The Spatial Race Between Labour and Capital: A Route to Turing Instability

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Long Abstract

Motivation and Research Questions

A long-standing question in spatial and growth economics concerns whether the free movement of factors of production across locations is a force for convergence or divergence in income and welfare. The neoclassical tradition, grounded in the assumption of decreasing marginal productivity, predicts that the mobility of labour and capital generates equalisation of factor returns and, ultimately, convergence in per-capita incomes across regions and countries. The new economic geography (NEG) literature, by contrast, has demonstrated that local increasing returns to scale — arising, for instance, from technological spillovers and thick-market externalities — can reverse this conclusion, generating persistent spatial inequality through the self-reinforcing logic of agglomeration (Fujita, Krugman, and Venables, 1999; Puga, 1999; Krugman, 1996). A particularly compelling arena for these forces is urban economics, where the local increasing returns to human capital attract skilled workers, raise wages, and thereby further stimulate in-migration (Ciccone and Hall, 1996), while rising congestion costs and rents provide the counteracting centrifugal force.

A notable limitation of much of the existing literature is that it typically analyses the spatial dynamics of only one factor at a time. NEG models, for example, focus predominantly on the spatial distribution of workers, while largely treating capital as either immobile or as a passive residual (Desmet and Rossi-Hansberg, 2014; Desmet, Nagy, and Rossi-Hansberg, 2018; Quah, 2002). This paper departs from that tradition by developing a unified spatial framework in which both physical capital and labour are mobile across a continuous set of locations, each following its own gradient-based reallocation rule. The central research question is: what spatial patterns emerge in the long run from the joint dynamic interaction of two mobile factors in the presence of agglomeration economies, congestion costs, and local capital accumulation? The paper's main answer is that this interaction can give rise to a phenomenon known as Turing instability — the spontaneous emergence of persistent, spatially heterogeneous patterns from an initially uniform distribution (Murray, 2007) — a result with significant implications for how economists understand the formation and persistence of regional inequality.

The Model

The model considers an economy populated by a continuum of locations indexed by $x \in \Omega$, where Ω is assumed to have the topology of a circle or torus (i.e., there are no borders). Two factors of production, labour $L(x,t)$ and physical capital $K(x,t)$, are distributed across space and evolve over time according to partial differential equations that capture, simultaneously, local accumulation dynamics and spatial reallocation flows.

Workers are assumed to derive instantaneous utility from their wage net of local living costs, which include both a housing component (increasing in the local wage) and a congestion cost (increasing in local population density), consistent with the spatial social interaction models of Mossay (2003) and Mossay and Picard (2011). Workers relocate across locations in order to maximise utility, with mobility governed by static and myopic expectations — that is, current utility differentials are treated as permanent. Mobility costs are convex in distance, which under standard regularity conditions yields a gradient-flow dynamics for the spatial evolution of labour:

$$\partial_t L = -\frac{1}{\gamma_L}\partial_x \left(L\, \partial_x u\right) + \sigma_L \partial_{xx} L + nL,$$

where γ_L is the marginal cost of labour mobility, $\sigma_L \geq 0$ captures idiosyncratic dispersion in location preferences (generating a diffusive term), and n is the net local population growth rate due to births and

deaths. The idiosyncratic component of workers' preferences provides a micro-founded rationale for spatial diffusion that does not rely on arbitrary smoothing assumptions.

Analogously, firms allocate physical capital by following the spatial gradient of the profit rate, also with myopic and static expectations and convex mobility costs:

$$\partial_t K = -\frac{1}{\gamma_K}\partial_x \left(K\, \partial_x r\right) + sY - \delta K,$$

where γ_K measures the marginal cost of capital mobility, s is the saving and investment rate, Y is local output, and δ is the capital depreciation rate. Local investment equals local saving, a behavioural assumption that mirrors the myopic decision-making attributed to households and firms throughout the model.

Production at each location is given by $Y(x,t) = A(x,t)F(K(x,t), L(x,t))$, where F is constant returns to scale and satisfies the standard Inada conditions. The local total factor productivity $A(x,t) = g(L(\cdot,t))$ is an increasing function of the spatial distribution of workers, capturing the idea — standard in the urban and spatial economics literatures (Ciccone and Hall, 1996; Puu, 1985, 1993) — that human density generates knowledge spillovers and agglomeration externalities. The micro-foundations for this specification are developed in Fiaschi and Ricci (2022). Factor markets are competitive and frictionless, so that wages and profit rates are set equal to their respective marginal products.

Equilibrium Analysis

A spatial equilibrium is defined as a configuration in which neither labour nor capital flows between locations, i.e., all spatial gradients in utility and profit rates are zero. The most natural candidate for such an equilibrium is the spatially uniform solution, in which both factors are evenly distributed across locations. In this case, the spatial dynamics collapses to a purely local (Solow-type) accumulation process whose properties are well understood.

However, a spatially uniform equilibrium does not necessarily imply that labour and capital are equally distributed: the presence of location-specific cost-of-living components implies that equal utility can arise from different combinations of wages and living costs, while spatially heterogeneous technological spillovers can sustain equal profit rates at unequal factor densities.

To make the analysis tractable, the paper focuses on a parametric specification in which production is Cobb-Douglas with capital share α, the cost of living comprises a housing component proportional to the wage and a congestion cost of the form $c_c L^{\phi}$, and technological progress is $g(L) = \beta L^{\mu}$. Under this specification, the equilibrium utility at a location, as a function of the local labour stock alone (after substituting the optimal capital-labour ratio), is hump-shaped in L: it rises initially due to the productivity-enhancing effect of denser labour — the agglomeration force — and then falls due to rising congestion. Formally, $\bar{u}(L)$ is concave in L under the parameter restriction $\alpha < \mu < \alpha + 1$ and $\phi > 1$, and reaches a peak at a threshold value $L^* = \left[C_h \beta(1-\alpha)(\mu-\alpha)K^{\alpha}/(C_c\phi)\right]^{1/(\alpha+\phi-\mu)}$, which is increasing in the local capital stock K.

Stability Theorems and Turing Instability

Two key theoretical results characterise the stability of the uniform spatial equilibrium. First, if the economy starts in a spatially uniform configuration, it remains spatially uniform for all time — the uniform solution is self-consistent. Second, and more importantly, the stability of this uniform equilibrium depends on the aggregate labour stock relative to the threshold $L^*(K(t))$: when $L(t) > L^*(K(t))$, the uniform distribution is stable; when $L(t) \leq L^*(K(t))$, it is unstable. Since $L^*(K(t))$ is increasing in the capital stock, a rising capital stock — which is the natural trajectory of an economy with positive saving — progressively widens the basin of instability. This is the core mechanism by which the joint dynamics of labour and capital can generate Turing-type spatial patterns.

The Turing instability, originally identified in mathematical biology as the mechanism by which spatial patterns spontaneously emerge in reaction-diffusion systems (Murray, 2007; Marciniak-Czochra, Karch, and Suzuki, 2017), operates here through a conceptually analogous but economically distinct channel. Prior

work has applied Turing mechanisms to economic-ecological systems (Brock and Xepapadeas, 2010), to economic-demographic dynamics (Zincenko et al., 2021), and to capital-labour models in a more stylised setting (Aly, 2012), as well as to the morphogenesis of urban systems (Friesen, Tessmann, and Pelz, 2019) and to new economic geography (Ohtake, 2024). The present paper contributes to this growing literature by showing that Turing instability can arise endogenously from the interaction between capital accumulation (which raises the instability threshold L^*) and labour mobility (which would otherwise smooth out spatial heterogeneities), without requiring differential diffusion rates as in the classical biological paradigm. The result is the spontaneous emergence of spatial clusters — locations with high concentrations of both labour and capital, separated by depopulated or economically marginal areas.

Numerical Explorations

The paper accompanies the theoretical analysis with a series of numerical simulations that progressively build up the complexity of the model. A baseline "dumb economy" — with no factor accumulation and no diffusion — exhibits the threshold dynamics clearly: below L^*, all factors concentrate in a single location in the long run; above L^*, the uniform distribution prevails. Adding labour diffusion to this baseline model preserves the qualitative dynamics but accelerates convergence to the uniform distribution. The introduction of population growth alone (with no capital accumulation) is also capable of restoring uniform spatial distribution in the long run.

The most revealing simulation is the full model with positive saving, depreciation, population growth, and labour diffusion ($s = 0.25$, $\delta = 0.1$, $n = 0.02$, $\sigma_L > 0$). In this case, the dynamics unfolds in two phases: an initial transitory convergence toward the uniform distribution, followed by a sharp and persistent divergence, with the emergence of distinct Turing spots — dense agglomerations separated by areas that become entirely depopulated. This two-phase dynamics captures, in stylised form, a pattern that is visible in long-run historical data on population density, such as the spatial evolution of population in the United States from 1492 to 2020, which is used as motivating evidence.

Contributions and Implications

The paper makes several contributions to the literature on spatial economics and economic geography. First, it extends the standard spatial growth framework by endogenising the joint dynamics of two mobile factors, rather than treating capital as exogenous or immobile, thereby complementing the tradition of continuous spatial modelling in economics (Puu, 1985, 1993; Brito, 2004; Desmet and Rossi-Hansberg, 2014). Second, it identifies a novel route to Turing instability in an economic model, driven not by differential diffusion rates — as in the biological paradigm (Murray, 2007) — but by the interaction between capital accumulation and labour mobility under agglomeration and congestion forces, contributing to the emerging literature on pattern formation in spatial economics (Brock and Xepapadeas, 2010; Aly, 2012; Zincenko et al., 2021; Ohtake, 2024). Third, it provides a formal characterisation of the conditions under which spatial inequality is a stable long-run outcome, and shows that these conditions become more likely to be satisfied as the economy accumulates capital, offering a dynamic and endogenous explanation for the persistence of regional disparities documented empirically in Fiaschi, Gianmoena, and Parenti (2018) and Mirza et al. (2021).

From a policy standpoint, the results suggest that interventions aimed at fostering spatial convergence — such as promoting labour mobility or reducing congestion costs — may be insufficient or even counterproductive if the underlying capital accumulation process continually shifts the economy into the unstable regime. A more effective strategy may need to directly address the spatial distribution of capital investment or the nature of the technological spillovers that generate agglomeration externalities.

Keywords: spatial economics, economic geography, Turing instability, factor mobility, agglomeration, capital accumulation, labour migration, reaction-diffusion systems, regional convergence.