Monopolistic competition, free entry and occupations in the city

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Abstract

Constant Elasticity of Substitution (CES) demand systems are building blocks in many economic fields. On top of its simplicity/tractability, these systems constitute the only case where the decentralized economy sustains the social allocation. In this theoretical article, we demonstrate that this feature is not robust to the inclusion of entrepreneurship and space. Toward that goal, we develop a city model with monopolistic competition, CES preferences, and where entry is determined through occupational choice. We provide a full analysis of the market outcome. Notably, we highlight the presence of general equilibrium linkages left out by traditional models. These new linkages generate a striking result that contradicts conventional wisdom. Even if preferences are CES, the market outcome does not deliver optimum product diversity, which makes the city size inefficient. In particular, the market outcome generates an under-provision of diversity, and urban growth is too large compared to what is socially desirable.

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1 Introduction

There are two notions of free entry in the economic literature. The standard notion is related to free entry in the market. Firms face a trade-off between entering and not entering an industry (see Parenti et al. (2017)). In a spatial context, "industrial selection" has been studied by Abdel-Rahman and Fujita (1990), Behrens and Murata (2009), Kanemoto (2013), Behrens et al. (2015), and Malykhin and Ushchev (2018). The other notion of free entry concerns occupations. Households can become either workers or entrepreneurs (see Lucas (1978)). In a spatial context, "occupational selection" has been investigated by Behrens et al. (2014, 2018) and Davis and Dingel (2019) (see Behrens and Robert-Nicoud (2015) for a review of the literature).

Traditional models largely rely on the first notion of free entry. Surprisingly, there is no general theory of a monopolistic competition model with an endogenous land market and with free entry into an occupation. A related question is whether the results obtained under free entry in the market remain valid under free entry into occupations. Notably, we gauge the possibility that the CES economy, the building block in many theories, can generate inefficient allocations.

In the present article, we reassess the tenet of free entry in monopolistic competition models. To that end, we build a parsimonious urban model with monopolistic competition, where (*ex ante*) homogeneous households are equipped with CES preferences. The developed setting encapsulates two new ingredients: that ownership is private and that entry is determined through occupational choice. Private ownership means that each firm is owned and managed by a single household called the entrepreneur. Occupational choice implies that each household is free to decide whether to become a worker or an entrepreneur.

Armed with this new framework, we provide a full characterization of the market outcome. We notably determine the condition(s) for optimality. We study both the optimum city size and the optimum number of varieties. We derive a version of the Henry George Theorem (HGT), which is slightly different from that determined by Behrens and Murata (2009). We also derive a new necessary and sufficient condition for that the market outcome to be socially optimal.

From this thorough analysis, we obtain two valuable results. First, we show that the new model encompasses two general equilibrium (GE) interactions left out by traditional models. On the one hand, the individual labor supply now enters directly the free-entry condition. On the other hand, the aggregate labor supply is determined by the free-entry condition.

Second, we point out a set of new results concerning optimum diversity. We demonstrate that the presence of these two GE linkages can offset some results established under free entry into the industry. We find that the CES economy does not provide optimum product diversity, implying that the city size becomes non-optimal. This striking result has an intuitive explanation. When some households decide to become entrepreneurs, the mass of firms in the economy is improved. But this also leads to a decrease in the number of employees in the labor market. This, in turn, affects the aggregate labor offer and hence the equilibrium quantities. The social planner takes into account the externalities of the occupational choice on the labor market. On the contrary, households disregard this linkage, causing inefficiency. As the market outcome is sub-optimal, the logical question is whether there is over- or under-provisioning of diversity. We show that the market outcome always triggers too few varieties. In turn, the inefficiency in the good market translates to the land market. Among others, it is shown that urban growth is too large compared to what is socially desirable. City size is therefore too large, a conclusion that concurs with Brueckner (2000). Last, as efficiency is not guaranteed, we highlight that optimality can always be restored through standard remedies. To eliminate inefficiency, local subsidies are sufficient (see Fujita and Thisse (2002) and subsidies as classical instruments to restore optimality in the city). These subsidies can be implemented and managed by a central government.

This article contributes to the literature of monopolistic competition. For a decade, the literature has expanded steadily. Zhelobodko et al. (2012) have studied the role of additive preferences. Parenti et al. (2017), and Thisse and Ushchev (2018) have attempted to provide a general and unified theory for monopolistic competition models. Traditional models have been extended to encompass income dispersion (Osharin et al. (2014)), input–output linkages (Kichko (2017)), networks (Ushchev and Zenou (2018)), growth theory (Boucekkine et al. (2017) and Etro (2019)), and intermediate goods (Bucci and Ushchev (2020)). The present article adds to this list by investigating how occupational choice modifies the market outcome in a city model with additive preferences, homogeneous workers, and monopolistic competition.

This article also contributes to the analysis of the optimal resource allocations in monopolistic competition models (Spence (1976), Dixit and Stiglitz (1977), Kuhn and Vives (1999), Nocco et al. (2014), Parenti et. (2017), and Dhingra and Morrow (2019)). When workers are homogeneous, the broadly shared consensus is that the CES utility function constitutes a rare case that leads to optimality. Such a result remains valid under firms' cost heterogeneity (see Dhingra and Morrow (2019)), as well as under spatial heterogeneity (see Fujita and Thisse (2002)). In a spatial setting, this cornerstone result also implies that city size is optimal. Abdel-Rahman and Fujita (1990), and Behrens and Murata (2009) state that the HGT holds at the first and second best as soon as preferences are CES. Contrary to common wisdom, the present model highlights that this conclusion can be overturned when individuals make occupational choices. Under CES, the decentralized economy no longer sustains the social allocation. Notably, it is demonstrated that the market outcome engenders too few varieties.

This article is finally related to the body of work on optimum city size. In the new setting, the centralized economy sets a smaller city than the one determined by the decentralized economy: city expansion is too large. In so doing, the article joins the debate on city expansion. Conventional wisdom states that cities are too large or oversized (see Brueckner (2000)). The present article points out the interplay of space and monopolistic competition in generating this feature.

The article is structured in the following manner: Section 2 presents the model, and Section 3 provides the conclusions.

2 Model

In this section, we develop an urban land-use model with two novelties: i)- firms' ownership is private and ii)- entry is determined through occupational choice. In Section 2.1, we describe the framework. In Section 2.2, we derive the market outcome. In Section 2.3, we operate a thorough comparison between the new market outcome and the standard market outcome with classical free entry. In Section 2.4, we tackle the question of optimality.

2.1 Environment

In this section, we build the setup.

2.1.1 Geography and population

Let $\mathcal{S} \subset \mathbb{R}_+$ be a linear city composed of a continuum of locations denoted by $s \in \mathcal{S}$. The city is assumed to be monocentric: $s = 0$ is the Central Business District (CBD), where all firms are exogenously located. Consequently, s represents both the distance to the city center and access to jobs. The market under study gathers a continuum of households with mass $h > 0$. Households differ notably according to their occupations. There are two possible occupations open to households: they can become either workers or entrepreneurs. Last, land intensity is particular in the city. For workers, land intensity is 1 in all locations. For entrepreneurs who are exogenously located in the CBD, land intensity is unbounded. As a consequence, the CBD is somehow a "skyscraper" composed of two stages. At the first stage, workers are uniformly distributed on $[0, 1]$. At the second stage, entrepreneurs are uniformly distributed on $[1, 1 + n]$ where n is the mass of entrepreneurs in the city. See Figure 1 for an illustration where ℓ is the mass of workers in the economy. The fact that workers are uniformly dispersed across space is explained by exogenous land consumption (see see Fujita and Thisse (2002)).

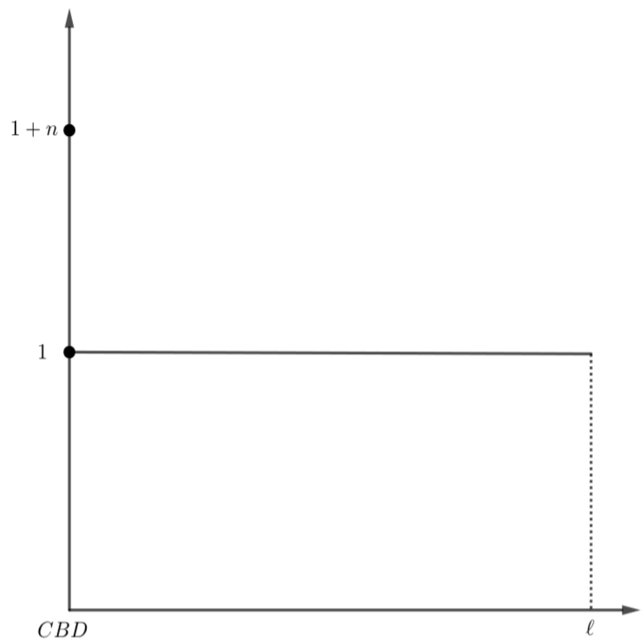

Figure 1: Geography

2.1.2 Households as consumers

As consumers, households (workers and entrepreneurs) derive utility $\mathcal{U}$ from consuming a continuum of varieties of a differentiated good produced in a single industry. In particular, we assume that preferences are CES (see Dixit and Stiglitz (1977), Krugman (1980), and Melitz (2003)):[1]

$$\mathcal{U}(\mathbf{x}) = \int_0^n x_k^\rho dk, \quad 0 < \rho < 1 \tag{1}$$

such that:

$$r_u(x) = -\frac{xu''(x)}{u'(x)} = 1 - \rho < 1, \quad r_u'(x) = 0, \quad r_{u'}(x) = 2 - \rho < 2$$

with x_k being the consumption of variety k, $\mathbf{x}$ being the consumption profile, n being the mass of varieties, ρ being a parameter and r_u being the inverse of the elasticity of substitution. As indicated by Zhelobodko et al. (2012), r_u captures the relative love for variety and corresponds to the analog of the Arrow–Pratt measure of relative risk aversion. The assumption $r_u < 1$ implies that the equilibrium price is strictly positive. The assumption $r_u' = 0$ means complete firms' pass-through.[2] The assumption $r_{u'} < 2$ is valuable as it implies the strict concavity of the profit function. In turn, the strict concavity

[1] A more standard version of equation (1) is the following: $\mathcal{U}(\mathbf{x}) = \left(\int_0^n x_k^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}}$ with $\sigma > 1$.

[2] By firms' pass-through, we mean how firms pass on cost increases.

of the profit function implies the uniqueness of a solution for the firm's problem. Last, note that we manipulate CES preferences because these preferences have become a workhorse model in economics (see Head and Mayer (2014)). Notably, CES demand is widely used in trade models (see Krugman (1980) and Melitz (2003)), in macroeconomic models (see new Keynesian models), and in spatial models (see Fujita and Thisse (2002)).

2.1.3 Workers

There are $\ell > 0$ identical workers in the economy. Workers commute to the CBD to work, incurring "iceberg" transportation costs (as in Behrens and Murata (2009)). They supply $\psi(s)$ units of labor in a competitive market for wage rate normalized to one $w = 1$. The effective labor supply of a household residing in s is given by:

$$\psi(s) = g(1 - 2\theta s), \quad g > 0, \theta < \frac{1}{2h}$$

Here θ is the efficiency loss in terms of unit of labor due to commuting. When $\theta = 0$, the effect of transportation costs are canceled out, and workers supply a constant number of g units of labor. It is as if land were eliminated in the setting. To ensure that $\psi(s) > 0$ irrespective of the (internal) spatial distribution of workers, we assume that $\psi(h) = g(1 - 2\theta h) > 0$, that is, $\theta < \frac{1}{2h}$. In addition, workers consume a single unit of land and pay a rent R. Last, workers and entrepreneurs own the houses of the economy. They hold identical shares in all houses and receive a fraction $\frac{1}{h}$ of the land rent in each location. Therefore, the revenue of the workers living in s is:

$$\mathcal{I}_W(s) = \psi(s) - R(s) + \frac{\mathcal{R}}{h}$$

with $\mathcal{R}$ being the aggregate land rent of the city, and the budget constraint of the workers residing in s is given by:

$$\int_0^n p_k y_k dk = \mathcal{I}_W(s) \tag{2}$$

$\mathbf{y}$ is the consumption profile of workers, and p_k is the price of variety k.

2.1.4 Labor market clearing condition

As the labor market is perfectly competitive (i.e. workers are perfectly mobile across firms), there is no unemployment in equilibrium, and the following constraint is satisfied:

$$\mathcal{G}(\ell) = \int_0^n l_k dk \tag{3}$$

Here l_k is the mass of workers employed in firm k, and $\mathcal{G}(\ell)$ is the aggregate labor supply. Owing to inelastic demand of land, workers are uniformly distributed across space in equilibrium, and city size, denoted by $\check{s}$, is simply ℓ. This also implies that the aggregate labor supply $\mathcal{G}(\ell)$ is given by:

$$\mathcal{G}(\ell) = \int_0^{\check{s}} \psi(s)ds = \int_0^{\ell} \psi(s)ds = g\ell\,(1 - \theta\ell) \tag{4}$$

In that case, equation (3) boils down to the following:

$$g\ell\,(1 - \theta\ell) = \int_0^n l_k dk \tag{5}$$

Equation (5) is the labor market clearing condition (LMCC), which stipulates that the supply of labor must be equal to the demand for labor.

2.1.5 Technology

The technology uses labor as the sole input, exhibits increasing returns to scale, and permits the production of a single variety of the differentiated good. For instance, producing q units of good needs $\mathcal{C}(q)$ units of labor such that:

$$\mathcal{C}(q) = cq + f$$

where c is the marginal cost, and f is the fixed cost. A direct consequence of (5) is that $l_k = \mathcal{C}(q_k)$, where q_k is the output of firm k.

2.1.6 Entrepreneurs

There are $n = h - \ell > 0$ identical entrepreneurs in the city. As owner-managers, they run a single firm (i.e. private ownership), hire workers, and earn profits as sole income:

$$\pi_k = p_k q_k - cq_k - f \tag{6}$$

with π_k being the profit of entrepreneur/firm k. Moreover, they live where firms are located, that is, in the CBD. In that case, the revenue of entrepreneur k is pinned down by the following:

$$\mathcal{I}_{E,k} = \pi_k - R(0) + \frac{\mathcal{R}}{h}$$

and the budget constraint of entrepreneur k is defined as:

$$\int_0^n p_k z_k dk = \mathcal{I}_{E,k} \tag{7}$$

where $\mathbf{z}$ is the consumption profile of the entrepreneurs.

2.1.7 Going back to the LMCC

As $\ell = h - n$, the mass of firms/entrepreneurs has now an effect on $\mathcal{G}(\ell)$ the aggregate labor supply.[3] In particular, the mass of entrants generates two conflicting forces. On the one hand, there is a pure "population effect." An increase in n naturally leads to a decrease in ℓ the labor force, which lowers the aggregate labor force. On the other hand, there is a "spatial effect." A decrease in ℓ also implies that $\check{s}$ the city size decreases (i.e. the city is less sprawl). Therefore, having to face lower transportation costs, workers are more productive. This improves the aggregate labor supply. To gauge the net effect, we compute the following (using equation (4)):

$$\mathcal{G}(\ell) = \mathcal{G}(h - n) = gh(1 - \theta h) - \left[g(1 - 2\theta h)n + g\theta n^2\right]$$

and we also note the following:

$$\frac{\partial \mathcal{G}(h-n)}{\partial n} = -\left[g(1 - 2\theta h) + 2g\theta n\right] < 0$$

as $g(1 - 2\theta h) > 0$. Clearly, the net effect is negative. An increase in the mass of varieties diminishes the aggregate labor supply. This means that the population effect always outweighs the spatial effect in the model. Such a new relationship is fundamental for optimality.

2.2 Toward a market outcome

In this section, we derive useful (and intermediate) results to define a market outcome.

2.2.1 Game

Within the present framework, households play the following three-step game.

1. They choose an occupation: "worker" or "entrepreneur".

2. They choose a level of consumption, and entrepreneurs/firms set prices and quantities

3. Workers choose a location in the city

The game is solved by backward induction in what follows.

[3] Note that this effect is not present in the standard model.

2.2.2 Urban equilibrium

The location choice made by households depends on their social status. When households are workers, they freely choose a location in the city and so bid for land. When households are entrepreneurs, they live where firms are located (i.e. in the CBD). As a consequence, they pay $R(0)$ the land rent in $s = 0$ but do not bid for land. $R(0)$ is treated as exogenous by entrepreneurs. Such an assumption is motivated by three arguments. First, the assumption is not an outlier in the literature. The literature traditionally assumes that the economic agents that are exogenously located in the city do not participate to the land market even if they have to pay land rents (see Zenou (2009), Zenou (2011) and Marchiori et al. (2022)). Second, the location choice of entrepreneurs is not detrimental for our main result: the inefficiency of the CES case. This is because the occupational choice implies a non-arbitrage condition in equilibrium that imposes identical revenues for workers and entrepreneurs. Third, even if $R(0)$ is considered as exogenous by entrepreneurs, entrepreneurship endogenously depends on land. When households face the trade-off between becoming workers or becoming entrepreneurs, they are aware of the following mechanism. If there is an increase in the mass of entrepreneurs/firms n, the mass of workers $\ell = h - n$ decreases and so the city size $\check{s} = \ell$ decreases too. As the city is less sprawl, workers on average make shorter commutes. This decreases land rents in the CBD, and the costs of living of entrepreneurs are diminished. In turn, this favors entrepreneurship.

To pin down the location of workers in the city, it is common to use the bid rent theory (see Fujita and Thisse (2002)). The theory is meant to determine the maximum rent that an economic agent can pay for living in a given location, and it assumes that land is allocated to the highest bid rent so that:

$$R(s) = max\left\{\Psi_W(s), 0\right\}$$

with Ψ_W being the bid rent function of workers. To have a consistent spatial equilibrium, it is also necessary to suppose that all workers reach the same revenue in equilibrium. In that case, the wage net of commuting costs/land rents are equalized in all locations:

$$\mathcal{I}_W = \mathcal{I}_W(s), \quad \forall s \in [0, \ell]$$

Here $\mathcal{I}_W$ is now the equilibrium revenue of workers.

Solving the workers' location problem gives:

$$R(s) = g(s) - g(\ell) = 2g\theta(\ell - s)$$

Plugging R into the expression of $\mathcal{R} = \int_0^\ell R(s)ds + n \times R(0)$ yields:

$$\mathcal{R} = g\theta(h^2 - n^2)$$

In turn, the revenue of workers collapses to:

$$\mathcal{I}_W = g(1-\theta h) + \frac{g\theta(2h-n)n}{h} \tag{8}$$

As usual in urban models, R is a decreasing function with respect to s. This is because workers face a trade-off between accessibility and land prices, when choosing their location. They want to live in the CBD to minimize their commuting costs. However, they also anticipate that more workers want to reside near the city center. This increases land prices. To avoid this, some workers have an incentive to live farther away.

2.2.3 Consumers' equilibrium

Workers (as consumers) aim to maximize (1) subject to (2) with $\mathcal{I}_W$ (defined in equation (8)). Since $\mathcal{U}$ is additive, the problem is a well-posed one and admits a unique solution:

$$p_k = \frac{\rho y_k^{\rho-1}}{\lambda_W}, \quad \forall k \in [0,\ell] \tag{9}$$

This is the standard inverse demand function where λ_W is the Lagrange multiplier of workers:

$$\lambda_W = \frac{\int_0^n \rho y_j^\rho dj}{\mathcal{I}_W}$$

λ_W acts as a scaling factor in the model. Similarly, entrepreneurs (as consumers) aim to maximize (1) subject to (7). This leads to:

$$p_k = \frac{\rho z_k^{\rho-1}}{\lambda_E}, \quad \forall k \in [0,n] \tag{10}$$

with

$$\lambda_E = \frac{\int_0^n \rho z_j^\rho dj}{\mathcal{I}_E}$$

λ_E is the Lagrange multiplier of entrepreneurs.

2.2.4 Firms' equilibrium

The product market clears such that:

$$q_k = \int_0^\ell y_k dk + \int_0^n z_k dk$$

Using equations (9)-(10) and assuming that firms do not discriminate consumers, each entrepreneur/firm $k \in [0,n]$ maximizes its profit function:

$$\max_{x_k} \left\{ \left(\frac{\rho x_k^{\rho-1}}{\lambda_W} - c \right) \int_0^\ell x_k dk + \left(\frac{\rho x_k^{\rho-1}}{\lambda_E} - c \right) \int_0^n x_k dk - f \right\}$$

where λ_W and λ_E are considered as parameters. Such a problem is well-posed and admits a unique solution:

$$p_k = \frac{c}{\rho} > c, \quad \forall k \in [0, n] \tag{11}$$

As usual, prices (and so markups) are constant under CES preferences.

2.2.5 Free entry into occupation

Households costlessly choose to become either workers or entrepreneurs. In equilibrium, the mass of entrepreneurs (firms) must equate $\mathcal{V}(\mathbf{p}; \mathcal{I}_W)$ the (indirect) utility of being a worker with $\mathcal{V}(\mathbf{p}; \mathcal{I}_E)$ the (indirect) utility of being an entrepreneur:

$$\mathcal{V}(\mathbf{p}; \mathcal{I}_W) = \mathcal{V}(\mathbf{p}; \mathcal{I}_E)$$

In that case, no entrepreneur is better off being a worker, and conversely, no worker is better off being an entrepreneur. It is verified if entrepreneurs and workers receive the same income $\mathcal{I}_E = \mathcal{I}_W$. The free-entry condition therefore boils down to:

$$\pi(n) - g = 0 \tag{12}$$

Equation (12) constitutes the new zero-profit condition (ZPC), stating that the revenue from entrepreneurship is equalized to the individual labor supply, which constitutes the outside option in the labor market. Notably, n now plays the role of a selection cut-off as it determines how households are split between entrepreneurs and workers.

2.2.6 New general equilibrium linkages

We last underline that the new framework we develop encapsulates GE channels that are left out by traditional models.

Comparing our setup with the standard definition of monopolistic competition models with urban land-use (see Appendix A), new linkages emerge as outcomes.[4] These effects transit through the interplay of the free-entry condition and the labor market such that:

$$\begin{cases} \pi(n) = 0 & \text{under standard free entry} \\ \pi(n) = g & \text{under free entry into occupation} \end{cases} \tag{13}$$

and

$$\begin{cases} gh(1 - \theta h) = \int_0^n (cq_k + f)dk & \text{under standard free entry} \\ g\ell(1 - \theta \ell) = \int_0^n (cq_k + f)dk & \text{under free entry into occupation} \end{cases} \tag{14}$$

In the model with free entry into occupation, the ZPC and the LMCC are intertwined according to two channels. On the one hand, the individual labor supply enters directly

[4]We show in Appendix A the analog definition of a market outcome in the standard model.

the free-entry condition, from 0 to $g > 0$ (see system (13)). On the other hand, the aggregate labor supply is determined by the free-entry condition (see system (14)). As entry is now pinned down by an occupational choice, the number of employees declines in the new model, from h to ℓ. This implies that the city is less sprawl, and the labor market has a lower aggregate labor offer, $g\ell(1-\theta\ell)$ instead of $gh(1-\theta h)$.[5]

2.3 The market outcome

In this section, we determine and then make a transparent comparison of the market outcome of the two models (the market outcome under free entry into occupation and the market outcome under standard free entry as in Appendix A).

2.3.1 Equilibrium consumption/quantity

Because workers and entrepreneurs share the same incomes and preferences, and also considering that $h = \ell + n$, the product market clearing condition is reduced to:

$$q(n) = x(n)h$$

Similarly, assuming symmetry and plugging the pricing rule (11) into the profit function gives:

$$\pi(n) = \frac{(1-\rho)chx(n)}{\rho} - f$$

This implies that the new ZPC becomes the following:

$$\frac{(1-\rho)chx(n)}{\rho} = f + g \Leftrightarrow x^* = \frac{\rho(f+g)}{(1-\rho)ch} > x^s = \frac{\rho f}{(1-\rho)ch} \tag{15}$$

x^* is the equilibrium level of consumption in the new model and x^s is the equilibrium level of consumption in the standard model. Under CES preferences, standard models tend to underestimate individual consumption compared to what prevails in the new model ($x^* > x^s$). In turn, the equilibrium output per firm are improved: $q^* > q^s$. The reason for this is easy to grasp. When free entry is determined through occupational choice, a new GE channel is integrated in the ZPC: $g > 0$, the outside option is higher in the new model than in the standard one. In addition, the left-hand sides of the two ZPCs are identical and increase strictly with respect to x. This implies that entrepreneurs produce more in the new model (See Figure 2 for an illustration of this mechanism).

2.3.2 Equilibrium mass of varieties

After simple algebra, we find the following.

[5]It can be readily verified that $g\ell(1-\theta\ell) < gh(1-\theta h)$.

Proposition 1 *There exists a unique equilibrium mass of entrepreneurs/firms n^* given by:*

$$n^* = \frac{-\left[f + g(1-2\theta h) + \frac{\rho(f+g)}{1-\rho}\right] + \sqrt{\left[f + g(1-2\theta h) + \frac{\rho(f+g)}{1-\rho}\right]^2 + 4g^2\theta h(1-\theta h)}}{2g\theta}$$

and such that:[6]

$$n^* < n^s = \frac{(1-\rho)gh(1-\theta h)}{f}$$

[6] n^s is the equilibrium mass of firms in the standard model.

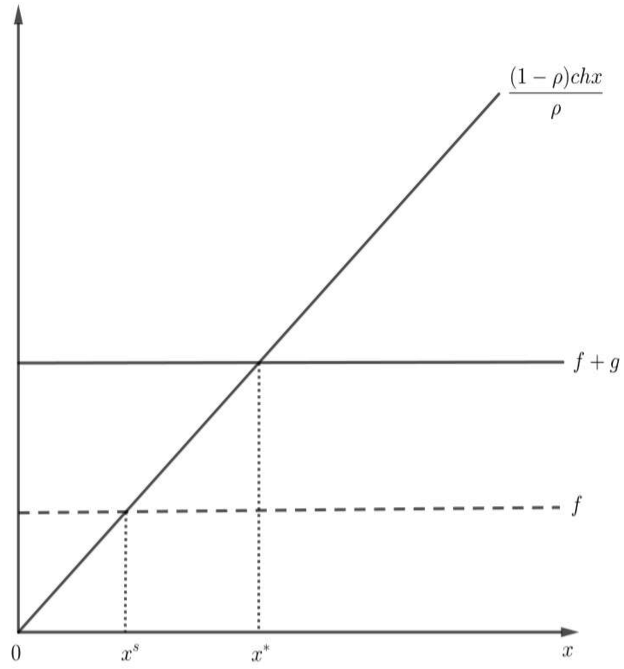

Figure 2: Equilibrium consumption

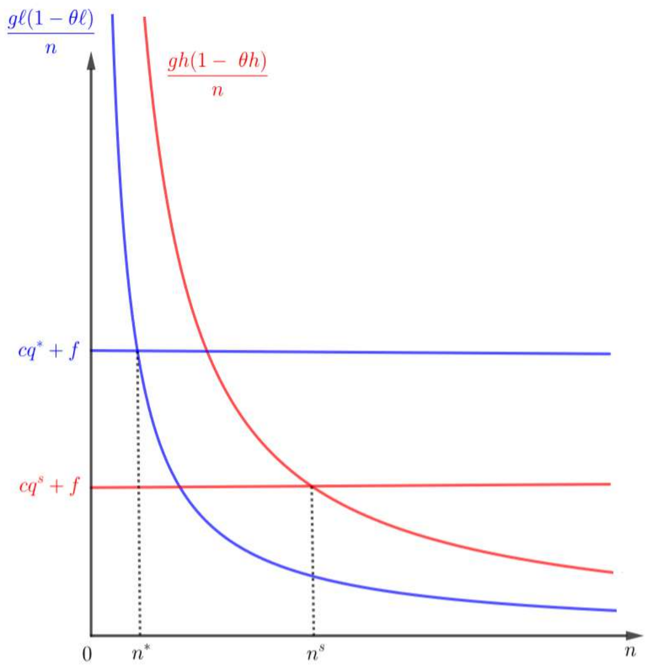

Figure 3: Equilibrium mass of firms

Intuitively, when free entry is determined through occupational choice, the ZPC changes and there is a growth of the cutoff from the zero level to the positive value (being equal to the wages). This prevents some workers from becoming entrepreneurs. This limits entry, and the long-run number of firms should fall $n^* < n^s$. In fact, there is another story behind the fact that $n^* < n^s$. Under additive preferences, x is determined by the ZPC whereas n is pinned down by the LMCC. Moreover, as $x^* > x^s$ and so $q^* > q^s$, firms need more labor in the new model than in the standard one. But when entry is determined by an occupational choice, the labor market has a lower aggregate labor offer. In other words, firms in the new model needs to produce more $q^* > q^s$ with less labor force $g\ell(1-\theta\ell) < gh(1-\theta h)$. As shown in Figure 3, this generates a smaller mass of firms: $n^* < n^s$.

2.4 Optimality

In this section, we clarify the optimization problem facing the social planner. First, we study the optimum city size and derive a new version for the HGT. We also show that the optimum city size is always higher in the new model than in the standard model. Second, we focus on the optimum mass of entrants. Interestingly, we find that CES preferences cannot sustain social allocation. Particularly, we prove that the market outcome systematically delivers an under-provision of diversity. We finally show that optimality can always be restored through local subsidies.

2.4.1 The social planner program

As workers and entrepreneurs share the same preferences, the social planner chooses h the city size (in terms of population), n the mass of varieties and $\mathbf{x}$ the consumption profile to maximize the utility of workers and entrepreneurs under technical constraints. The problem of the social planner is therefore as follows:

$$\max_{h,n,\mathbf{x}} \quad \int_0^n x_k^\rho dk$$

under the following resource constraints:

$$\begin{cases} h = \ell + n \\ q_k = x_k h \\ g\ell(1-\theta\ell) = \int_0^n (cq_k + f) dk \end{cases}$$

The first constraint is simply a population constraint. The second constraint links consumption to quantity. The third constraint is the labor balance condition.

Manipulating the constraints, and by symmetry, the social planner problem collapses to the following new intertwined program:

$$\max_{h,n} \quad n \times \left(\frac{1}{ch} \left\{ \frac{gh(1-\theta h)}{n} - [f + g(1-2\theta h) + g\theta n] \right\} \right)^{\rho} \tag{16}$$

and

$$\max_{h,n} \quad n \times \left(\frac{1}{ch} \left[\frac{gh}{n} - (f+g) \right] \right)^{\rho} \tag{17}$$

when $\theta = 0$. Surely, one can directly determine a solution for the social planner program. For the sake of simplicity, we split the problem into two sub-parts. First, we study the optimum city size (see Section 2.4.2). Second, we focus on the optimum mass of varieties (see Section 2.4.3). This method permits disentangling carefully the sources of non-optimality.

2.4.2 The optimum city size

For n given, the problem of the social planner boils down to:

$$\max_{h} \quad \left(\frac{1}{ch} \left\{ \frac{gh(1-\theta h)}{n} - [f + g(1-2\theta h) + g\theta n] \right\} \right)^{\rho}$$

The social criterion can be a hump-shaped function with respect to h. This means that the social planner faces a trade-off when choosing the number of inhabitants. In effect, there are two conflicting forces that interact to determine the optimum population city size. These two opposing forces affect the level of consumption. In turn, consumption is directly driven by aggregate labor efficiency. As population size increases, the number of workers improves, thereby increasing aggregate labor offer and consumption. This pushes toward a high value for h. However, as population size increases, the improvement in the number of workers also prompts workers to live farther away. They incur more transportation costs, and aggregate labor offer diminishes as an outcome. This lowers consumption and so pushes toward a low value of h.

Deriving the first-order condition of this problem, we find the following.

Proposition 2 *The optimum city size, denoted by $\tilde{h}$, is given by the new HGT:*

$$(f+g) \times n = \mathcal{R}$$

such that:

$$\tilde{h} = \sqrt{\frac{(f+g)n}{g\theta} + n^2} > h^s = \sqrt{\frac{fn}{g\theta}}$$

where $h^s = \sqrt{\frac{fn}{g\theta}}$ is the optimum city size in the standard model.

The optimum city size is larger in the model with free entry into occupation than in the standard model. The optimum city size is also determined by a new version of the HGT. The initial version of the HGT draws a link between aggregate land rent and public goods (see Arnott (1979), Stiglitz (1977), and Arnott and Stiglitz (1979)). For any level of the public good, when the city is efficient, aggregate land rent equals public expenditure. This implies that a single tax on land values is sufficient to finance public expenditure. The HGT is also used to design a test for optimal city size, and to estimate whether a city is over- or under-populated (see Kanemoto et al. (1996) and Arnott (2004)). It is also worth noting that the basic HGT can be extended. In a system of cities with homogeneous workers, urban land values balance agglomeration benefits. In an urban model with monopolistic competition, the HGT states that aggregate fixed costs coincide with aggregate land rents (see Behrens and Murata (2009)). In the present article, the HGT is slightly modified. The obtained HGT implies that the aggregate land rent is equalized to f the fixed cost augmented by g the outside option in the labor market.

2.4.3 The optimum product diversity

In this second part of the welfare analysis, we attempt to answer the following questions. Is the market outcome optimal? What induces efficiency? In case of sub-optimality, is there over- or under-provisioning of varieties? and how to restore efficiency?

For h given, the problem of the social planner is reduced to:

$$\max_{n} \quad n \times \left(\frac{1}{ch} \left\{ \frac{gh(1-\theta h)}{n} - [f + g(1-2\theta h) + g\theta n] \right\} \right)^{\rho}$$

The criterion can be a single-peaked function with respect to n. This means that the social planner faces a trade-off between quantity and product diversity when choosing the mass of varieties. On the one hand, there is a "love-of-variety welfare" effect. A high value for n improves the social criterion as it implies a large number of varieties. On the other hand, there is a quantity effect. A low value for n also increases the social criterion as it generates a high level of consumption. A decrease in n increases the labor force, and firms are able to exploit more their increasing return to scale and produce more.

After computing the first-order condition of the social problem, we obtain the following.

Proposition 3 *The market outcome does not sustain the social allocation. In particular, the decentralized economy triggers too few varieties: $n^* < \tilde{n}$ with $\tilde{n}$ being the social mass of varieties.*

The mass of varieties is not optimal. Based on Proposition 2, this also implies that the city size turns out to be inefficient.

The fact that the CES economy no longer provides optimum product diversity is in stark contrast with standard findings. We believe this is the first article in the literature to state that the CES economy can be sub-optimal. Under additive preferences and homogeneous workers, it is well established that the CES is the only utility for which the market outcome is socially optimal (see Thisse and Ushchev (2018) for a review of the literature). Such a result is also acknowledged to be robust to the inclusion of firms' heterogeneity. Dhingra and Morrow (2019) show that the decentralized economy with heterogeneous firms remains first best under CES demand. They point out that firms' heterogeneity does not induce additional distortions as long as preferences are CES. Such a finding is also known to hold despite differences in spatial location (see Fujita and Thisse (2002)). For example, Abdel-Rahman and Fujita (1990) and Behrens and Murata (2009) prove that CES uniquely sustains the social allocation at the first best and at the second best.

The fact that CES now undo the social equilibrium is explained by two arguments. On the one and, efficiency does not rely on the standard trade-off between quantity (tougher competition) and product diversity (love of variety). To see that, note that it is readily verified using Proof 3 that a market outcome is efficient if and only if the following condition is satisfied:

$$\frac{gh(1-\theta h)+g\theta n^2}{gh(1-\theta h)+g\theta(2h-n)n} \times \mathcal{E}_x(u) = 1 - r_u(x) \tag{18}$$

with $\mathcal{E}_x(u) = \rho$ and $r_u(x) = 1-\rho$. Then, note that, when land/distance is eliminated (i.e. when $\theta = 0$), the condition boils down to:

$$r_u(x) = 1 - \mathcal{E}_x(u) \tag{19}$$

which is always verified.[7] The "private markup" $r_u(x)$ always coincides with the "social markup" $1-\mathcal{E}_x(u)$ (see Kuhn and Vives (1999)). Households take into account consumers' marginal utility while the social planner pays attention to consumer utility. This difference in behavior encapsulates two channels that play in opposite directions (see Spence (1976)). There is a "business stealing effect" in the sense that new entrants may reduce the output of incumbent firms. This pushes toward excess variety. There is also an "incomplete appropriability effect" in the sense that new entrants cannot capture the entire benefit coming from the introduction of new variety in the economy. This does not yield much diversity. But with CES preferences, the inability of firms to appropriate the full consumer surplus exactly offsets the inability of firms to account for the business stealing effect.
On the other hand, there is an additional source of non-optimality. The CES case becomes inefficient due to the combination of the occupational choice and the location choice. This is captured by the spatial term $\frac{gh(1-\theta h)+g\theta n^2}{gh(1-\theta h)+g\theta(2h-n)n}$. The inclusion of the occupational choice made by households leads to an externality in the labor market that is not well evaluated. When a household decides to become an entrepreneur, this marginally improves the number of firms in the economy. However, this also marginally decreases the number of employees.

[7] This means that the decentralized economy sustains the social allocation when $\theta = 0$.

In turn, this affects the labor offer and quantities. The social planner cares about these externalities, whereas households disregard them.

When the market outcome is demonstrated to be sub-optimal, it is natural to gauge if there is over- or under-provisioning of diversity. The previous proposition unambiguously shows that the decentralized economy delivers too few entrepreneurs. The under-provisioning of varieties also translates to the land market. As city size is given by $\check{s} = \ell = h - n$, the centralized economy predicts a smaller city than the decentralized economy. Thus, the present article joins the debate on urban sprawl: are cities too big or too small? The common belief is that cities are oversized. Scholars and people think that city expansion is excessive (see Brueckner (2000)). This article concurs with this literature and points out the interplay of space and monopolistic competition in generating this feature.

Last, Proposition 3 gives way to the question as to whether economists have in their toolbox a policy to restore the optimality of the economy. A standard remedy in urban economics is to introduce subsidies by a central government (see Fujita and Thisse (2002)). Here, we explore the influence of such subsidies to restore the efficiency of the city. With that goal in mind, we assume the existence of a central government that sets subsidies $\varphi(s)$ that are location dependent. The subsidies are financed by a lump-sum tax that is applicable to all workers. This lump-sum tax is denoted by $\overline{\varphi}$ and is defined as follows:

$$\overline{\varphi} = \frac{1}{\ell} \int_0^{\ell} \varphi(s) ds$$

Under this new environment, we find the following:

Proposition 4 *If φ is given by:*

$$\varphi(s) = 4g\theta s$$

then the market outcome sustains the social allocation.

In line with Fujita and Thisse (2002), the use of linear local subsidies implemented by a government is sufficient to eliminate inefficiency.

3 Conclusion

It is well-acknowledged that CES demand systems constitute a workhorse model in economics. It is also well-established that a remarkable feature of these systems is that they deliver the optimality of the free-entry equilibrium. In this article, we show that this feature is not robust to the inclusion of entrepreneurship and space. In particular, we build a city model with monopolistic competition, (*ex ante*) homogeneous households, CES preferences, and where entry is determined through occupational choice. We show that the developed framework encapsulates general equilibrium interactions neglected by standard models. These linkages transit through the free-entry condition and the labor market. The omission of these general equilibrium linkages is significant as it can offset well-established results. In particular, the market outcome delivers too few varieties, generating urban sprawl. This striking result is explained by the fact that households overlook the new general equilibrium effects between the labor market clearing condition and the free-entry condition. Finally, efficiency can be restored at the city scale with the concurrence of a central government that provides local subsidies.

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A The standard model

Hereafter, we underline the main characteristics of a market outcome in the standard model, that is, with free entry in the market. In Section A.1, we display the standard definition of a market outcome. In Section A.2, we determine the market outcome. In Section A.3, we derive the condition(s) for optimality.

A.1 Definition

A (standard) market outcome satisfies the following:
i)- each entrepreneur k maximizes its profit function:

$$\max_{x_k} \left\{ \left[\frac{\rho x_k^{\rho-1}}{\lambda} - c \right] h x_k - f \right\}$$

where λ is considered as a parameter.
ii)- the labor market clearing condition holds:

$$gh(1-\theta h) = \int_0^n (cq_k + f) dk$$

vi)- the product market clearing condition is verified:

$$q_k = x_k h$$

vii)- the free-entry condition is satisfied:

$$\pi(n) = 0$$

A.2 Form

A standard market outcome denoted by (x^s, q^s, n^s) is given by the following:
- the equilibrium consumption x^s is pinned down by:

$$x^s = \frac{\rho f}{(1-\rho)ch}$$

- the equilibrium quantities q^s are:

$$q^s = hx^s$$

- the equilibrium mass of firms n^s is:

$$n^s = \frac{gh(1-\theta h)}{chx^s + f} = \frac{(1-\rho)gh(1-\theta h)}{f}$$

A.3 Optimality

For n fixed, the standard HGT is the following:

$$fn = \mathcal{R}$$

and the optimum city size is given by:

$$h^s = \sqrt{\frac{fn}{g\theta}}$$

For n fixed, the market outcome verifies the following:

$$1 - r_u(x) = \frac{chx}{chx + f}$$

The social outcome satisfies the following:

$$\mathcal{E}_x(u) = \frac{chx}{chx + f}$$

As a consequence, the decentralized and centralized economies coincide if and only if

$$1 - r_x(u) = \mathcal{E}_x(u)$$

The decentralized economy sustains the social allocation under CES as $\mathcal{E}_x(u) = \rho$ and $1 - r_x(u) = \rho$.

B Proofs

In what follows, we derive the proofs of the article

B.1 Proof of Proposition 1

Existence and uniqueness Manipulating the expression for the equilibrium level of consumption yields:

$$gh(1 - \theta h) - [f + g(1 - 2\theta h) + chx]\, n = g\theta n^2$$

Let $\Psi_1(n) = g\theta n^2$ that is defined over $n \in [0, \infty)$ such that:

$$\Psi_1(0) = 0$$

$$\Psi_1(h) = g\theta h^2 > 0$$

$$\Psi_1'(n) = 2g\theta n$$

$$\Psi_1''(n) = 2g\theta$$

Similarly, let $\Psi_2(n) = gh(1-\theta h) - [f + g(1-2\theta h) + chx]\, n$ that is defined over $n \in [0, \infty)$ such that:

$$\Psi_2(0) = gh(1-\theta h)$$

$$\Psi_2(h) = -(f - g\theta h + chx)h$$

$$\Psi_2'(n) = -[f + g(1-2\theta h) + chx] < 0$$

$$\Psi_2''(n) = 0$$

The equilibrium mass of firms denoted by n^* is the intersection of Ψ_1 and Ψ_2. As Ψ_1 is continuous and strictly increasing, Ψ_2 is continuous and strictly decreasing, $\Psi_1(0) < \Psi_2(0)$ and $\Psi_2(h) < \Psi_1(h)$, then there exists a unique n^* that solves the polynomial equation. In addition, it is easy to verify that such a solution n^* belongs to $(0, h)$.

Determination of $\mathbf{n}^*$ The discriminant of the polynomial equation is the following:

$$\Delta = [f + g(1-2\theta h) + chx^*]^2 + 4g^2\theta h(1-\theta h) > 0$$

as $1 - \theta h > 0$. After simple algebra, there exists a unique positive solution given by:

$$n^* = \frac{-[f + g(1-2\theta h) + chx^*] + \sqrt{[f + g(1-2\theta h) + chx^*]^2 + 4g^2\theta h(1-\theta h)}}{2g\theta} > 0$$

B.2 Proof of Proposition 2

In order to derive simple computations, we assume that $u(x) = x^\rho$. Then the social program is the following:

$$\max_h \quad n \times u\left(\frac{1}{ch}\left\{\frac{gh(1-\theta h)}{n} - [f + g(1-2\theta h) + g\theta n]\right\}\right)$$

The FOC of the previous problem is given by:

$$n \times \frac{\partial x}{\partial h} \times u'(x) = 0$$

with $\frac{\partial x}{\partial h} = -\left(\frac{g\theta}{cn} - \frac{f+g+g\theta n}{ch^2}\right)$, that is,

$$\frac{\partial x}{\partial h} = 0 \Leftrightarrow (f+g)n = g\theta(h^2 - n^2) = \mathcal{R}$$

which also gives:

$$\tilde{h} = \sqrt{\frac{(f+g)n}{g\theta} + n^2}$$

B.3 Proof of Proposition 3

Inefficiency In order to derive simple computations, we assume that $u(x) = x^\rho$. Then, the social program is the following:

$$\max_n \quad n \times u\left(\frac{1}{ch}\left\{\frac{gh(1-\theta h)}{n} - [f + g(1-2\theta h) + g\theta n]\right\}\right)$$

The FOC of the previous problem is given by:

$$u(x) + n \times \frac{\partial x}{\partial n} \times u'(x) = 0$$

with $\frac{\partial x}{\partial n} = -\left[\frac{g(1-\theta h)}{cn^2} + \frac{g\theta}{ch}\right]$. Noting that $\mathcal{E}_x(u) = \frac{xu'(x)}{u(x)}$, the FOC collapses to the following:

$$\mathcal{E}_x(u) = \frac{chxn}{gh(1-\theta h) + g\theta n^2}$$

Noting that $gh(1-\theta h) + g\theta(2h-n)n = (chx + f + g)n$ and after simple computations, the FOC can be reformulated as:

$$\frac{gh(1-\theta h) + g\theta n^2}{gh(1-\theta h) + g\theta(2h-n)n} \times \mathcal{E}_x(u) = \frac{chx}{chx + f + g}$$

On the other hand, in the decentralized economy, and after standard manipulations, we end-up with the following:

$$1 - r_u(x) = \frac{chx}{chx + f + g}$$

Thus, the market outcome is efficient if and only if:

$$\frac{gh(1-\theta h) + g\theta n^2}{gh(1-\theta h) + g\theta(2h-n)n} \times \mathcal{E}_x(u) = 1 - r_u(x)$$

The above equation is verified under CES preferences if and only if $n = 0$. This implies inefficiency.

Existence and uniqueness When preferences are CES, $\mathcal{E}_x(u) = \rho$, and $\tilde{n}$ the social mass of firms solves the following:

$$gh(1-\theta h)(1-\rho) - [f + g(1-2\theta h)]\, n = g\theta(1+\rho)n^2$$

Let $\tilde{\mu}_a(n) = g\theta(1+\rho)n^2$ and $\tilde{\mu}_b(n) = gh(1-\theta h)(1-\rho) - [f + g(1-2\theta h)]\, n$. As $\tilde{\mu}_a(0) = 0$, $\tilde{\mu}_a(h) = g\theta(1+\rho)h^2$ and $\frac{\partial \tilde{\mu}_a(n)}{\partial n} = 2g\theta(1+\rho)n > 0$, and $\tilde{\mu}_b(0) > \tilde{\mu}_a(0)$, $\tilde{\mu}_b(h) < \tilde{\mu}_a(h)$ and $\frac{\partial \tilde{\mu}_b(n)}{\partial n} = -[f + g(1-2\theta h)] < 0$, $\tilde{n}$ is the unique intersection point between $\tilde{\mu}_a$ and $\tilde{\mu}_b$, and belongs to $(0, h)$.

Under– or over–provision of diversity? Remind that the social mass of firms $\tilde{n}$ solves the following:

$$gh(1-\theta h)(1-\rho) - [f + g(1-2\theta h)]\, n = g\theta(1+\rho)n^2$$

and that the equilibrium mass of firms $\tilde{n}$ solves the following:

$$gh(1-\theta h) - [f + g(1-2\theta h) + 2g\theta\rho h]\, n = g\theta(1-\rho)n^2$$

Remind that $\tilde{\mu}_a(n) = g\theta(1+\rho)n^2$ such that:

$$\begin{aligned}
\tilde{\mu}_a(0) &= 0 \\
\tilde{\mu}_a(h) &= g\theta(1+\rho)h^2 > 0 \\
\frac{\partial\tilde{\mu}_a(n)}{\partial n} &= 2g\theta(1+\rho)n > 0 \\
\frac{\partial^2\tilde{\mu}_a(n)}{\partial^2 n} &= 2g\theta(1+\rho) > 0
\end{aligned}$$

Let $\mu_a^*(n) = g\theta(1-\rho)n^2$ such that:

$$\begin{aligned}
\mu_a^*(0) &= 0 \\
\mu_a^*(h) &= g\theta(1-\rho)h^2 < \mu_a^s(h) \\
\frac{\partial\mu_a^*(n)}{\partial n} &= 2g\theta(1-\rho)n < \frac{\partial\mu_a^s(n)}{\partial n} \\
\frac{\partial^2\mu_a^*(n)}{\partial^2 n} &= 2g\theta(1-\rho) < \frac{\partial^2\mu_a^s(n)}{\partial^2 n}
\end{aligned}$$

Note that $\tilde{\mu}_a(n) - \mu_a^*(n) = 2g\theta\rho n^2 > 0$ (see Figure 4). Similarly, remind that $\tilde{\mu}_b(n) = gh(1-\theta h) - [f + g(1-2\theta h)]\, n$ such that:

$$\begin{aligned}
\tilde{\mu}_b(0) &= gh(1-\theta h) > 0 \\
\frac{\partial\tilde{\mu}_b(n)}{\partial n} &= -\left[f + g(1-2\theta h)\right] < 0 \\
\frac{\partial^2\tilde{\mu}_b(n)}{\partial^2 n} &= 0
\end{aligned}$$

Let $\mu_b^*(n) = gh(1-\theta h) - [f + g(1-2\theta h) + 2g\theta\rho h]\, n$ such that:

$$\begin{aligned}
\mu_b^*(0) &= gh(1-\theta h) = \mu_b^s(0) \\
\frac{\partial\mu_b^*(n)}{\partial n} &= -\left[f + g(1-2\theta h) + 2g\theta\rho h\right] < \frac{\partial\tilde{\mu}_b(n)}{\partial n} \\
\frac{\partial^2\mu_b^*(n)}{\partial^2 n} &= 0
\end{aligned}$$

Note that $\tilde{\mu}_b(n) - \mu_b^*(n) = 2g\theta\rho h n > 0$ (see Figure 5). Last, note that $\tilde{\mu}_b(n) - \mu_b^*(n) > \tilde{\mu}_a(n) - \mu_a^*(n)$ as $n < h$. This implies that $n^* < \tilde{n}$ (see Figure 6).

B.4 Proof of Proposition 4

Under the new environment, the revenue of the worker living in s becomes:

$$\mathcal{I}_W(s) = \psi(s) + \varphi(s) - R(s) + \frac{\mathcal{R}}{h} - \overline{\varphi}$$

Following the strategy of the article gives:

$$\pi(n) = \psi(0) + \varphi(0) - \overline{\varphi}$$

Assume that $\varphi(s) = 4g\theta s$. Then this leads to:

$$\overline{\varphi} = \frac{1}{\ell}\int_0^{\ell} \varphi(s)ds = 2g\theta\ell$$

and

$$\pi(n) = g(1 - 2\theta\ell)$$

As $p^* = \frac{c}{\rho}$, we obtain the following:

$$\pi(n) = \frac{(1-\rho)chx(n)}{\rho} - f$$

This gives:

$$\pi(n) = g(1-2\theta\ell) \Leftrightarrow chx(1-\rho) = \rho\left[f + g(1-2\theta\ell)\right]$$

$$\Leftrightarrow \left\{\frac{gh(1-\theta h)}{n} - \left[f + g(1-2\theta h) + g\theta n\right]\right\}(1-\rho) = \rho\left\{f + g\left[1 - 2\theta(h-n)\right]\right\}$$

$$\Leftrightarrow gh(1-\theta h)(1-\rho) - \left[f + g(1-2\theta h)\right]n = g\theta(1+\rho)n^2$$

Using Proof 3, this means that optimality is restored.

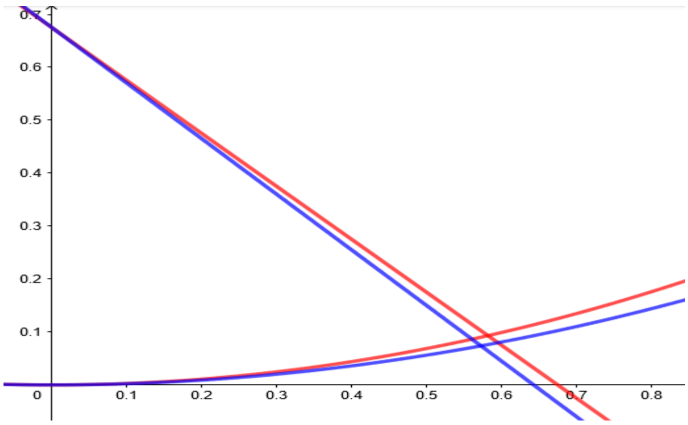

Figure 4: Non-optimality for n^*: Part 1 (blue lines = decentralized economy, red lines = centralized economy)

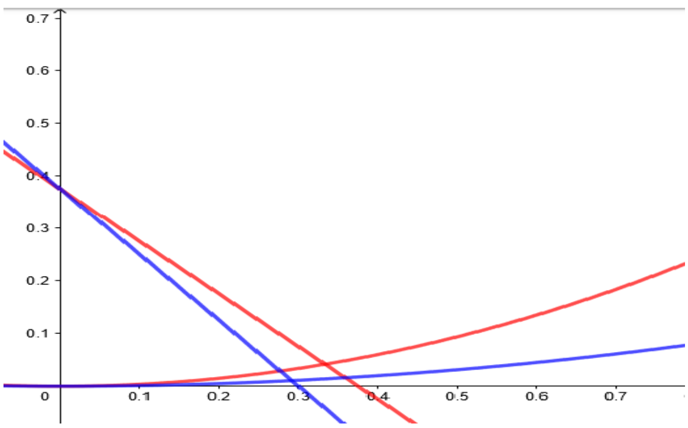

Figure 5: Non-optimality for n^*: Part 2 (blue lines = decentralized economy, red lines = centralized economy)

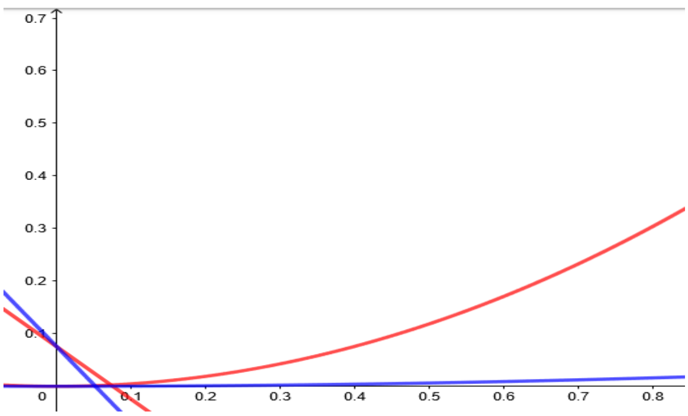

Figure 6: Non-optimality for n^*: Part 3 (blue lines = decentralized economy, red lines = centralized economy)