Corporate Income Tax, Formula Apportionment, and Multi-regional Firms

Santiago M. Pinto
Federal Reserve Bank of Richmond

February 18, 2026

Preliminary and incomplete draft
Comments are welcome

Abstract

We examine the determinants of the corporate income tax system when firms operate in multiple regions and local governments use a formula apportionment (FA) system to allocate the firms' corporate income across regions. When a firm, such as a multi-regional firm (MRF), has business activities in multiple regions, measuring income earned in each region raises a complex conceptual and administrative problem. In the FA regime, the MRF's taxable income is determined using a formula that considers the company's capital, sales, and labor shares at each location. The present paper develops a theoretical framework that generalizes the analysis of Pinto (2007). In our setup, a MRF has a presence in two regions, and regional governments strategically decide the weights in the formula. The MRF produces a homogeneous good in one or two regions, and the good can be sold domestically or shipped to the subsidiary in the other region. Domestic prices are endogenously determined. We derive the decentralized FA system when regional governments consider the formula's impact on the welfare of domestic consumers, tax revenue, and the MRF, and compare these outcomes with those of centralized solutions.

Keywords: corporate taxation, formula apportionment, strategic competition

1 Introduction

The last two decades have been characterized by a rising participation of multi-regional firms (MRFs) in global activity, along with the rise in foreign direct investment inflows. The growing mobility of capital and the concomitant importance of foreign direct investment have given a new meaning to fiscal competition, especially concerning corporate income taxes.[1] When an MRF has business activities established in multiple regions, the regional authority can levy a corporate tax on income generated in that region. In this context, MRFs can shift profit from a high tax jurisdiction to a low tax jurisdiction by shifting activities from one location to another. As a consequence, regional tax revenue will be affected.

Measuring income earned within each region raises a difficult conceptual and administrative problem. The current system of corporate taxation in the European Union, for example, requires firms to maintain different accounts for its activities in each region where it operates (Separate Accounting (SA)). The U.S.A. and Canada, on the other hand, have adopted a system of formula apportionment (FA) to allocate income across states.[2] The FA system used in the U.S.A. assumes that the proportion of a multi-regional firm's income earned in a given state is a weighted average of the proportion of the firm's total sales, property (or capital), and payroll in that state. Thus, the firm's activities in a specific region is approximated by the share of these factors in the region, so the firm is not required to keep different accounts. Specifically, let I denote the set of states where the firm operates. The tax due by a firm to state $i \in I$ is $T^i = t^i \gamma^i \Omega^i$, where t^i is state i's tax rate, γ^i is the share of total profits that are subject to taxation in state i according to the formula selected by that state, and Ω^i represents the firm's taxable income as defined by state i's tax law.[3] The share γ^i is defined by

$$\begin{aligned} \gamma^i &= m^{ki}\left(\frac{k^i}{\sum_{i \in I} k^i}\right) + m^{si}\left(\frac{s^i}{\sum_{i \in I} s^i}\right) + m^{wi}\left(\frac{w^i}{\sum_{i \in I} w^i}\right) \\ &= m^{ki}\lambda^{ki} + m^{si}\lambda^{si} + m^{wi}\lambda^{wi} \end{aligned} \tag{1}$$

[1]Recent literature has focused on tax competition between MRFs. For instance, Devereux and Griffith (1998) emphasizes the importance of effective marginal tax rates on the locational decisions of foreign investors. Additionally, DeMooij and Ederveen (2003) find that a 1 percentage point reduction in the host country tax rates leads to an increase of 3.3 percent in foreign direct investment inflows.

[2]Mintz and Smart (2004) provide a good description of the subnational corporate tax system in Canada. In Canada, Provinces use the same method of allocating income across jurisdictions. Specifically, the general formula is given by the sum of payroll and sales shares in a province divided by two.

[3]Regional governments may use different rules to define tax bases according to their tax codes. The present paper considers one way in which tax bases may differ across regions: the proportion of capital costs that can be deducted from the corporate taxable income.

Table 1: Formulas for tax year 2016

State	S	W	K	Manuf.	State	S	W	K	Manuf.
AK	33%	33%	33%		MT	33%	33%	33%	
AL	50%	25%	25%		NC	75%	13%	13%	
AR	50%	25%	25%		ND	33%	33%	33%	
AZ	50%	25%	25%	90% S, 5% W, 5% K	NE	100%	0%	0%	
CA	100%	0%	0%		NH	50%	25%	25%	
CO	100%	0%	0%		NJ	100%	0%	0%	
CT	100%	0%	0%	33% S 33% W, 33% K	NM	33%	33%	33%	70% S 15% W, 15% K
DC	100%	0%	0%		NV				
DE	33%	33%	33%		NY	100%	0%	0%	
FL	50%	25%	25%		OH	60%	20%	20%	
GA	100%	0%	0%		OK	33%	33%	33%	
HI	33%	33%	33%		OR	100%	0%	0%	
IA	100%	0%	0%		PA	100%	0%	0%	
ID	50%	25%	25%		RI	100%	0%	0%	
IL	100%	0%	0%		SC	100%	0%	0%	
IN	100%	0%	0%		SD				
KS	33%	33%	33%		TN	50%	25%	25%	
KY	50%	25%	25%		TX	33%	33%	33%	
LA	33%	33%	33%		UT	100%	0%	0%	
MA	50%	25%	25%	100% S	VA	50%	25%	25%	
MD	50%	25%	25%	100% S	VT	50%	25%	25%	
ME	100%	0%	0%		WA				
MI	100%	0%	0%		WI	100%	0%	0%	
MN	96%	2%	2%		WV	50%	25%	25%	
MO	33%	33%	33%		WY				
MS	100%	0%	0%						

Note: S: sales; W: payroll; K: property

where k^i, s^i, and w^i are property (or capital), sales, and payroll in state i, respectively; m^{qi} is the weight given to factor $q = k, s, w$ in the apportionment formula in state i, such that $0 \leq m^{iq} \leq 1$, and $m^{ki} + m^{si} + m^{wi} = 1$; and λ^{qi} is the share of factor q in state i. States in the U.S.A. do not follow the same principle when choosing the apportionment method. The Multi-state Tax Compact of 1967 recommended states to adopt a uniform, equally weighted formula (EWF), i.e., $m^{ki} = m^{si} = m^{wi} = 1/3$ for all regions i. Since the late 1970's, however, almost two-thirds deviated from the standard system and most of them adopted a system where the sales factor is double weighted (DWSF), i.e., $m^{si} = 1/2, m^{ki} = m^{wi} = 1/4$. Table 1 shows the actual formulas chosen by the states as of 2016.

Even though this method of apportionment is relatively easy to administer, it creates very complicated incentive effects. On one hand, firms operating in different regions react to different formulas by changing the allocation of property, sales and workers across regions. On the other hand, given that the tax policy chosen by different regional governments affects residents of other states, some kind of strategic interaction can be expected.

An additional problem arises when regions are allowed to choose their own FA systems. If they all adopt the same formula, exactly 100 percent of a firm's income will be apportioned across states.[4] Non-uniformity, however, can result in more or less than 100 percent of a

[4] In other words, $\sum_{i \in I} \gamma^i = 1$.

firm's income being subject to state income tax.[5]

It has been claimed that by deviating from a uniform formula to one that weighs more heavily the sales portion, tax authorities can offer tax breaks and help the economic development of the region. However, if more states pass such legislation, other states will be compelled to do the same, initiating a "race to the bottom", in which all states end up imposing the same (lower) tax liability. Anand and Sansing (2000) suggest in their paper that the choice between EWF and DWSF for the states depends on the desire of the states to tax immobile capital, such as agriculture and natural resources, rather than taxing mobile capital such as manufacturing. Pinto (2007), on the other hand, shows that the generalized shift towards a formula that predominantly weighs the sales shares can be justified as an attempt by regional governments to implement "higher levels" of local public goods.[6]

In this paper, we develop a theoretical framework that generalizes the analysis of Pinto (2007), focusing on the issues described above. In our setup, we assume that a MRF can operate in two countries or regions. Regional governments face the problem of deciding the specific apportionment formula used to allocate the MRF's income for tax purposes, in a context of strategic interaction.[7] The MRF produces a homogeneous good, and sells the good in different (segmented) markets. The domestic price of the good produced by the MRF depends on the domestic supply of goods. The MRF will shift production, investment, and, consequently profits, across locations depending on the domestic corporate income system in place. We derive the FA system chosen in equilibrium when governments consider the impact of the formula on the welfare of domestic consumers, tax revenue (or the government size), and the MRF. Since different formulas affect domestic prices, tax revenue, and MRF's net-profits, the paper provides a conceptual framework that can be used to explain which group is determinant in deciding the domestic FA system. We also compare these outcomes to the centralized solutions.

The main contribution of our paper is that it spells out how the MRF's decision to allocate capital and sales affects the price of the good in the respective regions. Firms react to policies adopted by the government by changing their capital and sales shares, which, in turn, affects the price of domestic goods. Though past research has looked into a vast number of theoretical models regarding the impact of a FA regime, this particular aspect

[5]However, if the definition of taxable income differs across states, i.e., Ω^i is not the same for all i, then it may not be true that 100 percent of the firm's income is apportioned when all states use the same apportionment formula.

[6]Other papers discussing the impact of taxation under FA system are Gordon and Wilson (1986), Gerard and Weiner (2003), Eggert and Schjelderup (2003), Eggert and Schjelderup (2005), Nielsen et al. (2003), and Sørenson (2004).

[7]The paper focuses on alternatives that employ capital (or property), sales shares, or a linear combination of the two, i.e., m^{iW} is assumed to be equal to zero for all regions.

has not been investigated. In Pinto (2007), prices are exogenous and the selection of the specific formula is driven by the assumption of decreasing returns to scale in production.[8] In the present paper, the latter effect is not present because we assume constant returns to scale. However, since in the current setup the MRF can reduce its tax burden and influence domestic prices by transferring capital or production across regions, domestic governments may be induced to adopt different formulas.

A few papers have formally studied the economic implications of using a FA system. In general, given the complexity of the issue under study, it has been difficult for the literature in this area to provide clear and unambiguous conclusions. The earlier papers by McLure (1980), McLure (1981), Goolsbee and Maydew (1998), and Mieszkowski and Zodrow (1985) first elucidated that formulary apportionment mostly transforms the state corporate income tax into three separate taxes on the factors in the apportionment formula. Gordon and Wilson (1986) examine the response of firms to a system of formula apportionment, restricting attention to cases in which all states use the same system, with different corporate tax rates. Most of their analysis is concerned with the component of the tax tied to the allocation of property (i.e., $m^{iF} = m^{iW} = 0$ for all regions i). The setup of our theoretical model is related to Pethig and Wagener (2003), Kolmar and Wagener (2004), and Mintz and Smart (2004).[9] All these papers, however, assume that the formulas are exogenously given. Anand and Sansing (2000) provide an explanation for why states may choose different apportionment methods, even though aggregate social welfare is maximized when states use the same formulas. They demonstrate analytically that "importing" states have incentives to increase sales factor weights, while "exporting" states have incentives to reduce the weights on productive factors. The results are explained by the fact that they assume that some inputs are completely immobile. [10] In the present work we endogenize the choice of the formulas in a context where the price of the good produced by the MRF responds to quantities sold domestically.

Section 2 sets up the model. Section 3 explores the properties of equilibrium and describes the various stages of the game. It also elaborates on the intuition of the model by adopting a numerical example that illustrates the properties of the equilibrium in greater detail. Section 4 describes the main results of the paper, and Section 8 concludes.

[8]Pinto (2007) shows that regions choosing a formula that only weighs the sales portion is a Nash equilibrium of the FA game. However, the results are explained by the fact that capital and sales respond differently to tax rates precisely due to the decreasing returns to scale assumption.

[9]Mintz and Smart (2004) focuses on FA systems that exclusively use capital or property shares, too.

[10]Nielsen et al. Nielsen et al. (2004) and Sørenson (2004) compare tax spillovers generated by national corporate tax systems which rely on separate accounting (SA) or FA. These papers, as some of the papers mentioned before, assume that the tax system is exogenously given and concerns a FA system that only employs capital or property shares.

2 The Model

Consider an economy with two regions a and b. A MRF operates through subsidiaries in both regions. It produces and sells a homogeneous good at both locations. The MRF has the ability to transfer, at a cost, part of its production from one region and sell it in the other. Capital is the only factor of production. Output is produced using a CRS technology. Let k^{ii} denote the level of capital in i used by the MRF to produce the good that is sold in region i, and k^{ji} the amount of capital used by the subsidiary in region j to produce the good for consumers in i. Thus, the total number of units sold in location i is $k^i = k^{ii} + k^{ji}$, for $i, j = a, b$ and $i \neq j$. We assume that the MRF can practice price discrimination between the two countries. When the MRF sells k^i units in i, the total revenue is $r(k^i) \equiv p(k^i)k^i > 0$, where $p(k^i)$ is the price function, and $r'(k^i) \equiv p(k^i) + p'(k^i)k^i$. We assume $p'(k^i) < 0, p''(k^i) \geq 0$, and $r''(k^i) \equiv 2p'(k^i) + p''(k^i) < 0$.[11] The cost per unit of capital at location i for the MRF (which is also the per unit production cost) is constant and given by c^i. When the MRF produces in j and sells in i (i.e., $k^{ji} > 0$), location i incurs a transportation cost τ^i per unit shipped to i.[12]

Domestic governments raise revenue by imposing a corporate income tax on the MRF. As in Pinto (2007), capital expenses can only be imperfectly deducted from the firm's taxable income. Specifically, the tax base in region i is the value of output minus a fixed share μ^i of the true capital cost, where $\mu^i \geq 0$ represents the proportion of capital expenses that can be deducted from taxable income. The deductibility of capital expenses can either be above or below its true costs. If a tax system allows only incomplete deduction of capital costs, then $\mu^i < 1$, which means that a positive tax is also levied on capital. If $\mu^i = 1$, the corporate tax falls only on pure profits, and when $\mu^i > 1$, capital is subsidized, i.e., firms can deduct more than the true capital expenditures from taxable income.[13]

2.1 Formula Apportionment

Suppose that both regions adopt a formula apportionment (FA) method to calculate the share of the firm's activities in each jurisdiction. Under this tax system, each region apportions the total taxable income of the MRF according to a given formula, which determines the proportion of the firm's total profits taxed by the region. Supposedly, the formula should

[11] We will later construct a numerical example using the linear demand function $p(k^i) = \alpha - \beta k^i$, which satisfies these assumptions for $k^i \leq \alpha/2\beta$.

[12] τ^i may also be interpreted as a tariff on imports entering region i. Under this interpretation, we would also need to account for the revenue generated by this policy instrument.

[13] Our subsequent analysis will assume that μ^i is exogenous and focus on the case where $\mu^i = 0, i = a, b$. In Pinto (2007), μ^i is an endogenous policy variable.

capture the proportion of the overall activities of the MRF performed in the region. Our interest is on FA tax regimes that consider sales, property, or a combination of the two factors as proxies of the firm's activities in each region.

Formally, the FA system can be described as follows. Let γ^i denote the share of the MRF's activities in region i as determined by the apportionment formula employed by the tax authority in that region. Then,

$$\gamma^i = m^{ki}\lambda^{ki} + m^{si}\lambda^{si}, \quad i = a, b, \tag{2}$$

where $0 \leq m^{ki}, m^{si} \leq 1$, $m^{ki} + m^{si} = 1$,

$$\lambda^{ki} \equiv \frac{k^{ii} + k^{ij}}{k}, \quad \lambda^{si} = \frac{k^{ii} + k^{ji}}{k}, \tag{3}$$

and $k = k^{ii} + k^{ji} + k^{jj} + k^{ij}$, is the total amount of capital used by the MRF. If region i exclusively uses capital (or property) shares in the formula, then $m^{ki} = 1$, and $\gamma^i = \lambda^{ki}$. If it only employs production shares, then $m^{ki} = 0$ and $\gamma^i = \lambda^{si}$. A formula that gives positive weights to both factors is represented by $0 < m^{ki} < 1$.

Next, we establish the specific relationship between taxable income and economic profits. The formula is applied to the firm's taxable income, which may differ from the firm's economic profits due to the imperfect deductibility of capital costs. The MRF's total economic profits are

$$\begin{aligned} \pi &= \pi^a + \pi^b \\ &= \left[r(k^a) - c^a(k^{aa} + k^{ab}) - \tau^a k^{ba}\right] + \left[r(k^b) - c^b(k^{bb} + k^{ba}) - \tau^b k^{ab}\right] \\ &= r(k^a) + r(k^b) - c^a(k^{aa} + k^{ab}) - c^b(k^{bb} + k^{ba}) - (\tau^a k^{ba} + \tau^b k^{ab}), \end{aligned} \tag{4}$$

where π^i is economic profit in region i. Total economic profits of the MRF are given by the sum of total revenues minus total production costs and total transportation costs. The revenue generated in location i is derived from the units sold by the firm in i, which are the sum of the units produced in i and the units produced in j and shipped to i. But then the firm faces the cost of transporting k^{ji} units from j.[14]

[14]Transfer pricing does not affect aggregate profits. When the subsidiary in i pays $\rho^i k^{ji}$ to the subsidiary in j for k^{ji} units of the good, the revenue in j increases in exactly that amount, so π remains unchanged. The only real costs are those associated with the transportation of goods from one region to another.

The relationship between taxable income and economic profits can be expressed as follows:

$$\begin{aligned}
\omega &= \omega^a + \omega^b \\
&= \left[r(k^a) - \mu^a c^a(k^{aa} + k^{ab}) - \tau^a k^{ba}\right] + \left[r(k^b) - \mu^b c^b(k^{bb} + k^{ba}) - \tau^b k^{ab}\right] \\
&= \pi + (1 - \mu^a)c^a(k^{aa} + k^{ab}) + (1 - \mu^b)c^b(k^{bb} + k^{ba}), \qquad (5)
\end{aligned}$$

where ω^i is taxable income in region i. Region i allows the MRF to deduct a proportion μ^i of capital expenses. Specifically, the MRF can deduct $\mu^i c^i(k^{ii} + k^{ij})$ from its taxable income in i. Thus, the firm's total taxable income is given by total economic profits plus two additional terms that depend on μ^a and μ^b and the MRF's capital allocation across countries. If $\mu^i < 1$, then as the firm employs more capital in i, its taxable income rises as well. On the contrary, when $\mu^i > 1$, capital employed in i is subsidized, so taxable income declines with higher levels of k^{ii} and k^{ij}.

The total taxes paid by the MRF are

$$T = T^a + T^b = t^a\gamma^a\omega + t^b\gamma^b\omega = (t^a\gamma^a + t^b\gamma^b)\omega,$$

so after-tax profits $N \equiv \pi - T$ are

$$\begin{aligned}
N &= \pi - (t^a\gamma^a + t^b\gamma^b)\omega \\
&= (1 - \bar{t})\pi - \bar{t}(1 - \mu^a)c^a(k^{aa} + k^{ab}) - \bar{t}(1 - \mu^b)c^b(k^{bb} + k^{ba}), \qquad (6)
\end{aligned}$$

where $\bar{t} = t^a\gamma^a + t^b\gamma^b$ is the effective (total) tax rate faced by the firm. Throughout the analysis, we assume that the corporate income tax rates t^i are strictly positive and exogenously given. In other words, we assume that there is already a given corporate income tax system in place and focus on the determination of the specific formula to apportion the firm's income.

Henceforth, we assume $\mu^a = \mu^b = \mu = 0, c^a = c^b = c, \tau^a = \tau^b = \tau$ and $t^a = t^b = t$.[15] Under these conditions, the MRF's net profits become

$$N = (1 - \bar{t})\pi - \bar{t}ck = \pi - \bar{t}(\pi + ck), \qquad (7)$$

[15] We will later examine the extent to which some of these assumptions affect the conclusions of the analysis.

where

$$\begin{aligned}
\pi &= r(k^a) + r(k^b) - ck - \tau(k^{ab} + k^{ba}), \\
\bar{t} &= t\left[1 - \Delta\left(\frac{k^{ab} - k^{ba}}{k}\right)\right], \\
T &= \bar{t}(\pi + ck) = \bar{t}\omega, \quad \text{and} & (8) \\
\Delta &= m^{kb} - m^{ka}. & (9)
\end{aligned}$$

2.2 Timing of events

In what follows, we examine the incentives of domestic governments to choose different FA systems. In doing so, it is assumed that domestic governments decide the formula system non-cooperatively. The FA game is modeled as a three-stage game as follows:

1. The governments simultaneously choose m^{ka} and m^{kb};
2. The MRF observes $\{m^{ka}, m^{kb}\}$, and decides the capital allocation across countries: $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\}$;
3. Payoffs at each location are determined.

We derive the subgame-perfect Nash equilibria of the FA game.

3 Characterization of the Equilibrium

In this section, we examine the properties of the equilibrium. We begin by studying the response of the MRF in the second stage of the game to different FA systems.

3.1 Second Stage: The MRF's Problem

When the MRF produces goods abroad and sell them domestically, it faces a transportation cost. So why would a MRF incur in such a cost? The answer is that the tax system, in this case the FA system, distorts the firm's decisions about where to produce and where to sell. Consider equations (7) and (8). Suppose $m^{kb} > m^{ka}$, i.e., the formula used by the tax authorities in b weighs capital shares relatively more than in a. If the firm produces more in b (by allocating more capital to that region), $\bar{t}$ would rise. Thus, the firm has an incentive to shift production to a. Since a weighs relatively more heavily the sales proportion of the formula, the MRF has incentives to ship and sell part of its production to region b (i.e., $k^{ab} > 0$). By choosing $k^{ab} > k^{ba}$, the MRF can reduce the effective tax rate and, consequently, taxes paid. In other words, by producing in one location and selling in the other, the MRF can obtain a tax benefit. However, the latter strategy also entails a transportation cost for the MRF. At the same time, the reallocation of capital and sales across countries affects the domestic price, and, consequently, profits. Consequently, when the countries employ different FA methods, the MRF can benefit by changing the allocation of capital across countries.[16]

Formally, the goal of the MRF is to choose the capital allocation $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\}$ that maximizes total after tax profits N. The following Kuhn-Tucker conditions characterize the solution of this problem as a function of $\Delta \equiv m^{kb} - m^{ka}$:

$$\begin{aligned}
\frac{\partial N}{\partial k^{aa}} &= (1-\bar{t})r'(k^a) - c - \frac{\partial \bar{t}}{\partial k^{aa}}(\pi + ck) \leq 0; && (10)\\
\frac{\partial N}{\partial k^{ab}} &= (1-\bar{t})r'(k^b) - c - (1-\bar{t})\tau - \frac{\partial \bar{t}}{\partial k^{ab}}(\pi + ck) \leq 0; && (11)\\
\frac{\partial N}{\partial k^{ba}} &= (1-\bar{t})r'(k^a) - c - (1-\bar{t})\tau - \frac{\partial \bar{t}}{\partial k^{ba}}(\pi + ck) \leq 0; && (12)\\
\frac{\partial N}{\partial k^{bb}} &= (1-\bar{t})r'(k^b) - c - \frac{\partial \bar{t}}{\partial k^{bb}}(\pi + ck) \leq 0; && (13)
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial \bar{t}}{\partial k^{aa}} &= \frac{\partial \bar{t}}{\partial k^{bb}} = \frac{t\Delta}{k^2}(k^{ab} - k^{ba}); && (14)\\
\frac{\partial \bar{t}}{\partial k^{ab}} &= \frac{t\Delta}{k^2}[(k^{ab} - k^{ba}) - k]; && (15)\\
\frac{\partial \bar{t}}{\partial k^{ba}} &= \frac{t\Delta}{k^2}[(k^{ab} - k^{ba}) + k]. && (16)
\end{aligned}$$

and the non-negativity and complementary slackness constraints $k^{ij} \geq 0, k^{ij}(\partial N/\partial k^{ij}) = 0, i, j = a, b$. Note that the solutions depend on the difference between the formulas Δ, and not the levels.

The MRF will produce in one region and sell in the other only if the tax benefits of such a strategy are sufficiently large. Specifically, if the difference between formulas Δ is sufficiently small, the MRF will produce only for the domestic market. The following proposition characterizes the possible solutions that can be attained depending on the value of Δ.[17]

[16] To the extent that $t^a = t^b = t$ and $\mu^a = \mu^b = \mu$, and $c^a = c^b = c$, this type of behavior does not arise in the context of a separate accounting (SA) method. In general, the SA method requires firms to maintain different accounts for their activities in each region where they operate. Firms would have an incentive to transfer income from one region to the other if the tax benefits compensate for the cost of engaging in such a transfer. However, the tax benefits disappear under the present assumptions.

[17] The Appendix shows two additional properties of the solution of the MRF's problem: (1) at a solution k^{ab} and k^{ba} cannot be simultaneously strictly positive; and (2) if $k^{aa} > 0$ and $k^{bb} > 0$ are part of a solution,

Proposition 1. *Let*

$$\Delta^* = \frac{\tau(1-t)}{tp(k^*)}, \text{ and } k^* = r'^{-1}\left[\frac{c}{(1-t)}\right] > 0, \tag{17}$$

or, in other words, k^* is implicitly defined by $(1-t)r'(k^*) - c$, such that $k^* \equiv k^*(t,c) > 0$. (i) If $\Delta \in [-\Delta^*, \Delta^*]$, then $k^{aa} > 0, k^{bb} > 0, k^{ab} = k^{ba} = 0$ is a solution. Moreover, if $\Delta^* \geq 1$, then $k^{ab} > 0$ and $k^{ba} > 0$ cannot be a solution.
Suppose $\Delta^* < 1$. (ii) If $\Delta \in [-1, -\Delta^*)$, then $k^{aa} \geq 0, k^{ab} = 0, k^{ba} > 0$, and $k^{bb} > 0$ is a solution. (iii) If $\Delta \in (\Delta^*, 1]$, then, $k^{aa} > 0, k^{ab} > 0, k^{ba} = 0$, and $k^{bb} \geq 0$ is a solution.

Proof. Part (i). Evaluated at $k^{aa} = k^{bb} = k^* > 0$, and $k^{ab} = k^{ba} = 0$, and if $-\Delta^* < \Delta < \Delta^*$, then equations (10) and (13) are satisfied with equality, while (11) and (12), respectively, become

$$\begin{aligned} -(1-t)\tau + t\Delta p(k^*) &< 0, &(18)\\ -(1-t)\tau - t\Delta p(k^*) &< 0. &(19) \end{aligned}$$

Thus, $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$ is a solution. Moreover, by substituting $\Delta = \Delta^*$ into the Kuhn-Tucker conditions, it can be shown that equations (10), (11), and (13) hold with equality, while (12) is strictly negative when the expressions are evaluated at $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$.[18] A similar reasoning can be applied to show that the previous solution also holds when $\Delta = -\Delta^*$. Thus, the proposition is satisfied when $-\Delta^* \leq \Delta \leq \Delta^*$.

Note that if $\Delta^* \geq 1$, then $k^{aa} = k^{bb} = k^*$, $k^{ab} = k^{ba} = 0$ is the only solution of the MRF's maximization problem since Δ is between -1 (when $m^{kb} = 0$ and $m^{ka} = 1$) and 1 (when $m^{kb} = 1$ and $m^{ka} = 0$). Again, evaluating equations (10) and (13) at $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$, these expressions are always negative since $-\Delta^* \leq -1 \leq \Delta \leq 1 \leq \Delta^*$.
Parts (ii) and (iii). If $\Delta^* < 1$, then there are values of m^{ka} and m^{kb} such that $\Delta > \Delta^*$ or $\Delta < \Delta^*$. The system of equations (10) - (13) evaluated at $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$ and $\Delta = \Delta^* + \varepsilon$, where $\varepsilon > 0$ gives $(\partial N/\partial k^{aa}) = (\partial N/\partial k^{bb}) = 0$, and

$$\begin{aligned} \frac{\partial N}{\partial k^{ab}} &= \varepsilon tp(k^*) > 0; &(20)\\ \frac{\partial N}{\partial k^{ba}} &= -\varepsilon tp(k^*) - 2(1-t)\tau < 0. &(21) \end{aligned}$$

Equation (20) states that k^{ab} should be raised, while (21) implies that $k^{ba} = 0$. An analogous

then $k^a = k^b$.

[18] In fact, at $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$, $\partial N/\partial k^{ba} = -2(1-t)\tau$, when $\Delta = \Delta^*$.

Case	Δ	Capital allocation				Units sold
		k^{aa}	k^{ab}	k^{ba}	k^{bb}	in i
(i)	$[-1, -\hat{\Delta})$	0	0	+	+	$k^a > k^b$
(ii)	$[-\hat{\Delta}, -\Delta^*)$	+	0	+	+	$k^a = k^b$
(iii)	$[-\Delta^*, \Delta^*]$	+	0	0	+	$k^a = k^b$
(iv)	$(\Delta^*, \hat{\Delta}]$	+	+	0	+	$k^a = k^b$
(v)	$(\hat{\Delta}, 1]$	+	+	0	0	$k^a < k^b$

"+" means strictly positive.

derivation can be followed to conclude that when $\Delta = -\Delta^* - \varepsilon$, $\partial N/\partial k^{ab} > 0$ and $\partial N/\partial k^{ba} < 0$ at $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*, k^*, 0, 0\}$, so $k^{ba} = 0$ and k^{ab} should be increased. $\square$

Henceforth, we assume that $\Delta^* < 1$ to allow for $k^{ab} > 0$ or $k^{ba} > 0$ in equilibrium.[19] Thus, when the difference between the tax formulas is small enough, specifically, when $-\Delta^* \leq \Delta \leq \Delta^*$, the MRF does not have an incentive to produce abroad, so the firm's production in region i is entirely directed to satisfy the local demand in i. The reason is that the tax benefits the firm can obtain from producing elsewhere, represented by the second term on the right-hand side of inequalities (18) or (19) do not compensate for the (net of tax) shipping costs, represented by the first term of the previous expressions. As m^{kb} increases for a given value of m^{ka} (i.e., Δ increases), the benefits of producing (selling) in region b decline (increase). Thus, the MRF shifts production from b to a and ships part of its production to b.

It is possible to characterize the solutions further by using additional results derived in the Appendix. First, to the extent that k^{aa} and k^{bb} are positive in equilibrium, then $k^a = k^b$. This follows from the fact that $\partial\bar{t}/\partial k^{aa} = \partial\bar{t}/\partial k^{bb}$, so if $k^{aa} >$ and $k^{bb} > 0$, then expressions (10) and (13) are identical. Second, when $k^{aa} > 0, k^{ab} > 0, k^{bb} = k^{ba} = 0$, it can be shown that $k^a < k^b$. Similarly, when $k^{bb} > 0, k^{ba} > 0, k^{bb} = k^{ba} = 0$, $k^a > k^b$. Third, let $\hat{\Delta}$ be defined as the value of Δ at which $k^{bb} = k^{ba} = 0$, and $k^{aa} = k^{ab} = \hat{k} > 0$.[20] Depending on the relationship between Δ^* and $\hat{\Delta}$, two different equilibrium configurations will be observed as Δ gets larger. For instance, if $\Delta^* < \hat{\Delta} < 1$, then the capital allocation across regions depends on Δ as follows:

However, if $\hat{\Delta} < \Delta^*$, we would not observe the intermediate stages at which $k^{aa} > 0, k^{ab} = 0, k^{ba} > 0, k^{bb} > 0$ or $k^{aa} > 0, k^{ab} > 0, k^{ba} = 0, k^{bb} = 0$, such as cases (ii) and (iii).

If Δ is relatively small (m^{kb} is small relative to m^{ka}, so that $\Delta < \Delta^* < 0$), then production would take place mostly in region b. In fact, for a sufficiently small values of Δ, $k^{aa} = k^{ab} = 0$

[19] In other words, we assume that $p(k^*) > \tau(1-t)/t$.

[20] Similarly, $-\hat{\Delta}$ be defined as the value of Δ at which $k^{aa} = k^{ab} = 0$, and $k^{bb} = k^{ba} = \hat{k} > 0$.

and the good would be entirely produced in b, i.e., $k^a = k^{aa}$ and $k^b = k^{ab}$. If Δ is sufficiently large (m^{kb} is large relative to m^{ka}, so that $\Delta > \Delta^* > 0$), then production would take place in region a. As before, when Δ is sufficiently large, $k^{bb} = k^{ba} = 0$ and $k^a = k^{ba}, k^b = k^{bb}$.

It follows from (17) that $\partial\Delta^*/\partial t < 0$ and $\partial\Delta^*/\partial\tau > 0$. In other words, for a given value of Δ, as t gets larger, it becomes more likely that the MRF will produce in one region and sell in the other (i.e., $k^{ij} > 0$) since the tax incentives get larger. However, the opposite effect takes place when τ increases: when τ becomes sufficiently large, the MRF will prefer to produce locally to satisfy the entire domestic demand since it is more costly to ship the good from the other region.

3.2 Effect of the Formula on Tax Revenue, Profits, and Consumer Surplus, and Tax Revenue

To the extent that the formulas chosen by the regional tax authorities affect the MRF's incentives to allocate capital across regions, they will also have an impact on other relevant variables. In particular, different values of Δ will affect domestic prices of the good, taxes collected by regional tax authorities, consumers' surplus, and the MRF's profits.[21]

First, consider the consumer surplus in i given by

$$CS^i \equiv \int_0^{k^{i*}} p(k^i)dk^i - p(k^{i*})k^{i*},$$

where $k^i = k^{ii} + k^{ji}$ is the total number of units sold in i. Then,

$$\frac{\partial CS^i}{\partial\Delta} \equiv -p'(k^{i*})k^{i*}\frac{\partial k^{i*}}{\partial\Delta}.$$

Hence, $\partial CS^i/\partial\Delta > 0$ if and only if $\partial k^i/\partial\Delta > 0$, since in this case the price paid by consumers in i, $p(k^{i*})$, declines. The next section develops a numerical example that illustrates these results.

Second, the effect of an increase in Δ on the MRF's after-tax profits N can be summarized as follows:

$$\frac{\partial N}{\partial\Delta} = \begin{cases} -t(k^{ba}/k)\omega < 0, & \text{if } \Delta \in [-1, -\Delta^*); \\ t(k^{ab}/k)\omega > 0, & \text{if } \Delta \in (\Delta^*, 1]; \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

Note that when $\{m^{ka}, m^{kb}\} = \{0, 1\}$, so that $\Delta = 1 > \Delta^*$, then $\partial N/\partial\Delta > 0$. However,

[21]The analysis performed here will be useful for the determination of the formulas in section 3.4.

since m^{ka} cannot be reduced any further and m^{kb} cannot be increased any further, N attains a maximum at the corner solution $\{m^{ka}, m^{kb}\} = \{0, 1\}$. A similar reasoning can be used to show that N also reaches a maximum at $\{m^{ka}, m^{kb}\} = \{1, 0\}$ or $\Delta = -1$. Hence, N is maximized when the regional governments adopt opposed formulas. The following proposition summarizes this result.

Proposition 2. *The MRF's net-profits are maximized when* $\{m^{ka}, m^{kb}\} = \{0, 1\}$ *(i.e.,* $\Delta = 1$*). or* $\{m^{ka}, m^{kb}\} = \{1, 0\}$ *(i.e.,* $\Delta = -1$*).*

Finally, the effect of an increase in m^{ki} (for a fixed m^{kj}) on T^a, T^b, and consequently on T, is more complicated and depends on several factors.[22] For instance, the tax revenue collected in region i, T^i, is affected directly by both Δ and the level of m^{ki}, and indirectly by the effect of Δ on the capital allocation across regions. As an illustration, consider the case $\Delta < \Delta^*$. Then, in equilibrium $k^{bb} > 0, k^{ba} > 0, k^{aa} \geq 0, k^{ab} = 0$. If, additionally, $m^{ka} = 1$ and $k^{aa} = 0$, i.e., when the formula in a exclusively weighs capital shares, production takes place only in b, then taxes collected in region a would be zero.[23] Total taxes levied on the MRF, however, depend on Δ (both directly and indirectly through its effect on the equilibrium capital allocation), but not on the levels of m^{ka} and m^{kb}.[24] Specifically, the effect of m^{ki} on T is given by

$$\frac{\partial T}{\partial \Delta} = t\omega \frac{d\gamma}{d\Delta} + \bar{t}\frac{\partial \omega}{\partial \Delta}, \tag{23}$$

where $\gamma = \gamma^a + \gamma^b$, and $d\gamma/d\Delta$ denotes the total effect of Δ on γ. When Δ becomes sufficiently large, the MRF is induced to produce in a and ship part of its production to b. By engaging in this type of strategy, the firm is able to reduce its effective tax rate (i.e., $\bar{t} < t$), so total tax revenue will tend to decrease. At the same time, a higher Δ also affects taxable profits.

3.3 A Numerical Example

In this section, we construct a numerical example to examine the solution of the MRF's problem for different FA systems chosen by the regional governments. Consider the case of linear demand functions $p(k^i) = \alpha - \beta k^i$, with $\alpha, \beta > 0$. In this case, the consumer surplus in

[22] We will examine these effects in more detail in a numerical example developed in the following section.

[23] Taxes collected in a are

$$T^a = t\left[\frac{(k^{aa} + k^{ba})}{k} + m^{ka}\frac{(k^{ab} - k^{ba})}{k}\right]\omega.$$

[24] Obviously, the values of $\Delta = m^{kb} - m^{ka}$ depend, for a given m^{kb}, on m^{ka}. In other words, for a given $0 \leq m^{kb} \leq 1$, $\Delta \in [m^{kb} - 1, m^{kb}]$, since $0 \leq m^{ka} \leq 1$.

i is given by $CS^i \equiv (1/2)\beta {k^i}^2$. The equilibrium values k^*, and $p(k^*)$, and Δ^* are, respectively, given by[25]

$$k^* = \frac{1}{2\beta}\left[\frac{\alpha(1-t)-c}{(1-t)}\right] > 0, \quad p(k^*) = \frac{1}{2}\frac{[(1-t)\alpha + c]}{(1-t)}, \quad \text{and} \quad \Delta^* = \frac{2\tau(1-t)^2}{t\left[\alpha(1-t)+c\right]}. \quad (24)$$

The example uses the following parameter values: $\alpha = 10, \beta = 1, c = 2.50, \tau = 0.85$, and $t = 0.30$. The values of these exogenously given parameters determine $\{\Delta^*, k^*\} = \{0.2923, 3.2143\}$, and $\{\hat{\Delta}, \hat{k}\} = \{0.3212, 3.1900\}$. In table 2, we consider values of m^{ka} and m^{kb} ranging from 0 to 1 in intervals of 0.1.[26] The numerical example allows us to investigate the impact of different FA systems on the MRF's capital allocation, good prices at the two locations, tax revenues generated by different formulas, consumer surplus, and the MRF's net profits. The table show the values of these variables as a function of the FA chosen in region a for different values of m^{kb}. Since the countries are symmetric (with the exception of the FA system), similar results hold for region b.

The numerical example reveals a few interesting observations. First, as Δ increases from -1 to 1, the MRF's capital allocation changes following the pattern explained earlier, shifting from cases 1 through 3. Specifically, for $-\Delta^* \leq \Delta \leq \Delta^*$, $k^{aa} = k^{bb} = k^*$, and $k^{ab} = k^{ba} = 0$, and for $\Delta > \Delta^*$, it becomes profitable for the MRF to produce in region b and sell in a, i.e., $k^{ba} > 0$. Note that when $\Delta^* < \Delta \leq \hat{\Delta}$, the equilibrium allocations are characterized as follows: (i) $k^a = k^{aa}, k^b = k^{bb} + k^{ab}$ and $k^a = k^b$; and (ii) as Δ increases, k^{ba} rises, k^{aa} falls, and both $k^a = k^b$ decrease. And when Δ increases and $\Delta > \hat{\Delta}$, $k^a = k^{ba}$ rises and $k^b = k^{bb}$ falls. A similar pattern is observed when $-\Delta \leq -\Delta^*$ and Δ increases, but with $k^a = k^{aa} + k^{ba}$, and $k^b = k^{bb}$ for $-\hat{\Delta} \leq -\Delta < -\Delta^*$, and $k^a = k^{ba}$ and $k^b = k^{bb}$ for $-\Delta < -\hat{\Delta}$.

Second, since prices depend negatively on total units sold domestically, then $p(k^a)$ does not change as Δ changes.

Third, as shown in proposition 2, the MRF reaches its highest profit when the countries choose diametrically opposite formulas, i.e., N is largest when $m^{ki} = 0$ and $m^{kj} = 1$. Third, as the capital allocation changes, the domestic price of the good is affected. In particular, when m^{ka} rises and the formula gives a lower weight to the sales share in a, the amount of goods sold in a increases, and the price at that location declines. Fourth, consumer surplus at location a is highest when the amount sold in a is large, and the price at a is small. As described earlier, the latter occurs as m^{ka} gets larger.

[25] We assume $\tau(1-t) < t[\alpha(1-t)+c]/[2(1-t)]$, so that $\Delta^* < 1$, and $\alpha(1-t) > c$, so that $k^* > 0$.

[26] The intervals could be made finer without affecting the conclusions of the analysis. Three values of m^{kb} are presented here ($m^{kb} = 0.0, 0.5, 1.0$) to save space.

3.4 First Stage: The Government's Problem

At the first stage of the game, the governments simultaneously decide their respective FA systems, anticipating the MRF's reaction in the second stage. We assume that regional tax authorities implement domestic policies that maximize the welfare of certain groups:[27] government, consumers, or the MRF. Specifically, the regional government may wish to implement a formula that maximizes only total tax revenue in region i, T^i. Alternatively, the government of region i may wish to maximize the well-being of domestic consumers U^i. If we assume that the tax revenue collected by taxing the MRF is distributed back to domestic consumers, then U^i would be given by the sum of consumer surplus CS^i and tax revenue T^i in i, i.e., $U^i = CS^i + \phi T^i$, where $\phi \geq 0$ is a constant. Finally, the government may just be concerned about the well-being of the MRF, so it will implement a formula that maximizes total net profits N. Considering the previous alternatives, the tax authorities in $i = a, b$ will choose the FA system m^{ki} that maximizes the objective function

$$W^i = \mathbb{1}_T^i \, T^i + \mathbb{1}_C^i \, U^i + \mathbb{1}_N^i \, N, \tag{25}$$

where $\mathbb{1}_h^i$ is an indicator function which is equal to one when the government in i maximizes $h = C, T, N$, and zero otherwise. Different formulas would be used by the domestic government depending on the values $\mathbb{1}_h^i$.[28] The following section characterizes the equilibrium achieved for different government's objectives using the numerical example developed earlier.

4 Results

In this section, we summarize the sub-game perfect Nash equilibria reached under different values of $\mathbb{1}_h^i$ in stage one of the FA game. The equilibria, which are derived based on the standard assumption that governments act non co-operatively, depend on the welfare maximizing objectives of the domestic government. We consider three alternatives: (i) $\mathbb{1}_T^i = 1$, (ii) $\mathbb{1}_C^i = 1$, and (iii) $\mathbb{1}_N^i = 1$. Figures 2, 3, and 4 summarize the results from the previous table, focusing on the consumer surplus, tax revenue, and the MRF's net profits.

[27] These groups can be thought of as organized pressure groups that lobby regional governments for policies that benefit the group.

[28] Throughout the analysis, we will assume that regional governments have the same objectives, i.e., $\mathbb{1}_h^a = \mathbb{1}_h^b$. In addition, the subsequent analysis will focus exclusively on the determination of the formula given that a corporate income tax system is already in place with $t > 0$.

4.1 Tax Revenue: $\mathbb{1}_T^i = 1$

Suppose that governments maximize tax revenues, or $\mathbb{1}_T^i = 1$. From figure 2 (and table 2), it can be concluded that when region b chooses $m^{kb} = 0$, a best response by the tax authorities in region a is to choose $m^{ka} = 0$. Similarly, a best response by region b to $m^{ka} = 0$ is $m^{kb} = 0$, since the problem is equivalent to the one faced by location a. As a result, $\{m^{ka}, m^{kb}\} = \{0, 0\}$ is a Nash equilibrium. In fact, when $m^{kb} = 0$, the solution will be unchanged as long as $\Delta < \Delta^*$ (by proposition 1). In this example , T^a reaches its maximum ($T^a = 2.53$) at $m^{ka} = \{0.0, 0.1, 0.2\}$ (see table 2). As a result, there is a range of values that constitute a Nash equilibrium, and the set of equilibria can be characterized as follows:

$$\{(m^{ka}, m^{kb}) : 0 \leq m^{ka} < 0.3, 0 \leq m^{kb} < 0.3 \text{ and } -\Delta^* \leq \Delta \leq \Delta^*\}.$$

In other words, when governments maximize domestic tax revenue, the equilibrium FA systems tend to weigh more heavily the sales portion of the formula. The equilibria include the case of assigning full weight to the sales portion of the formula. Furthermore, note that since in equilibrium $-\Delta^* \leq \Delta \leq \Delta^*$, the MRF does not have incentives to produce abroad: the firm's production in region i is entirely directed to satisfy local demand, i.e., case 2 above holds.

The intuition of this result is straightforward. Location a has an incentive to always choose a lower value of m^{ka} relative to m^{kb} in order to attract more capital and more production to location a, and, hence, increase taxes collected in a. However, location b reacts in the same way, leading to the equilibria summarized above. This result is consistent with the "race-to-the-bottom" argument present in the traditional literature on tax competition.

4.2 Utility of Domestic Consumers: $\mathbb{1}_C^i = 1$

Next, suppose that governments maximize the welfare of domestic consumers, or $\mathbb{1}_C^i = 1$, and they choose the formulas that maximize $CS^i + \phi^i T^i$ in a decentralized way. Consider for the moment the case in which $\phi^i = 0$. In other words, regional governments simply maximize CS^i.[29] Figure 3 (and table 2) shows that when $m^{kb} = 1$, a's best response is to choose $m^{ka} = 1$ as well. The same argument holds for region b. Thus, $\{m^{ka}, m^{kb}\} = \{1, 1\}$ is a Nash equilibrium of the FA system game. As in the previous case, there are additional

[29] Throughout this exercise, the regional authorities take the corporate tax system as given with $t > 0$ and $\mu = 0$, so that regions can only adjust their formulas. We make this clear because in the present setup, CS^i would be maximized when $t = 0$.

Nash equilibria described as follows:

$$\{(m^{ka}, m^{kb}) : 0.7 < m^{ka} \leq 1, 0.7 < m^{kb} \leq 1 \text{ and } -\Delta^* < \Delta < \Delta^*\}.$$

Even though the equilibrium formulas give higher weights to the capital shares, the MRF still has incentives to produce locally to satisfy the domestic demand because both tax authorities are choosing the same formula. As before, case 2 holds.

To understand this result, recall that, for a given m^{kj}, $\partial CS^i/\partial m^{ki} = \beta(\partial k^i/\partial m^{ki})$. In other words, the impact of m^{ki} on CS^i depends on the effect of m^{ki} on the amount of goods sold in i (or, equivalently, its effect on $p(k^i)$). If $m^{ki} > m^{kj}$, then the MRF can reduce part of its tax burden by reallocating its sales to region i (and reducing its production in i). A higher volume of sales in i reduces the price paid by consumers in i, and raises CS^i. As a result, the best response by region i to m^{kj} is $m^{ki} > m^{kj}$. However, region j faces the same problem, resulting in Nash equilibria with values of m^{ki} and m^{kj} close to 1.

4.3 Net Profits: $\mathbb{1}^i_M = 1$

Finally, suppose that governments maximize the MRF's aggregate profits, i.e., $\mathbb{1}^i_M = 1$. Figure (4) shows that when region b chooses high values of $m^{kb} = 0$, then a's best response is to choose $m^{ka} = 1$. When region b chooses $m^{kb} = 1$, then a's best response is to choose $m^{ka} = 0$. Since the problem is symmetrical, region b responds similarly. Thus, there are two possible Nash equilibria in this case: (1) $\{m^{ka}, m^{kb}\} = \{1, 0\}$; and (2) $\{m^{ka}, m^{kb}\} = \{0, 1\}$. At both Nash equilibria, domestic governments choose opposed formulas. As a result, the MRF produces in one single location, sells part of its production to domestic consumers, and ships the rest abroad (cases 1 or 3 above hold). It follows from proposition 2 that the MRF net-profits are maximized at these equilibria.

5 Centralized Solution

How do the equilibria from sections 4 differ from those reached in the centralized case? Suppose that the formulas are chosen cooperatively by the regions.[30] Based on the previous numerical example, the following conclusions would follow.

First, consider the solutions attained by a central authority that maximizes total taxes $T = T^a + T^b$. Table 2 shows that T reaches its maximum when both locations choose the same formula, i.e., $0 \leq m^{ka} = m^{kb} \leq 1$. Note that any value of $\{m^{ka}, m^{kb}\}$ such

[30] The rest of the corporate income tax system is, as before, exogenously given with $t > 0$.

that $-\Delta^* < \Delta < \Delta^*$ also maximizes T. When the difference between the formulas is sufficiently large, the MRF can reduce its tax burden by shifting capital from one region to the other. In this case, the effective tax rate $\bar{t}$ is lower than the statutory tax rate t. To the extent that $m^{ka} = m^{kb}$, regardless of the values of m^{ka} and m^{kb}, $\bar{t} = t$, so the MRF will not be able to escape the tax. The set of equilibria reached in the decentralized case, $\{(m^{ka}, m^{kb}) : 0 \leq m^{ka} < 0.3, 0 \leq m^{kb} < 0.3 \text{ and } -\Delta^* < \Delta < \Delta^*\}$, is in fact a subset of the set of solutions reached in the centralized case. Figure 5 compares both cases.

Second, consider the formulas that maximize total consumer surplus $CS = CS^a + CS^b$. Table 2 shows that CS reaches its maximum when m^{ka} and m^{kb} take diametrically opposite values: if $\{m^{ka}, m^{kb}\} = \{0, 1\}$ or $\{m^{ka}, m^{kb}\} = \{1, 0\}$, then $CS = 2.87$.

And third, note that the solutions that maximize CS coincide with the ones that maximize net profits N (as stated earlier in proposition 2). The reason is that when $|\Delta| = 1$, the MRF will respond by choosing the levels of k^{ab} and k^{ba} that minimize the effective tax rate $\bar{t}$. Since in our setup the corporate tax is essentially a tax on capital (since a proportion of the capital costs cannot be deducted from taxable income), the MRF will choose a higher level of aggregate capital k when $|\Delta| = 1$ because the effective tax rate $\bar{t}$ would be lowest. In turn, a higher k would imply a lower price paid by consumers and a higher CS.[31] Figure 5 summarizes and compares the solutions reached in each case.

6 Implications of the Assumptions

We evaluate in this section the extent to which some of the assumptions made earlier affect the conclusions of the analysis. So far, we have assumed that $\mu = 0$ and that regions impose the same corporate tax rates. Now, we examine the implications of assuming $\mu > 0$, so that a positive fraction of capital expenses can be deducted from taxable income, and we analyze the case in which $t^a = 0$ and $t^b = t > 0$.

[31] If $m^{ki} = 1$ and $m^{kj} = 0$, the MRF produces in region i and sells part of its production in j. The price of the good will tend to be lower in j than in i, so $CS^j > CS^i$. But overall, consumer surplus will be higher in this case relative to other alternatives.

6.1 Effect of μ on Δ^*

Suppose that $\mu > 0$. Then, proposition 1 still holds, but the values of k^* and Δ^* are now defined by

$$(1-t)r'(k^*_\mu) - (1-t\mu)c = 0, \quad \text{or} \quad k^*_\mu = r'^{-1}\left[\frac{(1-\mu t)c}{(1-t)}\right], \quad \text{and} \tag{26}$$

$$\Delta^*_\mu = \frac{(1-t)\tau}{t\left[p(k^*_\mu) - \mu c\right]}. \tag{27}$$

It is straightforward to show that since $\partial k^*_\mu/\partial\mu > 0$ (as long as $\Delta \in [-\Delta^*_\mu, \Delta^*_\mu]$), then $\partial\Delta^*_\mu/\partial\mu > 0$. If μ increases, the MRF can deduct a larger proportion of domestic capital expenses from taxable income. Hence, it becomes beneficial for the firm to produce domestically in order to satisfy the local demand rather than ship the goods from another region. Since this effect holds for both regions (given that $\mu = \mu^a = \mu^b$ in the present version of the model), the equilibrium values of k^{aa} and k^{bb} will tend to rise. At the same time, Δ^*_μ tends to rise as μ increases, so it would be less likely to observe cases where $k^{ij} > 0$.[32]

Even when economic and taxable profits coincide, i.e., $\mu = 1$, the FA system may still distort the MRF's decision to allocate capital across regions. If $\mu = 1$, then $\Delta^*_\mu = \tau(1-t)/\{t[p(k^*_\mu) - c]\} > 0$, so as $|\Delta|$ gets larger,[33] the MRF will eventually find it profitable to produce only in one location, which means that either $k^{aa} = 0$ or $k^{bb} = 0$.[34]

[32]For the linear demand function $p(k^i) = \alpha - \beta k^i$,

$$\Delta^*_\mu = \frac{2(1-t)^2\tau}{t\left\{\alpha(1-t) + c[1-\mu(2-t)]\right\}}, k^*_\mu = \frac{1}{2\beta}\left[\frac{\alpha(1-t) - c(1-\mu t)}{(1-t)}\right], p(k^*_\mu) = \frac{1}{2}\frac{\alpha(1-t) - c(1-\mu t)}{(1-t)},$$

when $\Delta \in [-\Delta^*_\mu, \Delta^*_\mu]$. Note that $\Delta^*_\mu > 0$ for all $0 \le \mu < \hat{\mu} \equiv [1/(2-t)][\alpha(1-t)/c + 1]$, $\Delta^*_\mu < 0$ for $\mu > \hat{\mu}$, and Δ^*_μ is not defined for $\mu = \hat{\mu}$. Also, since it was assumed that $\alpha > \alpha(1-t) > c$, then $\hat{\mu} > 1$.

[33]Or as t gets larger or τ smaller for a given Δ.

[34]Characterizing the equilibrium under a linear demand function $p(k^i) = \alpha - \beta k^i$ requires addressing specific technical constraints, particularly regarding the interiority of the solutions. Assume, as before, that $2\tau(1-t)^2 < t[\alpha(1-t)+c]$ (so that $\Delta^*_\mu < 1$ when $\mu = 0$). Now, as $\mu^+ \to \hat{\mu}$, $\Delta^*_\mu \to +\infty$. Moreover, as $\mu^- \to \hat{\mu}$, $\Delta^*_\mu \to -\infty$, and as $\mu \to \infty$, $\Delta^*_\mu \to 0$. Hence, there are values of μ such that $\Delta^*_\mu = 1$ and $\Delta^*_\mu = -1$, respectively defined as $\underline{\mu} \equiv \{t[\alpha(1-t)+c] - 2\tau(1-t)^2\}/(2-t)tc$ and $\overline{\mu} \equiv \{t[\alpha(1-t)+c] + 2\tau(1-t)^2\}/(2-t)tc$. From the definitions, it is clear that $\underline{\mu} < \hat{\mu} < \overline{\mu}$. Note that if $\mu \in [\underline{\mu}, \hat{\mu}) \cap (\hat{\mu}, \overline{\mu}]$, then $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^*_\mu, k^*_\mu, 0, 0\}$ for all Δ since $|\Delta^*_\mu| \ge 1$. As μ gets closer to $\hat{\mu}$ from below, $k^{ab} = k^{ba} = 0$ since Δ^*_μ will eventually become greater than one. Note that evaluated at $k^{aa} = k^{bb} = k^*_\mu$, total taxable income ω becomes zero when $\mu = \hat{\mu}$. We do not consider the solutions when $\mu \ge \hat{\mu}$ here, since in this case $T \le 0$. The derivations of the equilibria that arise in this case are available upon request.

6.2 Different Tax Systems Across Regions

Suppose $t^a = 0$, in which case m^{ka} becomes irrelevant, and $t^b = t > 0, \mu^b = 0$, and $0 \leq m^{kb} \leq 1$. The effective tax rate becomes

$$\bar{t} = \frac{t}{k}\left[k^{bb} + k^{ab} - m^{kb}(k^{ab} - k^{ba})\right]. \tag{28}$$

Two results are worth emphasizing in this case.[35] First, in equilibrium, the MRF will never produce in b and ship to a. In other words, there does not exist a value of m^{kb} such that $k^{ba} > 0$. Second, for values of m^{kb} less than or equal to

$$m^{kb*} \equiv \frac{\tau}{tp(k^{bb*})}\frac{[k^{aa*} + (1-t)k^{bb*}]}{k^{bb*}}, \tag{29}$$

$k^{aa} > 0, k^{bb} > 0$ and $k^{ab*} = 0$, so there is no shipment of goods from a to b. If $m^{kb*} \geq 1$, then the latter is the only solution. But if $m^{kb*} < 1$, then for values of $m^{kb} > m^{kb*}$, then k^{ab} becomes positive. This means that when the weight on the property (sales) portion of the formula is large (small) enough, then the firm will find it beneficial to produce in a instead of b, and ship part of its production to b. It may be the case that for values of m^{kb} sufficiently large, $k^{bb} = 0$ and $k^{ab} > 0$, which means that production only takes place in region a. Additionally, if $k^{bb} = 0$ and $k^{ab} > 0$ when $m^{kb} = 1$, then taxes collected in b would be zero since $\bar{t} = 0$. In other words, the formula weighs exclusively on property in region b, but production takes place elsewhere.

Which value of m^{kb} would be chosen by the tax authorities in b under the present circumstances? As in the previous section, the value of m^{kb} chosen in b would depend on the objective function. First, as before, $\partial CS^i/\partial m^{kb} > 0$ if and only if $\partial k^{ii}/\partial m^{kb} > 0$. Second, note that

$$\frac{\partial N}{\partial m^{kb}} = -\left[r(k^{bb}) - \tau k^{ab}\right]\frac{\partial \bar{t}}{\partial m^{kb}}, \quad \text{where} \quad \frac{\partial \bar{t}}{\partial m^{kb}} = -\frac{tk^{ab}}{k}. \tag{30}$$

So to the extent that in equilibrium $k^{ab} > 0$, net profits increase as m^{kb} rises because in this case the effective tax rate faced by the MRF falls. And finally, changes in m^{kb} affect tax revenue in $T^b = \bar{t}[r(k^{bb}) - \tau k^{ab}]$ through both its effect on $\bar{t}$ and on taxable income. These effects will be examined in more detail using a numerical example.

[TO BE COMPLETED]

[35]These results are derived in Appendix B.

7 The FA Game in the Presence of Domestic Firms

Suppose that a domestic firm produces and sells the same homogeneous good produced by the MRF in each region. We assume the domestic firm produces only for the local market (i.e., it cannot export the good), has a per unit production cost c, faces the corporate income tax rate t, is allowed to deduct a proportion $\mu = 0$ of capital expenses (same as the MRF), and determines the level of production and sales after observing the MRF's decisions.[36] The net profits of a domestic firm in i is

$$N_D^i = (1-t)\left[p(k^i + K^i) - c\right] K^i - tcK^i, \tag{31}$$

where $k^i = k^{ii} + k^{ji}$ denotes the units sold in i by the MRF and K^i the units sold by the domestic firm in i. The domestic firm's reaction function $K^i(k^i)$ is implicitly determined by the FOC

$$\frac{\partial N_D^i}{\partial K^i} = (1-t)\left[p'(k^i + K^i)K^i + p(k^i + K^i)\right] - c \leq 0, \quad K^i \geq 0. \tag{32}$$

Similar conclusions as those examined earlier for the MRF follow in the present case, but the value of Δ^* is now defined by

$$\Delta_D^* = \frac{(1-t)\tau}{tp(k_D^* + K^*)}, \tag{33}$$

where $k^{aa} = k^{bb} = k_D^*$ is the MRF's equilibrium output level when $k^{ab} = k^{ba} = 0$, and K^* the domestic firm's equilibrium output level. To the extent that $k_D^* + K^* > k^*$, then $p(k^*) > p(k_D^* + K^*)$ and $\Delta_D^* > \Delta^*$. In other words, when the MRF faces competition in the domestic market, its ability to engage in a strategy that involves importing the good from the branch located in the other region is significantly diminished. As a result, the equilibrium $k^{aa} = k^{bb} = k_D^* > 0$ and $k^{ab} = k^{ba} = 0$ (i.e., the equilibrium where the MRF produces exclusively to satisfy the local demand) would be observed for a larger set of values of Δ, relative to the case without local competition.

The formula implemented in region i affects the domestic firm's profits indirectly through k^i. Using the envelope theorem,

$$\frac{\partial N_D^i}{\partial m^{ki}} = (1-t)p'(k^i + K^i)K^i \frac{\partial k^i}{\partial m^{ki}}, \tag{34}$$

[36] This setup essentially grants the MRF the advantage of moving first. Additionally, when the domestic firm decides its level of output, it treats the domestic firm's decision in region j as given.

which means that changes in the formula that lead to an increase in the MRF's sales in region i, k^i, would reduce the domestic firm's profits.

For the linear demand function $p(k^i + K^i) = \alpha - \beta(k^i + K^i)$, the domestic firm's reaction function is

$$K^i = \frac{1}{2\beta}\frac{[\alpha(1-t)-c]}{(1-t)} - \frac{(k^{ii}+k^{ji})}{2}. \tag{35}$$

The values of k^* and Δ^* are given, in this case, by

$$k^*_D = \frac{1}{2\beta}\frac{[(1-t)\alpha - c]}{(1-t)}, \quad \text{and} \quad \Delta^*_D = \frac{4\tau(1-t)^2}{t\,[\alpha(1-t)+3c]}. \tag{36}$$

Note that $k^* = k^*_D$ and $k^*_D + K^* = (3/2)k^*$, which means that $p(k^*) > p(k^*_D + K^*)$. Specifically,[37]

$$p(k^*) > p(k^*_D + K^*) = \frac{1}{4}\frac{[(1-t)\alpha + 3c]}{(1-t)}, \tag{37}$$

so $\Delta^*_D > \Delta^*$, as stated earlier.

Characterization of the Solutions

Next, we use the previous numerical example to examine how the equilibrium values of m^{ka} and m^{kb} change in the presence of a domestic firm. In addition to the three possible cases considered earlier (the MRF, consumers, and tax revenue), we also describe the equilibria arising when local governments maximize the profits of the local firm N^i_D.

[TO BE COMPLETED]

8 Conclusion

In this paper, we explore the impact on domestic welfare of choosing different FA systems. Each specific formula will induce MRFs to change their activities across regions. Further, the model also points out that the formula chosen to tax a MRF in a particular location has an impact on the pricing strategy of the MRF. As firms change their capital and sales shares in reaction to the policies adopted by the government, it affects the price of domestic goods.

The paper characterizes several equilibria under different objective functions. It concludes that when governments choose the formulas that maximize domestic tax revenue, they will tend to implement formulas that weigh the sales portion of the formula more heavily; when

[37] The inequality follows as long as $(1-t)\alpha > c$, which is required for quantities to be positive in equilibrium.

governments maximize the MRF's net profits, they will end up choosing diametrically opposite formulas; and when they value consumers' well-being, they will adopt formulas that weigh more heavily capital shares.

The paper also shows that these formulas depart from those chosen cooperatively by the two regions. For instance, suppose that a central authority wants to choose the formulas that maximize total tax revenue paid by the MRF. Then, the paper shows that the formulas should be the same. If the goal is to maximize total consumer surplus, then it should induce regions to implement diametrically opposite formulas.

A Results

Proposition 3. *$k^{ab} > 0$ and $k^{ba} > 0$ cannot be part of a solution.*

Proof. Suppose that at a solution $k^{ab} > 0$ and $k^{ba} > 0$. For the latter to be the case, we need

$$(1-\bar{t})r'(k^a) - c = (1-\bar{t})\tau - \frac{\partial \bar{t}}{\partial k^{ba}}(\pi + ck)\,; \tag{38}$$

$$(1-\bar{t})r'(k^b) - c = (1-\bar{t})\tau - \frac{\partial \bar{t}}{\partial k^{ab}}(\pi + ck)\,. \tag{39}$$

Additionally,

$$(1-\bar{t})r'(k^a) - c \leq \frac{\partial \bar{t}}{\partial k^{aa}}(\pi + ck)\,; \tag{40}$$

$$(1-\bar{t})r'(k^b) - c \leq \frac{\partial \bar{t}}{\partial k^{bb}}(\pi + ck)\,. \tag{41}$$

Combining

$$\left(\frac{\partial \bar{t}}{\partial k^{aa}} - \frac{\partial \bar{t}}{\partial k^{ba}}\right)(\pi + ck) \geq (1-\bar{t})\tau \Leftrightarrow \frac{t\Delta}{k}(\pi + ck) \geq (1-\bar{t})\tau; \tag{42}$$

$$\left(\frac{\partial \bar{t}}{\partial k^{bb}} - \frac{\partial \bar{t}}{\partial k^{ab}}\right)(\pi + ck) \geq (1-\bar{t})\tau \Leftrightarrow -\frac{t\Delta}{k}(\pi + ck) \geq (1-\bar{t})\tau; \tag{43}$$

but the two inequalities cannot be simultaneously satisfied, leading to a contradiction. □

Proposition 4. *If $k^{aa} > 0$ and $k^{bb} > 0$ are part of a solution, then $k^a = k^b$.*

Proof. If $k^a > 0$ and $k^b > 0$ are part of a solution, then

$$(1-\bar{t})r'(k^a) - c = \frac{\partial \bar{t}}{\partial k^{aa}}(\pi + ck)\,; \tag{44}$$

$$(1-\bar{t})r'(k^b) - c = \frac{\partial \bar{t}}{\partial k^{bb}}(\pi + ck)\,. \tag{45}$$

But since $(\partial \bar{t}/\partial k^{aa}) = (\partial \bar{t}/\partial k^{bb})$, the result follows. □

Proposition 5. *If in equilibrium $k^{aa} = k^{ab} = 0, k^{ba} = k^a > 0$, and $k^{bb} = k^b > 0$, then $k^a > k^b$. Similarly, if in equilibrium $k^{bb} = k^{ba} = 0, k^{aa} = k^a > 0$, and $k^{ab} = k^b > 0$, then $k^a < k^b$.*

Proof. When $k^{aa} = 0$ and $k^{bb} > 0$ are part of the equilibrium, then

$$(1-\bar{t})r'(k^a) - c < \frac{\partial \bar{t}}{\partial k^{aa}}(\pi + ck)\,;$$

$$(1-\bar{t})r'(k^b) - c = \frac{\partial \bar{t}}{\partial k^{bb}}(\pi + ck)\,.$$

Since

$$\frac{\partial \bar{t}}{\partial k^{aa}}(\pi + ck) = \frac{\partial \bar{t}}{\partial k^{bb}}(\pi + ck)\,,$$

and $k^{aa}=0$, then $r'(k^a)<r'(k^b)$, which implies $k^b<k^a$ since $r''(\cdot)<0$. The same reasoning can be used to prove the second part of the proposition. $\square$

Proposition 6. *Let $\{\hat{k},\hat{\Delta}\}$ be implicitly defined by*

$$(1-t)r'(k)-c=\frac{t\Delta}{2}\left[\frac{r(k)}{k}-\frac{\tau}{2}-r'(k)\right], \quad \textit{and} \tag{46}$$

$$\Delta=\frac{(1-t)\tau}{t[p(k)-\tau]}. \tag{47}$$

Then,
(i) If $\Delta=\hat{\Delta}$, then $k^{aa}=k^{ab}=\hat{k}>0, k^{bb}=k^{ba}=0$ is a solution to the MRF's optimization problem;
(ii) If $\Delta=-\hat{\Delta}$, then $k^{bb}=k^{ba}=\hat{k}>0, k^{aa}=k^{ab}=0$ is a solution to the MRF's optimization problem;
(iii) $k^*>\hat{k}$;
(iv) $\Delta^*\leq\hat{\Delta}$; and

Proof. (i) Using $\Delta=\hat{\Delta}$, and evaluating the system of equations (10)-(13) at $k^{aa}=k^{ab}=\hat{k}$, and $k^{bb}=k^{ba}=0$, gives $\partial N/\partial k^{aa}=\partial N/\partial k^{ab}=\partial N/\partial k^{bb}=0$, and $\partial N/\partial k^{ba}<0$, confirming that the latter is a solution to the MRF's optimization problem.

(ii) Similar reasoning as in (i) above.

(iii) Recall that $r'(\cdot)>0$, $r''(\cdot)<0$, and $(1-t)r'(k^*)-c=0$. We will prove that $\hat{k}<k^*$ by showing that $(1-t)r'(\hat{k})-c>0$. At the equilibrium $k^{aa}=k^{ab}=\hat{k}, k^{bb}=k^{ba}=0$, the FOCs with respect to k^{aa} and k^{ab} become

$$\begin{aligned}(1-t)r'(\hat{k})-c &= \frac{t\Delta}{2}\left[\frac{r(\hat{k})}{\hat{k}}-\frac{\tau}{2}-r'(\hat{k})\right], &(48)\\ (1-t)r'(\hat{k})-c &= -\frac{t\Delta}{2}\left[\frac{r(\hat{k})}{\hat{k}}-\frac{\tau}{2}+r'(\hat{k})\right]+(1-t)\tau+\frac{t\Delta\tau}{2}. &(49)\end{aligned}$$

Adding (48) and (49) gives

$$(1-t)r'(\hat{k})-c=\frac{(1-t)\tau}{2}-\frac{t\Delta}{2}\left[r'(\hat{k})-\frac{\tau}{2}\right]. \tag{50}$$

Equalizing the RHS of (48) and (50)

$$\frac{r(\hat{k})}{\hat{k}}-\frac{\tau}{2}=\frac{(1-t)\tau}{t\Delta}+\frac{\tau}{2}. \tag{51}$$

Consider once again the expression on the RHS of (50):

$$\begin{aligned}\frac{(1-t)\tau}{2}-\frac{t\Delta}{2}\left[r'(\hat{k})-\frac{\tau}{2}\right] &> \\ \frac{(1-t)\tau}{2}-\frac{t\Delta}{2}\left[\frac{r(\hat{k})}{\hat{k}}-\frac{\tau}{2}\right] &= 0.\end{aligned}$$

The inequality holds because $r(\hat{k})/\hat{k}>r'(\hat{k})$, given that $r''(\cdot)<0$. The equality is obtained by substituting (51) into the squared-brackets. Hence, $(1-t)r'(\hat{k})-c>0$.

(iv) For $k^{aa} = k^{ab} = \hat{k} > 0, k^{bb} = k^{ba} = 0$ to be a solution, we need $\hat{\Delta} \geq \Delta^*$. Otherwise, from Proposition 1, $k^{aa} = k^{bb} = k^* > 0$ and $k^{ab} = k^{ba} = 0$. □

B No corporate tax in region a

Suppose that there is no corporate tax in region a, while $\mu^b = 0, t^b = t > 0$ and $0 \leq m^{kb} \leq 1$. In this case, the MRF's profit is given by

$$N = \pi^a + (1 - \bar{t})\pi^b - \bar{t}c(k^{bb} + k^{ba}), \tag{52}$$

where

$$\bar{t} = \frac{t}{k}\left[k^{bb} + k^{ab} - m^{kb}(k^{ab} - k^{ba})\right] \tag{53}$$

The FOCs become

$$\begin{aligned}
\frac{\partial N}{\partial k^{aa}} &= r'(k^a) - c - \frac{\partial \bar{t}}{\partial k^{aa}}\left[\pi^b + c(k^{bb} + k^{ba})\right] \leq 0; && (54)\\
\frac{\partial N}{\partial k^{ab}} &= (1 - \bar{t})\left[r'(k^b) - \tau\right] - c - \frac{\partial \bar{t}}{\partial k^{ab}}\left[\pi^b + c(k^{bb} + k^{ba})\right] \leq 0; && (55)\\
\frac{\partial N}{\partial k^{ba}} &= r'(k^a) - c - \tau - \frac{\partial \bar{t}}{\partial k^{ba}}\left[\pi^b + c(k^{bb} + k^{ba})\right] \leq 0; && (56)\\
\frac{\partial N}{\partial k^{bb}} &= (1 - \bar{t})r'(k^b) - c - \frac{\partial \bar{t}}{\partial k^{bb}}\left[\pi^b + c(k^{bb} + k^{ba})\right] \leq 0; && (57)
\end{aligned}$$

in addition to the non-negativity and complementary slackness constraints $k^{ij} \geq 0, k^{ij}(\partial N/\partial k^{ij}) = 0, i, j = a, b$, where

$$\begin{aligned}
\frac{\partial \bar{t}}{\partial k^{aa}} &= -\frac{t}{k^2}[k^{bb} + k^{ab} - m^{kb}(k^{ab} - k^{ba})]; && (58)\\
\frac{\partial \bar{t}}{\partial k^{bb}} &= \frac{t}{k^2}[k^{aa} + k^{ba} - m^{kb}(k^{ab} - k^{ba})]; && (59)\\
\frac{\partial \bar{t}}{\partial k^{ab}} &= \frac{t}{k^2}[(1 - m^{kb})(k^{aa} + k^{ba}) - m^{kb}(k^{bb} + k^{ba})]; && (60)\\
\frac{\partial \bar{t}}{\partial k^{ba}} &= -\frac{t}{k^2}[(1 - m^{kb})(k^{bb} + k^{ab}) - m^{kb}(k^{aa} + k^{ab})]. && (61)
\end{aligned}$$

The following proposition states that under the present conditions, the MRF will never produce in b and ship to a, i.e., $k^{ba} = 0$ is always part of a solution to the system $(54) - (57)$. The reason is that there are no tax benefits for the MRF of producing in b and shipping to a (in fact, a positive value of k^{ba} would increase the effective tax rate faced by the MRF), but the firm would still incur the per unit transportation cost τ.

Proposition 7. *For all values of $0 \leq m^{kb} \leq 1$, $k^{ba*} = 0$.*

Proof. First, note that

$$\frac{\partial \bar{t}}{\partial k^{ba}} = \frac{\partial \bar{t}}{\partial k^{aa}} + \frac{\bar{t}m^{kb}}{k}. \tag{62}$$

Next, suppose that $k^{aa} > 0$, so that $\partial N/\partial k^{aa} = 0$. Then, using (62) it follows from (54) that $\partial N/\partial k^{ba} < 0$, which means that $k^{ba} = 0$. For this same reason, $k^{aa} > 0$ and $k^{ba} > 0$ cannot simultaneously hold. Now, suppose that $k^{ba} > 0$ and $k^{aa} = 0$, which means that $\partial N/\partial k^{ba} = 0$ and $\partial N/\partial k^{aa} \leq 0$. However, using again (62) and the FOC (56), it follows that $\partial N/\partial k^{aa} > 0$, contradicting $k^{aa} = 0$ and $k^{ba} > 0$. $\square$

Proposition 8. *Let k^{aa*} and k^{bb*} be implicitly defined by*

$$r'(k^{aa*}) = c - r(k^{bb*})\frac{tk^{bb*}}{(k^{aa*} + k^{bb*})^2}, \tag{63}$$

$$r'(k^{bb*}) = \frac{c}{(1-\bar{t})} + r(k^{bb*})\frac{tk^{aa*}}{(k^{aa*} + k^{bb*})^2}, \tag{64}$$

and

$$m^{kb^*} = \frac{\tau}{tp(k^{bb*})}\frac{[k^{aa*} + (1-t)k^{bb*}]}{k^{bb*}}. \tag{65}$$

If $0 \leq m^{kb} \leq m^{kb^*}$, then $\{k^{aa}, k^{bb}, k^{ab}, k^{ba}\} = \{k^{aa*}, k^{bb*}, 0, 0\}$ is a solution, with $k^{aa*} > k^{bb*}$.

Proof. Evaluated at $m^{kb} = m^{kb^*}$ and $k^{aa} = k^{aa*}, k^{bb} = k^{bb*}, k^{ab*} = 0$, (54) and (57) become equal to zero and (55) strictly negative. Since $r'(\cdot) > 0, r''(\cdot) < 0$, and given that from (63) and (64), $r'(k^{aa*}) < r'(k^{bb*})$, then $k^{aa*} > k^{bb*}$. $\square$

References

Anand, B. and Sansing, R. (2000), 'The weighting game: formula apportionment as an instrument of public policy', *National Tax Journal* **53**, 183–199.

DeMooij, R. A. and Ederveen, S. (2003), 'Taxation and foreign direct investment: A synthesis of empirical research', *International Tax and Public Finance* **10**(6), 673–693.

Devereux, M. P. and Griffith, R. (1998), 'Taxes and the location of productoin: Evidence from a panel of us multinationals', *Journal of Public Economics* **68**, 335–367.

Eggert, W. and Schjelderup, G. (2003), 'Symmetric tax competition under formula apportionment', *Journal of Public Economic Theory* **5**, 439–446.

Eggert, W. and Schjelderup, G. (2005), 'Corporate tax systems and cross country proft shifting: Formula apportionment versus separate accounting', *Working Paper* .

Gerard, M. and Weiner, J. M. (2003), 'Cross-border loss offset and formulary apprtionment: How do they affect multijurisdictional firm investment spending and interjurisdictional tax competition?', *CESifo Working Paper, No. 1004* .

Goolsbee, A. and Maydew, E. L. (1998), 'Coveting thy neighbor's manufacturing: the dilemma of state income apportionment', *NBER Working Papers, Working Paper 6614* .

Gordon, R. and Wilson, J. (1986), 'An examination of multijurisdictional corporate income taxation under formula apportionment', *Econometrica* pp. 1357–1374.

Kolmar, M. and Wagener, A. (2004), 'The role of the tax base in tax competition with formula apportionment', *mimeo* .

McLure, C. E. (1980), The state corporate income tax: Lambs in wolves' clothing, *in* 'The Economics of Taxation', Brookings Institution, Washington, DC.

McLure, C. E. (1981), 'The elusive incidence of the corporate income tax: the state case', *Public Finance Quarterly* **9**(4), 395–413.

Mieszkowski, P. M. and Zodrow, G. R. (1985), 'The incidence of a partial state corporate income tax', *National Tax Journal* **38**, 489–96.

Mintz, J. and Smart, M. (2004), 'Income shifting, investment, and tax competition: Theory and evidence from provincial taxation in canada', *Journal of Public Economics* **88**, 1149–68.

Nielsen, S. B., Raimondos-Meller, P. and Schjelderup, G. (2004), 'Company taxation and tax spillovers: separate accounting versus formula apportionment', *Economic Policy Research Union, Paper 01-07, Copenhagen Business School* .

Nielsen, S., Raimondos-Mller, P. and Schjelderup, G. (2003), 'Formula apportionment and transfer pricing under oligopolistic competition', *Journal of Public Economic Theory* **5**, 419–437.

Pethig, R. and Wagener, A. (2003), 'Profit tax competition and formula apportionment', *CESifo Working Paper No. 1011* .

Pinto, S. M. (2007), 'Corporate profit tax, capital mobility, and formula apportionment', *Journal of Urban Economics* **62**(1), 76–102.

Sørenson, P. (2004), 'Company tax reform in the european union', *International Tax and Public Finance* **11**, 91–115.

Figure 1: **The Model**

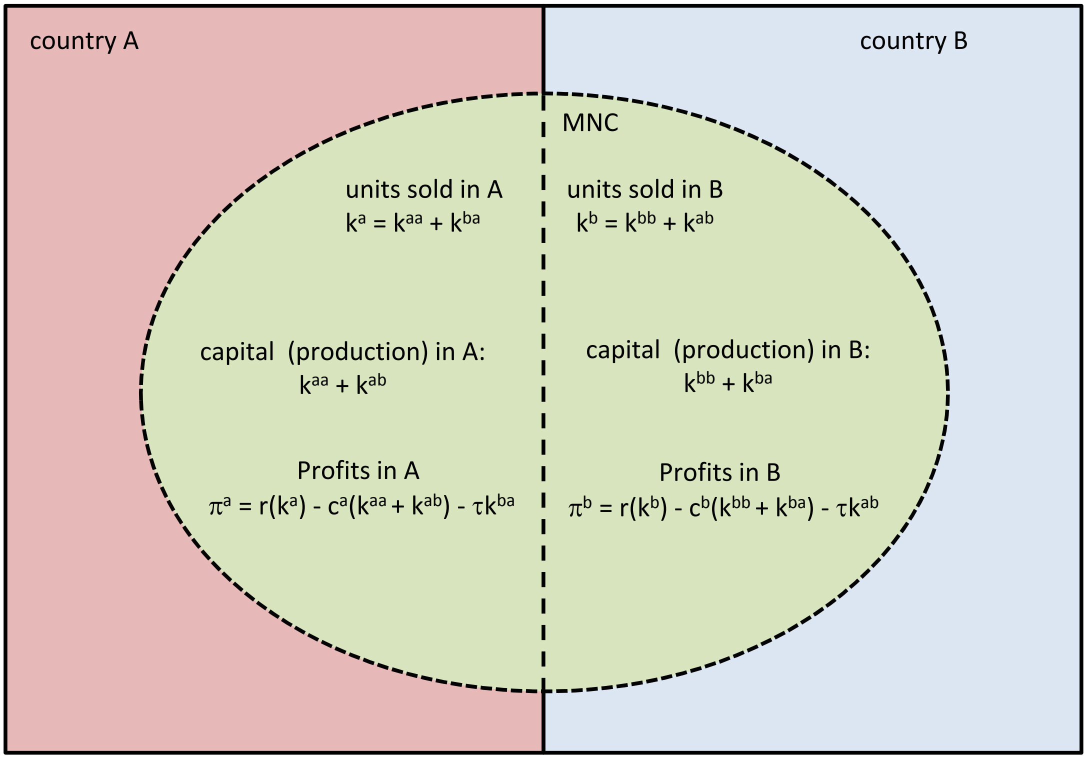

Figure 2: **Tax Revenues in a as a function of m^{ka} for different values of m^{kb}**

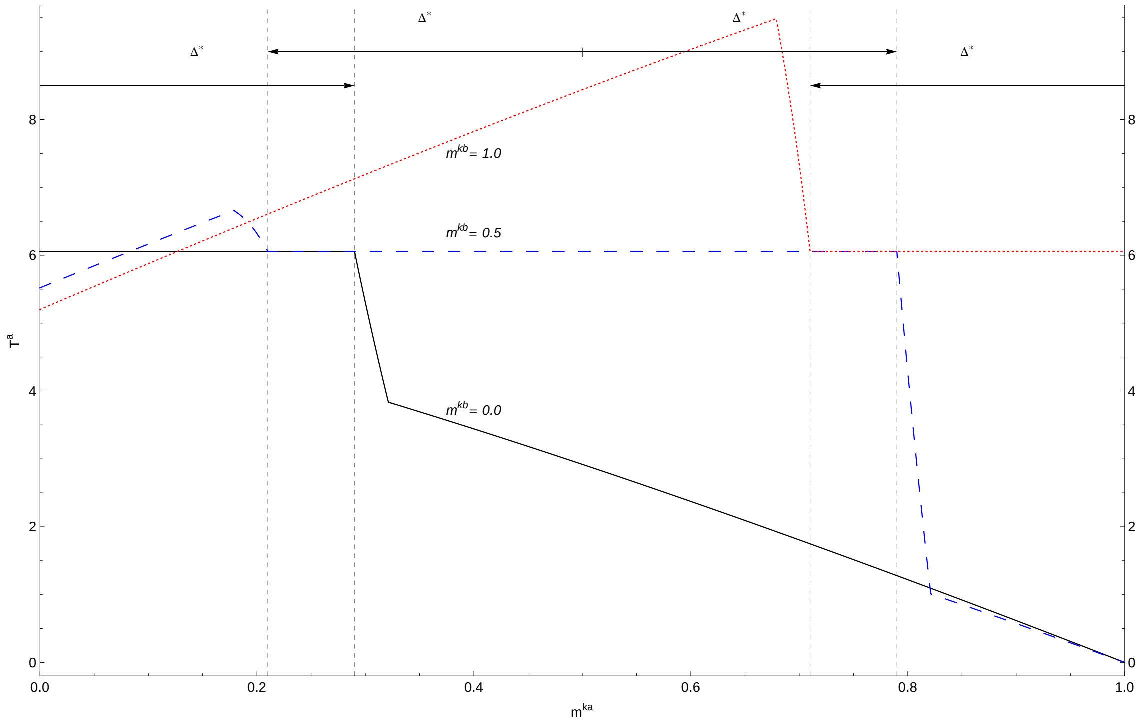

Figure 3: **Consumer Surplus in a as a function of m^{ka} for different values of m^{kb}**

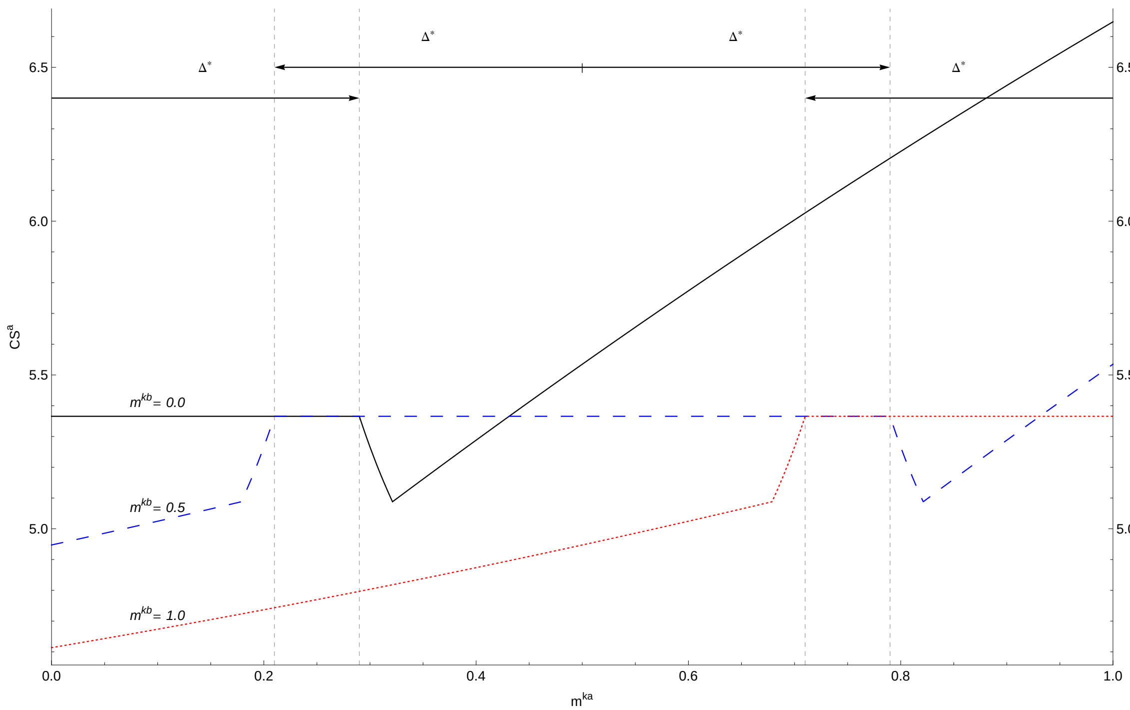

Figure 4: **After-Tax profits as a function of m^{ka} for different values of m^{kb}**

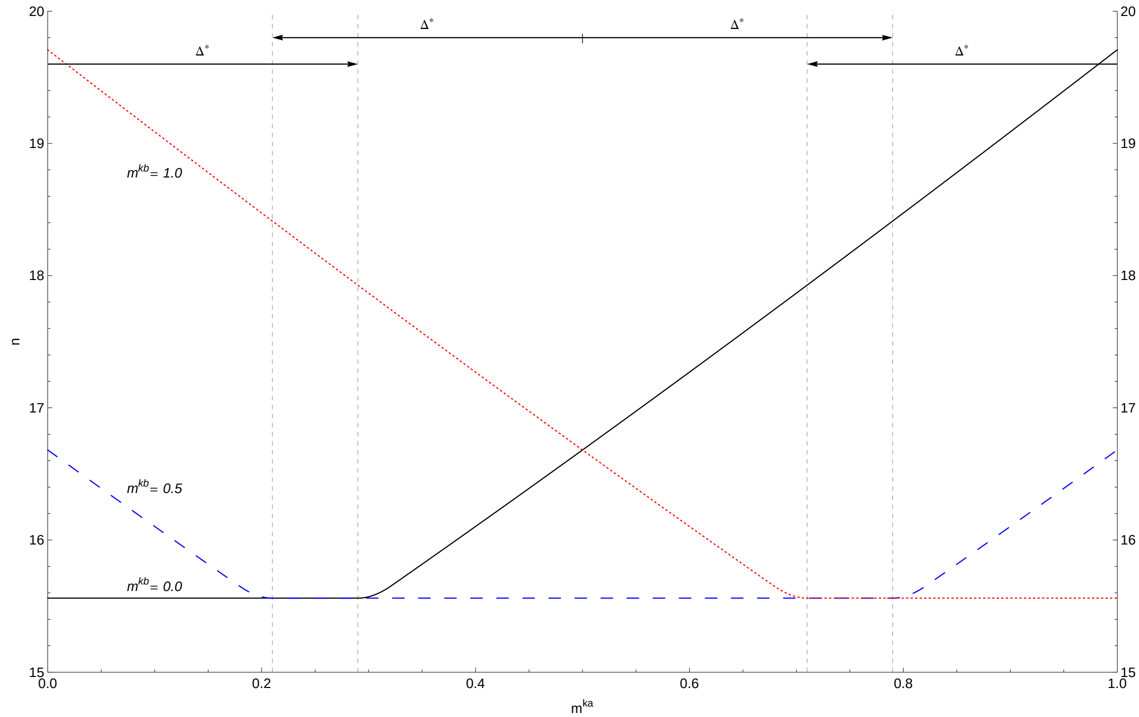

Figure 5: **Centralized and Decentralized Solutions: Summary of Results**

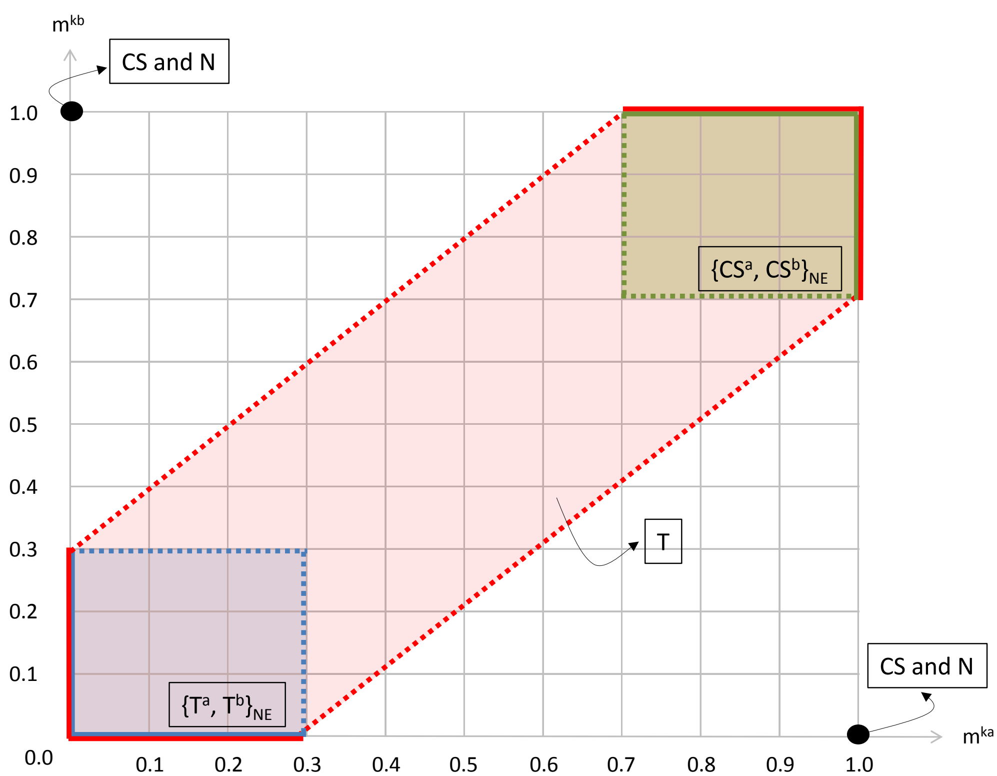

Table 2: Equilibrium values for different combinations of m^{ka} and m^{kb}

	m^{ka}	k^{aa}	k^{ab}	k^{ba}	k^{bb}	k^a	k^b	k	$p(k^a)$	$p(k^b)$	T^a	T^b	T	CS^a	CS^b	CS	N	π	$\bar{t}$
$m^{kb}=0.0$	0.0	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.1	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.2	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.3	2.128	0.000	1.118	3.245	3.245	3.245	6.491	6.755	6.755	5.304	5.915	11.218	5.266	5.266	10.532	15.571	26.789	0.261
	0.4	0.000	0.000	3.252	3.170	3.252	3.170	6.422	6.748	6.830	3.442	5.591	9.033	5.288	5.025	10.313	16.102	25.135	0.219
	0.5	0.000	0.000	3.327	3.146	3.327	3.146	6.473	6.673	6.854	2.919	5.520	8.439	5.535	4.947	10.482	16.681	25.120	0.204
	0.6	0.000	0.000	3.398	3.122	3.398	3.122	6.520	6.602	6.878	2.373	5.451	7.824	5.774	4.874	10.647	17.270	25.093	0.189
	0.7	0.000	0.000	3.465	3.100	3.465	3.100	6.565	6.535	6.900	1.806	5.384	7.190	6.004	4.804	10.808	17.867	25.058	0.173
	0.8	0.000	0.000	3.529	3.078	3.529	3.078	6.607	6.471	6.922	1.220	5.321	6.541	6.226	4.737	10.963	18.473	25.014	0.157
	0.9	0.000	0.000	3.589	3.057	3.589	3.057	6.646	6.411	6.943	0.617	5.260	5.877	6.441	4.674	11.115	19.087	24.965	0.141
	1.0	0.000	0.000	3.646	3.038	3.646	3.038	6.684	6.354	6.962	0.000	5.202	5.202	6.648	4.614	11.262	19.708	24.910	0.125
	m^{ka}	k^{aa}	k^{ab}	k^{ba}	k^{bb}	k^a	k^b	k	$p(k^a)$	$p(k^b)$	T^a	T^b	T	CS^a	CS^b	CS	N	π	$\bar{t}$
$m^{kb}=0.5$	0.0	3.146	3.327	0.000	0.000	3.146	3.327	6.473	6.854	6.673	5.520	2.919	8.439	4.947	5.535	10.482	16.681	25.120	0.204
	0.1	3.170	3.252	0.000	0.000	3.170	3.252	6.422	6.830	6.748	6.165	2.868	9.033	5.025	5.288	10.313	16.102	25.135	0.219
	0.2	3.245	1.118	0.000	2.128	3.245	3.245	6.491	6.755	6.755	6.322	4.896	11.218	5.266	5.266	10.532	15.571	26.789	0.261
	0.3	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.4	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.5	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.6	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.7	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.8	2.128	0.000	1.118	3.245	3.245	3.245	6.491	6.755	6.755	4.285	6.933	11.218	5.266	5.266	10.532	15.571	26.789	0.261
	0.9	0.000	0.000	3.252	3.170	3.252	3.170	6.422	6.748	6.830	0.574	8.459	9.033	5.288	5.025	10.313	16.102	25.135	0.219
	1.0	0.000	0.000	3.327	3.146	3.327	3.146	6.473	6.673	6.854	0.000	8.439	8.439	5.535	4.947	10.482	16.681	25.120	0.204
	m^{ka}	k^{aa}	k^{ab}	k^{ba}	k^{bb}	k^a	k^b	k	$p(k^a)$	$p(k^b)$	T^a	T^b	T	CS^a	CS^b	CS	N	π	$\bar{t}$
$m^{kb}=1.0$	0.0	3.038	3.646	0.000	0.000	3.038	3.646	6.684	6.962	6.354	5.202	0.000	5.202	4.614	6.648	11.262	19.708	24.910	0.125
	0.1	3.057	3.589	0.000	0.000	3.057	3.589	6.646	6.943	6.411	5.877	0.000	5.877	4.674	6.441	11.115	19.087	24.965	0.141
	0.2	3.078	3.529	0.000	0.000	3.078	3.529	6.607	6.922	6.471	6.541	0.000	6.541	4.737	6.226	10.963	18.473	25.014	0.157
	0.3	3.100	3.465	0.000	0.000	3.100	3.465	6.565	6.900	6.535	7.190	0.000	7.190	4.804	6.004	10.808	17.867	25.058	0.173
	0.4	3.122	3.398	0.000	0.000	3.122	3.398	6.520	6.878	6.602	7.824	0.000	7.824	4.874	5.774	10.647	17.270	25.093	0.189
	0.5	3.146	3.327	0.000	0.000	3.146	3.327	6.473	6.854	6.673	8.439	0.000	8.439	4.947	5.535	10.482	16.681	25.120	0.204
	0.6	3.170	3.252	0.000	0.000	3.170	3.252	6.422	6.830	6.748	9.033	0.000	9.033	5.025	5.288	10.313	16.102	25.135	0.219
	0.7	3.245	1.118	0.000	2.128	3.245	3.245	6.491	6.755	6.755	7.340	3.878	11.218	5.266	5.266	10.532	15.571	26.789	0.261
	0.8	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	0.9	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275
	1.0	3.276	0.000	0.000	3.276	3.276	3.276	6.552	6.724	6.724	6.058	6.058	12.115	5.366	5.366	10.731	15.560	27.675	0.275

Parameter values: $\alpha = 10, \beta = 1, c = 2.50, t = 0.275, \tau = 0.74, \Delta^* = 0.29$.