Long-run Zipf's and Gibrat's Laws Deviation: Spanish Population Impact

The analysis of power laws and, specifically, the study of the potential validity of the Zipf's and Gibrat's Laws, holds a long tradition as scientific field inside of the Urban Economics (Clauset, Shalizi, and Newman 2009; Fang, Pang, and Liu 2017; González-Val, Ximénez-de-Embún, and Sanz-Gracia 2024). Zipf proposes a linear stable relationship between the rank and the population of cities, called as Zipf's law. On the other hand, Gibrat postulates a law of proportional growth, or Gibrat's law, where the population growth rate of cities is independent from their initial population. For a extensive review of literature, see Arshad, Hu, and Ashraf (2018) and Nitsch (2005). The present paper follows González-Val, Ximénez-de-Embún, and Sanz-Gracia (2024) study but with the following differences. First, the present paper deals with Spain and its NUTS-3 administration level as geographical reference, the provinces, with few literature on the topic as indicated.

Second, in addition to applying the methodology from González-Val, Ximénez-de-Embún, and Sanz-Gracia (2024), this paper goes beyond them and other papers and this is the first time, to the best of our knowledge, that a paper analyzes, by modern advanced Difference-in-Differences techniques for multiple time periods, the impact of the fulfillment of Zipf's and Gibrat's laws on the population size of the provinces. Finally, it is remarkable highlighting the wider sample used. Whereas other authors as the previous ones stop the analysis in 2010, this article goes beyond until data of 2020.

At the sub-national level there is scarce literature. Some examples include studies for the German case (Giesen and Súdekum 2010), an Indian province (Ganapathy and Arumugam 2014; Subbarayan 2009), Colombia (Pérez Valbuena and Meisel Roca 2014), eight Chinese subregions (Ye 2011), 26 provinces from China (Ziqin 2016), analyzed sub-national Chinese administrative areas (Ye 2011), Turkish provinces (Kundak and Dókmeci 2018), Pakistan (Arshad, Hu, and Ashraf 2019) and the United States (González-Val, Ximénez-de-Embún, and Sanz-Gracia 2024). For the Spanish case there is a lack of papers on the sub-national level, an example includes with an analysis with regions (González-Val 2023). This author does not apply Zipf for provinces but urban growth is analyzed.

There is empirical evidence of a constant factor of the inverse proportional relationship between the rank of a city (municipality in Spain) and its population (Auerbach and Ciccone 2023). There is also an enriched formulation Zipf based on the statistical regularity of slope equal to -1 when regressing the logarithm of the rank of the city with respect to the logarithm of its population (Wan et al. 2020; Wang 2021; Wei et al. 2021). The formulation known as Zipf's law (Soo 2005) is one of the most conspicuous empirical facts in economics (Ciaschini et al. 2025; Gabaix 1999b). This law is intended to be an effective tool for the analysis among different metropolitan hierarchies (Berezkin et al. 2020; Ciaschini et al. 2025; Nitsch 2005; Peña and Sanz-Gracia 2021; Pérez-Campuzano, Guzmán-Vargas, and Angulo-Brown 2015). Zipf's law mostly holds for cities above a certain population size (Bergs 2018; Gabaix 1999a; Gabaix 1999b). Other laws include rural-urban elements as the Taylor's power law (Leroy R. Taylor 1961), applied to animal and human populations (L. R. Taylor, Woiwod, and Perry 1978).

Eeckhout (2004) proposed that the Pareto exponent in a standard Zipf equation where the Pareto coefficient decreases as more cities are added to the sample. This paper will additionally try to solve this issue for the last available census of Spain, by means of a rolling sample regressions approach. Similar exercise was conducted by (González-Val and Sanz-Gracia 2024) but United States (US; 2000 and 2010 censuses), Spain (2001 and 2011 censuses) and Italy (2001 and 2011 censuses). Nonetheless, the present paper enlarges and updates de sample, comparing for 1900 and 2020. They find that Eeckhout's proposition is only valid once the distribution of data starts to be lognormal; for the Pareto-distributed upper-tail, the estimated exponent does not change with sample size. As Peña and Sanz-

Gracia (2021), the former authors estimate by employing rolling sample regressions (Peng 2010). As the present results will show, the upper-tail will show a raise when the sample enlarges. This paper largely focuses on studying the validity of Zipf's and Gibrat's laws considering each one of provinces (NUTS 3 level) from 1900 to 2010, since testing both laws requires long time (Gabaix and Ioannides 2004). Nonetheless, the present paper updates the sample by other authors (González-Val and Sanz-Gracia 2024; González-Val, Ximénez-de-Embún, and Sanz-Gracia 2024) by focusing on Spain and also considering the 2020 year. Finally, the main novelty of this paper is obtaining the impact of the municipal Zipf's and Gibrat's non-compliance on total provincial population by an advanced and recently proposed Difference-in-Differences econometric technique for multiple time periods developed by Callaway and Sant'Anna (2021). The event is considered when there is a first rejection of these laws. As the result sections will show, it is found an increase/reduction after the (non)compliance of Zipf's/Gibrat's laws. Robustness checks are provided.

References

Ahundjanov, Behzod B. and Sherzod B. Akhundjanov (2019). Gibrat's law for CO2 emissions. Physica A: Statistical Mechanics and its Applications 526 (2019), p. 120944.

Arshad, Sidra, Shougeng Hu, and Badar Nadeem Ashraf (2018). Zipf's law and city size distribution: A survey of the literature and future research agenda. Physica A: Statistical Mechanics and its Applications 492 (2018), pp. 7592.

(2019). Zipf's law, the coherence of the urban system and city size distribution: Evidence from Pakistan. Physica A: Statistical Mechanics and its Applications 513 (2019), pp. 87-103.

Auerbach, Felix and Antonio Ciccone (2023). The Law of Population Concentration. Environment and Planning B: Urban Analytics and City Science 50.2 (2023), pp. 290298.

Berezkin, D., A. Goryachev, D. Gritsenko, A. Lychkina, and A. Zolotarev (2020). Zipf's

Law for Russian Cities: Analysis of New Indicators. Economy of Regions 16.3 (2020),

pp. 935947.

Bergs, Rolf (2018). Segmentation of Urban and Non-Urban Space in the Netherlands

Testing Zipf's Law with Nocturnal Satellite Imagery. Review of Regional Research 38.2

(2018), pp. 111140.

Callaway, Brantly and Pedro H.C. Sant'Anna (2021). Difference-in-differences with multiple time periods. Journal of Econometrics 225.2 (2021), pp. 200-230.

Ciaschini, Clio, Rosanna Salvia, Margherita Carlucci, and Luca Salvati (2025). Testing the Zipf's law under heterogeneous urban-rural hierarchies with spatial quantile regressions. Applied Economics 57.21 (2025), pp. 26652678. doi: 10.1080/00036846.

2024.2331027.

Clark, J. Stephen and Jack C. Stabler (1991). Gibrat's Law and the Growth of Canadian Cities. Urban Studies 28.4 (1991), pp. 635639.

Clauset, Aaron, Cosma Rohilla Shalizi, and M. E. J. Newman (2009). Power-Law Distributions in Empirical Data. SIAM Review 51.4 (2009), pp. 661703.

Eeckhout, Jan (2004). Gibrat's Law for (All) Cities. American Economic Review 94.5

(Dec. 2004), pp. 14291451.

Fang, Chuanglin, Bo Pang, and Haimeng Liu (2017). Global city size hierarchy: Spatial patterns, regional features, and implications for China. Habitat International 66 (2017), pp. 149-162.

Gabaix, Xavier (1999a). Zipf's Law and the Growth of Cities. The American Economic Review 89.2 (1999), pp. 129-132.

(1999b). Zipf's Law for Cities: An Explanation. The Quarterly Journal of Economics 114.3 (Aug. 1999), pp. 739-767.

Gabaix, Xavier and Rustam Ibragimov (2011). Rank 1/2: A Simple Way to Improve the OLS Estimation of Tail Exponents. Journal of Business Economic Statistics 29.1 (2011), pp. 24-39.

Gabaix, Xavier and Yannis M. Ioannides (2004). Chapter 53 - The Evolution of City Size Distributions. In: Cities and Geography. Ed. by J. Vernon Henderson and Jacques-Francois Thisse. Vol. 4. Handbook of Regional and Urban Economics. Elsevier, 2004, pp. 23412378.

Ganapathy, Kumar and Subbarayan Arumugam (2014). The temporal dynamics of regional city size distribution: Andhra pradesh (1951-2001). Journal of Mathematics and Statistics 10 (Apr. 2014), pp. 221230.

Giesen, Kristian and Jens Súdekum (2010). Zipf's law for cities in the regions and the Country. Journal of Economic Geography 11.4 (May 2010), pp. 667686.

González-Val, Rafael (2023). Parametric, semiparametric and nonparametric models of urban growth. Cities 132 (2023), p. 104079. issn: 0264-2751. doi: https://doi.org/10.1016/j.cities.2022.104079. url: https://www.sciencedirect.com/science/article/pii/S0264275122005182.

González-Val, Rafael, Luis Lanaspa, and Fernando Sanz-Gracia (2014). New Evidence on Gibrat's Law for Cities. Urban Studies 51.1 (2014), pp. 93115. doi: 10.1177/0042098013484528. eprint: https : / / doi . org / 10 . 1177 / 0042098013484528. url: https://doi.org/10.1177/0042098013484528.

González-Val, Rafael and Fernando Sanz-Gracia (2024). A test of the relationship between the Pareto exponent and sample size. Investigaciones Regionales - Journal of Regional Research (Nov. 2024).

González-Val, Rafael, Domingo P. Ximénez-de-Embún, and Fernando Sanz-Gracia (2024). A long-term, regional-level analysis of Zipf's and Gibrat's laws in the United States. Cities 149 (2024), p. 104946.

Kundak, Seda and Vedia Dókmeci (2018). A Rank-Size Rule Analysis of The City System at The Country and Province Level in Turkey. ICONARP International Journal of Architecture and Planning 6.1 (June 2018), pp. 7798.

Nitsch, Volker (2005). Zipf zipped. Journal of Urban Economics 57.1 (2005), pp. 86

100.

Peña, Guillermo (2025). Wildfire impacts on Spanish municipal population. Journal of

Environmental Management 377 (2025), p. 124504. issn: 0301-4797.

Peña, Guillermo, Miguel Puente-Ajovín, and Fernando Sanz-Gracia (2025). Gibrat's law for CO2 emissions revisited. Applied Economics Letters 32.6 (2025), pp. 825829.

Peña, Guillermo and Fernando Sanz-Gracia (2021). Zipf's exponent and Zipf's law in the BRICS: a rolling sample regressions approach. Economics Bulletin 41.4 (None 2021), pp. 25432549.

Peng, Guohua (2010). Zipf's law for Chinese cities: Rolling sample regressions. Physica A: Statistical Mechanics and its Applications 389.18 (2010), pp. 38043813. issn: 0378-4371. doi: https://doi.org/10.1016/j.physa.2010.05.004.

Pérez Valbuena, Gerson Javier and Adolfo Meisel Roca (2014). Ley de Zipf y de Gibrat

para Colombia y sus regiones: 1835-2005. Revista de Historia Economica - Journal

of Iberian and Latin American Economic History 32.2 (2014), pp. 247286.

Pérez-Campuzano, Enrique, Lev Guzmán-Vargas, and F. Angulo-Brown (2015). Distributions of city sizes in Mexico during the 20th century. Chaos, Solitons Fractals 73 (2015), pp. 6470. issn: 0960-0779.

Sanso-Navarro, Marcos and María Vera-Cabello (2023). The impact of terrorism at the municipality level in Colombia: A new dierence-in-dierences approach. Tech. rep. Social Sciences Research Network, 2023. doi: 10.2139/ssrn.4130613.

Soo, Kwok Tong (2005). Zipf's Law for cities: a cross-country investigation. Regional Science and Urban Economics 35.3 (2005), pp. 239263. issn: 0166-0462.

Subbarayan, A. (2009). The size distribution of cities in Tamilnadu (1901-2001). In-

ternational Journal of Agricultural and Statistical Sciences 5.2 (2009). Cited by: 7,

pp. 373382. url: https://www.scopus.com/inward/record.uri?eid=2- s2.0-

78049344274&partnerID=40&md5=366848f35fefdfb23c829f4b62d6b1b5.

Taylor, L. R., I. P. Woiwod, and J. N. Perry (1978). The Density-Dependence of Spatial

Behaviour and the Rarity of Randomness. Journal of Animal Ecology 47.2 (1978),

pp. 383406. issn: 00218790, 13652656. url: http://www.jstor.org/stable/3790

(visited on Sept. 19, 2025).

Taylor, Leroy R. (1961). Aggregation, Variance and the Mean. Nature 189 (1961), pp. 732-735. url: https://api.semanticscholar.org/CorpusID:4263093.

Wan, Guanghua, Dongqing Zhu, ChenWang, and Xun Zhang (2020). The size distribution of cities in China: Evolution of urban system and deviations from Zipf's law. Ecological Indicators 111 (2020), p. 106003. issn: 1470-160X. doi: https://doi.org/10.1016/j.ecolind.2019.106003. url: https://www.sciencedirect.com/science/article/pii/S1470160X19309987.

Wang, Qiuping A. (2021). Principle of least eort vs. maximum eciency: deriving Zipf-

Pareto's laws. Chaos, Solitons Fractals 153 (2021), p. 111489. issn: 0960-0779. doi:

https://doi.org/10.1016/j.chaos.2021.111489. url: https://www.sciencedirect.

com/science/article/pii/S0960077921008432.

Wei, Jing, Jianjun Zhang, Bofeng Cai, Ke Wang, Sen Liang, and Yuhuan Geng (2021).

Characteristics of carbon dioxide emissions in response to local development: Empirical explanation of Zipf's law in Chinese cities. Science of The Total Environment 757 (2021), p. 143912. issn: 0048-9697. doi: https://doi.org/10.1016/j.scitotenv. 2020 . 143912. url: https : / / www . sciencedirect . com / science / article / pii / S004896972037443X.

Ye, Xinyue (2011). Re-examination of Zipf's law and urban dynamic in China: A regional approach. Annals of Regional Science 49 (Aug. 2011), pp. 122. doi: 10.1007/s00168-011-0442-8.

Ziqin, Wen (2016). Zipf law analysis of urban scale in China. Asian Journal of Social

Science Studies 1 (Mar. 2016), p. 53. doi: 10.20849/ajsss.v1i1.21.