

# **Digitization and contextual factors in Emilia-Romagna municipalities: A cluster and poset based approach**

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*[Preliminary version. Please do not quote]*

## **Abstract**

The degree of digitization has increased in several sectors across Europe, especially since the Covid-19 pandemic crisis. To encourage this process, a lot of funds have been allocated in the Recovery and Resilience Plan. As the literature shows, implementing digitization is important for improving development and quality of life. However, digital in itself is not a determinant of development. It is necessary to consider the relationships between it and the various factors with which it must coexist, such as institutional framework, human and organizational capital. The study aims to analyze the territory of Emilia-Romagna through the indicators of digitization and development of complementary factors. A cluster analysis and a poset methodology are proposed to identify municipal areas where this link is present or needs to be strengthened. It turns out that there is a correlation between the level of digitization and the development of complementary factors. The municipalities where this relationship is strongest are in the top 10 of the ranking and are classified as urban poles, according to SNAI classification. The region is characterized by strong spatial differences between and within clusters. A policy suggestion also emerges: action is needed for municipalities that score low in only one dimension, especially if that dimension is an impact factor among those identified in the sensitivity analysis.

Keywords: Digitalization; Development; Contextual factors; Composite index; Cluster; Poset; Regional policy; SNAI.

JEL classification: O20, R10, R58

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# 1. Introduction

The Covid-19 pandemic crisis has accelerated the digital transition in many sectors of the economy, causing organizational change and business redefinition (Fletcher and Griffiths, 2020; Martin and Barbero, 2020; McKinsey Digital, 2020; UN E-Government, 2020; Hantrais et al., 2021). Eurostat surveys confirm the digitalization trend with a higher growth rate in the adoption of digital technologies by citizens and businesses (López Peláez et al., 2021). When we talk about digital transformation, we refer to a comprehensive concept that encompasses the culture, organization, and relationships that can create value for citizens and organizations (Mergel et al., 2019). While digitization is the transformation that socioeconomic systems undergo because of the wide adoption of digital technologies (Katz et al., 2014).

In the European context, digital progress has been monitored by the European Commission with the Digital Economy and Society Index (DESI) since 2014. Recently, the Commission modified the DESI to align it with the four cardinal points defined in the '*Pathway to the Digital Decade*'. The four dimensions (human capital, connectivity, digital technology integration, and digital public services) are in turn organized into sub-dimensions. In total it includes eleven indicators to assess progress.

The DESI Report 2020 (European Commission, 2020) highlighted the increasing trend in the use of digital solutions during the Covid-19 pandemic. DESI report 2022 results show further progress in digital transformation for most member states (European Commission, 2022). However, business adoption of technologies and digital skills are insufficient. For example, only 54 percent of people have at least basic digital skills, there is still a large gap in connectivity between rural and urban areas, and only 55 percent of small and medium-sized enterprises have reached at least the basic level in digital technology adoption.

Looking at the four dimensions, Italy is below the European average in two out of four dimensions (human capital and digital public services). This places it below the global average. Looking at the progress made by member states over the past five years, however, we can see that Italy has increased its DESI score more than the convergence curve predicted (European Commission, 2022).

It is therefore necessary to continue along this path. The Recovery and Resilience Plan (RRP) represents a huge opportunity for member states to invest in their digital transformation. Under this facility, each member state must assign at least 20 percent of RRP resources to contribute to the digital transition. Italy has decided to dedicate 25 percent of spending to digital objectives (European Commission, 2022).

In line with the European strategy, the Emilia-Romagna Region has developed a strategic plan that rethinks society digitally to accelerate digital transformation at the regional level as well. The strategy is named '*Data Valley Bene Comune*' (DVBC), is divided into eight challenges and aims at the full participation of all regional society in the opportunities offered by digital.

The DVBC strategy is a clear policy choice that identifies digital as a cross-cutting element that can strengthen the existing ecosystem formed by business, research, associations, and public administration. In this sense, digital is an enabler for development.

This link is confirmed by the literature, in fact there is a broad consensus among that digitization is correlated with positive macro-economic outcomes (Vasetskaya and Gaevskaia, 2019; Habibi and Zabardast, 2020; Myovella et al., 2020) and with quality of life and social well-being (Kryzhanovskij et al., 2021; Elmassah and Hassanein, 2022). For these reasons, digitization has also been a relevant

issue in the context of EU cohesion policy, especially in remote rural areas, where vulnerable populations, such as the elderly, are located.

Digitization, however, is not an enabling factor in itself. So, it is necessary to focus on its relationship with the factors with which it must coexist (OECD, 2008). Other factors like human capital, high-skilled occupations, organizational capital, institutional framework, an adequate educational system and other intangibles assets (Crandall et al., 2007; Katz et al., 2010; Mack and Faggian, 2013; Matteucci, 2020; Torres and Augusto, 2020) are fundamental to take full advantage of the potential impact of digitization. All these elements form the context in which digitization processes are developed. In this essay they are called complementary factors or contextual factors. Thus, the goal of this work is to study the territory of Emilia-Romagna through several indicators that measure the degree of digitization and the degree of development of complementary factors. The unit of analysis is the municipality because it is the main responsible actor that can guide local policies for performance in many of the contextual factor's indicators considered.

Through cluster analysis and the construction of rankings using the poset methodology, I analyze the relationships between the degree of digitization and contextual factors, at municipal level. This makes possible to identify areas where this relationship exists, is stronger or needs to be strengthened. Therefore, this new reading of the territory is also intended to offer a tool to support regional policymaking.

Indicators related to digital progress are constructed basing them on DESI-ER (the DESI for the Emilia-Romagna region, based on European DESI) using data for the 330 municipalities in the region. Additionally, I identify a set of indicators that can provide information on complementary factors for Emilia-Romagna municipalities. To the best of my knowledge, there are no references in the literature that clearly indicate which factors to consider. So, for the selection of indicators, I choose to rely on the composite indices of well-being and quality of life because they may be a good reference when it is necessary to synthesize information in multidimensional contexts.

To identify the areas where are located the best and worst performing municipalities based on their scores in the various dimensions, I proposed a cluster analysis. This is particularly useful for obtaining homogeneous groups of municipalities by jointly considering the dimensions of digitization and context factors.

For the paper's goal, the construction of a composite index may allow a better comprehension of complex phenomena. In general, it lets to observe a phenomenon in a broad and general sense, considering the various indicators that influence it (Nardo et al., 2008). Although the communicative advantages of composite indicators are obvious, their development involves several pragmatic choices that have been criticized as highly subjective, arbitrary, potentially misleading, and prone to obscure essential information (Freudenberg, 2003; Saltelli, 2007; Cherchye et al., 2007; Fattore, 2016; Barclay et., 2019). The validity of a composite index depends heavily on how the components are aggregated and weighted. However, numerous alternatives exist to address the shortcomings of composite indexes (Albo et al., 2019; Aparicio and Kapelko, 2019; Ruiz et al., 2020; Smirlis, 2020). One of these, little explored empirically in the literature, is the use of partially ordered set analysis (poset), the use of which has also been advocated by Amartya Sen (1970; 2018). Partial orderings-or partial rankings or partial comparability-serve to establish minimum standards of comparability that can be affirmed without contradicting other rankings.

Recognizing incompatibilities, partial ordering is less attractive than a composite index, but it succeeds in offering an information panel of a set of indicators that is coherent and more accurate

than a composite index. The main merits of the methodology are that it is based on well-established mathematical concepts, makes full use of all the information in the data, and considers the purely ordinal nature of the data, avoiding subjective choices such as the assignment of weights and thus maintaining a high standard of objectivity. For these reasons I decide to rank municipalities by using the poset methodologies.

The paper is structured as follows. It first reviews the literature about linkages between digitalization and economic growth, well-being and quality of life. Section 3 focuses on data and provide descriptive statistics and a cluster analysis of the Emilia-Romagna municipalities. Section 4 examines poset methodology and its applications. Section 5 summarizes the results and Section 6 presents some robustness checks. Section 7 provides discussion and conclusion.

## **2. Literature review**

Challenge 7 of the DVBC project ('From Marginal Settings to Digital Communities') aims to develop digital technologies and digitize territories to improve quality of life and counter depopulation. Investing in digitization should have precisely this goal. From this perspective, digital is not just a tool, but a factor that can impact and change social development.

Digital technologies have a potential impact on quality of life and human development (OECD, 2004). Internet use and broadband deployment, for example, have a positive effect on economic growth. The literature makes several contributions to support this thesis (Koutroumpis, 2009; Czernich et al., 2011; Ghosh, 2017) and it is also confirmed by the empirical literature (World Bank, 2009). For example, starting from Barro's (1997) economic growth equation, Choi and Hoon Yi (2009) estimate the effect of the Internet on economic growth using a panel data. The Internet variable is the ratio of Internet users to total population. Using the investment rate, public consumption rate and inflation as controls, they verified that the Internet plays a positive and significant role on GDP. In addition, broadband availability has a significant positive effect on growth in many sectors (Shideler and Badasyan, 2007), on the number of firms (Lehr et al. 2006) and in increasing labor productivity (Najarzadeh et al., 2014).

These positive effects are also relevant in a local context. For instance, broadband expansion has a positive causal effect on local economic growth (Kolko, 2012) and local employment (Lehr et al., 2006; Stenberg et al., 2009) in the United States. In the Brazilian local context Jung and López-Bazo (2020) finds that these benefits are greatest for less developed regions, suggesting a regional convergence role for digitization. Thus, digitization practices are associated with regional productivity (even in the long-run) and furthermore these effects increase over time (Tranos et al., 2021; Crespo Cuaresma and Lutz, 2021). Regional digital capital (measured through a cross-regional panel data that assesses four dimensions: users' access to digital technology, users' digital practices, human capital, and innovation capacity) is also a good predictor for all phases of labor market resilience (Reveiu et al., 2022).

In some cases, broadband infrastructure facilitates the development of poor regions, enhancing some degree of territorial equilibrium (Suriñach et al., 2007). In addition, for more isolated areas, ICT diffusion can be a means of zeroing out distances (Cairncross, 2001). Peripheral regions, therefore, could benefit from previously inaccessible opportunities (Bonaccorsi et al., 2005).

Although GDP and economic growth are often used as a summary measure of national economic well-being, they should not actually be understood as such. They are simply a measure of the marketable output of an economy. In measuring quality of life, more attention should be paid to

subjective and objective measures of well-being that better reflect the heterogeneity of contexts (Aitken, 2019). Quality of life is measured through a set of indicators that can reflect the satisfaction of people's needs.

The relationships between digitization and quality of life has recently been studied in the literature and can be traced to two strands of research.

The former argues for a positive correlation of digitization on quality of life. Osipova and Naumova (2020) define the level of quality of life of citizens in the Irkutsk region (Russia) through five dimensions, each consisting of a set of indicators. Through regression and correlation analysis, they study the relationships between these dimensions and the level of digitization. The study demonstrates that quality of life directly affects the access to digitization.

This relationship is not unique. In fact, the second strand of research argues that digitization might also leads to negative effects such as unemployment, inequality, and problems with sustainability and well-being (Acemoglu, 2002; Autor and Dorn, 2013; Frey and Osborne, 2017; Linkov et al., 2018). Through a panel database for 67 countries, Maiti and Awasthi (2019) study the relationships between well-being and progress and level of ICT exposure through the construction of two composite indices. They find that a high degree of digitization is accompanied by higher exposure to cyber threats, unemployment, digital inequality, privacy issues, and sedentariness. Although the level of ICT exposure may have negative effects at the disaggregate level, at the aggregate level it positively and significantly affects well-being and progress.

It seems that the negative aspects arising from digitization are offset in those contexts where considerable importance is attached to education and training, culture, civic activities, health, and equal development opportunities. Digitalization is not a determining factor in improving quality of life. Many researchers argue that the positive or negative influence of digitization on quality of life depends on the political and socioeconomic context. To take full advantage of its potential impact, it is necessary to develop all those complementary factors that are critical to community development.

According to the OECD (2008), it is not ICT and the digitization process per se that have an impact on the economy and society, but how they are used to transform organization, processes, and behavior. ICT and broadband expansion can contribute to productivity and economic growth if they are accompanied by complementary inputs such as human capital, high-skilled occupations, organizational capital, institutional framework, an adequate education system, and other intangibles (Crandall et al., 2007; Mack and Faggian, 2013; Katz et al., 2010; Matteucci, 2020; Torres and Augusto, 2020).

### **3. Data, descriptive and cluster analysis**

#### **3.1 Data**

The dataset collects indicators from different sources. They were collected by Art-ER during the implementation of Challenge 7 of the DVBC project. For each indicator, data from the last available year are accessible for all 330 municipalities in the Emilia-Romagna region. Indicators are continuous or dichotomous variables. Continuous indicators not expressed in terms of population are weighted according to municipal population, normalized through a min-max normalization procedure, and multiplied by 100. In this way, all indicators have a value between 0 and 100, where 0 indicates a very bad performance and 100 an excellent performance. Dichotomous indicators, on the other hand, have a rewarding role. They take the value 1 to indicate whether a particular service or infrastructure

is present in that municipality. The value 1 is replaced with  $100/n$ , where  $n$  is 55 for digitization and 37 for context factors.

Indicators are then organized to provide an overview of the state of digitization and the quality of complementary factors for territorial development. To do this, are grouped into dimensions. Dimensions are constructed through a simple average<sup>1</sup> of continuous indicators to which the dummy-premium is added, if it exists for that dimension. Each indicator has the same weight.

Indicators to describe the digital transformation process are selected based on those chosen for the construction of DESI-ER, the composite index promoted by the Digital Agenda of Emilia-Romagna. This is created based on DESI, the digitalization index developed by the European Commission. The indicators are further skimmed following a correlation analysis. In fact, while the original version contains 60 indicators, the version presented here collects 55<sup>2</sup>. The indicators are grouped into the same four dimensions found in the European DESI: human capital, connectivity, integration of digital technologies, digital public services.

Indicators related to the status of complementary factors are selected from a dataset constructed by Art-ER for the digital community project in Emilia-Romagna territories. The indicators selected for that project are suitable for providing information on the socioeconomic context and measuring the degree of development of the complementary factors reported in the literature. A set of 37 indicators is defined, selected based on representative consistency, quality of data collection, and correlation between indicators. The indicators are grouped into six dimensions: foreigners, environmental quality, services, culture, gender, income and employment. The definition of the six dimensions is based on those identified by the OECD (2014) for measuring regional and local well-being. According to data availability, the 12 dimensions proposed by OECD are reduced to the six presented below. Nevertheless, the defined framework can provide information on both the material conditions and quality of life of municipalities in the region.

For a complete list of indicators with sources and reference years, see the tables in the Appendix A.

To check the internal consistency of model dimensions I resort to Cronbach's alpha coefficient (Cronbach, 1951). This is the most widely used method in the literature (Green et al., 1977; Hattie, 1985; Feldt et al., 1987; Cortina, 1993; Miller, 1995; Raykov, 1998). The C-alpha measures the

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<sup>1</sup> It is important to note that the aggregation by simple average imply that there is an offsetting relationship between the indicators contained in each dimension. This strong assumption implies that each indicator has equal importance in defining the overall score for each dimension.

<sup>2</sup> Having a data set with only 330 observations, I eliminated variables with a correlation greater than 0.7. The five removed variables are:

- N. of literacy/training digital courses (PEI) per 100 excluding digitals. It is correlated with the n. of trainees trained with PEI courses out of the total n. of excluded with a correlation of 0.81. Both are solid indicators; I remove the first because the n. of trainees is in percentage.
- Female graduates in STEM disciplines. It is correlated with the n. of published datasets with a correlation of 0.71. I remove it because it is an estimated indicator.
- % of civic covered by 30 mbps band. It is correlated to the % of civic covered by 2-30 mbps band by construction (corr=0.81), because if the civic is covered by 30 mbps band, it is also covered by 2-30 mbps band. Therefore, I decided to keep the indicators of 2-30 mbps band and 100 mbps band.
- Ultra-wideband subscriptions (% of resident population). It is correlated to the n. of local apps with a correlation of 0.92. I removed the first because is an estimated indicator.
- Total interactive services detected by the municipal territory. It is correlated with the interactivity index of municipal services with a correlation of 0.93. They are basically the same thing. I eliminate the former because its measurement is based on a universe that progresses over time, while the latter is based on a definite maximum that does not change.

portion of the total sample variability of individual indicators due to the correlation of those indicators. If the correlation is high, it is evident that the individual indicators measure the same underlying construct. Therefore, high c-alpha indicates that the individual indicators measure the latent phenomenon well. According to Nunnally (1978), a sufficiently high c-alpha threshold is 0.7. It can be seen from *Table 1* that all dimensions meet this requirement.

*Table 1. Cronbach Coefficient Alpha of dimensions*

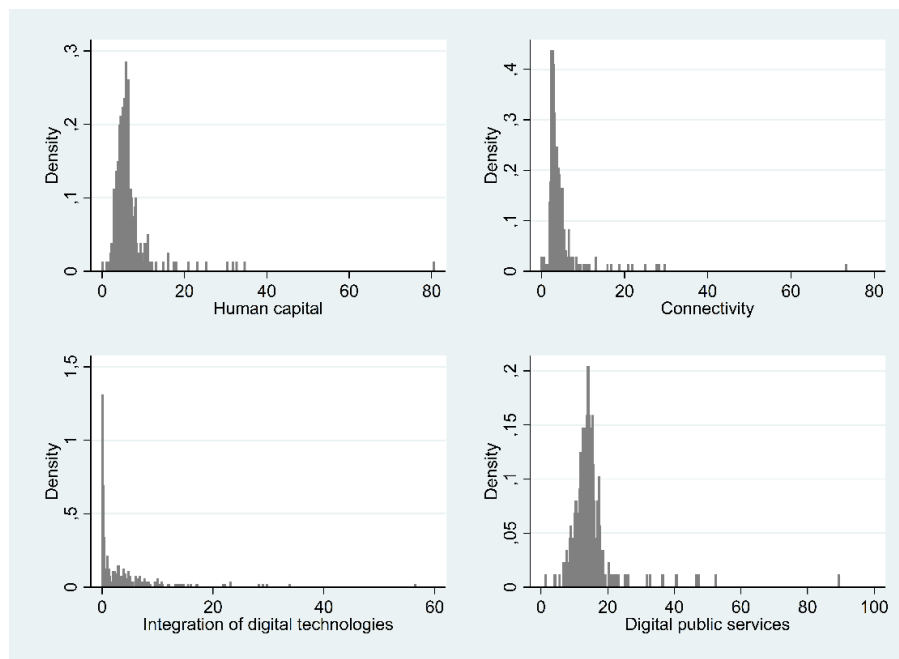
	Dimension	C-alpha
Digital	Human capital	0.9119
	Connectivity	0.8505
	Integration of digital technologies	0.7773
	Digital public services	0.9159
Complementary factors	Foreigners	0.9833
	Environmental quality	0.9941
	Services	0.9499
	Culture	0.9419
	Gender	0.9970
	Income and employment	0.9931

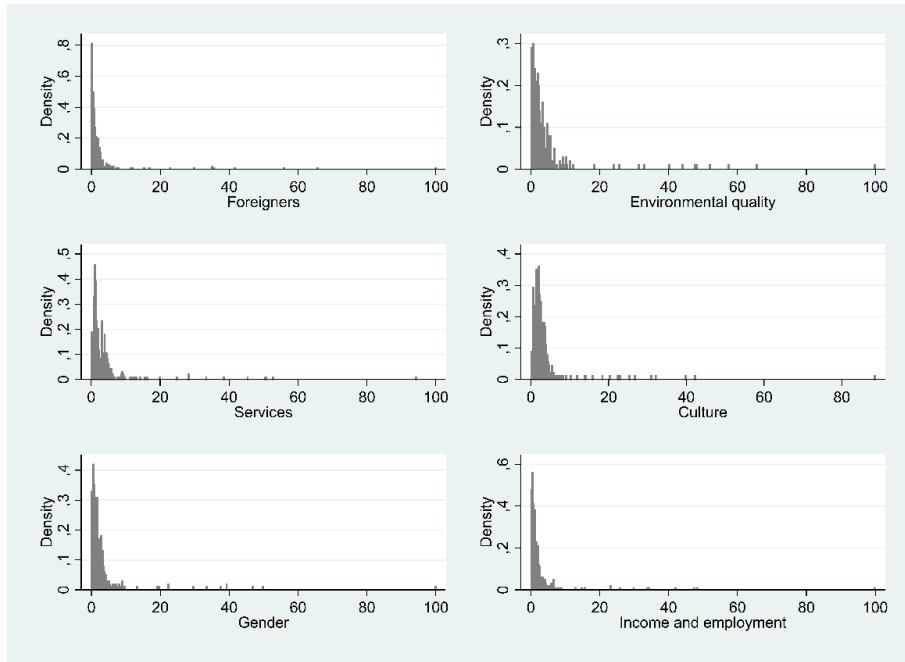
*Source: author's elaborations*

### 3.2 Descriptive analysis

After defining the dimensions, it is useful to look at the distribution of scores. The histograms show that, with few exceptions, most municipalities score very low on both digitization dimensions (i.e. human capital, connectivity, digital technology integration, digital public services) and context factor dimensions (i.e. foreigners, environmental quality, services, culture, gender, income and employment). This is especially true for the digital dimension ‘integration of digital technology’ and for the complementary factors dimension ‘foreigners’. In contrast, the dimensions in which they score relatively better are ‘digital public services’ for the digital dimension and ‘services’ for the complementary factors dimension.

*Figure 1. Distribution of scores for the digital dimension (above) and for the complementary factors dimension (below)*

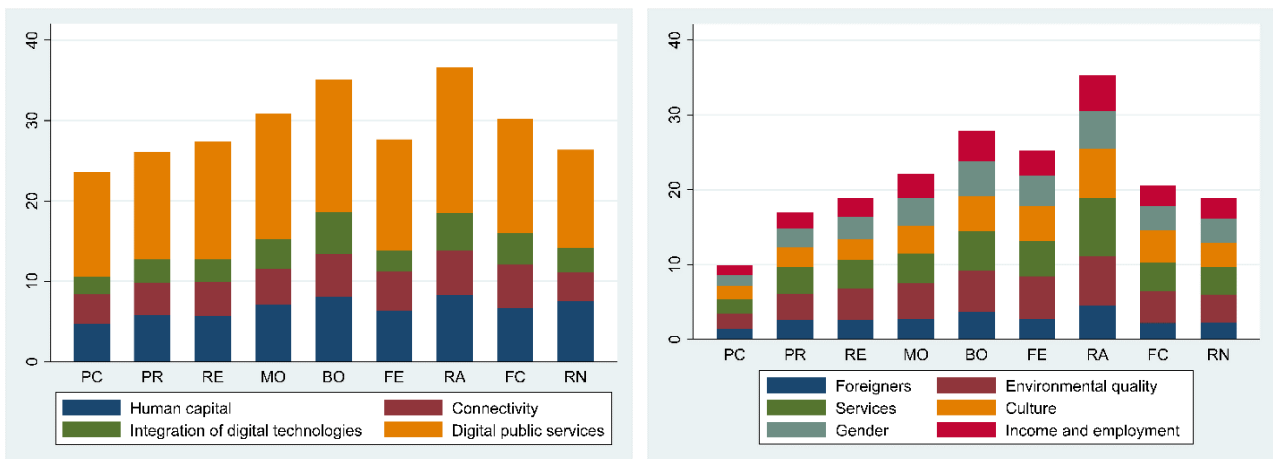




Source: author's elaborations

The overall degree of digitization can be considered as the sum<sup>3</sup> of the scores obtained in each digital dimension. The same applies to the degree of quality of context factors. Having said that, it is possible to observe the overall performance and scores obtained in each dimension at NUTS-3 level. From *Figure 2* it is evident that scores are less heterogeneous for digitization than for the quality of complementary factors. For the latter, differences between the scores are very high. The best performing province is Ravenna (obtaining a score equal to 36,7 in digitization and 35,3 for the contextual factors), followed by Bologna (that scores 35,1 in digitization and 27,9 for the quality of complementary factors). Instead, the least performing province is Piacenza for both digitization (23,6) and for the quality of complementary factors (9,9).

Figure 2. Scores for the digital dimension (left) and the complementary factors dimension (right), NUTS-3 level



Source: author's elaborations

<sup>3</sup> The overall performance thus defined implies that each dimension contributes equal weight. Conscious that the multidimensional nature of the phenomenon may be compromised, an alternative method of performance evaluation is proposed in *Section 4*.

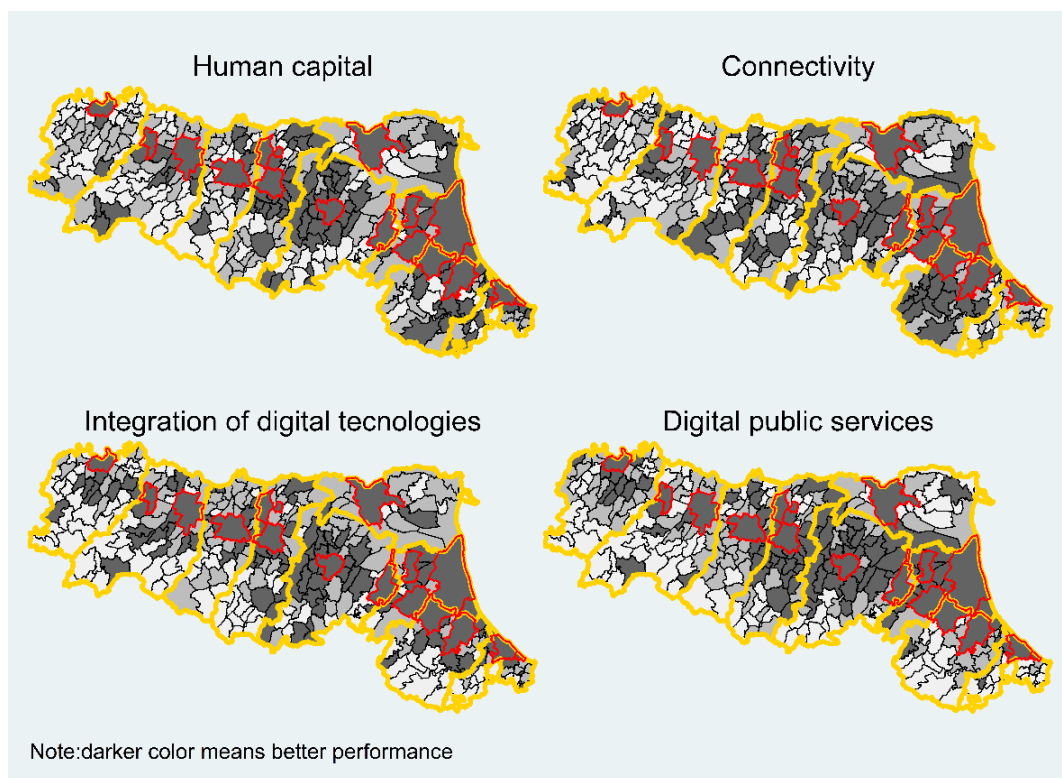


Figure 3 shows the spatial distribution of municipalities divided by terciles according to their score in each dimension. Municipality's darker color means better performance. The yellow lines represent the NUTS-3 boundaries.

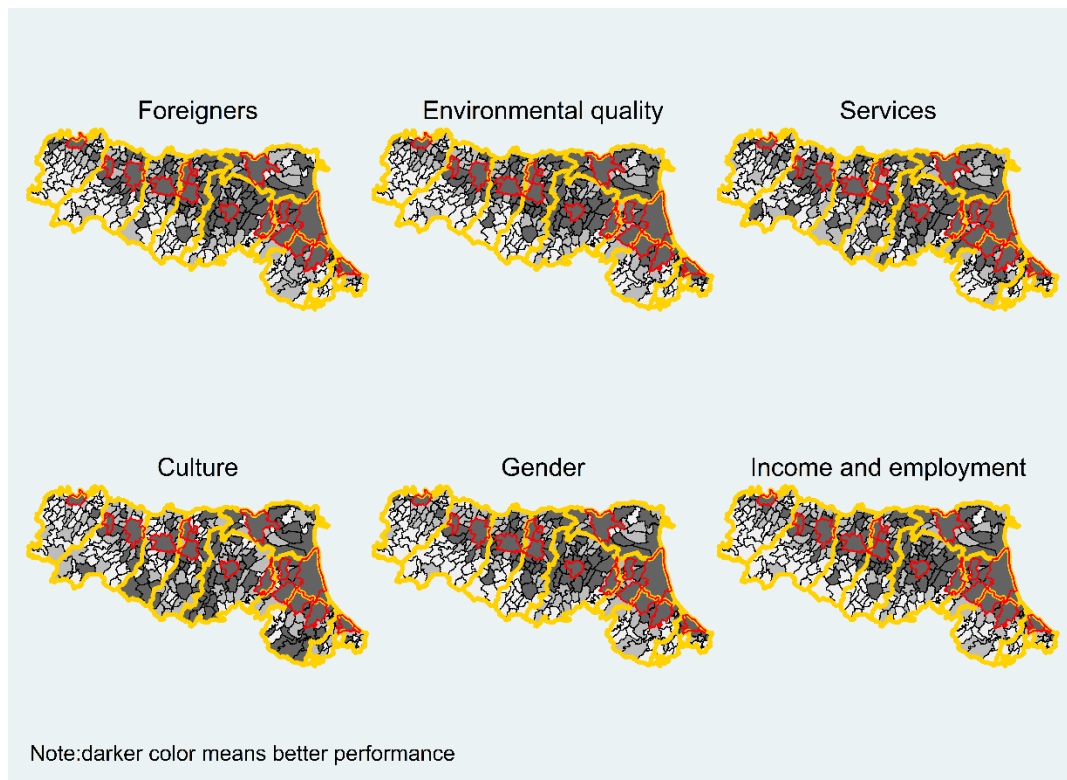
The resulting localization of the best/worst-performing municipalities recalls the SNAI national classification (National Strategy for Inner Areas)<sup>4</sup>. In this context two concepts are of particular interest: urban poles and inner areas. Urban poles are those municipalities that can offer some essential services such as a high school, a hospital facility with an emergency department and a small/medium-sized train station. Conversely, municipalities that are located at considerable distance from the main urban centers and thus suffer from a limited supply of essential services are considered 'inner areas'.

Note that the municipalities with the lowest scores (marked in light gray) are at a distance from the urban poles (marked with red borders). They belong to the most extreme inner areas, i.e., peripheral and ultra-peripheral areas. NUTS-3 regions with a larger share of peripheral and ultra-peripheral areas are the worst performers. Instead, the best-performing municipalities are close to each other and are concentrated near the region's urban poles. This is particularly true for digital dimensions 'human capital' and 'digital public services' and for contextual factors dimensions 'foreigners', 'environmental quality', 'gender' and 'income and employment'.

Figure 3. Spatial distribution of municipalities by score terciles for the digital dimension (above) and for the complementary factors dimension (below)



<sup>4</sup> The SNAI classification was introduced by the Italian Minister for Economic Development in 2014 and distinguishes six types of areas: urban poles, inter-municipal poles, outlying areas, intermediate areas, peripheral areas, and ultraperipheral areas. Intermediate, peripheral and ultra-peripheral areas are considered 'inner areas'. To better understand and learn more about the strategy for inland areas, see UVAL (2014)



*Source: author's elaborations*

### 3.3 Cluster analysis

To obtain homogeneous groups of municipalities based on the performance of different dimensions, a cluster analysis is proposed below. The two main clustering techniques are partitional clustering and hierarchical clustering (Everitt et al., 2011).

For partitional clustering it is necessary to know ex-ante the number of clusters to be identified. There are such methods as the elbow-method or the silhouette method that can help to make a choice. In this case, analysis using the elbow and silhouette methods did not lead to an unambiguous and accurate result. Therefore, I choose hierarchical cluster analysis.

Using hierarchical clusters amounts to assume that the clusters are overlapping. This assumption is not far-fetched because descriptive analysis suggests that there is a hierarchy among municipalities based on their scores in the different dimensions. The hierarchical analysis returns as output the dendrogram, a branching diagram that represents the relationships of similarity among a group of entities. Through the dendrogram it is possible to cut the graph at the appropriate dissimilarity level, which corresponds to a number of clusters. In addition, the study of the dendrogram is useful for understanding the similarity relationships between groups.

As an algorithm for clustering, I chose Ward's method. Ward's method is an agglomerative clustering method<sup>5</sup>, i.e., it follows a bottom-up approach: each observation starts in its own cluster, and cluster pairs are joined as you move up the hierarchy. Ward's method clusters items according to the minimum variance rule. In essence, its purpose is to minimize the variance within clusters. I chose this method because it is less susceptible to outliers and noise.

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<sup>5</sup> Alternatively, clustering can be done using the division method. In this case, a top-down approach is followed: all observations begin in a cluster and divisions are performed recursively as one moves down the hierarchy.

The dendrogram derived from the cluster analysis is very consistent with the previous descriptive analysis. This is true both when I cluster by considering the dimensions of digitization and the dimensions of complementary factors.

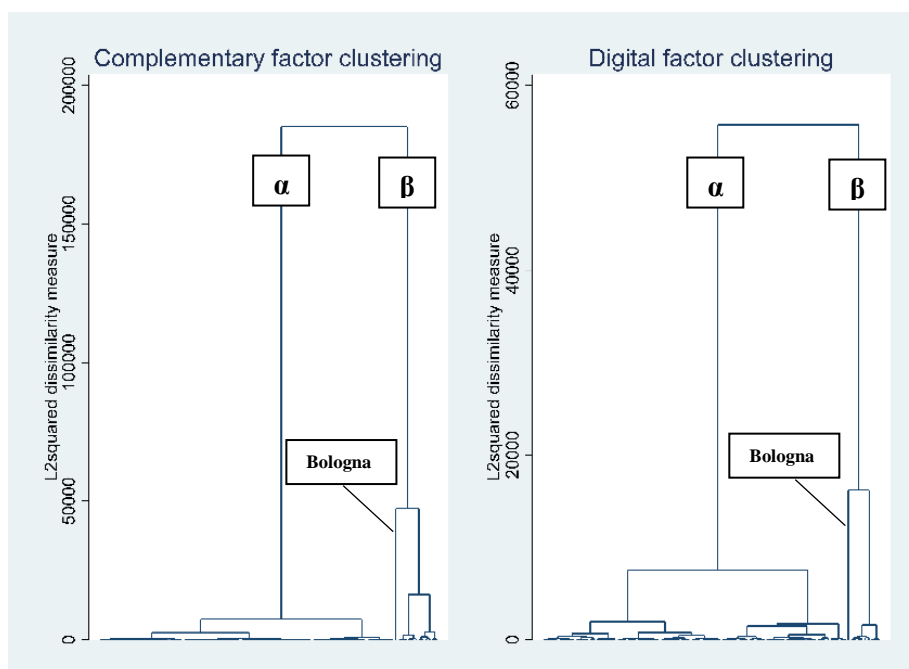
There are two ways to read a dendrogram: from top to bottom or from bottom to top. The first method allows to identify large-scale groups. The second identifies the presence of similarities among individual chunks. Having so many observations and since the goal is to identify large homogeneous groups, a top-down approach is more appropriate.

If we start reading the dendrogram from top to bottom (*Figure 4*), it is evident that in the first branch, for both digitization and complementary factors, there is a clear division into two very long clades,  $\alpha$  and  $\beta$ . This means that the Euclidean distance is considerable, indicating a strong dissimilarity between the two clusters. The one on the right (indicated as cluster  $\beta$ ) contains part of the municipalities classified as urban poles and the one on the left (indicated as cluster  $\alpha$ ) the rest of the municipalities. Cluster  $\beta$  for digitization contains 10 municipalities, all of which are also part of cluster  $\beta$  for complementary factors, which contains 13.

Cutting to a level of dissimilarity such that three clusters are obtained, it is observed that there is a single-leaf clade that contains only Bologna. This means that Bologna is completely separate from other municipalities; it is an outlier. The other branch includes the rest of the municipalities contained in  $\beta$ .

The dissimilarities of the groups contained in cluster  $\beta$  are high when compared with those in cluster  $\alpha$ . Several bifurcations are noticeable in  $\alpha$ , but the dissimilarities are small, indicating that the municipalities are very similar to each other.

*Figure 4. Hierarchical clustering dendrogram*



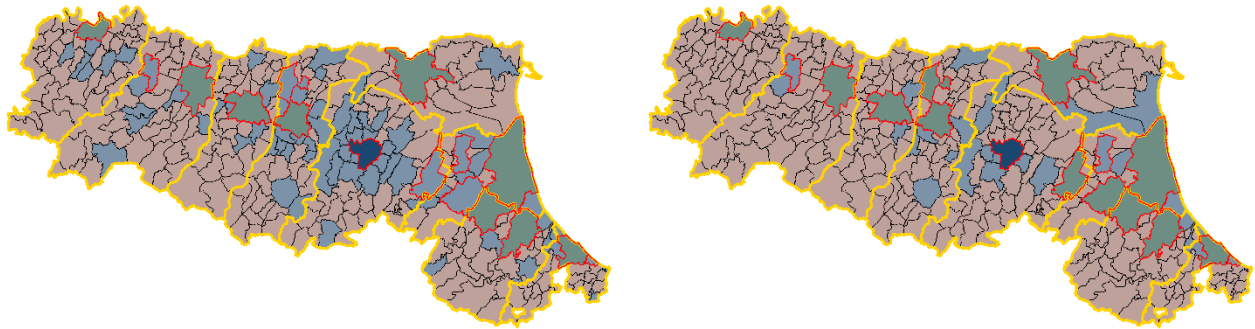
*Source: author's elaborations*

Given the high numerosity of cluster  $\alpha$ , I decided to perform further analysis within it. Again, there is a first main bifurcation that divides the cluster into two branches with a very large Euclidean distance. I then identified two additional clusters, one significantly less numerous than the other. For digitization, the smaller cluster contains 78 municipalities, while for complementary factors it

contains 28 municipalities. Of the latter, 26 municipalities are in common with the digitization cluster which contains 78.

To better understand which clusters each municipality belongs to, a spatial representation is proposed. The resulting regional spatial configuration is depicted in the maps.

Figure 5. Spatial representation of clusters of municipalities for digitalization (left) and for contextual factors (right)



Source: author's elaborations

Bologna (dark blue municipality) forms a cluster of its own. The gray municipalities are all urban poles, but not all urban poles are colored in gray. The urban poles of Fidenza, Lugo and Riccione do not belong to this cluster for both the digital and complementary factor dimensions. For the digital dimension, Carpi, Imola and Faenza are also excluded. These six municipalities all belong to the cluster whose municipalities are colored light blue. This cluster is more numerous for digitization dimension, and municipalities that are part of it are concentrated in the vicinity of the municipality of Bologna and, with a few exceptions, near urban poles. This suggests a possible spillover effect driven by geographical proximity. The largest cluster is the one to which the municipalities marked in pink belong. Note how the NUTS-3 regions identified as least performing in the descriptive analysis (Piacenza, Parma, Rimini) have the largest number of municipalities belonging to this cluster. The contrary happens for the higher-performing ones.

Cluster analysis suggests that performance on digitization dimensions is similar and goes hand in hand with performance on complementary factors. This is also confirmed by the correlation matrix.

Table 2. Correlation matrix of the scores

		Digital dimensions			
		Human capital	Connectivity	Integration of digital technologies	Digital public services
Complementary factors dimensions	Foreigners	0.9292	0.9197	0.8194	0.9232
	Environmental quality	0.9239	0.8944	0.8515	0.9406
	Services	0.9262	0.9026	0.8412	0.9318
	Culture	0.9289	0.9156	0.8317	0.8986
	Gender	0.9511	0.9223	0.8487	0.9421
	Income and employment	0.9532	0.9293	0.8516	0.9390

Source: author's elaborations

## 4. Method

In the previous section, indicators are grouped into dimensions in order to describe homogeneous phenomena. It is important to remember that underlying this is an important assumption: there is an offsetting relationship between the indicators contained in each dimension. In this sense, all indicators

have equal importance in determining the average score, and a good (bad) performance on one of them will improve (worsen) the score obtained in that dimension. To consider overall performance as the sum of scores obtained in the dimensions is to downplay the multidimensional nature of the phenomenon. Therefore, a ranking by aggregating dimensions by average would not be correct.

In this section an alternative procedure is proposed. Thanks to this is possible to rank municipalities by avoiding the aggregation procedure, leaving the multidimensional nature of the phenomenon intact.

Some mathematical definitions and their application follow.

#### 4.1 Mathematical definitions

Consider a set of elements  $x, y, z, w$  as belonging to a matrix  $M$  such that  $M = \{x, y, z, w\}$ . In the case under consideration, the elements represent the municipalities of the set  $M$ . Each municipality is evaluated in a space  $Q$ , consisting of  $q_1, q_2, q_3, q_4$ , etc. In the present case, these represent the dimensions of digitization and complementary factors defined above. Since all data are continuous,  $q_i(x)$  is the row of values of each indicator for municipality  $x$ ,  $q_i(y)$  is the row of data for municipality  $y$ , etc. Axioms that govern these relationships are:

- Reflexivity:  $x \leq x$  every object can be compared with itself;
- Anti-symmetry: if it is true that  $x \leq y$  and  $y \leq x$  then it is true that  $x = y$ ;
- Transitivity: if it is true that  $x \leq y$  and  $y \leq z$  then it is true that  $x \leq z$ .

To clarify further, municipality  $x$  will be ranked higher than municipality  $y$  if it is true that  $q_i(x) \geq q_i(y)$  and for at least one indicator it is true that  $q_i(x) > q_i(y)$ . If it is true for all indicators that  $q_i(x) = q_i(y)$ , then the two municipalities are equivalent.

Whenever  $x \leq y$  or  $y \leq x$  then  $x$  and  $y$  are acquirable, that is, I can establish ordinal relationships between the two. When this does not occur, then  $x$  and  $y$  are said to be incomparable ( $x \parallel y$ ). As reported in Fattore et al. (2012), the first case is called a partial-order chain  $P$  and the second case is called an anti-chain. The chains have a length given by the number of elements in them. The number of elements in the largest chain is called the poset height. The number of anti-chain elements is the greater the width of the poset. Amplitude is a parameter for understanding the level of incomparability between elements. When all elements are comparable, we have the special case of complete orders. Incomparabilities can be analyzed through the construction of the anti-chain matrix. The indicators that generate the most incomparabilities are those that have the greatest impact, since their introduction or elimination creates more changes in the position of the elements in the ranking.

Chains and anti-chains can be represented by Hasse's diagram. The Hasse diagram shows all the comparable and incomparable objects in the poset. If one object is connected to another then it is comparable, while if it is not then it is said to be incomparable. By transitivity, all connected items in a chain are comparable.

To better understand poset theory, a clarifying example of the methodology, Hasse diagram and incomparability matrix is given in the Appendix B.

In the case of composite indices, incomparabilities are not adequately considered. This penalizes the multidimensional aspect of the phenomenon being analyzed. Therefore, posets are more suitable in a world where total comparability is an exception. The poset also avoids arbitrary and subjective methodological choices, such as the choice of weights and aggregative methods.

To construct the final ranking, a score is assigned to each element. The method used here is local partial order models (LPOM), computed with PyHasse.com software, which consists of an approximation of the average score. Specifically, given a poset P on a population X, three subsets are identified:

- Down Set  $D(x) = \{y \in P: y \leq x\}$
- Up Set  $U(x) = \{y \in P: y \geq x\}$
- Incomparable  $I(x) = \{y \in P: y \parallel x\}$

The LPOM formula to compute the final score is:

$$H(x) = D(x) \frac{n + 1}{n + 1 - I(x)}$$

Where  $D(x)$  is the number of items ranked such that  $y \leq x$  and  $n$  is the total number of items.

When the goal is to produce a ranking (as in this case), poset is a good solution because it creates orderings. In the case where you want to establish cardinal relationships between objects, however, it is not the best solution. In other words, with poset you can only establish ordinal relationships, but you cannot quantitatively measure the distance separating two objects.

## 4.2 Applications

The original matrix consists of 330 municipalities and 55 indicators for digitization and 37 indicators for complementary factors. The poset results with such large data matrices are difficult to interpret. Therefore, I again use the organization of indicators into dimensions proposed in *Section 3.1* (4 for digitization and 6 for complementary factors). Three matrices are thus obtained:

- A 330x4 for a total of 1320 observations for digitization;
- A 330x6 for a total of 1980 observations for complementary factors;
- A 330x10 for a total of 3300 observations which includes the dimensions of digitization and complementary factors.

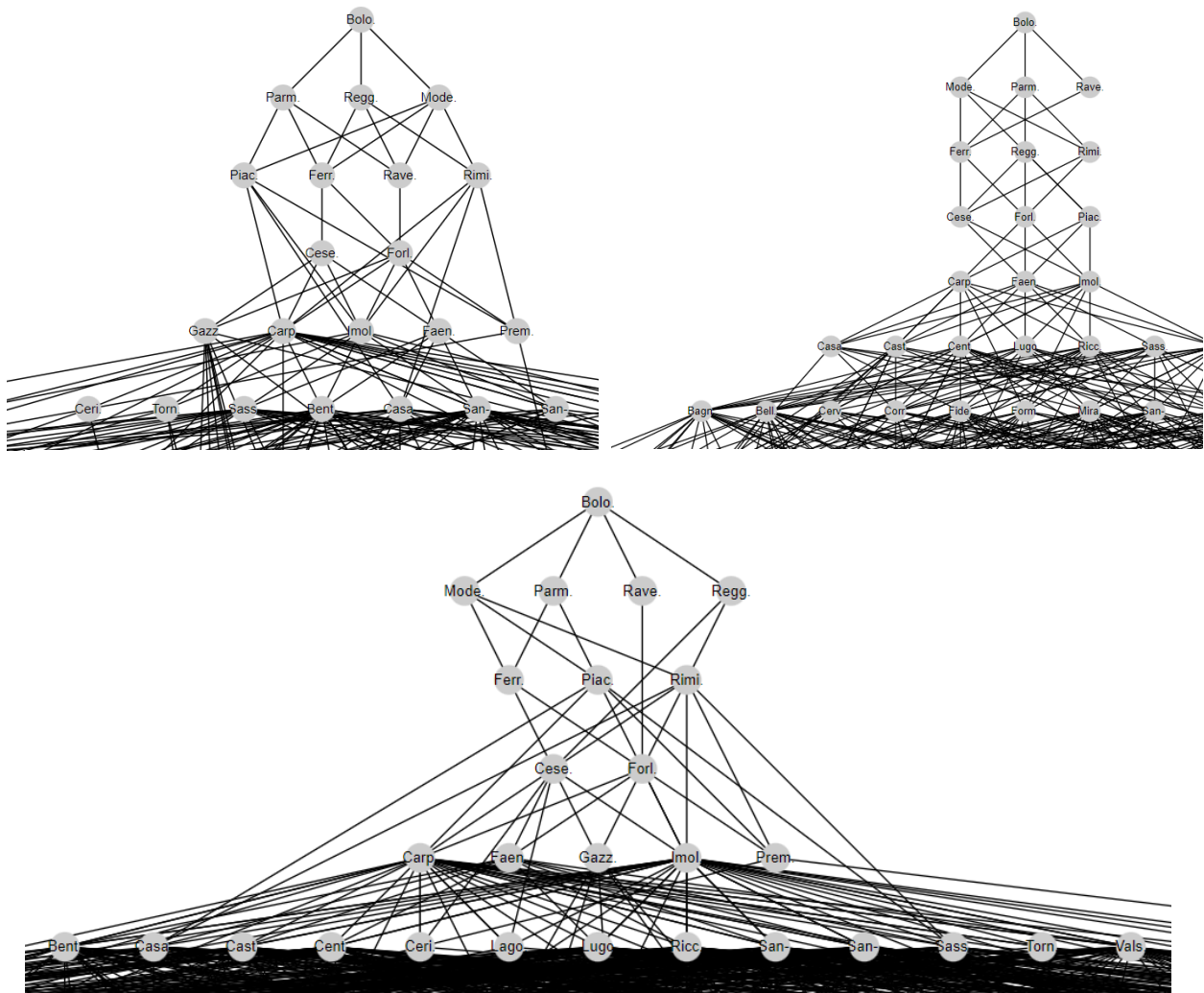
## 5. Results

The index developed analyzes, first separately and then jointly, the key dimensions inherent in digitization and the key dimensions related to complementary factors. The resulting index does not have a name, because the goal is not to create another composite index, but to investigate the relationships between the dimensions. For this reason, a synthetic aggregator is not used. This section presents the main results of the analysis performed using the poset method proposed in the previous section.

### 5.1 The Hasse diagram

First, the results derived from the Hasse diagram are presented. The complete Hasse diagrams are given in Appendix C. Partial sections of them are given here.

Figure 6. Partial section of the Hasse diagram for the digital dimension (left), for the complementary factor dimension (right) and for both dimensions (bottom)



Source: author's elaborations

The ranking levels are organized according to the incomparabilities between the different dimensions and are differentiated by item: there are 20 for digitization, 23 for complementary factors, and 13 for the two dimensions combined. This suggests a much more fragmented subdivision of territory than any other. Incomparabilities are not homogeneous along the diagram. For all three analyses, in fact, it is evident that the anti-chains are narrowed at the top of the diagram (this indicates little incomparability) and then reach maximum breadth in the middle, narrowing again at the end. Predictably, in the first 4 levels all the NUTS-3 capitals are present, albeit with different connections. In the last levels, however, the situation is more heterogeneous. For digitization, the last three levels are occupied by twelve municipalities, all of which are also present in the last two levels of the Hasse diagram that includes all dimensions. For complementary factors, however, the last four levels are occupied by fifteen municipalities, of which only ten are also found in the last levels of the Hasse diagram for all dimensions.

Notice how no one municipality stands out from the others. This means that there is no municipality that is totally incomparable with others.

The connection lines indicate the comparability of one municipality with another. If one municipality is connected and ranks higher than another, it means that it performs better in all dimensions. Bologna,

for example, in each Hasse diagram results alone at the top of the chain and is connected only to municipalities placed lower in the ranking.

## 5.2 The rank

The ranking of municipalities is obtained by calculating the downset  $D(x)$  and incomparabilities  $I(x)$  for each of them in order to calculate the final score using the Local Partial Ordered Model (LPOM) formula  $H(x)$ . The municipalities occupying the first and last ten positions in the ranking are presented below. As is obvious, the top ten municipalities have high downsets and few incomparabilities. In contrast, the bottom ten have low downsets and many incomparabilities. For all rankings, the top ten positions are occupied by the NUTS-3 capital. For the bottom ten positions, however, the variability is greater.

Table 3a. The top 10 and bottom 10 municipalities by digitalization scores, ranked by poset

Rank	Municipality	Digitalization score	Rank	Municipality	Digitalization score
1	Bologna	330,0	321	Riolunato	3,3
2	Modena	329,0	322	Coli	3,1
3	Parma	329,0	323	Piozzano	2,9
4	Reggio nell'Emilia	329,0	324	Sassofeltrio	2,6
5	Piacenza	326,9	325	Bore	2,6
6	Rimini	326,9	326	Terenzo	2,5
7	Ferrara	326,0	327	Varsi	2,5
8	Ravenna	325,9	328	Valmozzola	1,5
9	Cesena	324,9	329	Farini	1,2
10	Forlì	323,9	330	Montecopiolo	1,0

Table 3b. The top 10 and bottom 10 municipalities by complementary factor scores, ranked by poset

Rank	Municipality	Complementary factors score	Rank	Municipality	Complementary factors score
1	Bologna	330,0	321	Piozzano	2,2
2	Parma	329,0	322	Valmozzola	2,1
3	Modena	329,0	323	Pellegrino-Parmense	1,2
4	Ravenna	329,0	324	Farini	1,2
5	Reggio-nell'Emilia	328,0	325	Sassofeltrio	1,1
6	Rimini	327,0	326	Besenzone	1,1
7	Ferrara	326,0	327	Coli	1,1
8	Piacenza	325,9	328	Zerba	1,1
9	Forlì	323,0	329	Corte-Brugnatella	1,1
10	Cesena	322,9	330	Casteldelci	1,0

Table 3c. The top 10 and bottom 10 municipalities by both scores, ranked by poset

Rank	Municipality	Both scores	Rank	Municipality	Both scores
1	Bologna	330,0	321	Ottone	2,1
2	Modena	329,0	322	Maiolo	1,8
3	Parma	329,0	323	Coli	1,6
4	Ravenna	329,0	324	Piozzano	1,5
5	Reggio-nell'Emilia	329,0	325	San-Pietro-in-Cerro	1,5



6	Rimini	327,9		326	Valmozzola	1,5
7	Ferrara	327,0		327	Terenzo	1,4
8	Piacenza	326,9		328	Bore	1,4
9	Cesena	324,9		329	Farini	1,3
10	Forli	323,9		330	Montecopiolo	1,2

Source: author's elaborations

Figure 7 shows the scatter plot of the relationship between the score of municipalities in digitization and in complementary factors. The green line represents the regression line. Notice how there is a strong correlation between the two factors in the tails. Municipalities that perform excellently in one of the two items also perform well in the other. Similarly, municipalities that perform poorly in one of the two factors also perform poorly in the other. The coefficient of determination ( $R^2$ ) equal to 56% indicates a strong positive relation between complementary factors score and the digitalization score. Nevertheless, several municipalities lie far from the correlation line.

Red lines represent the median score. The median score is 169.96 for the complementary factors and 123.005 for digitization. In 78 percent of cases, if a municipality is above (or below) the median for digitization, it is also above (or below) the median in the score obtained on context factors. There are 37 municipalities with scores above the median in digitization and below the median in complementary factors and 37 municipalities below the median in digitization and above the median in complementary factors, that represent an exception.

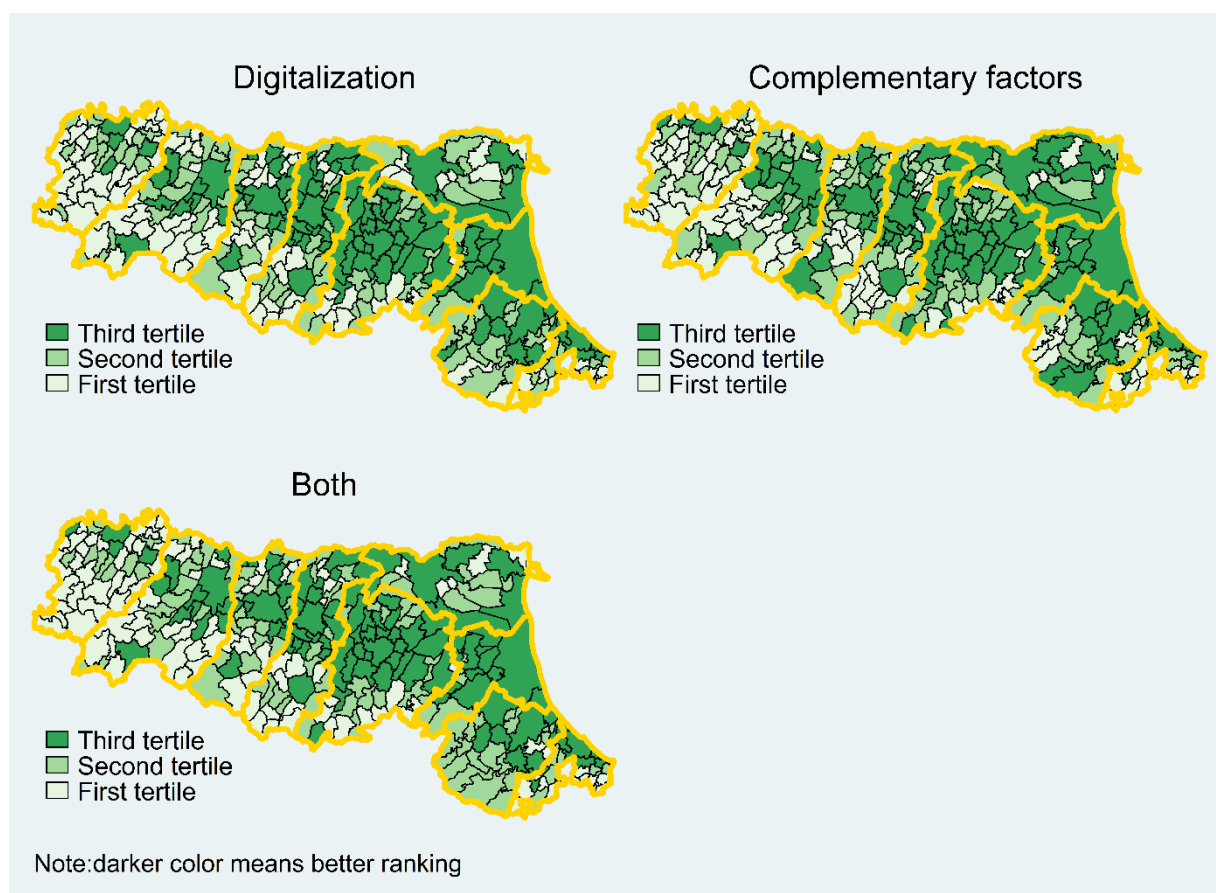
Figure 7. Scatter plot of municipalities scores



Source: author's elaborations

Figure 8 shows the distribution by tertiles of municipalities according to the poset score. For all cases presented municipalities in the third tertile (dark green) are characterized by a high average density (greater than 400 inhabitants per square kilometer) and an average population of more than 30,000. It contains all the urban poles. For municipalities in the first tertile (light green), the average population density is very low (between 70 and 80 inhabitants per square kilometer) and the average population is between 2,500 and 3,000. From this it is clear that small and less densely populated municipalities score low in the ranking by poset.

Figure 8. Spatial distribution of municipalities ranked by tertiles of poset



Source: author's elaborations

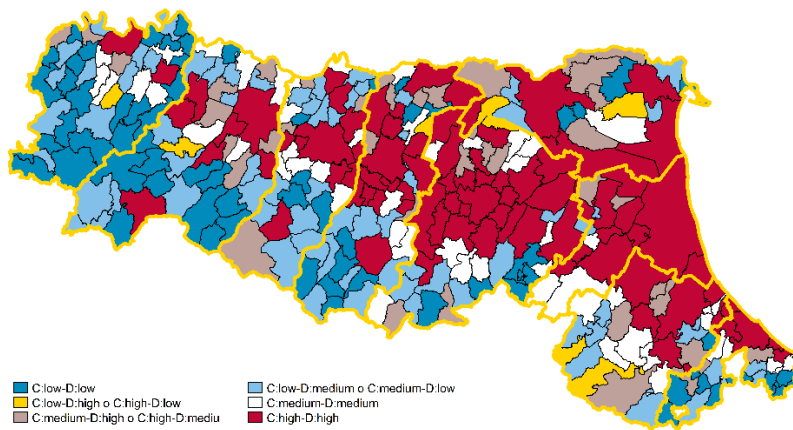
As mentioned above, there is positive correlation between the score obtained in digitization and the score obtained in the quality of complementary factors. *Figure 9* confirms this relationship. If a municipality belongs to a certain tertile in digitization, it often belongs to the same tertile in context factors. By cross-referencing the tertile membership, it is possible to verify that it often occurs. The red (blue) municipalities are those belonging to the first (third) tertile of the digitization and complementary factors rankings. These are the most numerous cases. In fact, 87 (69) municipalities belong to the third (first) tertile for both items. Also, the white ones are in the middle tertile of the ranking for both items. This is the case of 53 municipalities.

28 percent of the red-colored municipalities are classified as 'inner areas' according to the SNAI classification. Of these 24 municipalities, 6 belong to peripheral 'inner areas'. They are Borgo Val di Taro, Castelnovo ne' Monti, Cento, Codigoro, Pavullo nel Frignano and Vergato. All six are municipalities with a population significantly higher than the average for municipalities classified as peripheral. The lowest performing municipalities (colored in blue) are 70 percent 'inner areas'. The 30% that do not belong to this category are part of the so-called outlying areas. They have a significantly lower population than the average of municipalities in the same category. Once again, therefore, the size in terms of population of a municipality seems to be a determining factor in its performance in digitization or context factors.

The two middle colorings (pink and light blue) represent municipalities characterized by good performance in complementary factors and average performance in digitization and vice versa. The 8 yellow municipalities represent the most unique cases. 5 of these belong to the first tertile for complementary factors and the third for digitization. The opposite is true for the other 3. They are all

‘inner areas’. The 5 all have a significantly lower population than the average of the municipalities in the same category. These are the municipalities of Galliera, Ponte dell'Olio, Portico and San Benedetto, Ravarino, and Varano de' Melegari. The remaining 3, on the other hand, have a population around or above the average for municipalities in the same category. These are Fiscaglia, Santa Sofia, and Terre del Reno.

Figure 9. Distribution of municipalities ranked by poset score by cross-referenced tertiles



		Tertiles of digitization score		
		I	II	III
Tertiles of complementary factors score	I	69	36	5
	II	39	53	18
	III	3	20	87

Source: author's elaborations

### 5.3 Sensitivity analysis

The third stage involves sensitivity analysis. Sensitivity analysis can provide information about the impact that individual dimensions have on the definition of the score in the poset ranking. In other words, they tell us which dimension is most important in defining the partial order structure.

Sensitivity analysis compares the distances between the poset containing all dimensions and the posets in which one dimension is excluded. When the distance of the excluded dimension is high, it means that this dimension is important for the ranking structure. We can therefore call this analysis "dimension-related sensitivity analysis."

The sensitivity analysis for the digital dimension is depicted in *Table 3a*. The first row compares the distance of the excluded dimension "human capital" from the other three dimensions of digitization. The final impact of the dimension is given by the sum of the distances from all dimensions. This is also true for the subsequent dimensions. For digital dimensions, there is no substantial difference between the distances. However, the maximum distance is given by the ‘connectivity’ dimension. This dimension is the most important, i.e., the one that has the greatest impact in defining the ranking.

Table 4a. Distance matrix for digitization dimensions

	Human capital	Connectivity	Integration of digital technologies	Digital public services	Distance
Human capital		18492	16957	16913	52362
Connectivity	18492		21737	19290	59519
Integration of digital technologies	16957	21737		15942	54636
Digital public services	16913	19290	15942		52145

Source: author's elaborations

The distance matrix for the complementary factors is shown in *Table 3b*. In this case it is clear which dimension has the greatest distance. The ‘culture’ dimension has a significantly greater impact than

the other dimensions, so it is the most important in defining the final position of a municipality in the rank.

Table 4b. Distance matrix for complementary factor dimensions

	Foreigners	Environmental quality	Services	Culture	Gender	Income and employment	Distance
Foreigners		5121	10195	15906	5475	5977	42674
Environmental quality	5121		9877	15275	2373	3601	36247
Services	10195	9877		14918	9567	10337	54894
Culture	15906	15275	14918		14783	14935	75817
Gender	5475	2373	9567	14783		3748	35946
Income and employment	5977	3601	10337	14935	3748		38598

Source: author's elaborations

When all dimensions are considered together, the impact of ‘connectivity’ is greater than that of ‘culture’ in defining the ranking that considers both aspects.

The analysis is repeated on the clusters identified in *Section 3.3*, obviously not considering the cluster formed by the municipality of Bologna alone. In the clusters obtained by considering the dimensions of complementary factors, the ‘culture’ dimension has a greater impact in defining the ranking. The distance of the dimension ‘foreigners’ is slightly higher than that of the dimension ‘culture’ only in the cluster containing urban poles. This means that, for these municipalities, both dimensions have great importance in defining the ranking. For the clusters obtained by considering the digitization dimensions, ‘connectivity’ is the dimension of greatest impact for only one cluster (colored pink in *Figure 5*). For the other two, the dimension with the greatest distance is ‘integration of digital technologies’.

Distances are on the order of tens of thousands in the clusters of municipalities marked in pink, while they are on the order of a few tens for the cluster containing the municipalities marked in gray. This means that the inclusion or exclusion of a dimension has a much greater impact in defining the ranking of municipalities belonging to the first cluster than for those belonging to the second.

## 6. Robustness check

In this section, some robustness checks are reported to provide further validation of the rankings. Starting from the sensitivity analysis proposed in the previous section, the rankings of the municipalities are recalculated by removing the lowest and highest impact variables.

For digitization, the ranking is recalculated once by removing the ‘digital public services’ dimension (the least impactful) and once by removing the ‘connectivity’ dimension (the most impactful). As expected, eliminating the most impactful dimension creates greater imbalances in the ranking. Only twelve municipalities maintain the same position. The municipality that loses the most positions (142) is Lagosanto and the one that gains the most (128) is Gropparello. This is because in relation to the score for the other dimensions, that obtained in ‘connectivity’ is low for Gropparello and high for Lagosanto. Calculating the ranking for ‘digital public services’ there are twenty-four municipalities that maintain the same position, nineteen of which occupy the top 20 positions. This means that these municipalities in the other dimensions get relatively similar scores.

While for digitization in the sensitivity analysis no significantly different distances emerged, for the complementary factors the dimension ‘culture’ has a significantly greater distance than the other dimensions. In other words, while for digitization each dimension contributes more or less similarly and only ‘connectivity’ seems to have a slightly greater impact, for the complementary factors it is clear which dimension contributes more to define the ranking.

This is also evident in this context. To analyze the robustness of the ranking, the poset methodology is applied by removing once the least impactful dimension (‘gender’) and once the most impactful dimension (‘culture’). In this case, since the distances are very different, the contribution of each dimension in drawing up the ranking is even more evident. In fact, drawing up the ranking without considering the ‘gender’ dimension, no major shifts are noticeable. One hundred forty-nine municipalities hold the same position, with a maximum loss of 9 positions and a maximum gain of 13 positions. The situation changes considerably if the most impactful dimension is removed. In this case, only eighteen municipalities maintain the same position in the ranking, including 9 in the top 13 positions. The municipality that loses the most positions (123) is Lizzano in Belvedere, which has a relatively high score in ‘culture’ compared to that obtained in the other dimensions. The municipality that gains the most positions (153) is Torrile, which, however, has a low score in ‘culture’ compared to that obtained in the other dimensions.

The ranking is very robust for the top positions, which remain virtually unchanged regardless of the elimination of some dimension. This is because these municipalities have homogeneous scores in all dimensions. The greater the inhomogeneity of scores across dimensions, the greater the positions lost or gained by a municipality if a dimension is eliminated. This is even more true when the dimension eliminated is the most impactful for ranking purposes.

Note also that in the case of a ranking produced by aggregating dimensions through a simple average, relatively high (low) scores compensate (penalize) the municipality. Each dimension has equal weight, resulting in a flattening of performance. Therefore, the elimination of a variable considered to have a high (low) impact in the poset results in fewer (greater) upsets in ranking positions than in the case where the ranking is produced by poset methodology.

## **7. Discussion and conclusion**

After the Covid-19 pandemic, the degree of digitization has increased for several sectors of the economy. This positive trend has characterized all of Europe, which has been monitoring the state of digitization of member countries through the DESI composite index since 2014. Through the Recovery and Resilience Plan, several funds have been allocated to help the digital transition process. This is justified by the fact that there are several contributions in the literature that consider digital as an enabler for development and quality of life (Vasetskaya and Gaevskaia, 2019; Myovella et al., 2020; Habibi and Zabardast, 2020; Kryzhanovsky et al., 2021; Elmassah and Hassanein, 2022).

In line with the European strategy, the Emilia-Romagna Region has developed a strategic plan to rethink society digitally, called “*Data Valley Bene Comune*”.

However, digital is not by itself a determinant of development. It is necessary to consider the relationships between it and several factors (here called complementary factors) with which it must coexist (Crandall et al., 2007; OECD, 2008; Katz et al., 2010; Mack and Faggian, 2013; Matteucci, 2020; Torres and Augusto, 2020).

Therefore, the study aims to analyze the territory of Emilia-Romagna through indicators of digitization and development of complementary factors to identify areas where this link is present or needs to be strengthened.

The indicators are selected based on data collected by Art-ER during the implementation of Challenge 7 of the DVBC project. There are 55 (grouped into 4 dimensions) for digitization and 37 (grouped into 6 dimensions) for complementary factors. The dimensions are obtained by simple average following a min-max normalization procedure.

Descriptive analysis shows that both digitization dimension scores and complementary factor scores are highly concentrated in the lower tail of the distribution. Looking at the overall performance, the resulting spatial localization of the best/worst-performing municipalities recalls the SNAI national classification. This affects the performance at NUTS-3 level, namely NUTS-3 regions with a larger share of peripheral and ultra-peripheral areas are the worst performers. Instead, the best-performing municipalities are close to each other and are concentrated near the region's urban poles.

This is confirmed by the hierarchical cluster analysis carried out using Ward's method. This identifies four groups, one of which consists exclusively of Bologna (representing an outlier). The cluster analysis suggests that performance on digitization dimensions goes hand in hand with performance on complementary factors. This is also confirmed by the correlation matrix.

A poset analysis is proposed to produce a ranking of municipalities that avoided penalizing the multidimensionality of the phenomenon. This led to three types of results.

1. Hasse's diagram suggests an alternative territorial division. While the municipalities at the top of the ranking (the urban poles) have a low degree of incomparability, these grow in the middle of the ranking and then decline again. This means that the top 10 ranking contains municipalities that are much more similar to each other than those in the middle part of the ranking. No municipality remains completely isolated, that is, no municipality is totally incomparable with the others.
2. Using the Local Partial Order Model method, it is possible to score each element, producing a final ranking that takes incomparabilities into account. The correlation between the score obtained for digitization and the score obtained in the complementary factors is very strong in the extremes. The positive relationship is also confirmed by a high coefficient of determination. While in 78 percent of cases a municipality scores above (or below) the median score in both digitization and contextual factors, there are some municipalities that represent a hybrid case. In fact, there are 37 municipalities that score above the median in digitization and below the median in complementary factors and 37 municipalities that score below the median in digitization and above the median in complementary factors. The top-rated municipalities are characterized by a population above 30,000 with high density. Breaking down the municipalities according to their scores in the digitization ranking and in the complementary factors ranking by tertiles and crossing the memberships confirms the positive relationship. Again, the reference to the SNAI classification is confirmed. Among the best (lowest) performing municipalities, only a part (not) falls within the inner areas. These are municipalities with a higher (lower) than average population than those in the same category. Eight municipalities represent a very special case: they belong to the first tertile for one category and to the third for another.
3. Sensitivity analysis provides information on which dimensions have the greatest impact in determining the score and thus the ranking. While for digitization no substantial differences in impact emerge, for the complementary factors it is clear which dimension has the greatest

impact, namely ‘culture’. When considering all dimensions together, it is clear that the impact of ‘connectivity’ is greater than that of ‘culture’ in determining the ranking. Repeating the analysis on the clusters identified in *Section 3.3*, we see that the distances in the cluster marked in pink are much larger than those in the cluster marked in gray. This means that for the former the inclusion or exclusion of a dimension has a greater impact than for the latter.

Finally, some robustness checks are performed. These confirm that the ranking is robust for municipalities in the top positions that obtain homogeneous scores. The greater the inhomogeneity of scores among dimensions, the greater the positions lost or gained by a municipality if a dimension is eliminated. This is even more true when the dimension eliminated is the most impactful for ranking purposes.

In conclusion, there is a correlation between the level of digitization and the development of complementary factors. The dimensional aspect is very important in determining performance. The municipalities where the relationship is strongest are in the top 10 of the ranking. They are high populated and are characterized by high housing density. Cluster analysis groups them into the gray cluster (plus the blue Bologna) in *Figure 5*.

In general, a regional picture emerges characterized by strong spatial differences. There are differences between clusters but also within clusters themselves. It is evident which municipalities perform better and which perform worse, as suggested by the cluster analysis and the rank per poset. But even within the same cluster there are substantial differences. These are less pronounced at the two extreme poles of the ranking, where the incomparabilities are smaller due to homogeneous scores across dimensions. For municipalities occupying the innermost positions, however, there are substantial differences in the scores obtained across dimensions. As suggested by the robustness check, policies should target these municipalities. Particularly on those municipalities that score low in only one dimension. An increase in their performance would lead to an improvement in their ranking, especially if that dimension is an impact factor among those identified in the sensitivity analysis.

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## Appendix A – Indicators, sources, and reference year

Dimension	Indicators of the degree of digitization	Source	Year
Human capital	N. of trainees trained with PEI courses out of the total n. excluded	Emilia-Romagna region	2022
	N. of women trained through PEI courses out of the total n. trained	Emilia-Romagna region	2021
	Participants in ESF training courses with at least one digital module	Emilia-Romagna region	2021
	Female participants in ESF training courses with at least one digital module	Emilia-Romagna region	2021
	Female STEM students out of total STEM students (high school)	MIUR	2020/21
	N. of locations with digital assistance services per 100 excluding digital	Emilia-Romagna region	2022
	N. of graduates per 1000 residents > 8 years old	Istat	2020
	% employees in innovative local units related to the digital economy out of total employees	Asia	2019
	% employees in the ICT sector of total employees	Asia	2019
	N. of services delivered in innovation spaces per 1000 residents	Art-ER	2022
	N. of fans of Facebook accounts of municipalities per 100 residents	ADER	2022
	EGov: Social PA Index	ADER	2022

	Presence of at least one Facebook account on tourism and/or cultural events DUMMY	ADER	2022
	N. of local apps	ADER	2021
	% online payment of fines out of total municipal fines	Emilia-Romagna region / Lepida	2021
	N. of participants in PEI ON LINE courses per 1000 residents	Emilia-Romagna region	2020
<b>Connectivity</b>	% civics covered by 100 mbps bandwidth	Lepida	2022
	N. of connected manufacturing areas out of total industrial areas surveyed	Lepida	2022
	Schools connected to Lepida out of total schools with BUL 1 Gbps	Emilia-Romagna region / Lepida / ARES	2022
	Lepida network access points per sq. km	Lepida	2022
	Population covered by 4G mobile phone services	Istat	2018
	% civics covered by bandwidth 2-30 mbps	Lepida	2022
	Housing units available in FTTH saleability per 100 residents	Lepida	2022
	N. of public wi-fi access points per 1000 inhabitants ER-WIFI network	Lepida	2022
	Municipalities connected by fiber DUMMY	Lepida	2022
	PaIoT and sensornet network sensors per sq. km.	Lepida	2022
<b>Integration of digital technologies</b>	N. of innovation spaces per 1.000 residents-excluding libraries and PEI points	Art-ER	2022
	% Innovative Local Units related to the digital economy out of total LU	Asia	2019
	% Local Units in the ICT sector of total LU	Asia	2019
	Emilia-Romagna individuals who are members of at least one CLUST-ER for every 100 residents	Art-ER	2021
	Laboratories and high-tech network centers per 1.000 residents	Art-ER	2021
	Total number of start-ups per 100 residents	Registro imprese	2022
	N. of female start-ups on total start-ups by municipality	Registro imprese	2022
	Mln € of contributions/Mln € of approved investments related to interventions with S3 funding (all years available) by municipality	Emilia-Romagna region	2014/2021
	N. of innovative SMEs per municipality per 100 residents	Registro imprese	2022
	N. of women's SMEs out of total innovative SMEs by municipality	Registro imprese	2022
<b>Digital public services</b>	Interactivity index of municipal services	ADER	2022
	Service on appIO	io.italia.it	2021/22
	N. of municipal interactive services with SPID out of total municipal interactive services activated	ADER	2022
	N. of municipal interactive payment services with PAYER or other p@gopa-compliant platform out of total municipal interactive online payment services activated	ADER	2022
	% online services to businesses (ratio of potential universe of online services targeting businesses to those that have them)	ADER	2022
	N. of published datasets	ADER	2022
	Audio - video of municipal sessions DUMMY	ADER	2022

	Digitalized SUAP (both submission and payment online)	ADER	2022
	Municipalities with open data sections and allowing API queries, SPRQL or BULK downloading	ADER	2022
	Index of municipal library as a digital knowledge dissemination point	ADER	2022
	Utilization index of interactive services	Lepida	2020
	SPID users (issued by LepidaID) per 100 adult residents	Lepida	2021
	Average use of SUAP practices	Emilia-Romagna region	2021
	SPID LepidaID recognition counters for every 1.000 residents of legal age	Lepida	2022
	Number of thematic communities participated in by the municipality (directly or through union) out of the 11 COMTEMs activated	Lepida	2022
	Municipalities with digital transition manager DUMMY	Indice PA	2022
	% of tenders conducted electronically out of total tenders	Art-ER	2020/21
	Index of "digital museum"	Istat	2019
	Assisted with active ESF per 100 residents	Emilia-Romagna region	2021

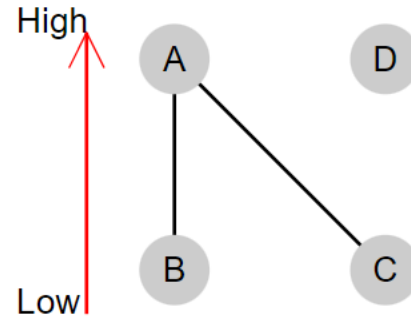
<b>Dimension</b>	<b>Indicators of the degree of quality of complementary factors</b>	<b>Source</b>	<b>Year</b>
<b>Foreigners</b>	% foreign residents	Emilia-Romagna region	2022
	% foreign active enterprises to total active enterprises	SMAIL	2019
	Foreign children enrolled in daycare as a percentage of total enrollment	Emilia-Romagna region	2020/21
<b>Environmental quality</b>	% KG waste separation of total waste generated	ISPRA	2022
	Non-consumption of land (% sq. km not consumed per sq. km of land area)	Snp ambiente	2021
<b>Services</b>	LPT stops per sq. km.	Lepida	2021
	N. of general practitioners per 1.000 population	SOLE project	2022
	N. of health houses per 1000 inhabitants	ReportER	2022
	Socio-assisted living facilities for the elderly per 100 elderly residents	ReportER	2022
	N. municipal or public kindergartens per 100 residents 0-2 years old	Emilia-Romagna region	2022
	N. municipal or public non-state preschools per 100 residents 3-5 years old	Emilia-Romagna region	2022
	N. state elementary school per 100 residents 6-10 years old	Emilia-Romagna region	2022
	N. state secondary schools per 100 residents 11-13 years old	Emilia-Romagna region	2022
	N. state secondary schools per 100 residents 14-18 years old	Emilia-Romagna region	2022
	Bike and/or car sharing DUMMY	ADER	2022
<b>Culture</b>	N. sports associations per 1000 residents	Emilia-Romagna region	2022

	N. cultural associations per 1000 residents	Emilia-Romagna region	2022
	N. environmental associations per 1000 residents	Emilia-Romagna region	2022
	N. tourism associations per 1000 residents	Emilia-Romagna region	2022
	N. welfare associations per 1000 residents	Emilia-Romagna region	2022
	N. of (municipal) libraries per 1000 residents	ADER	2022
	N. of museums per 1000 residents	Emilia-Romagna region	2022
	N. of cinemas per 1000 residents	Emilia-Romagna region	2021
	N. of theaters per 1000 residents	Emilia-Romagna region	2022
	Billboard events per 1000 residents	Emilia-Romagna region	2021
	Tourist attendance per 100 average residents	Emilia-Romagna region	2021
<b>Gender</b>	% women graduates of total graduates by municipality of residence	Istat	2020
	% Active female enterprises out of total active enterprises	SMAIL	2019
	% women hired out of total hires	SILER	2021
	% women elected to city council out of total number of councilors	Ministry of internal affairs	2022
<b>Income and employment</b>	N. of SMEs per 100 inhabitants	SMAIL	2019
	N. of large enterprises per 100 inhabitants	SMAIL	2019
	N. of employment contracts activated per 100 residents 15-64 years old	Employment agency	2021
	Average total IRPEF income	MEF	2020
	College students per 100 residents 19-34 years old	Istat	2017

## Appendix B - Clarifying example of poset methodology

A ranks better than B and C because for each  $q_i$  it scores higher than or equal to B and C. Because of its better performance it positions at the top of the Hasse diagram and is connected to B and C. B and C are not comparable to each other because they are not connected. This is because B scores higher than C in  $q_2$ ,  $q_3$  and  $q_4$ , but lower in  $q_1$ . D is not comparable with all other items because in no case does it score higher/worst or equal to another item in all categories. In Hasse's diagram, in fact, it is isolated from all others. The height of this poset is equal to two because two are the items in the longest chain. Also the width of the poset, or incomparability number, is equal to two.

	q1	q2	q3	q4
A	5	3	5	4
B	2	3	4	4
C	3	2	2	1
D	0	5	5	2



Finally, using the LPOM formula, the scores for each object can be calculated. This results in the following:

$$H(A) = 3 \frac{4 + 1}{4 + 1 - 1} = 3,75$$

$$H(B) = 1 \frac{4 + 1}{4 + 1 - 2} = 1,67$$

$$H(C) = 1 \frac{4 + 1}{4 + 1 - 2} = 1,67$$

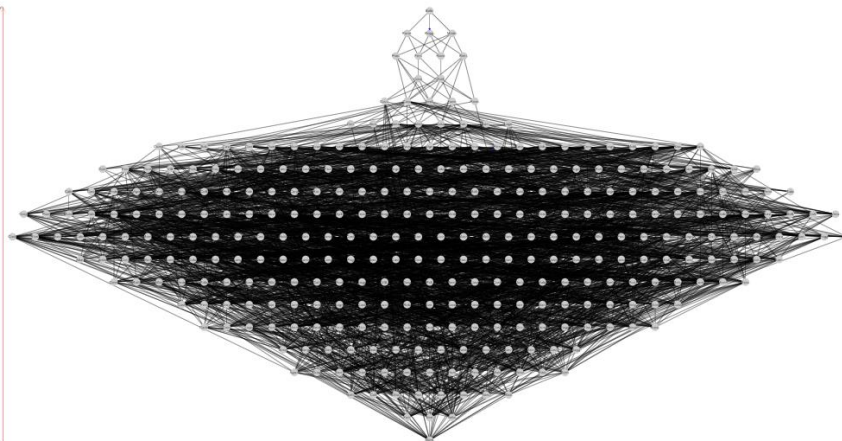
$$H(D) = 1 \frac{4 + 1}{4 + 1 - 3} = 2,50$$

Average score		LPOM score	
A	4,25	A	3,75
B	3,25	D	2,50
D	3	B	1,67
C	2	C	1,67

The ranking by poset is A,D,B=C. Note how it change consistently if compared to a ranking obtained with the arithmetic mean.

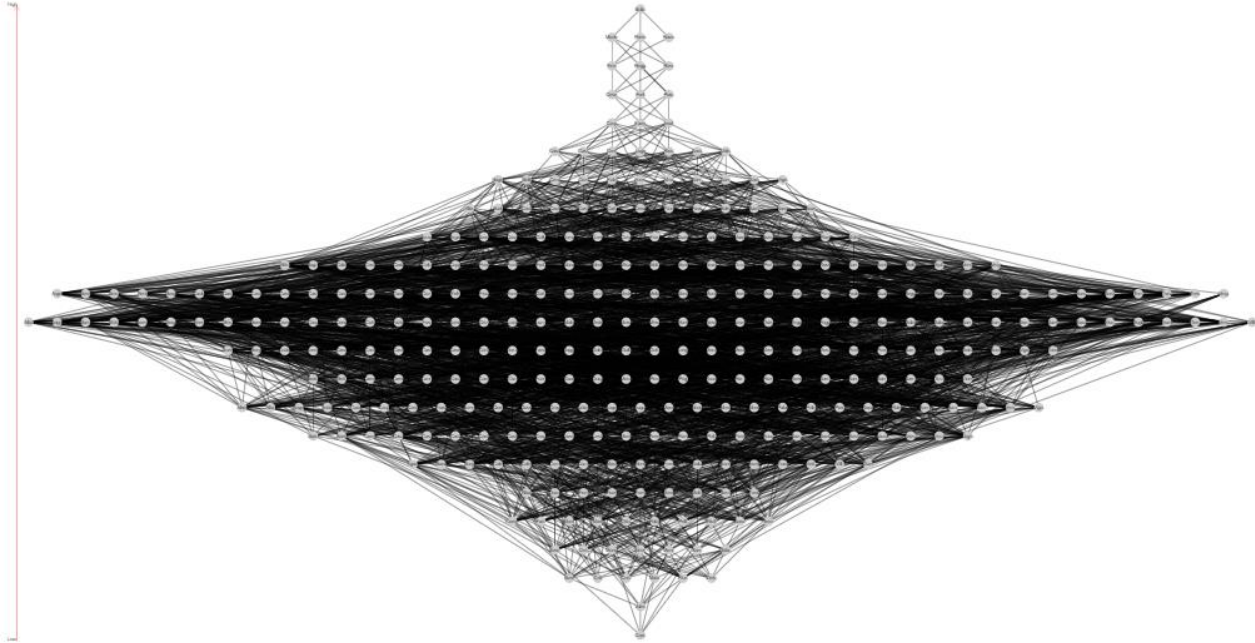
## Appendix C – Complete Hasse diagrams

Digitalization's Hasse diagram





Complementary factors' Hasse diagram



Hasse diagram for both dimensions

