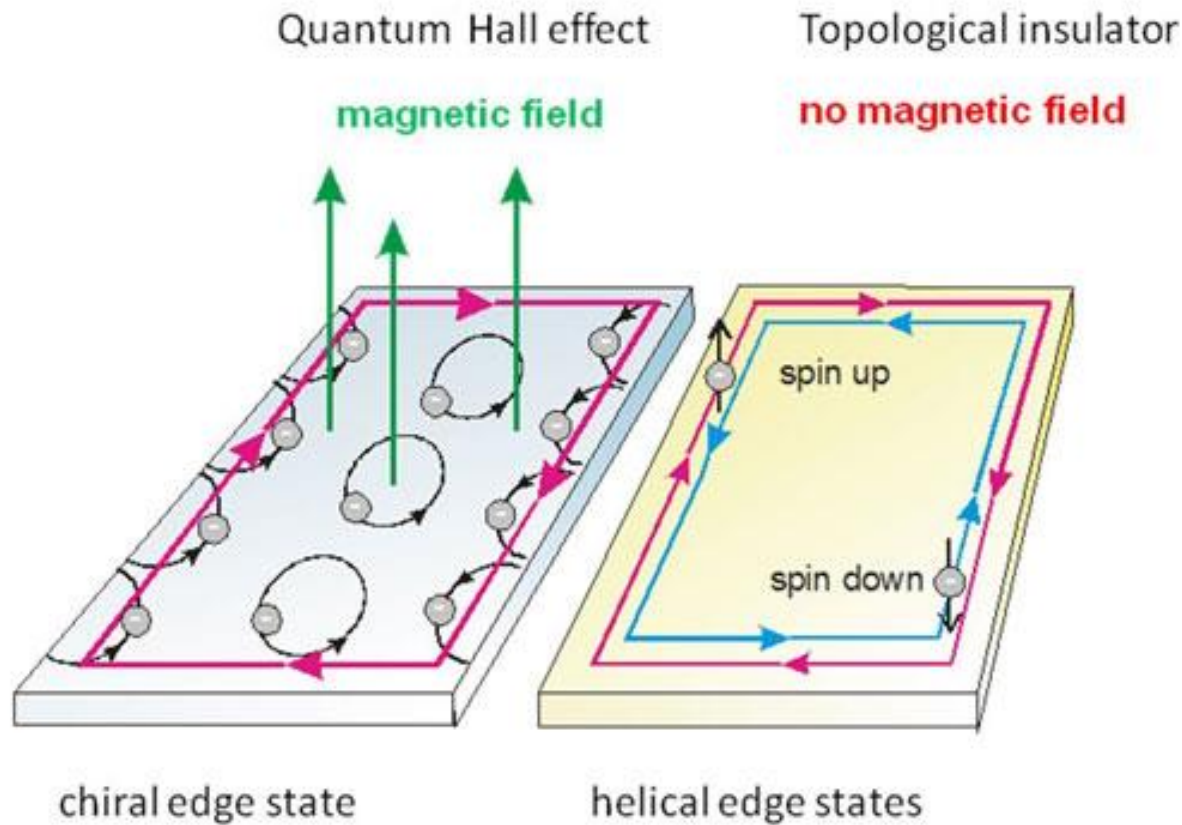
Curve I: normal band structure, Curves II, III, & IV, inverted (topologically non-trivial) band structure

Spin-locking prevents back scattering and provides ballistic channels with $G = 2e^2/h$

Quantum Hall Effect vs. Quantum Spin Hall Effect



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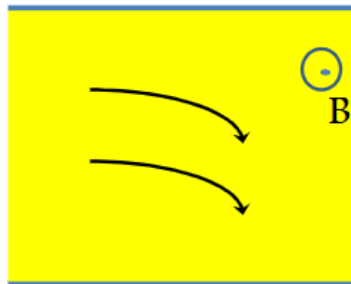


Summary

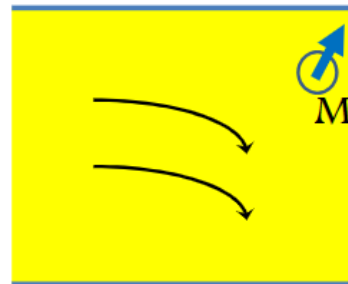


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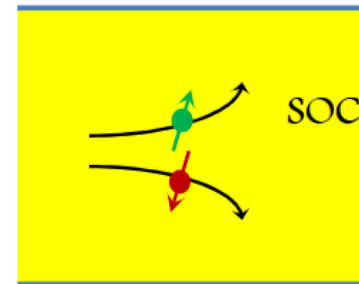
(a) Hall effect



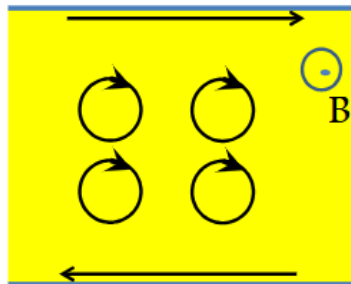
(b) Anomalous Hall effect



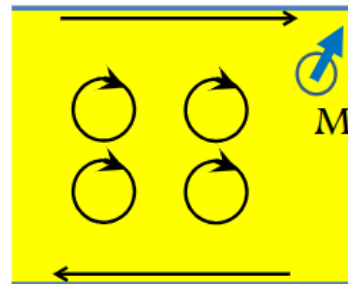
(c) Spin Hall effect



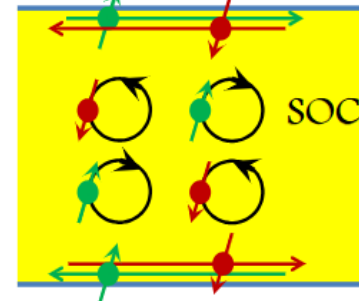
(d) Quantum Hall effect



(e) Quantum anomalous Hall effect



(f) Quantum spin Hall effect





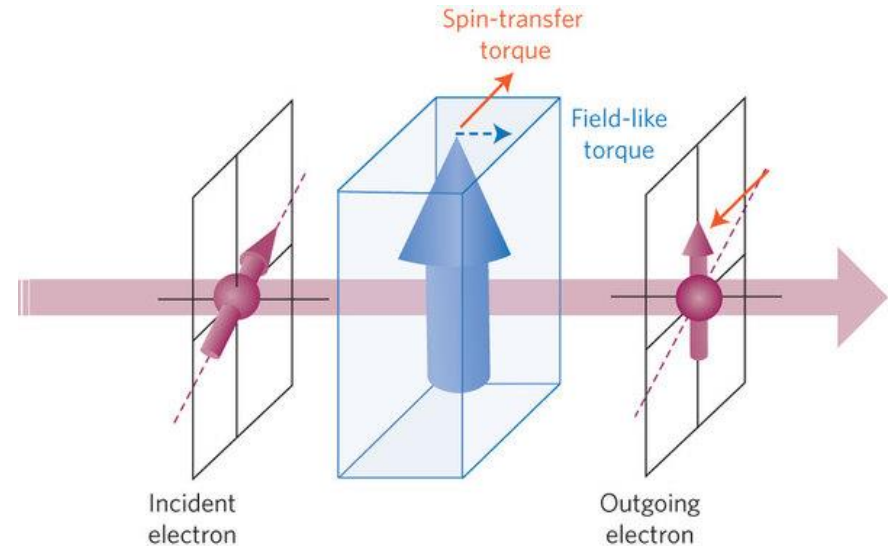
Current-Driven Torques

Spin-transfer torque - Concept



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- Magnetisation affects current: GMR, TMR etc.
- Newton's 3rd law => current affects magnetisation
- Theory:
 - Slonczewski, JMMM 1996
 - Berger, Phys. Rev. B 1996
- Transverse component of spin current is absorbed by layer
- Rate of change of angular momentum is a torque



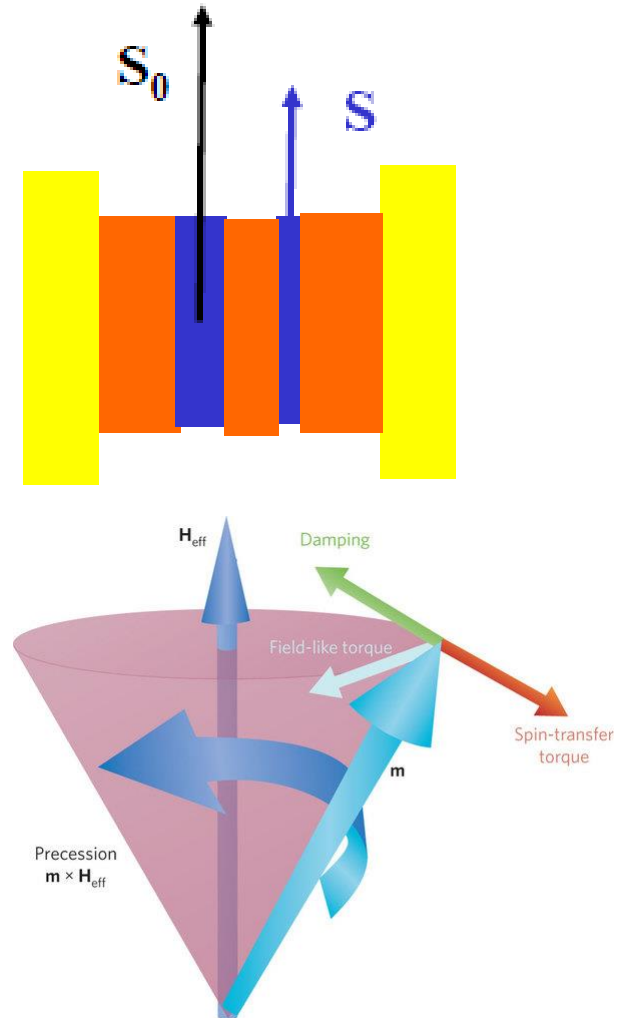
Brataas, Kent, and Ohno, Nature Materials **11**, 372 (2012).

$$\text{Torque} = \hbar \frac{\partial \mathbf{S}}{\partial t}$$

Form of spin-transfer torque

- Consider nanopillar geometry, e.g. Cu/Co/Cu/Co/Cu
- Thick Co fixed
- Thin Co feels torques
- Adiabatic relaxation of spins over $l_{sf} \Rightarrow$

$$\text{Torque} \propto J_s \mathbf{S} \times (\mathbf{S} \times \mathbf{S}_0)$$

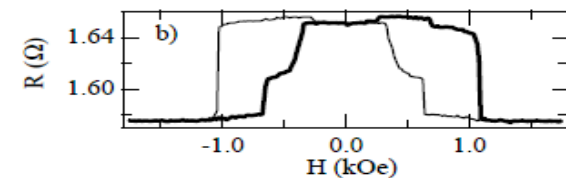
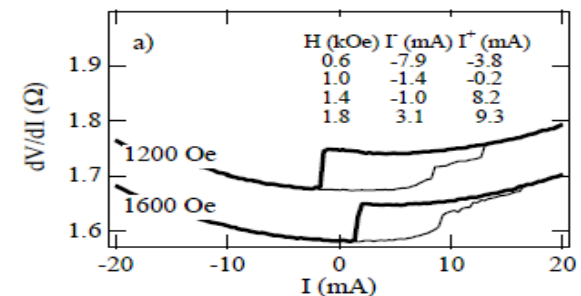
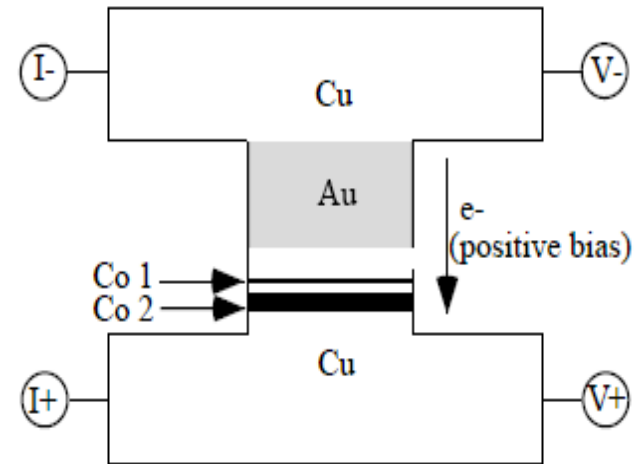


Current Driven Switching in Nanopillars



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- 130 nm diameter nanopillars patterned by e-beam lithography
- Devices must be **very small**
 - Oersted field
 - Current density (1 mA \Rightarrow 6×10^{10} A/m²)
- Probe reversal with GMR – reversible with current direction
- Parabolic background due to heating
- Katine *et al.* Phys. Rev. Lett. (2000)





STT switching & microwave generation

IrMn(6nm)/ CoFe(5nm)/ Cu(4nm)
/ CoFe(3nm)
(Grown at Leeds)

Junction size= 230 nm × 120 nm

Asymmetric switching currents (H=0)

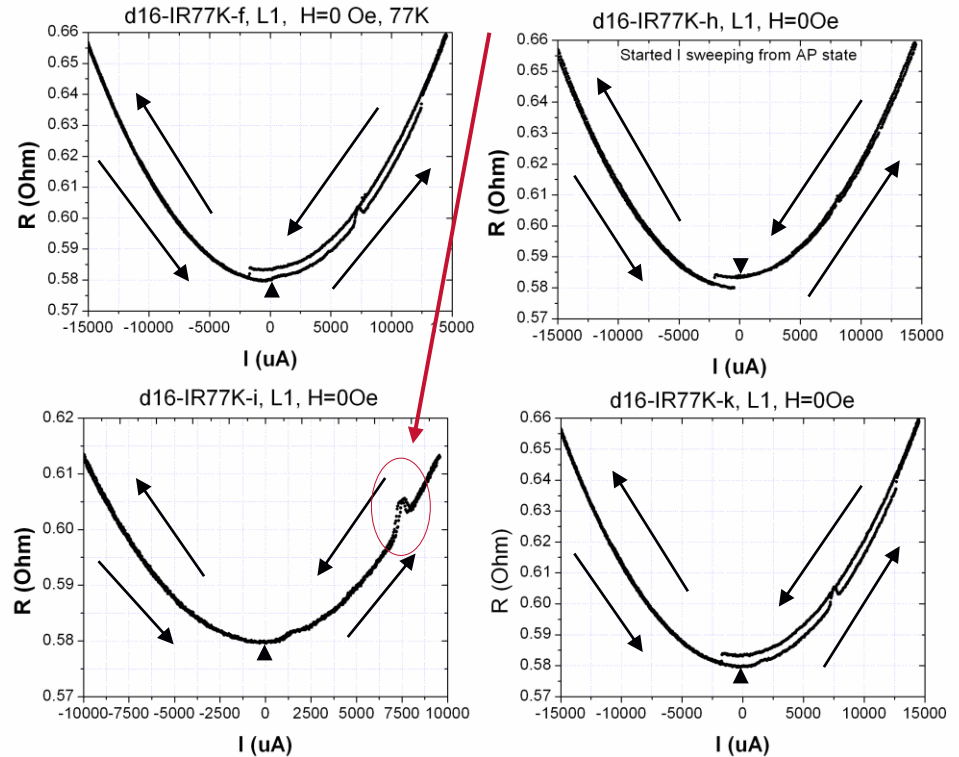
$I_{AP \rightarrow P} : J_c \sim 0.4 \times 10^7 \text{ A/cm}^2$

$I_{P \rightarrow AP} ; J_c \sim 2.5 \times 10^7 \text{ A/cm}^2$

Reversible peaks are only observable when electrons flows from free layer to fixed layer in the parallel state

Atif Aziz, Mark Blamire

Microwave generation peak

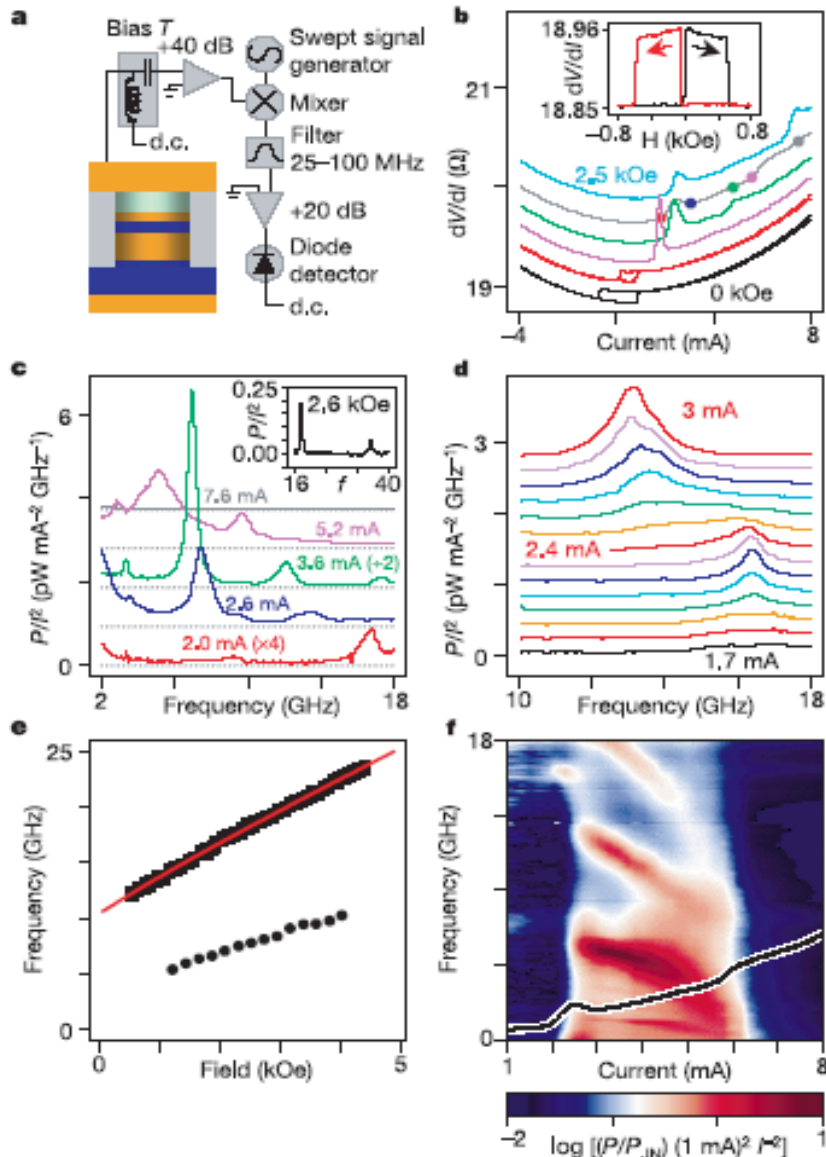


-ve current → electrons flow from fixed to free layer
+ve current → electrons flow from free to fixed layer

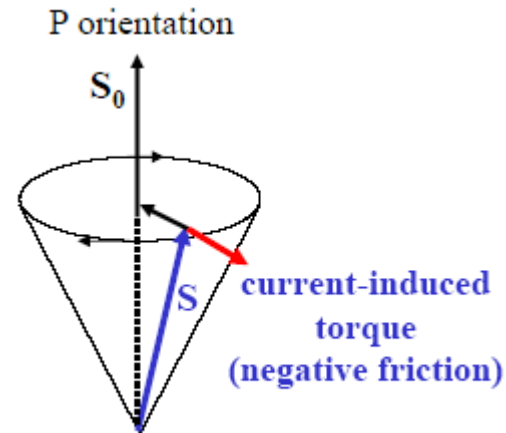
Spin-torque oscillators: stable dynamical states



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- Current induced torque opposes damping – if large enough oscillation can set in.
- **dc current** converted into **ac power**
- GHz frequency – tunable with field or current
- Kiselev *et al.*, Nature (2003)

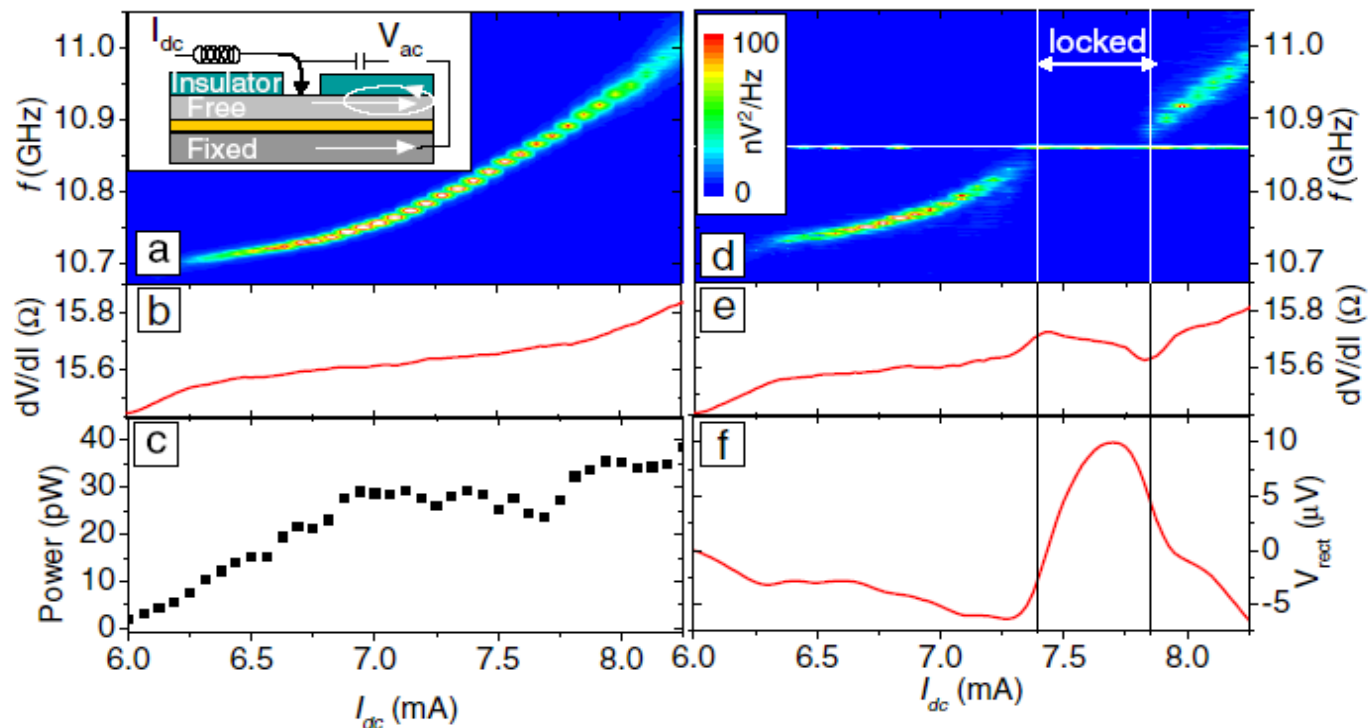


Injection locking



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- Pump with ac component of current
- Magnetisation dynamics **locked** to ac pump current



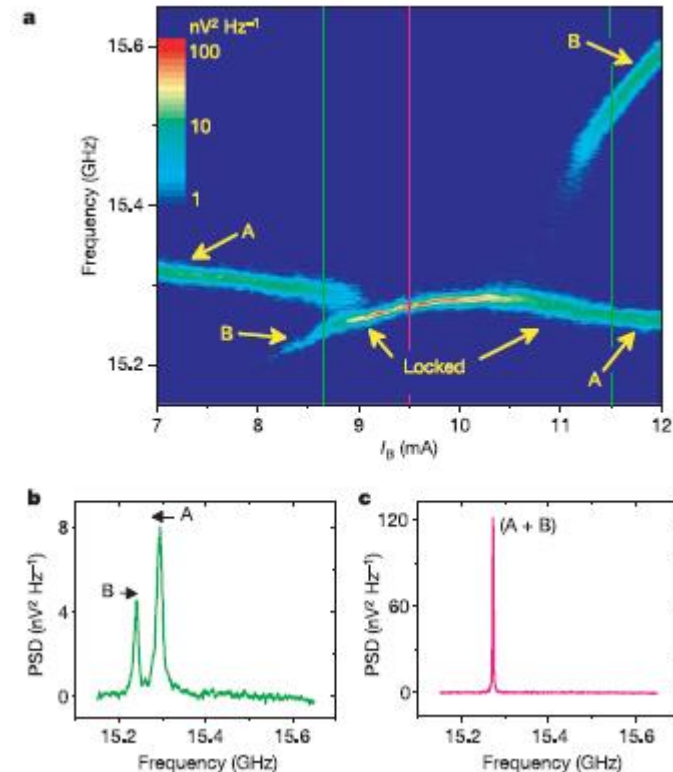
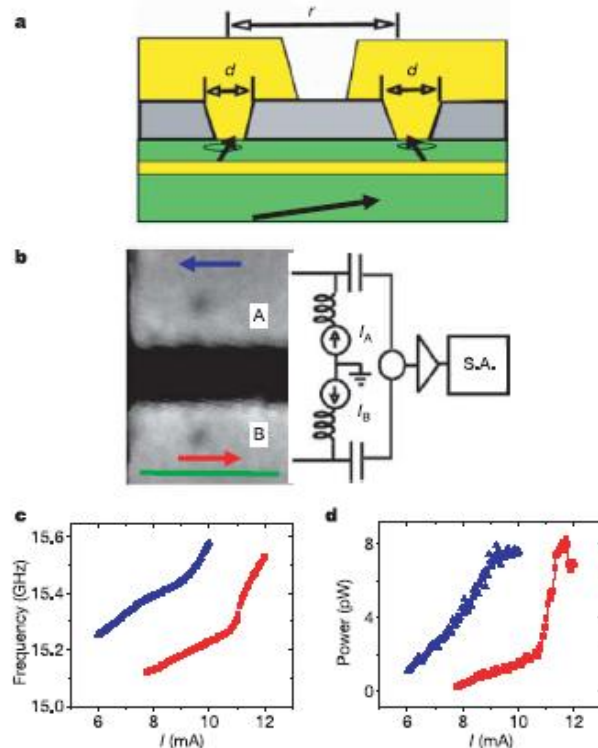
Rippard et al., Phys. Rev. Lett. (2005)

Mutual Phase-Locking



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Two point contacts in close proximity can exchange spin-waves and phase lock – much higher power available ($P \sim N^2$)



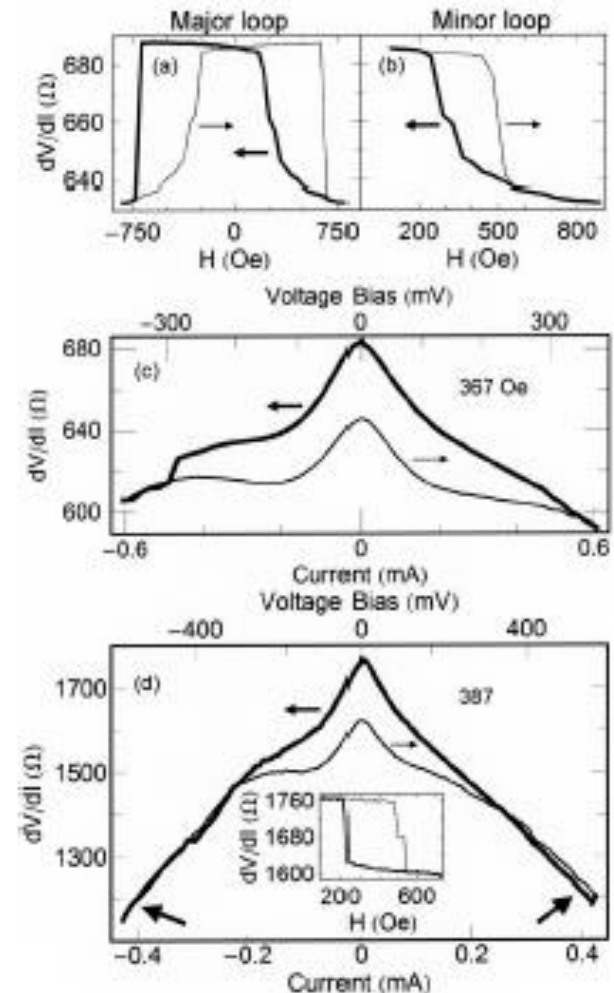
Kaka et al., Nature (2005)

Current-driven switching of MTJ



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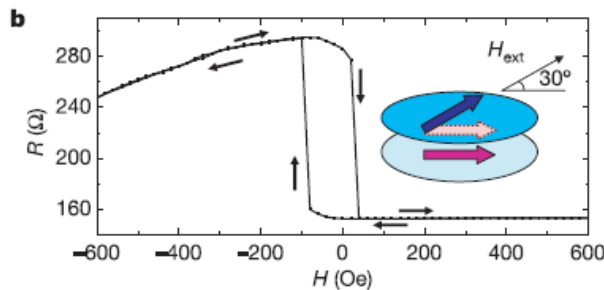
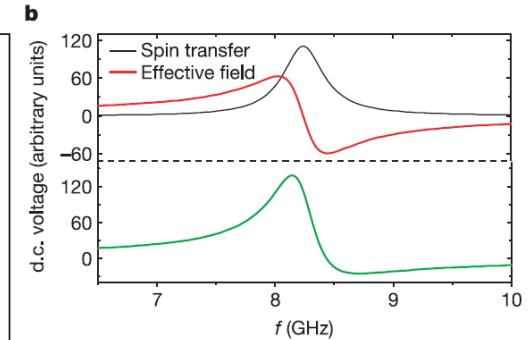
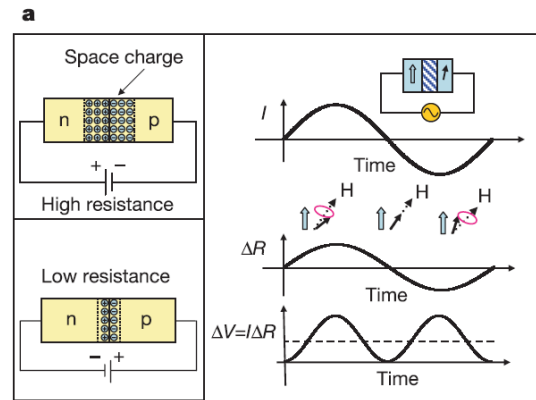
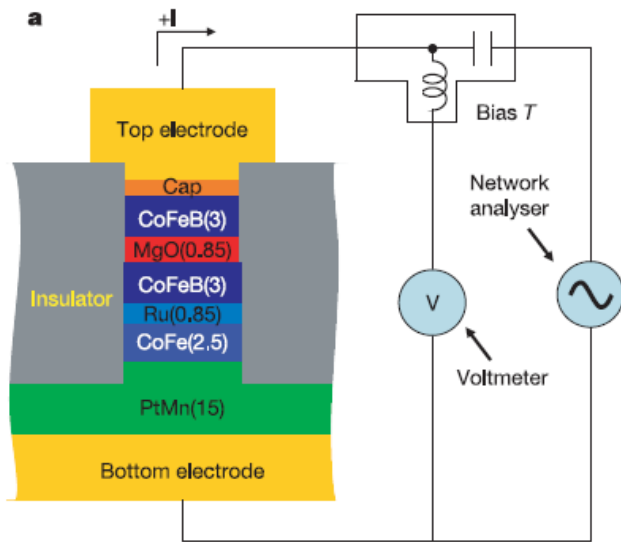
- Also possible to reversibly switch a tunnel junction
- Barrier must be **ultrathin** to allow enough current density
- Useful for writing MRAM data
- Large effects now available in MgO based low RA junctions
- Fuchs et al., Appl. Phys. Lett. (2004)



MgO-based MTJ Spin-Torque Diode



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- Injection of RF current I causes RF motion of free layer moment
- Device resistance ΔR varies through TMR
- Mixing of I and ΔR leads to output dc voltage

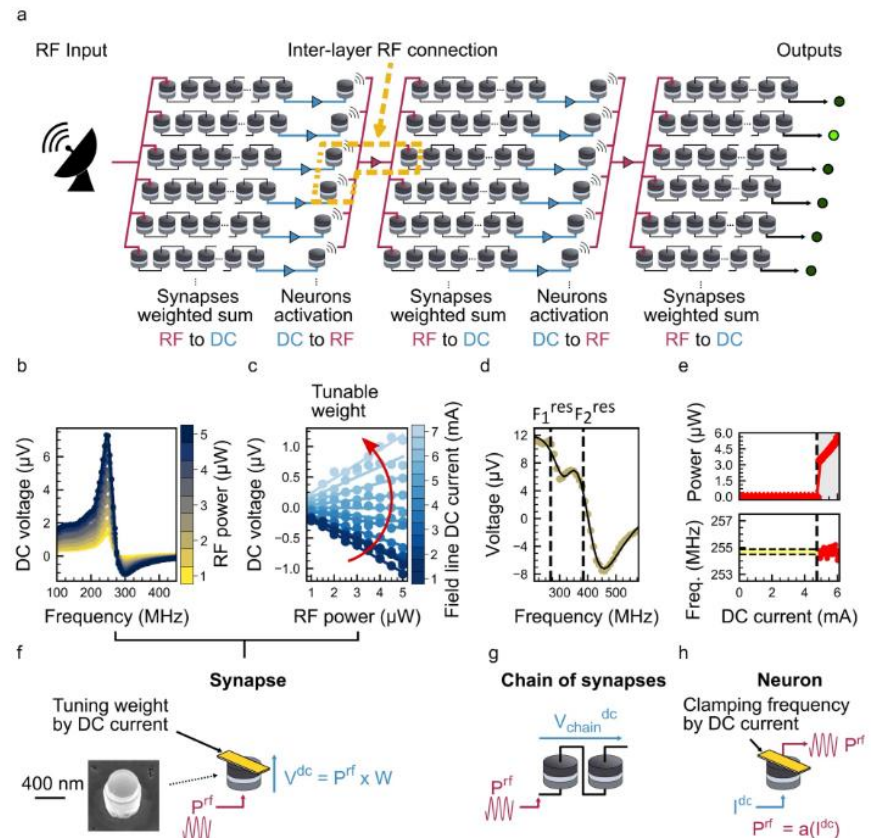
Tulapurkar *et al.*, Nature (2006)

RF connections used for MTJ-STO-based neural network



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- Classifies nonlinearly-separable RF inputs with an accuracy of 97.7%
- Can identify drones by WiFi emissions with few mW power (10,000x more efficient than in software)
- A. Ross & N. Leroux, et al. arXiv:2211.03659 [cs.ET]

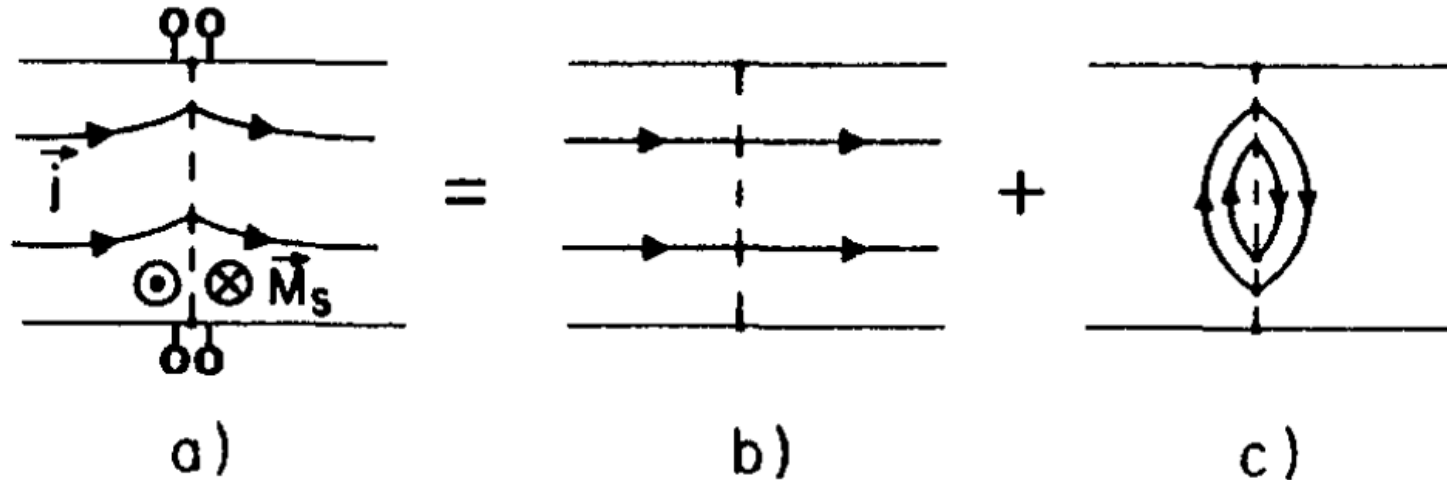


- Naturally occurring magnetic micro or nano structures
- Separate regions magnetised in different directions
- Thickness depends only on material parameters: exchange stiffness (A) and anisotropy (K)



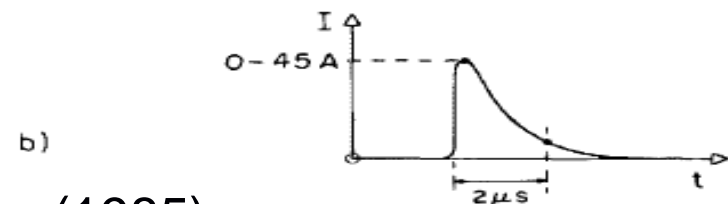
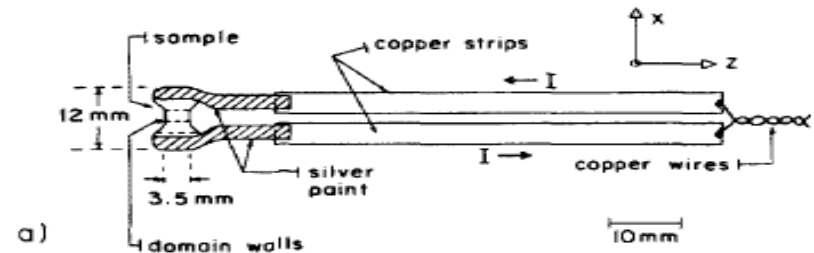
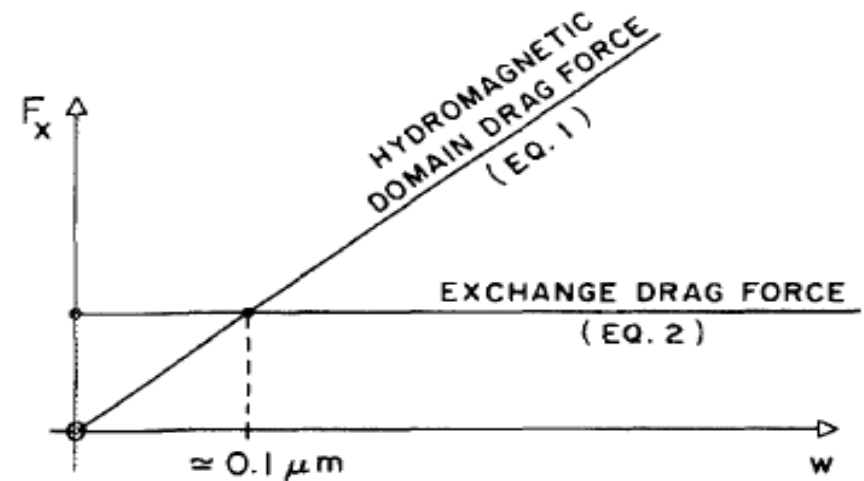
$$D = \pi \sqrt{\frac{A}{K}}$$

- Several papers by Berger and various co-authors in 1970s and 1980s
- Both theory and experiment
- Two mechanisms found: first is hydromagnetic drag
- Hall effect causes eddy current loops \rightarrow wall motion
- Only significant in films thicker than a few 10s of nm



Early work

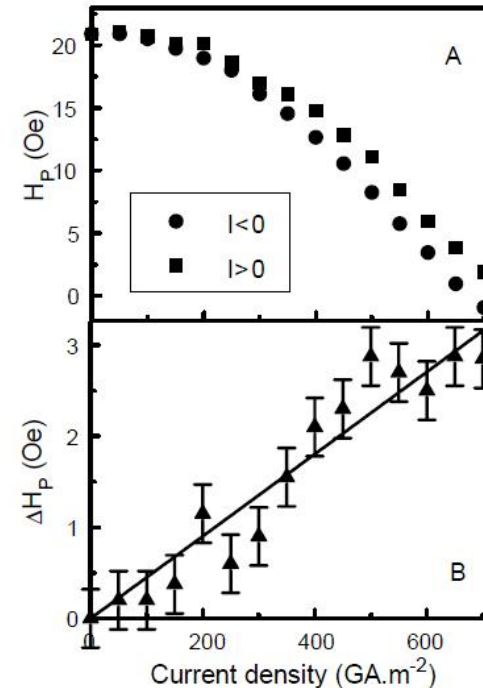
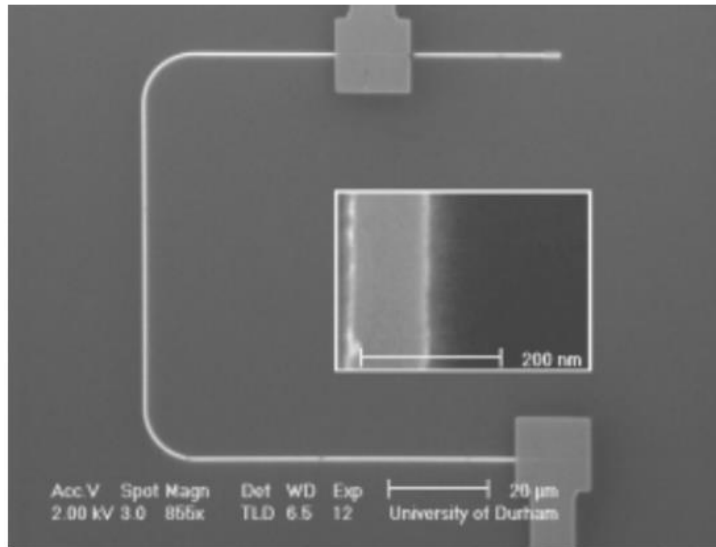
- 2nd mechanism – ‘*s-d* exchange force’, now known as spin transfer effect
- Walls moved by high current density pulses
- Observed by Kerr microscopy – wall motion depends on current direction



Wall motion in magnetic wires



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- U-shaped permalloy wire with injection pad
- Wall positioned in corner with vector field
- Dc current applied – pinning field drops due to heating, but difference between current directions proportional to current density: spin transfer effects move wall (or at least assist depinning)

Vernier *et al.*, Europhys. Lett. (2004)

Spin Transfer Torques



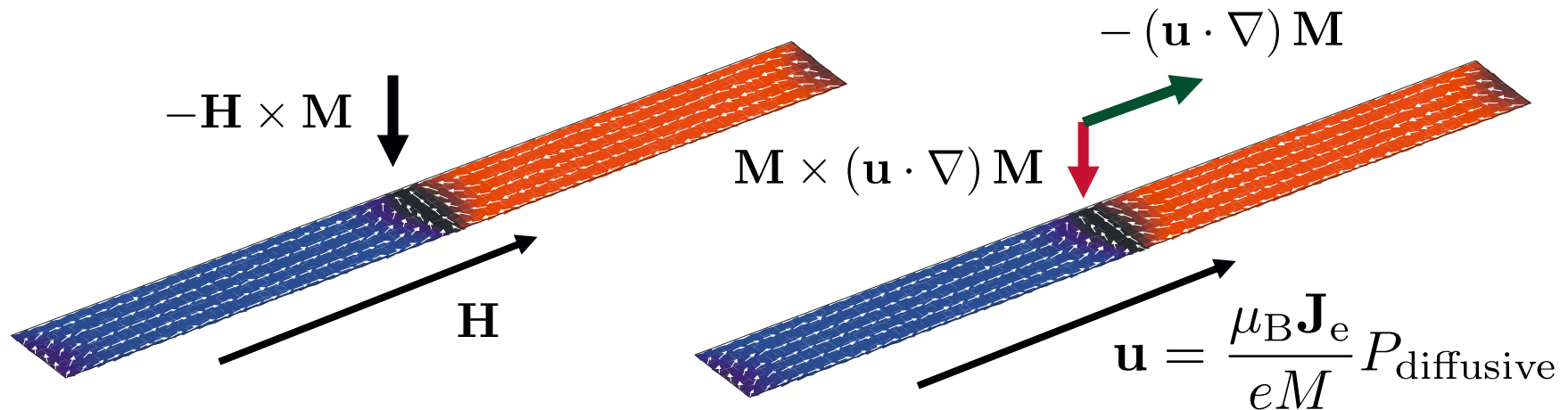
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Modified LLG equation given by A. Thiaville *et al.*, Europhys. Lett. **69** 990 (2005)

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{H} \times \mathbf{M} + \alpha \mathbf{M} \times \frac{d\mathbf{M}}{dt} - \underbrace{(\mathbf{u} \cdot \nabla) \mathbf{M}}_{\text{green}} + \underbrace{\beta \mathbf{M} \times (\mathbf{u} \cdot \nabla) \mathbf{M}}_{\text{red}}$$

γ_0 = gyromagnetic ratio, α = Gilbert damping constant, β = 'non-adiabaticity' parameter

Implemented in OOMMF code from Antoine Vanhaverbeke <http://www.zurich.ibm.com/st/magnetism/spintevolve.html>

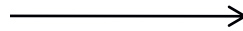


Sample Design

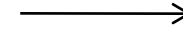


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Wall nucleates in
elliptical injection pad

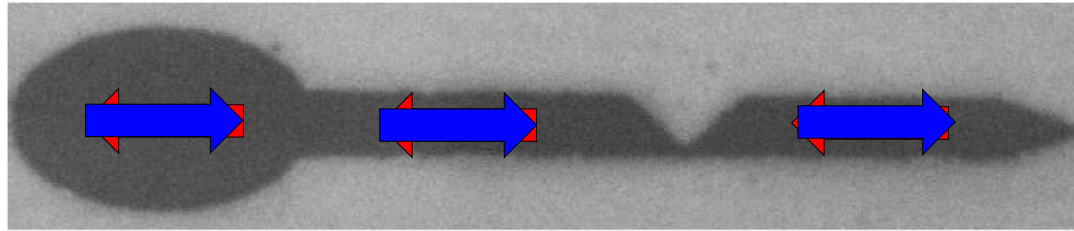


Wall pins in
patterned notch



Wall annihilates
at pointed end

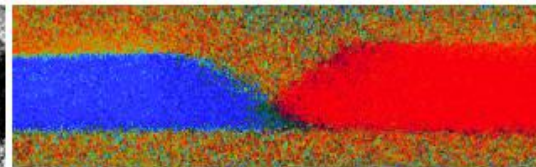
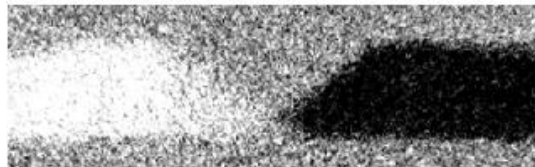
SEM



XMCD PEEM

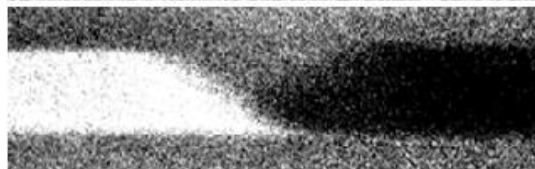


Raw PEEM



Vector Map

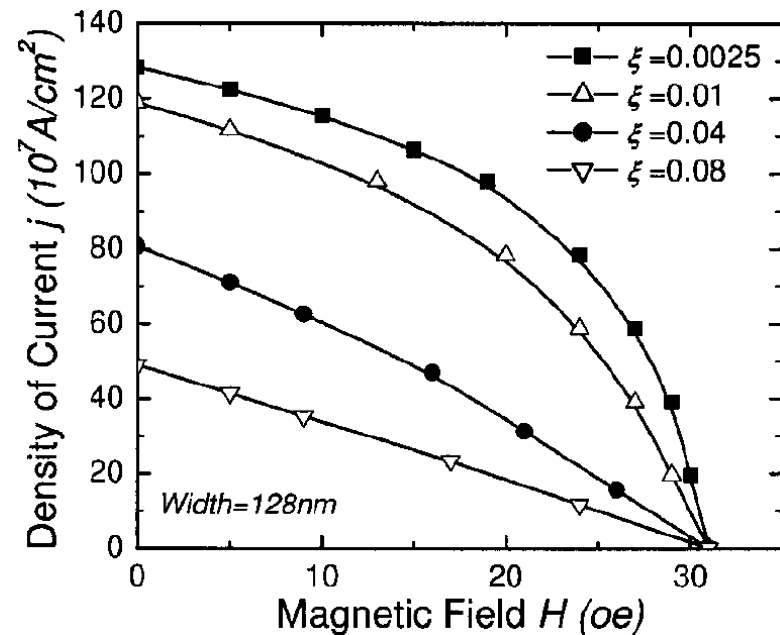
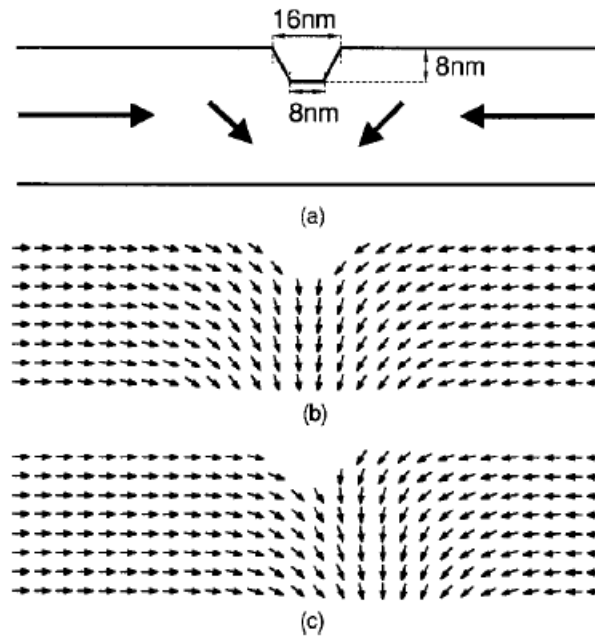
Raw PEEM



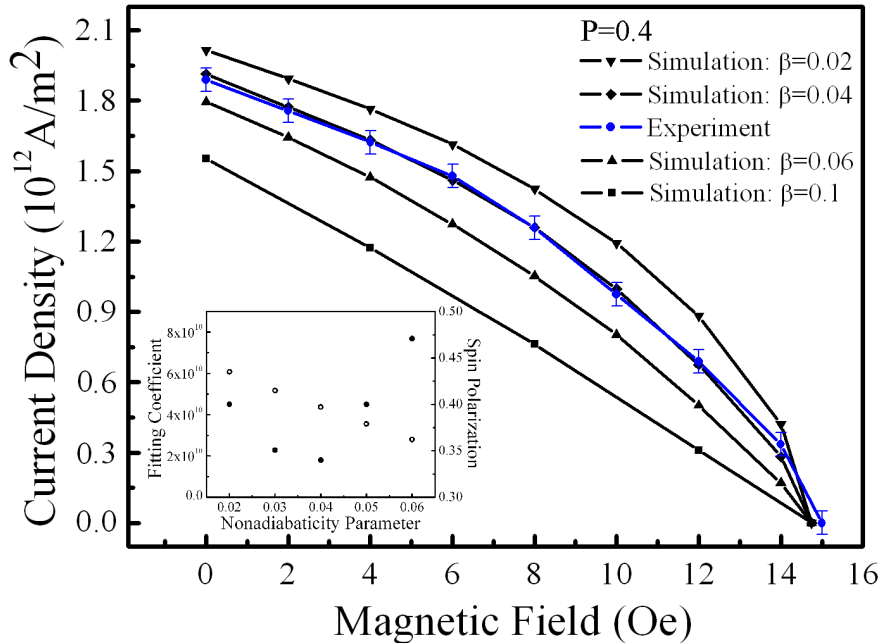
Simulation

Typical injection fields: 10-15 Oe Typical depinning fields: 15-20 Oe

Electron beam lithography, sputter ~20 nm Permalloy, 1-2 nm Al or Au cap, lift off



- Depinning boundary as a function of field
- Shape depends on strength of non-adiabatic 'field-like' term
- Nonadiabaticity parameter, $\xi = \beta$, obtained by curve-fitting results from micromagnetics simulations.



- Micromagnetic simulations including the adiabatic and nonadiabatic spin torque terms in the LLG equation (code from Antoine Vanhaverbeke at IBM, <http://www.zurich.ibm.com/st/magnetism/spintevolve.html>)
- Starting state as obtained from XMCD imaging

$$\frac{d\vec{M}}{dt} = -|\gamma| \vec{H}_{eff} \times \vec{M} + \alpha \vec{M} \times \frac{d\vec{M}}{dt} + u \vec{M} \times \left(\vec{M} \times \frac{\partial \vec{M}}{\partial x} \right) + \beta u \vec{M} \times \frac{\partial \vec{M}}{\partial x}$$

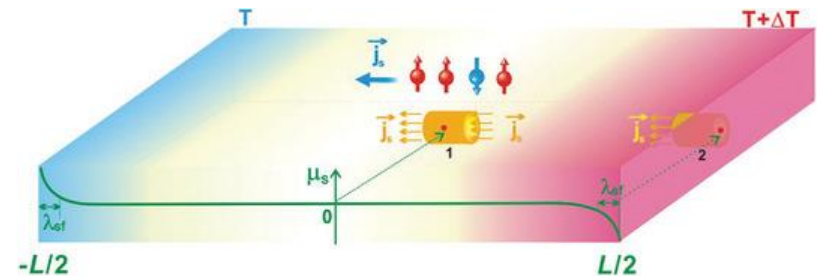
- Experimental depinning boundary as a function of field compared with simulated depinning boundaries for different values of β – best fit gives **$\beta = 0.040 \pm 0.005$, $P = 0.40 \pm 0.02$.**

- Seebeck effect

- any metal
- ∇T drives a charge current (in closed circuit)

- Spin-dependent Seebeck effect

- Ferromagnetic metal
- ∇T drives a pure spin current (equal and opposite flows of spin- \uparrow and spin- \downarrow electrons)
- This spin current exerts a spin-transfer torque, driving DW motion.



$$v = \frac{\beta}{\alpha} \frac{\gamma \hbar}{2eM_s} J_s$$

$$J_s = -\frac{2\sigma_{\uparrow}\sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} (S_{\uparrow} - S_{\downarrow}) \nabla T$$

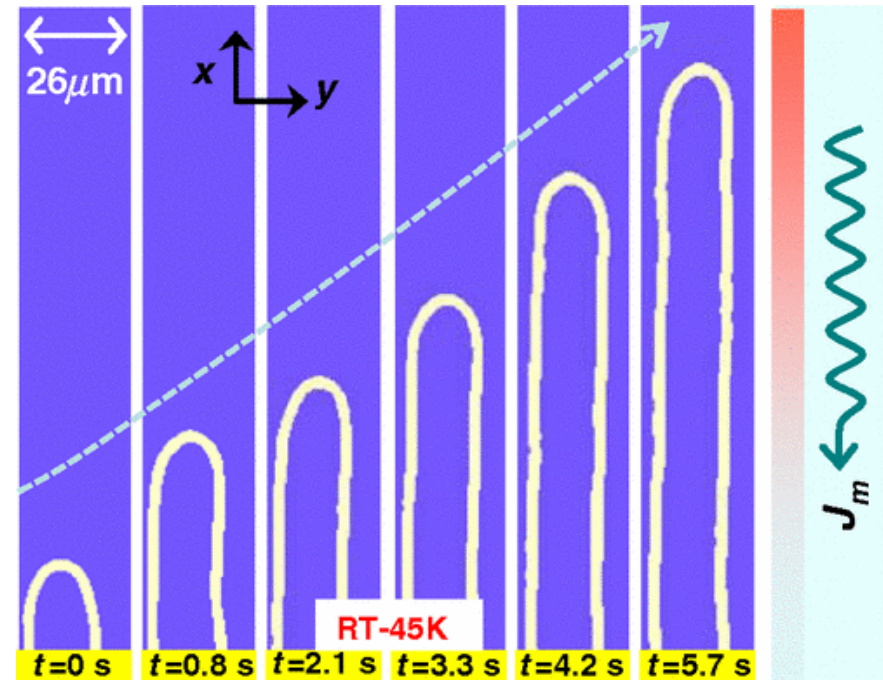
Spin Seebeck torque



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- SSE takes place in metals and insulators
- ∇T leads to a magnon current
- Magnons carry spin $S=1$
 - => this is a spin current
- Spin current exerts a spin-transfer torque on a DW
- DW will move towards hot region
- λ is thermal magnon wavelength

Observed in YIG film



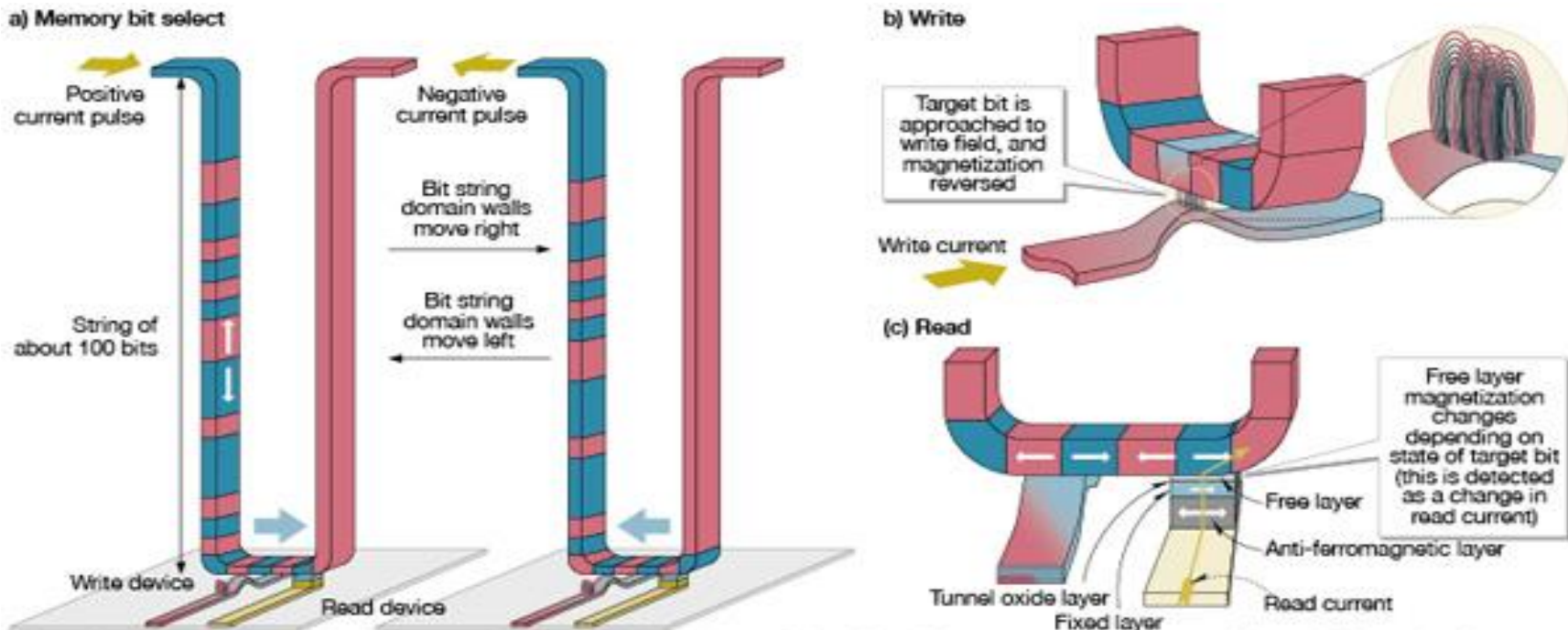
$$v = -\frac{\beta \gamma \hbar}{\alpha M_s} J_m \quad J_m = -\frac{k_B \nabla T}{6\pi^2 \lambda \hbar \alpha} F_0$$

Racetrack memory



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- Combine MTJ reader with CIDW motion to produce 3D 'storage class' memory



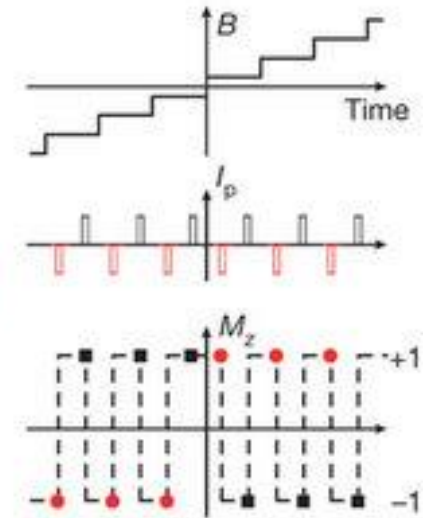
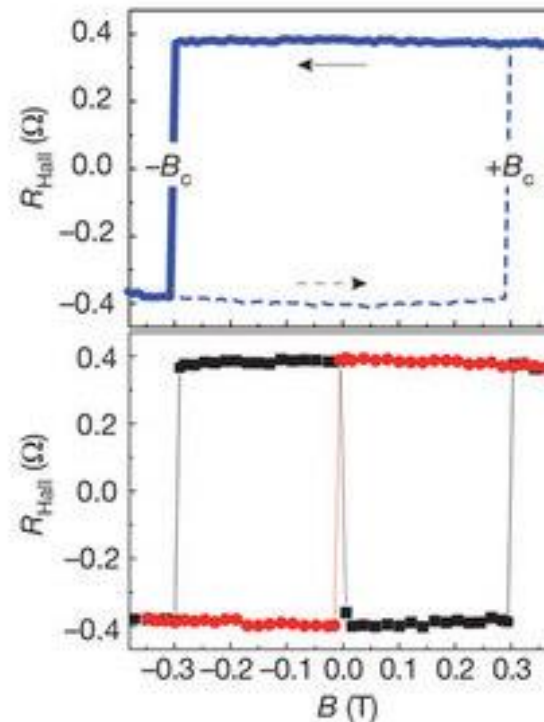
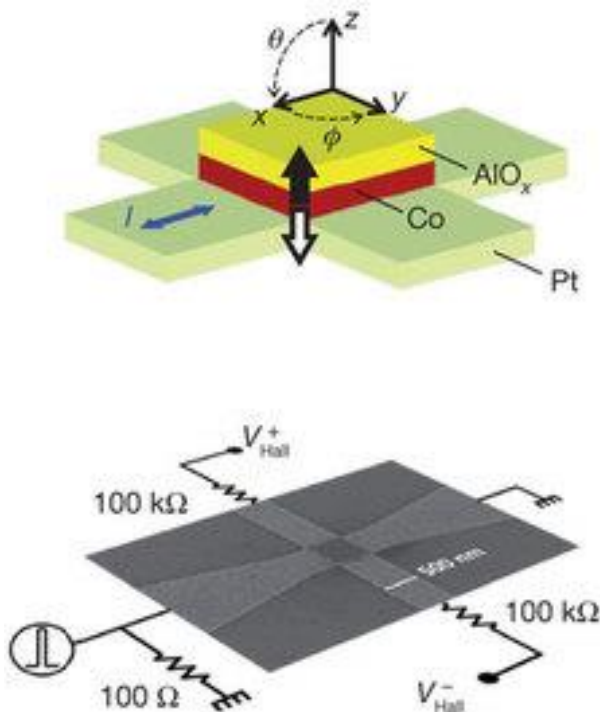
Using Current Pulses to Move Magnetic Domain Walls Magnetic Race-Track Memory records a string of about 100 bits of information perpendicularly to the Si substrate for each read/write device. This means information density is about 100 times higher than MRAM. Operating principles are shown for access memory bit select (a), write (b) and read (c). When selecting the memory bit for access a current pulse is applied to the magnetic material, causing the magnetic domain wall to move. A positive current pulse will move the wall to the right in the diagram, and a negative one to the left. The quantity of pulses can be controlled for random access, with data read executed after the target bit has been accessed. (Diagram by *Nikkei Electronics* based on material courtesy IBM)

Spin-Orbit Torques in Bilayers



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Out-of-plane magnetisation can be switched by in-plane current in adjacent layer

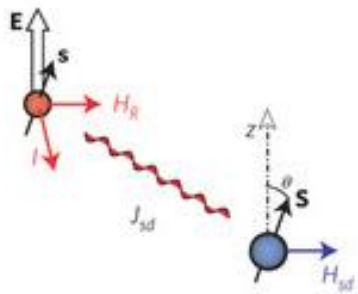
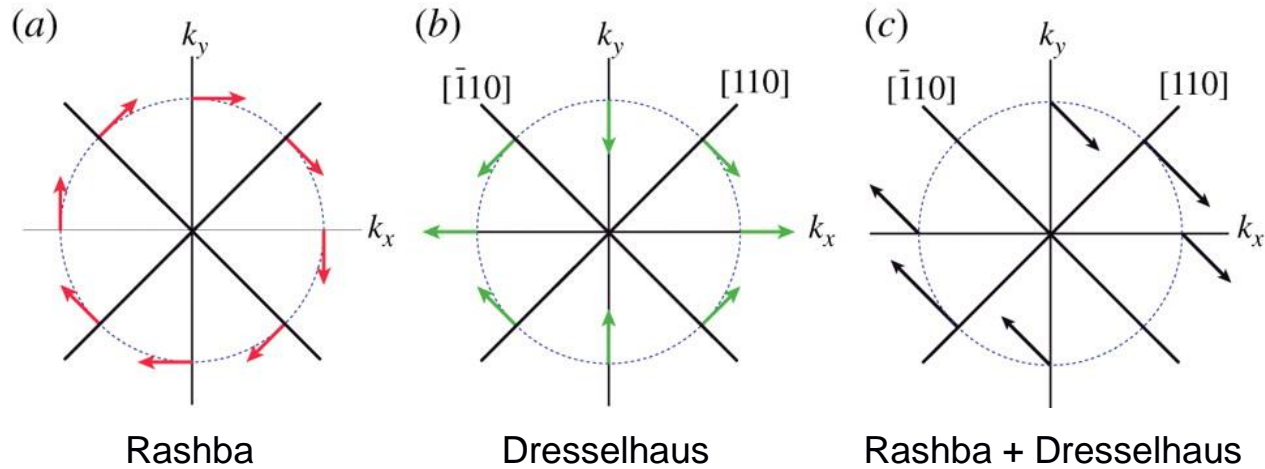


Rashba Mechanism

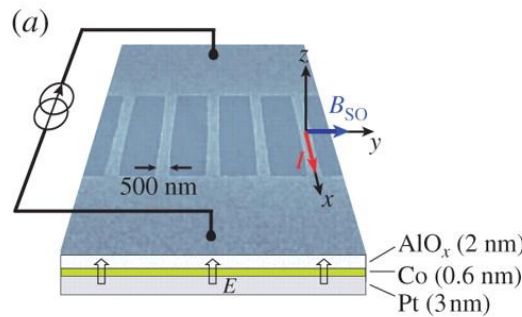


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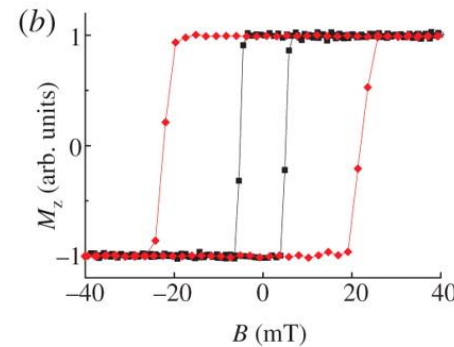
Orientation of the SO-induced magnetic field (arrows) as a function of current direction (solid lines).



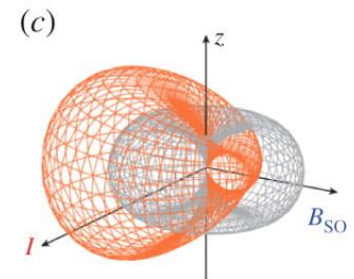
Moving spins are tilted by SO field.



Inversion asymmetric sample



Out-of-plane magnetisation.

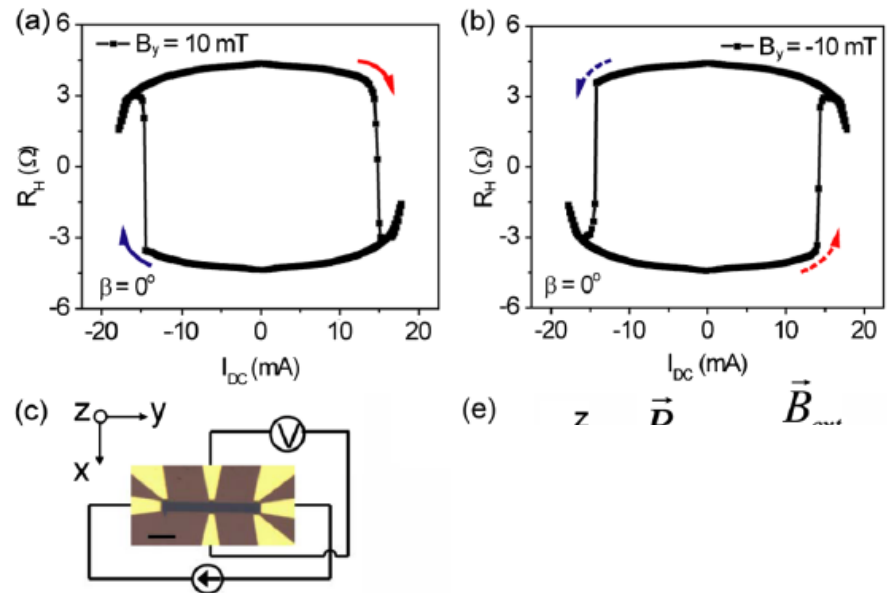
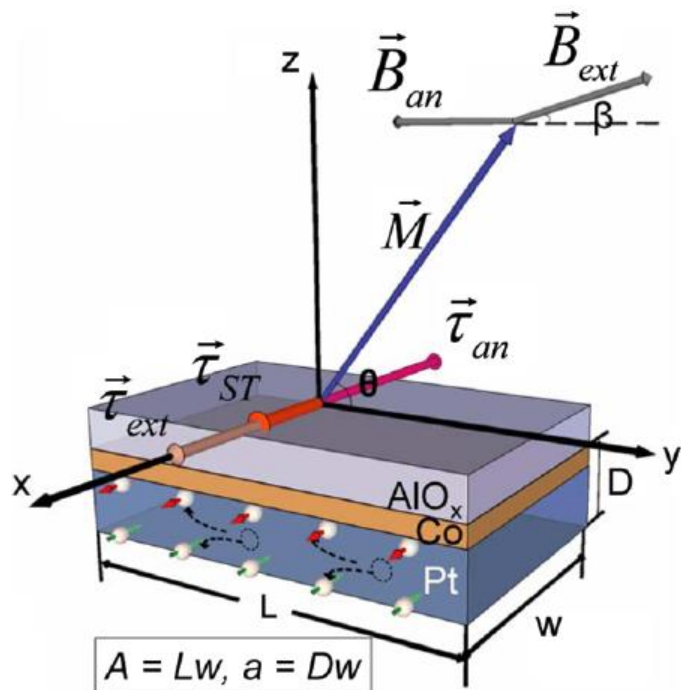


Energy isosurface with and without current flow

Spin Hall Mechanism



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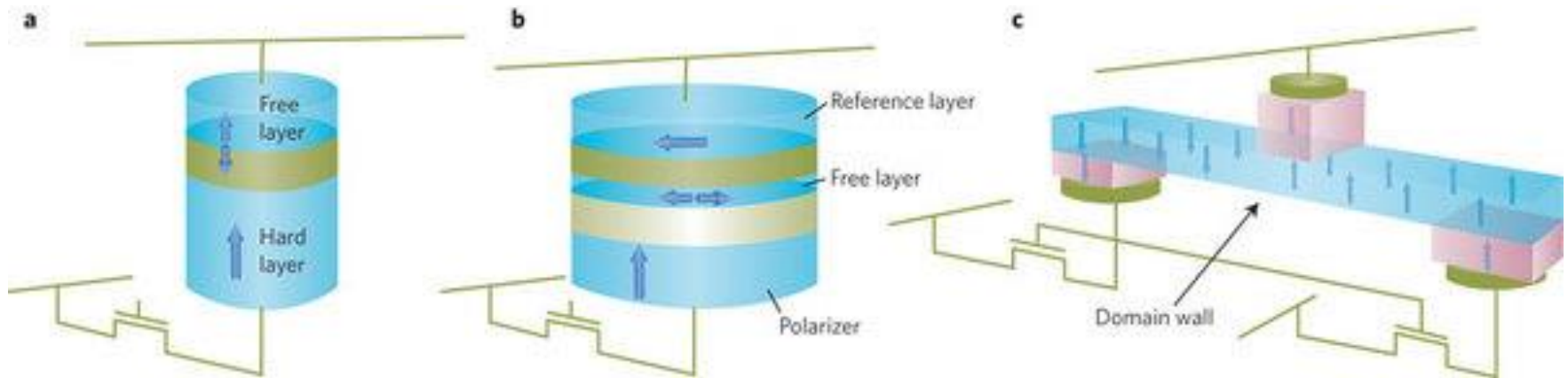


Spin current driven vertically into FM layer (through A) by horizontal charge current (passing through D).

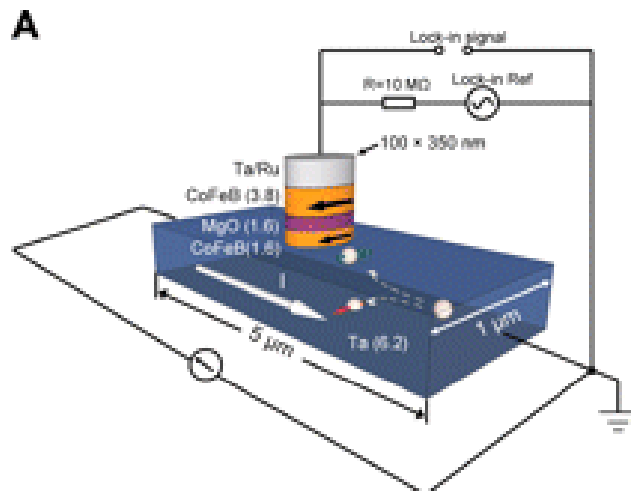
MRAM architectures



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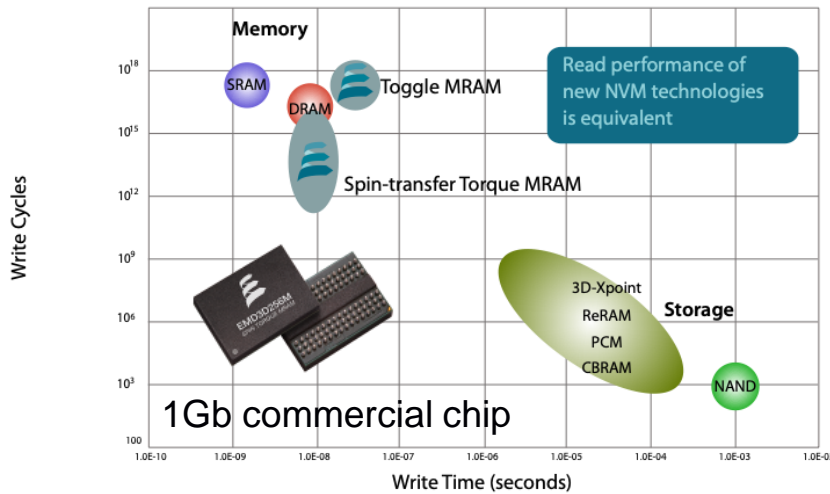
Brataas, Kent, and Ohno, Nature Materials **11**, 372 (2012).



Switching using current driven torques will dramatically reduce write energy per bit (from 100s of pJ to a few pJ).

Liu et al., Science **336**, 555 (2012)

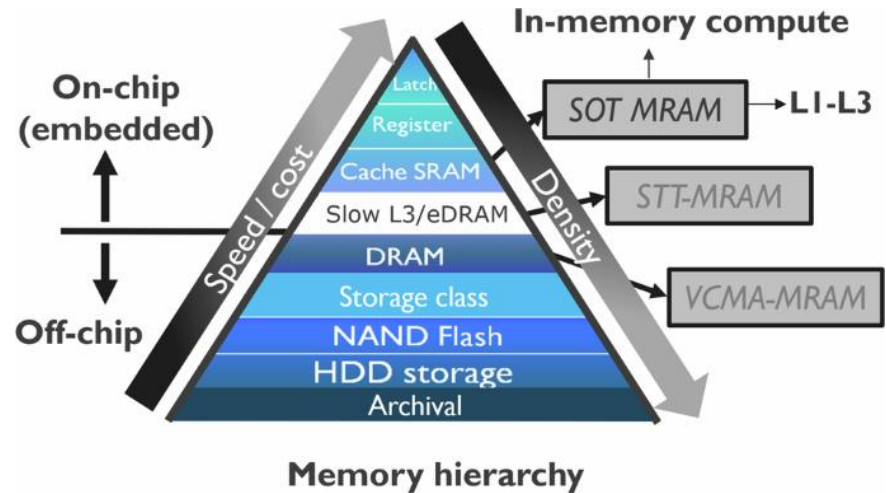
STT MRAM commercially available



- Fast write times
- High endurance
- Low power
- Radiation Hard

<https://www.everspin.com/spin-transfer-torque-mram-technology>

SOT MRAM under intense development



- Different memories suitable for different different points in the memory hierarchy (at least in the near future)

Nguyen et al., npj spintronics **2**, 48 (2024)

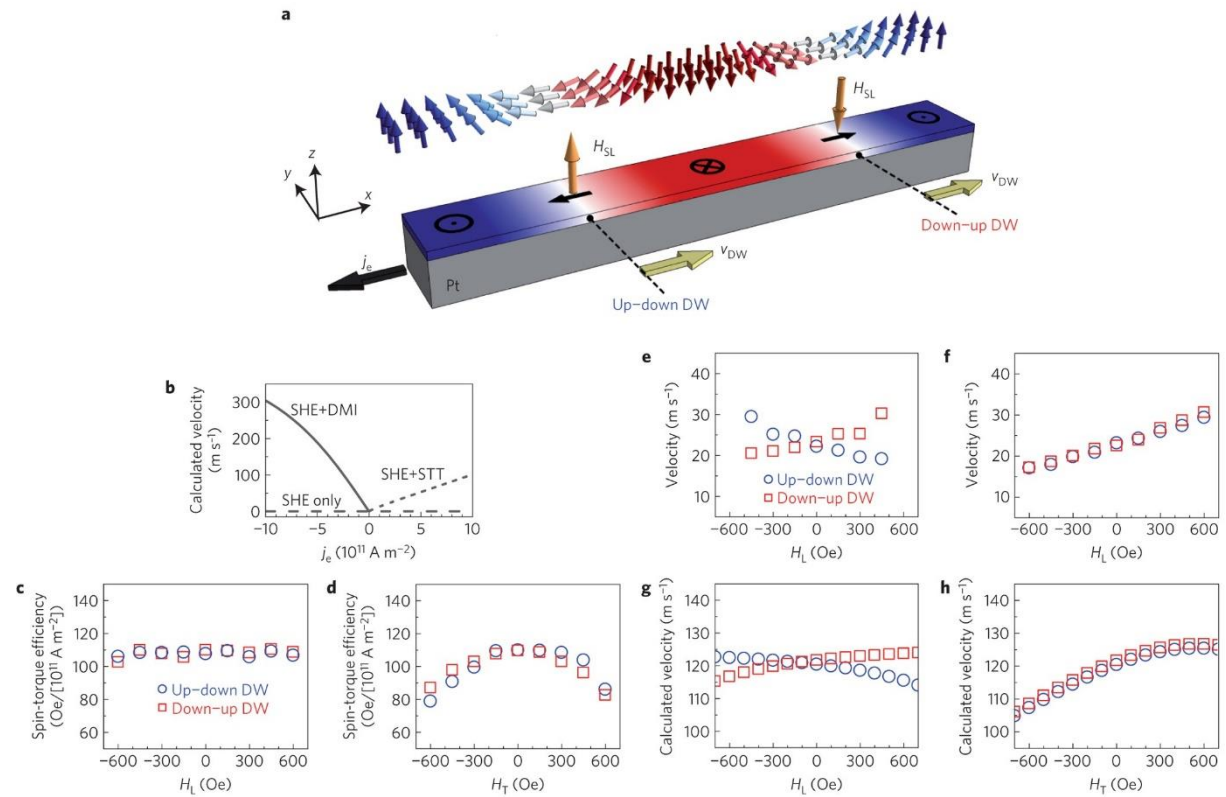
SOT-driven DW motion



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- Spin current from SO layer exerts torques DW
- Requires Néel wall in PMA material: direction of motion set by wall chirality
- Torque can be modelled as an effective field given by

$$H_{SHE} = \frac{\hbar\theta_{SHE}J_e}{2\mu_0eM_s t_F}$$
- θ_{SHE} is the spin Hall angle, which converts charge to spin current



Data for Ta/CoFe/MgO

Emori et al., Nature Materials **12**, 611 (2013).

Racetrack generations

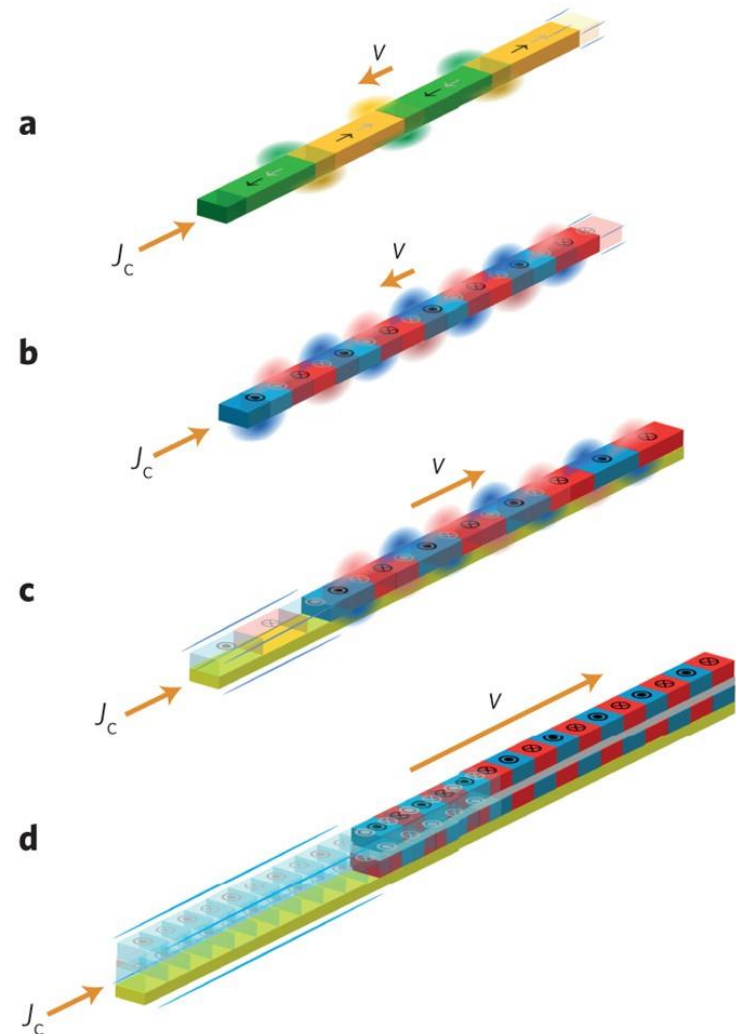


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- 1.0 – in-plane, STT
- 2.0 – PMA, STT
- 3.0 – PMA, SOT
- 4.0- PMA synthetic antiferromagnet, SOT

DW velocities ~ 1km/s in version 4.0

Parkin & Yang, Nature Nanotech **10**, 195–198 (2015)





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The End.