Optimal Population Distribution When Regional Economies, Disasters, and Utility Levels are Intercorrelated: Applying Portfolio Theory

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Extended Abstract

In Japan, where a fifth of the world's large (≥magnitude 6) earthquakes occur, public and business sectors are often decentralized geographically in order to diversify the "risk." [1] In Yonemoto (2025), the author has characterized the optimal level of diversification applying the traditional portfolio theory. This study extends it theoretically by introducing the correlation in utility levels as well as other externalities and empirically by using Bulgarian data.

There are many advanced preceding studies that have explored risk mitigations, such as the ones summarized in Brebbia (1998-), Okuyama and Chang (2004), Tatano and Takagi (2005), Jorissen and Stallen (2013), and Hagiwara and Asahi (2016), mostly given the population distribution or the system of cities. In contrast, this study considers the long-term or "ideal" distribution as a foundation of the discussions on the theories of agglomeration, which has been emphasizing the advantage of human/industry clusters[2].

For long-term optimality, the classical portfolio theory, which has been developed since the seminal work of Markowitz (1959), can be the first step as it takes both fluctuation (disaster damages) and return (economic growth) into account. This paper applies the portfolio theory, based on Markowitz (1959), Adachi and Saito (1992), and Hibiki and Tanabe (2005), straightforwardly to the allocation of the population.

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[1] For example, National Resilience Promotion Office, Cabinet Secretariat, Japan (2018) states "excessive population concentration status should be alleviated in the places that have high risk of occurrence of earthquakes and effective measures of boosting development of an autonomous, decentralized and coordinated nation should be considered and conducted in order to promote risk diversification" in its Fundamental Plan for National Resilience.

[2] Some researches have considered the locations under disaster risk but from different point of views. For example, Tatano et al. (2004) has analyzed locations within a city.

The Basic Model

Similarly to Yonemoto (2025), the following simple (linear) mean variance model of Portfolio Theory is employed to allocate the population. Referring to Hibiki and Tanabe (2005), formulate the maximization problem as:

$$\max_{\boldsymbol{w}} \boldsymbol{r}^T\boldsymbol{w} - \gamma\boldsymbol{w}^T\boldsymbol{V}\boldsymbol{w} \quad \text{s.t.} \quad \mathbf{1}^T\mathbf{w} = 1,$$
$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_I \end{bmatrix}, \quad \boldsymbol{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_I \end{bmatrix}, \qquad \boldsymbol{V} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1I} \\ \vdots & \ddots & \vdots \\ \sigma_{I1} & \cdots & \sigma_{II} \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \tag{1}$$

where w_i is the population share of region i, r_i is the expected return (economic growth rate) of i, σ_{ij} is the covariance between i and j (for $i = j$, the one represents the variance instead,) and γ is the coefficient of risk aversion, which is arbitrary chosen in this study, depending on the planner's objective. Then the corresponding Lagrangian is,

$$L = \boldsymbol{r}^T\boldsymbol{w} - \gamma\boldsymbol{w}^T\boldsymbol{V}\boldsymbol{w} + \lambda(1 - \mathbf{1}^T\mathbf{w}), \tag{2}$$

where λ is the Lagrangian multiplier. The first-order condition for maximization is,

$$\boldsymbol{r} - 2\gamma\boldsymbol{V}\boldsymbol{w} - \lambda\mathbf{1} = \mathbf{0},$$
$$\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{3}$$

Solving (3) for $\boldsymbol{w}$,

$$\boldsymbol{w} = \frac{1}{2\gamma}\boldsymbol{V}^{-1}(\boldsymbol{r} - \lambda\mathbf{1}), \tag{4}$$

Using the constraint in (1), λ is derived as follows:

$$\lambda = \frac{\mathbf{1}^T\boldsymbol{V}^{-1}\boldsymbol{r} - 2\gamma}{\mathbf{1}^T\boldsymbol{V}^{-1}\mathbf{1}}. \tag{5}$$

If there are non-negativity constraints on w_is,

$$w_i \geq 0 \quad \forall i, \tag{6}$$

The following Kuhn-Tucker conditions are added:

$$\frac{\partial L}{\partial w_i} \leq 0, \quad w_i \geq 0, \quad \text{and} \quad w_i\frac{\partial L}{\partial w_i} = 0. \tag{7}$$

In general, an algebraic solution (such as (4) in the case without non-negativity constraints) is not explicitly derived and the result is obtained numerically.

Externalities

This study extends the aforementioned model considering other types of externalities. For example, as Mui (1995) summarizes, since Veblen (1934) and Duesenberry (1949), various economic researches have been conducted assuming explicitly that the utility depends on some relative conditions or variables. It may be characterized by the comparison to the others, his/her state in the past or any other "reference" points. The related early works include Galbraith (1958), Sen (1966), Easterlin (1974) and Stigler and Becker (1977). The succeeding developments including the concepts of "altruism" and/or "envy" have been summarized by Kolm (1972), Abel (1990), Elster (1991), Carroll et al. (1997), Bowles (1998), Clark and Oswald (1998), and Dufwenberg et al. (2011). Yonemoto (2021a, b) has introduced state-dependency of utility into regional modelling. For example, one can consider the utility function of the following type:

$$u_{nt} = u\left(c_{nt}; c_{1t}, \cdots, c_{(n-1)t}, c_{(n+1)t}, \cdots, c_{Nt}\right), \tag{8}$$

where u_{nit} indicates the utility level of individual n in period t, which depends on the consumption levels of the other people (some located in other regions) $1 \cdots, n-1, n+1, \cdots, N$ and affected (lowered) by the disasters as they occur. Then, variance-covariance matrix $\boldsymbol{V}$ are modified taking those effects into account.

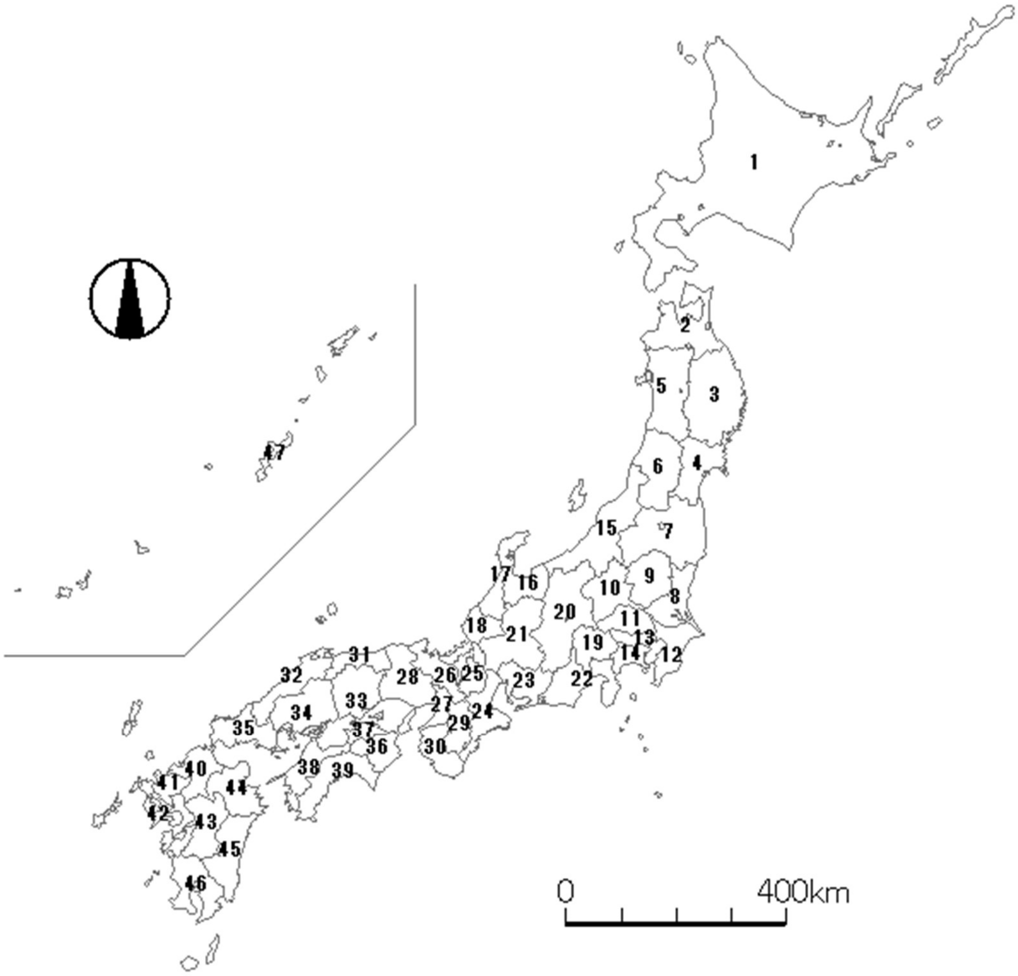
Japanese Case: Data and Methods

This study also presents numerical examples using Japanese data (mostly on earthquake) and Bulgarian data (mostly on flooding.)

Historic Earthquake Data

Japanese earthquake data are based on Seismic Intensity Database, Japan

Meteorological Agency. As the observation system started in 1919 and changed dramatically after 1995, the Great Hanshin-Awaji Earthquake, this study uses data between them (1919-1995.) Collect all data whose seismic intensities are 4 or higher, and the highest one in each prefecture (see Figure 1 for their locations) is used to represent its effect. There have been 1,286 samples during this period including 16 samples whose intensity is 6 (the highest-realized scale before 1995.) Figure 2 depicts the epicenters of the last 10 years (1986-1995) of the data; it can be confirmed they are dispersed nationwide.



1	Hokkaido	13	Tokyo	25	Shiga	37	Kagawa
2	Aomori	14	Kanagawa	26	Kyoto	38	Ehime
3	Iwate	15	Niigata	27	Osaka	39	Kochi
4	Miyagi	16	Toyama	28	Hyogo	40	Fukuoka
5	Akita	17	Ishikawa	29	Nara	41	Saga
6	Yamagata	18	Fukui	30	Wakayama	42	Nagasaki
7	Fukushima	19	Yamanashi	31	Tottori	43	Kumamoto
8	Ibaraki	20	Nagano	32	Shimane	44	Oita
9	Tochigi	21	Gifu	33	Okayama	45	Miyazaki
10	Gunma	22	Shizuoka	34	Hiroshima	46	Kagoshima
11	Saitama	23	Aichi	35	Yamaguchi	47	Okinawa
12	Chiba	24	Mie	36	Tokushima		

Figure 1: Japanese Prefectures

(Made by Author with MANDARA GIS software)

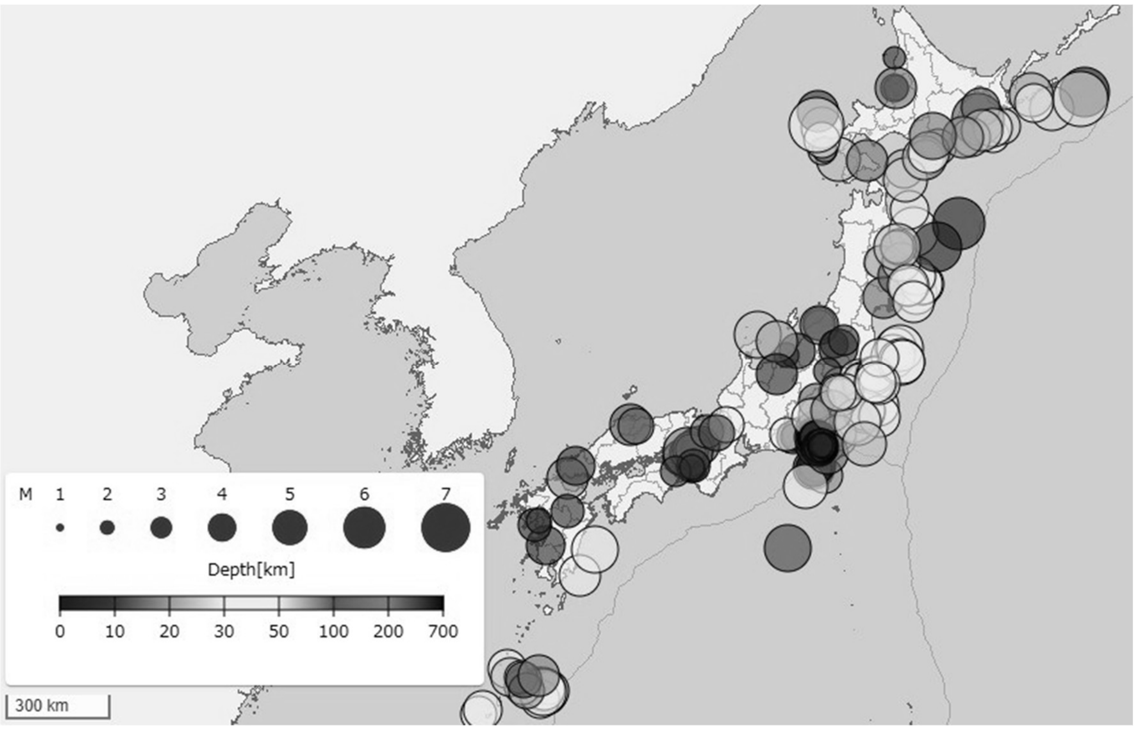


Figure 2: Example of Epicenters (≥ Intensity 4, Jan. 1986 - Dec. 1995)

Seismic Intensity Database, Japan Meteorological Agency

Converting Intensity to the Annual Rate of Damage

Since the seismic intensity is far from proportional to the degree of damage, first, it is converted to the acceleration (gal) by the following conventional equation[3]:

$$A = 10^{-047+\frac{SI}{2}}. \tag{9}$$

Then, by summing up for each year, annual A is obtained. It is converted to the rate of damage by using coefficient β estimated through the following regression (without a constant term):

$$\%DAMAGE_n = \beta A_n + e_n, \tag{10}$$

in which the original damage estimates (in monetary terms) are based on cui et al. (2017) and Toyoda et al. (2020)[4].

[3] (9) is a part of the formulae used in "instrumental seismic intensity," Japan Meteorological Agency.

[4] The prefectural GDPs, which are used as numerators to calculate the "rates," are the ones of 2011,

No.	prefecture	avg growth	avg damage	No.	prefecture	avg growth	avg damage
1	Hokkaido	3.3%	4.5%	25	Shiga	5.0%	0.7%
2	Aomori	4.0%	2.2%	26	Kyoto	3.8%	1.1%
3	Iwate	4.1%	2.7%	27	Osaka	4.7%	0.5%
4	Miyagi	4.4%	1.9%	28	Hyogo	3.5%	2.1%
5	Akita	3.6%	0.8%	29	Nara	4.0%	0.7%
6	Yamagata	3.9%	0.9%	30	Wakayama	2.9%	0.9%
7	Fukushima	4.3%	3.4%	31	Tottori	3.5%	1.1%
8	Ibaraki	4.5%	3.3%	32	Shimane	3.6%	0.3%
9	Tochigi	4.6%	2.8%	33	Okayama	3.7%	0.6%
10	Gunma	4.2%	1.1%	34	Hiroshima	4.3%	0.9%
11	Saitama	5.8%	2.6%	35	Yamaguchi	3.5%	0.7%
12	Chiba	5.6%	3.1%	36	Tokushima	4.1%	0.6%
13	Tokyo	3.9%	7.0%	37	Kagawa	4.2%	0.6%
14	Kanagawa	5.2%	2.5%	38	Ehime	3.4%	0.6%
15	Niigata	3.9%	0.8%	39	Kochi	3.2%	0.7%
16	Toyama	3.9%	0.3%	40	Fukuoka	3.9%	0.2%
17	Ishikawa	4.2%	0.4%	41	Saga	3.7%	0.2%
18	Fukui	3.9%	1.0%	42	Nagasaki	3.5%	0.9%
19	Yamanashi	4.3%	2.6%	43	Kumamoto	3.8%	1.6%
20	Nagano	4.5%	3.2%	44	Oita	3.8%	0.8%
21	Gifu	3.9%	0.6%	45	Miyazaki	3.9%	1.3%
22	Shizuoka	4.2%	3.8%	46	Kagoshima	3.6%	1.8%
23	Aichi	4.7%	0.6%	47	Okinawa	4.6%	2.1%
24	Mie						

Table 1: Estimated Average Growth and Damage Rates[5]

Expected Return and Variance-Covariance Matrix

(Gross) returns are represented by the average growth rates of prefectural GDPs in 1955-2009[6]. Subtracting (9), 77 (years) × 47 (prefectures) matrix of returns is constructed and the variance-covariance ($\boldsymbol{V}$ in (1)) is derived from it. And the row expected values of the matrix are $\boldsymbol{r}$s in (1). The average rates are presented in Table 1.

Prefectural Economic Accounts, Cabinet Office, Government of Japan. β has been estimated to be 0.043.

[5] High damage rate of Tokyo might be reflecting earthquake swarms around its territory of southern islands. Some geographically long or large prefectures, such as Tokyo, should be divided into several parts taking their particular characteristics into account (in future researches.)

[6] They are based on the time-series data of Prefectural Economic Accounts, Cabinet Office, Government of Japan. Because the data for 1975-1984 are not consistent with the ones for the other periods, they are simply extrapolated from 1985-1999 figures. Data for 1919-1954 are not officially released and thus not used. Note that annual growth rates are available but not used; the average rates are used for all periods in order to characterize the fluctuations due to the disasters but not by the business cycle.

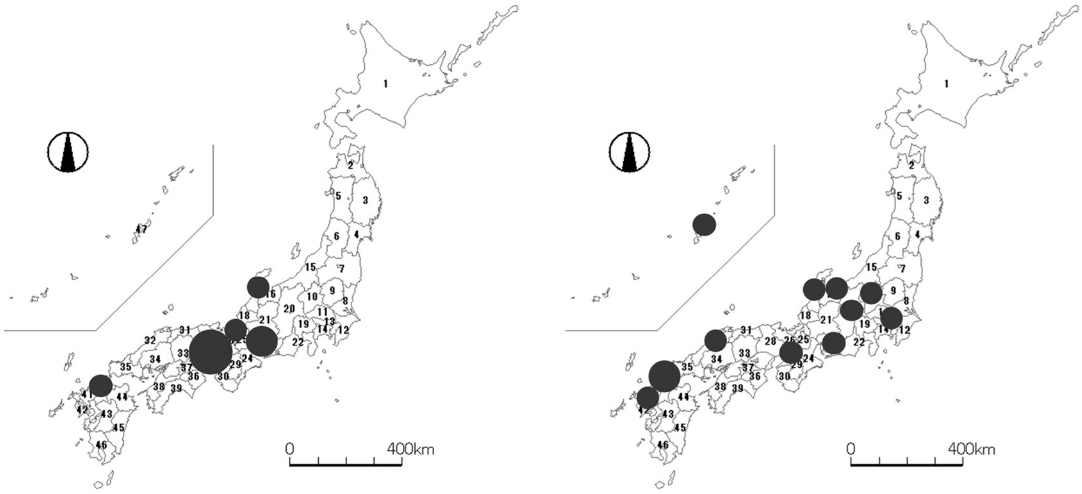


Figure 3: Japanese Results (Left: $\gamma=0.5$, Right: $\gamma=10$)

Non-negativity Constraints

If w_is are derived to be non-negative without any constraints, those solutions are explicitly obtained from (4). However, if not, they need to be numerically calculated. This study uses Mathematica software (Wolfram 14.3) to obtain them.

Japanese Case: Results

For $\gamma=0.5$, the resulting allocation of the population is 50.7% for No.27 (Osaka), 24.0% for No.23 (Aichi), 15.2% for No.40 (Fukuoka), 8.0% for No.17 (Ishikawa), and 2.1% for No.25 (Shiga) (and almost zero for all the other prefectures.) With low risk aversion, people are to be located in relatively growing regions with considerably fewer earthquakes. Note that, even then, the population needs to be somewhat dispersed.

For $\gamma=10$, the result is 47.5% for No.40 (Fukuoka), 14.4% for No.41 (Saga), 10.9% for No.17 (Ishikawa), 8.6% for No.16 (Toyama), 7.7% for No.27 (Osaka), 7.1% for No.32 (Shimane), 3.3% for No.23 (Aichi), 0.3% for No.47 (Okinawa), 0.09% for No.13 (Tokyo), 0.08% for No.10 (Gunma), and 0.04% for No.20 (Nagano) (and almost zero for all the other prefectures.) Now the population is more diversified mostly in the western Japan, the risk is taken more into account and even less-developing prefectures receive some part.

To perform a sensitivity analysis of externalities, for each covariance larger than 1, it is raised by 0.5%. Then, the result of the simulation indicates slight increases in the shares of No.41 (Saga), No.16 (Toyama), No.23 (Aichi), and No.20 (Nagano), and decreases in No.27 (Osaka), No.32 (Shimane), No.47 (Okinawa), No.13 (Tokyo), and No.10 (Gunma).

Bulgarian Case: Data and Methods

This study also conducts a similar analysis using Bulgarian flooding data. Flooding

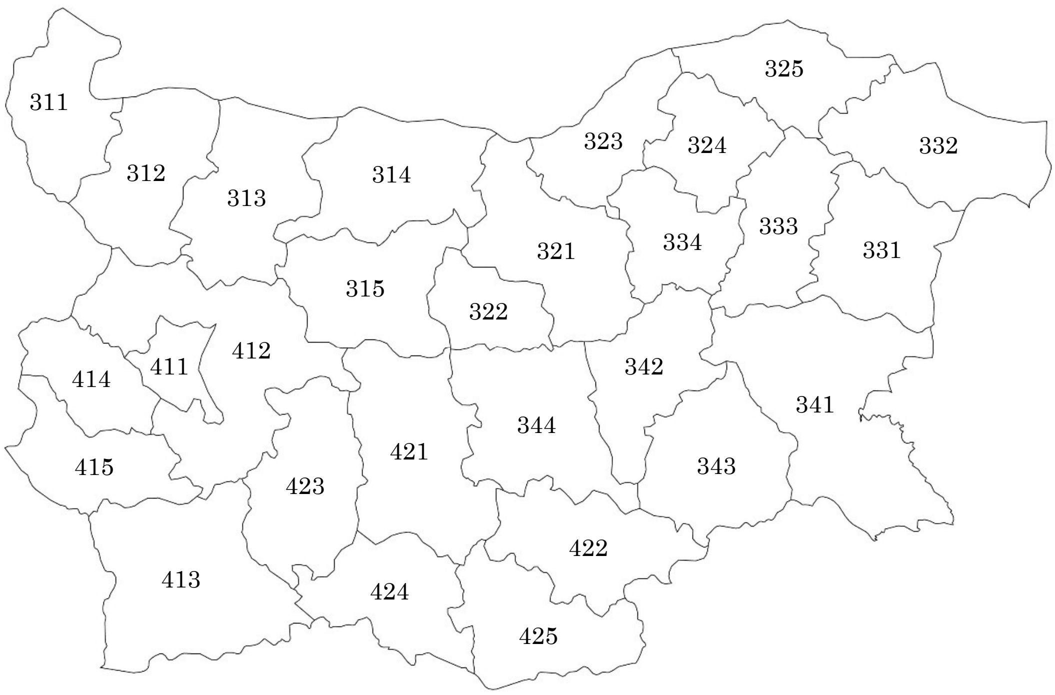


Figure 1: Bulgarian Oblasts

(Made from the original blank map by vepmaps.com)

oblast		disaster damage (in 2025 price)											econ. growth
No.	name	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2010-2020
311	Vidin	0	85	165	0	324	0	0	0	0	0	0	4.4%
312	Montana	339	321	226	9	7,305	12	0	3	3,476	3,584	506	5.8%
313	Vratsa	0	148	0	3	16,950	417	0	0	0	0	0	6.2%
314	Pleven	676	508	12	0	909	1,700	8	956	0	0	190	6.0%
315	Lovech	283	3	3	0	108	0	0	0	0	670	0	5.4%
321	V. Tarnovo	138	196	1,208	526	15,653	1,352	85	2,133	3,766	1,221	1,093	5.4%
322	Gabrovo	86	50	400	56	4,482	0	1,817	0	0	0	55	6.2%
323	Ruse	157	557	89	108	873	2,724	1,856	419	200	471	1,155	6.7%
324	Razgrad	16	0	28	1,018	202	0	1	0	0	0	0	5.6%
325	Silistra	0	268	0	1,689	0	0	0	0	0	0	96	5.4%
331	Varna	1,609	843	810	912	24,823	2,328	3,035	444	8	223	498	6.4%
332	Dobrich	249	320	1	144	5,158	1,280	22	531	0	146	0	4.5%
333	Shumen	7	6	147	35	1,908	968	0	0	0	0	0	6.3%
334	Targovishte	33	1	0	0	0	0	0	386	171	0	0	5.6%
341	Burgas	303	90	254	9	10,642	89,307	525	78,351	1,651	1,729	737	3.7%
342	Sliven	755	461	0	0	41	876	115	23	89	546	159	5.3%
343	Yambol	745	0	0	0	183	0	0	0	0	0	0	5.4%
344	St. Zagora	159	0	556	0	5,027	1,106	0	0	0	58	0	5.7%
411	Sofia-capital	138	0	0	0	0	0	0	0	0	0	0	7.2%
412	Sofia	279	121,862	528	272	3,768	4,619	0	229	1,190	498	1	7.8%
413	Blagoevgrad	7,443	680	40	77	420	252	49	0	211	284	1,035	6.4%
414	Pernik	549	14	1	9	6	1	1	50	1	1,122	1,226	4.3%
415	Kyustendil	1,384	279	298	201	0	76	7	450	36	179	1	4.4%
421	Plovdiv	2,736	0	100	130	11,534	929	5,402	1,862	1,837	1,032	1,220	7.5%
422	Haskovo	168	277	2,856	0	2,785	148	110	0	0	808	658	5.9%
423	Pazardzhik	596	226	0	15	661	0	0	664	14	0	0	5.2%
424	Smolyan	4,980	5,203	5,302	4,215	2,237	6,067	3,354	710	2,778	1,501	2,771	5.1%
425	Kardzhali	101	144	776	753	641	1,586	3,538	2,810	3,955	834	530	9.5%

Table 2: Bulgarian Average Growth and Damage Amounts (in million leva, BGN)

data are based on "crisis events" data of National Statistical Institute of Bulgaria, summarized by Vladev and Solakova (2025). Economic (including Consumer Price Index) data are from "macroeconomic data," National Statistical Institute of Bulgaria.

Figure 1 indicates Bulgarian regions (oblasts) and Table 2 is the data (after deflating) used in the analysis.

Conclusion at the Time of Submission of the Extended Abstract

In general, as people are more risk averse, the population needs to be dispersed. An increase in covariance results in either concentration or dispersion depending on the locations.

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