The Welfare Analysis of the Distribution Effect on Carbon Taxation

Shin-Kun Peng
Institute of Economics, Academia Sinica, Taipei, Taiwan

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Abstract

In this paper, we complement the literature by focusing on the examination and analysis the optimal tax rate as well as the optimal allocation schemes of the carbon taxation revenue with respects to the household welfare within a theoretical framework. We discuss three alternative schemes of fiscal policies associated with carbon taxation, respectively. First, if the share of taxation revenue to finance the local public goods is exogenously given by a specific political policy consideration, the optimal tax rate is increasing in pollution externality, but decreasing in either in the consumption weight of the aggregate manufacturing good or local public good. Second, we determine the optimal distribution share of the taxation revenue to finance the local public goods when the carbon tax rate is given by the government with a specific political policy consideration. It is shown that the optimal distribution share is decreasing in the share of carbon emissions in the production process of manufacturing goods. Finally, we examine the more general scheme when both the share and tax rate endogenous determining by the government simultaneously. It documents that the optimal tax rate is independent of the share of carbon emissions in the production process, and is increasing in the negative externality of pollution and the elasticity of consumption manufacturing goods, and decreasing in the weight of public good consumption. As the optimal distribution share is decreasing either in weight or elasticity of consumption manufacturing goods, or the share of carbon emissions in the production process of manufacturing goods.

JEL Codes: H20, H23, H41, Q50, Q58.

Keywords: Carbon emission, Pollution taxation, Fiscal policy, Local public good, Negative externality.

1 Introduction

The manufacturing sector is one of the primary sources of carbon emissions (Salem 2025; Sharma 2011). In general, production activities inevitably generate various levels of carbon pollution as a by-product. Such kinds of emissions will induce negative externalities on the household in the surrounding communities, particularly by deteriorating air quality and environmental conditions, which in turn usually emerge significant risks to the public health for residents (Ghasemi *et al.* 2023). Consequently, the governments usually need to adopt a few efficient environmental policies, such as the cap-and-trade system or carbon pollution taxation on carbon dioxide emissions (Pearce 1991), to regulate greenhouse gas emissions. These policies are designed to mitigate the adverse effects of climate change and the associated with health damages to the residents in the neighborhood communities near by the manufacturing firms (Shapiro and Walker, 2018). The imposition of a carbon tax on firms' dioxide emissions is thus regarded as an effective regulatory instrument for reducing carbon output (Aghion *et al.*, 2016; Anderson, 2009, 2019). For example, the British Columbia carbon tax scheme, introduced in 2008, required fuel producers to pay a levy per unit of carbon emissions (Yamazaki, 2017). Simulation and empirical analyses by Murray and Rivers (2015) document that this policy reduced emissions in British Columbia by approximately $5\% - 15\%$, demonstrating its effectiveness in improving local environmental quality. Hence, firms' pollution emissions can be viewed as one kinds of negative local public goods, and the local governments have both the responsibility and authority to implement some kinds of efficient policies to reduce carbon emissions and thereby enhance overall environmental quality for the residents.

When governments impose pollution taxes on the firms based on their amount of pollution emissions, they will generate the fiscal revenues from these levies taxation (Martin *et al.*, 2014; Metcalf and Stock 2020; Rausch and Reilly 2012). This raises an important and ongoing debate issue on how to efficiently utilize carbon tax revenues within the economy (Kollmann and Reichi, 2015; Mathur and Morris, 2012; Marron and Toder, 2013). The literature suggests several alternative schemes use such fiscal revenues (Metcalf *et al.* 2008). One common approach is to equally redistribute the taxation revenue to households, either as income transfers or social welfare subsidies (Goulder, 1995; Callen *et al.*, 2009; Jorgenson *et al.*, 2013; Berliant *et al.*, 2014; Goulder *et al.*, 2019; Anderson and Arkinson, 2020; Bourgeois *et al.*, 2021; Parry *et al.*, 2010; Rausch *et al.*, 2010; Rausch and Reilly, 2012). Another fiscal policy is to allocate the pollution taxation revenue to finance the local public goods which is regarded as a required consumption good and have positive externality and benefit on the residents, and thus to smooth the negative externality from carbon pollution emissions (Peng and Wang, 2005; Aydin and Esen, 2018; Rustico and Dimitrov, 2022), such kinds of policy like to set-up a clean-air infrastructure and environmental improvement facilities (Yenipazarli and Vakharia, 2015). Therefore, how to determine the optimal tax rate as well as the allocation of carbon tax revenue with respect to the overall social welfare of the residents, thus remains an important and interesting environmental as well as fiscal policy-relevant issue (Benchekroun and Van Long 1998; Bourgeois *et al*. 2021; Parry and Williams III 2010; Najjar and Cherniwchan, 2021).

Taiwan intends to adopt a pollution taxation that it functions as a source of fiscal revenue. Governments may use this revenue to fund local public projects or redistribute parts of it to neighborhood households, particularly treat as a subsidy for low-income families, thereby serving a role as social welfare subsidy purpose. However, Taiwan's policy framework differs slightly in terminology and administration. rather than imposing a "carbon tax," the government plans to implement a "carbon fee"- a charge administered by the Ministry of Environment-to improve overall environmental quality, effectively play a role as the positive local public good within the context of public economics. This carbon fee will has been implemented in 2025, the levy taxation will be based on the amount of the firms' carbon emission levels. Fundamentally, the carbon tax and carbon fee document the similar economic properties; the distinction lies primarily in which government institution has the authority to levy the carbon emissions and decide how the collected taxation revenues are utilized. Therefore, the optimal employment of pollution-related fiscal revenue remains a highly relevant and debatable issue for Taiwan government.

In this study, we consider the government can adopt to combine both the income subsidy and the local public goods provision policies by using the pollution tax revenue. Thus we complement the literature by focusing on the examination and analysis the optimal tax rate as well as the optimal allocation schemes of the carbon taxation revenue with respects to the household welfare within a theoretical framework. Section 2 sets up the theoretical model with respects to households, firms, and government, respectively. Section 3 develops the autarky economy framework. Section 4 examines the equilibrium and optimization based on the various schemes, and make a comparative static analysis to interpret their properties and their associated economic intuitions. Finally, section 5 concludes the findings of this study and documents the future possible extension study.

2 Model Setting

We develop a general equilibrium model involving three agents, including households, firms, and government, then examines how the different fiscal policy schemes to employ the carbon taxation revenue, and discuss its effect with respects to the overall social welfare.

2.1 Consumers

The welfare level of a representative household is specified as[1]

$$U = Q^{\alpha} G^{\beta} - \gamma E, \tag{1}$$

and Q is denoted by the composited manufacturing good with CES form of various manufacturing varieties produced in this economy as

$$Q = \left\{ \int_{i \in \Omega} \left[q\left(i\right) \right]^{\frac{\sigma-1}{\sigma}} di \right\}^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where Q, G, E, denotes the total consumption of manufacturing goods, the consumption of local public good provided by the government, and the total amount of the carbon emissions generated by all the manufacturing firms in the economy. For the simplicity, we assume that the provision of local public good is equal to the consumption of public good, and the production of local public good is completely determined by the finance of government fiscal source from the carbon emission taxation revenue. where the Ω is the mass of available consumption goods, $q\left(i\right)$ is the quantity consumption good for each variety i; σ denotes the elasticity of substitution between any two varieties; $0 < \alpha, \beta < 1$, and $\gamma > 0$ implies the degree of the negative externality on the health of the residents in this economy.

The price index of the aggregate consumption goods (**??**is given by

[1]The utility function specified by Egger *et al.* (2021) only consider the consumption of manufacturing good and doesn't include the consumption of local public good and negative externality souceing from pollution emission.

$$P = \left[\int_{i \in \Omega} p(i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}, \tag{3}$$

where $p(i)$ is the price of variety i.

The budget constraint for the household is given by

$$\int_{i \in \Omega} p(i) \, q(i) \, di = I = wL + (1-\eta) T = PQ, \tag{4}$$

where L is the amount of labor supply in this economy; w is the wage rate; T denotes the total pollution taxation revenue levied by the government, and $0 < \eta < 1$ represents the government decision on the proportion of the pollution taxation revenue to finance the local public good G. In equation (4), if η approaches 1 implies that the government decides to allocate almost whole the pollution taxation revenue to finance the local public goods G, and if η equals 0, from budget constrain equation(4), it is shown that the pollution taxation revenue would be equally lump sum transfer to the households as a subsidy. Thus, $(1-\eta)T$ implies that the parts of taxation revenue to be re-distributed to households as an income compensation in term of the social welfare subsidy. The aggregate price level is served as the numeraire $P = 1$.

2.2 The firms

Based on frontier model developed by Melitz (2003) and his followers (Melitz and Trefler, 2012), we specify the framework that before firms enter the market, they have to invest f_e units of final manufacturing goods to learn their capability $\phi \in [1, \infty)$, which is draw from a Pareto distribution $G(\phi) = 1 - \phi^{-\kappa}$ with density function $g(\phi) = \kappa \phi^{-(\kappa+1)}$. Without loss of generality, we set the lower bound of productivity equal to 1 ($\phi > 1$). Firms generate extra fixed entry costs f_d while entering the domestic market.

Given that all firms of type-ϕ apply identical optimal pricing rules, we can index each variety by the productivity, which allows us to express aggregate consumer demand in this closed economy as follows:

$$q(\phi) = Q \left[\frac{p(\phi)}{P} \right]^{-\sigma} = \frac{I}{P} \left[\frac{p(\phi)}{P} \right]^{-\sigma}, \tag{5}$$

In general, the firms produce manufacturing good generates carbon emissions. Firms can use labor, l, both in their production process and in reducing emissions, e. Follow Egger *et al.* (2021), an individual firm's production technology is given by $y(\phi) = \phi\xi(\phi) l(\phi)$, where $\xi(\phi)$ is the endogenous share of variable labor input that is used in production. The setting of firm-level carbon emissions follows Copeland and Taylor (2003), which is given by

$$e(\phi) = [\xi(\phi)]^{\frac{1}{\lambda}} l(\phi), \tag{6}$$

with $\lambda \in (0, 1)$. From (6), it means that emissions are increasing and convex in the share of variable labor input $\xi(\phi)$, and the marginal efficiency of abatement technology is decreasing in the labor share used in abatement, $1 - \xi(\phi)$. The production function can be rewritten as

$$y(\phi) = \phi [e(\phi)]^{\lambda} l(\phi)^{(1-\lambda)}, \tag{7}$$

thus, the ϕ denotes the exogenous level of total productivity, and emissions can be interpreted as an input of a Cobb-Douglas production technology. With exogenous emission tax t per unit of generated emissions, the $(1 - \eta)$ shares of tax will be redistributed to the all individuals. Firms minimize their production costs, and the cost function is derived from

$$\min_{e(\phi), l(\phi)} w \cdot l + t \cdot e \tag{8}$$

$$\text{s.t } y(\phi) = \phi [e(\phi)]^{\lambda} l(\phi)^{(1-\lambda)}, \tag{9}$$

then we obtain the amount of emissions and the labor demand, respectively, as

$$e(\phi) = \left[\frac{w}{t} \frac{\lambda}{(1-\lambda)}\right]^{(1-\lambda)} \frac{y(\phi)}{\phi}, l(\phi) = \left[\frac{t}{w} \frac{(1-\lambda)}{\lambda}\right]^{\lambda} \frac{y(\phi)}{\phi}, \tag{10}$$

therefore, the unit production costs is given by

$$c(\phi) = \Phi \frac{(w)^{1-\lambda} (t)^{\lambda}}{\phi}, \tag{11}$$

where $\Phi \equiv \lambda^{-\lambda} (1-\alpha)^{\lambda-1}$. The share of variable labor input used for production as

$$\xi(\phi) = \left[\frac{\lambda}{(1-\lambda)} \frac{w}{t}\right]^{\lambda}, \tag{12}$$

and the firm emission intensity is defined as

$$a(\phi) \equiv \frac{e(\phi)}{y(\phi)} = \frac{1}{\phi}\left[\frac{w}{t} \frac{\lambda}{(1-\lambda)}\right]^{(1-\lambda)}. \tag{13}$$

Since all firms of type ϕ adopt the identical pricing rules, we can index each variety by the productivity. The firms maximize their profit under monopolistic competition. Therefore, in autarky, firms optimal prices is $p(\phi) = [\sigma/(\sigma-1)]\, c(\phi)$, and the gross profits is given by

$$\pi(\phi) = \frac{1}{\sigma}\left[\frac{\sigma}{\sigma-1} c(\phi)\right]^{1-\sigma} I(P)^{\sigma-1} = \frac{1}{\sigma}\left[\frac{\sigma}{\sigma-1} c(\phi)\right]^{1-\sigma} Q(P)^{\sigma}. \tag{14}$$

3 Autarky Economy

In this paper, we will start to discuss the optimal fiscal policy for the pollution emission on the autarky framework, then move to examine the open economy in the future extension. For simplicity, we assume that before firms enter the market, f units of manufacturing good is required, the marginal firm with productivity ϕ_d is determined by the zero profit condition by free entry assumption, that is

$$\frac{1}{\sigma}\left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} [c(\phi_d)]^{1-\sigma} Q(P)^{\sigma} = wf, \tag{15}$$

With costly productivity draw, the expected profit $(\overline{\pi} - f)$ of entering the market equals to the fixed entry fee, and the corresponding variables are

$$\overline{\pi} = \int_{\phi_d}^{\infty} \pi(\phi) \frac{dG(\phi)}{1 - G(\phi_d)} = wf \left(\frac{\widetilde{\phi}}{\phi_d} \right)^{(\sigma-1)}, \tag{16}$$

$$\widetilde{\phi} = \left[\int_{\phi_d}^{\infty} \phi^{(\sigma-1)} \frac{dG(\phi)}{1 - G(\phi_d)} \right]^{\frac{1}{(\sigma-1)}} = \left(\frac{\kappa}{\kappa - \sigma + 1} \right)^{\frac{1}{(\sigma-1)}} \phi_d. \tag{17}$$

Then free entry condition implies that

$$[1 - G(\phi_d)] (\overline{\pi} - f) = wf_e. \tag{18}$$

Combine (15) and (18), the production threshold is

$$\phi_d = \left(\frac{\sigma - 1}{\kappa - \sigma + 1} \frac{f}{f_e} \right)^{\frac{1}{k}}. \tag{19}$$

It documents that the production threshold is not be affected by the level of emissions and the magnitude of tax. The mass of firms M is derived from the full-employment condition for labor as follows

$$L = \frac{M}{1 - G(\phi_d)} f_e + Mf + M\widetilde{l}, \tag{20}$$

and

$$\widetilde{l} \equiv \int_{\phi_d}^{\infty} l(\phi) \frac{dG(\phi)}{1 - G(\phi_d)} = (1 - \lambda) \frac{\kappa(\sigma - 1)}{\kappa - \sigma + 1} f, \tag{21}$$

and

$$M = \frac{\kappa - \sigma + 1}{\kappa [1 + (1 - \lambda)(\sigma - 1)]} \frac{L}{f}, \tag{22}$$

By price index $P = 1$, the wage rate is determined by

$$w = \left[\frac{1}{[1 + (1 - \lambda)(\sigma - 1)]} \frac{L}{f} \right]^{\frac{1}{(1-\lambda)(\sigma-1)}} \left(\frac{\sigma - 1}{\sigma} \frac{\phi_d}{\Phi} \right)^{\frac{1}{(1-\lambda)}} (t)^{-\frac{(\lambda)}{(1-\lambda)}}. \tag{23}$$

Total emissions in autarky are

$$E = M\widetilde{e}, \tag{24}$$

with

$$\frac{e(\phi)}{l(\phi)} = \frac{\widetilde{e}}{\widetilde{l}} = \left[\frac{w}{t}\frac{\lambda}{(1-\lambda)}\right], \tag{25}$$

and we know that

$$\widetilde{e} = \left[\frac{w}{t}\frac{\lambda}{(1-\lambda)}\right]\widetilde{l} = \frac{\lambda\kappa(\sigma-1)}{\kappa-\sigma+1}\left(\frac{w}{t}\right)f, \tag{26}$$

The emissions-labor ratio is related to the wage-tax ratio. Thus rewrite the total emissions as

$$E = M\widetilde{e} = \frac{\lambda(\sigma-1)L}{[1+(1-\lambda)(\sigma-1)]}\left(\frac{w}{t}\right). \tag{27}$$

The role of the government is to levy the pollution tax, collects the total tax revenue, distributes a part of the taxation revenue to finance the local public good, and employs other parts to subsidy the households as a income transfer. And we don't consider the government has the fiscal deficit or surplus. Thus, total emissions in autarky are

$$E = M\widetilde{e}, \tag{28}$$

with (25), we know that

$$\widetilde{e} = \left[\frac{w}{t}\frac{\lambda}{(1-\lambda)}\right]\widetilde{l} = \frac{\lambda\kappa(\sigma-1)}{\kappa-\sigma+1}\left(\frac{w}{t}\right)f, \tag{29}$$

Since the emissions-labor ratio is related to the wage-tax ratio, thus we can rewrite the total emissions as

$$E = M\widetilde{e} = \frac{\lambda(\sigma-1)L}{[1+(1-\lambda)(\sigma-1)]}\left(\frac{w}{t}\right), \tag{30}$$

and then the total tax revenue is given by

$$T = tM\widetilde{e} = \frac{\lambda(\sigma-1)}{[1+(1-\lambda)(\sigma-1)]}wL. \tag{31}$$

In turn, the total income I of a representative household is given by

$$I = wL + (1-\eta)T = \left[\frac{1+(1-\lambda\eta)(\sigma-1)}{1+(1-\lambda)(\sigma-1)}\right]wL, \tag{32}$$

in which the households income include the subsidy by the government sourced from the pollution revenue besides the wage income, and in turn, we can derive the distribution parts of taxation revenue on the finance of public goods as[2]

$$G = \eta T = \frac{\eta\lambda(\sigma-1)}{[1+(1-\lambda)(\sigma-1)]}wL. \tag{33}$$

Because under the monopolistic competition and free entry for the firms, the firms profits are zero. Given $\alpha+\beta=1$, thus the total welfare function (i.e., indirect utility) is obtained by

$$U = \frac{L}{1+(1-\lambda)(\sigma-1)}\left\{[1+(1-\lambda\eta)(\sigma-1)]^{\alpha}[\eta\lambda(\sigma-1)]^{\beta}w - \gamma\lambda(\sigma-1)\left(\frac{w}{t}\right)\right\}, \tag{34}$$

with the wage is derived as follows

$$w = \left[\frac{\sigma}{(\sigma-1)}\frac{1}{[1+(1-\lambda)(\sigma-1)]}\frac{L}{f}\right]^{\frac{1}{(1-\lambda)(\sigma-1)}}\left(\frac{\phi_d}{\Phi}\right)^{\frac{1}{(1-\lambda)}}(t)^{\frac{-\lambda}{(1-\lambda)}}. \tag{35}$$

[2]For simplicity, we specify that the production of local public good equals to the finance of local public good.

4 Optimization and Equilibrium

An explanation of both of the maximization of households and firms as well as the role of the government as the previous section, when we consider the households need to consume the local public good besides the convention setting with the manufacturing goods and agricultural goods consumption only[3]. The local public good is produced and provided by the government, and we don't consider the fiscal surplus or deficit from pollution taxation revenue on firms' emissions, then the equilibrium is defined by

Definition 1 *A competitive equilibrium is a list of prices {p, w, t}, and quantities on the decisions variables {q, l, y, e, η} for households, firms and government, respectively, such that*

(i), the households utility is maximized;
(ii), the firm's profit is maximized;
(iii), zero profit for the firms with free entry;
(iv), the government budget is balanced.

Since under the monopolistic competition and free entry specification for the firms, the profits of firms are zero in equilibrium, and also we specify that all the taxation revenues by the government is using parts of it to completely equal distribute to the households as a part of income for households as subsidy, and employs the residual parts of taxation revenue to finance the local public goods, thus there is not any taxation revenue surplus or deficit for the government. Therefore, with the maximization of social welfare is equivalent to the maximization of the represent household. And there are two decision variables for government, the choice of optimal carbon tax rate t, and determination of the optimal distribution share η of carbon taxation revenue to finance the local public goods. We will examine three different equilibrium schemes, respectively.

4.1 The allocation share of taxation revenue is given

First, we assume that the distribution share to finance on local public goods η is given by the government under a specific political policy consideration, by law or its authority, then we just can determine the optimal taxation rate t to maximize the social welfare (i.e.,

[3]We don't consider the consumption of agricultural good as the similar setting as Meltiz (2003) and Eggar *et al* (2021).

indirect utility). To differentiate the indirect utility function with respect to the tax rate t[4], we have

$$\frac{\partial U}{\partial t} = -\frac{(\lambda)}{(1-\lambda)}\left[\left[1+(1-\lambda\eta)(\sigma-1)\right]^{\alpha}\left[\eta\lambda(\sigma-1)\right]^{\beta}(t)^{-\frac{1}{1-\lambda}} - \gamma(\sigma-1)(t)^{\frac{-2+\lambda}{(1-\lambda)}}\right] = 0, \tag{36}$$

and it yields

$$t^{*} = \frac{\gamma(\sigma-1)}{\left[1+(1-\lambda\eta)(\sigma-1)\right]^{\alpha}\left[\eta\lambda(\sigma-1)\right]^{\beta}}, \tag{37}$$

Since the tax rate must be less than one, thus from the previous optimization tax rate t, the parameter sets (β, σ,λ) must be bounded by $\frac{\beta}{\lambda}\frac{\sigma}{\sigma-1} \leqslant 1$. Under this constrain condition, we have

$$\frac{\partial t^{*}}{\partial \eta} \gtreqless 0 \text{ if } \eta \gtreqless \frac{\beta}{\lambda}\frac{\sigma}{\sigma-1} \text{ with } \frac{\beta}{\lambda}\frac{\sigma}{\sigma-1} \leqslant 1. \tag{38}$$

From the previous obtained result for the optimization tax rate and its comparative analysis, we have

Proposition 1 *(i), it is shown that optimization tax rate t depends on the parameter sets (α, β, γ, σ,λ, η);*
(ii), the optimal tax rate t is increasing in γ, but decreasing either α or β, and the comparative static with
respect to η is increasing (decreasing) in η when η is larger (smaller) than $\frac{\beta}{\lambda}\frac{\sigma}{\sigma-1}$.

In addition, we need to check the extreme cases (i.e., corner solutions) for either tax rate is equal 0 or 100% tax on the carbon emission. From the optimal taxation rate equation (37), we know that if $t = 0$, then $\gamma = 0$, from (1), it implies that there is not any negative externality of the pollution on the households, and also there is not any taxation revenue to finance the local public goods since the tax rate is zero, in turn, there is not any provision of local public good by the government, then based on the specification of the

[4] Around the optimal tax rate, the second order condition is negative as shown in the Appendix, it guarantees the local maximum property.

utility function (1), the equilibrium utility level would be equal zero. Next, if $t = 100\%$, then

$$t = \frac{\gamma(\sigma-1)}{[1+(1-\lambda\eta)(\sigma-1)]^{\alpha}[\eta\lambda(\sigma-1)]^{\beta}} = 1, \tag{39}$$

It implies the parameter sets (α, β, γ, σ,λ, η) must bounded by the equation (39), since it is very difficult to match six parameters to satisfy this condition. Thus, we exclude this extreme case. The economic explanation of this proposition is intuitive, it implies that the firms must pay the higher carbon tax rate when the negative externality of carbon emissions is higher, and the firms levy the lower tax rate when either the consumption of the manufacturing goods or local public good is relatively more important for the households. And if the share of the taxation revenue on the local public goods, η, is relative higher (lower), then the firms must be levied more carbon tax rate when this share become higher, otherwise pay less.

4.2 The tax rate t is given

Next, we want to discuss how to determine the optimal allocation share η of the taxation revenue to finance the local public goods when the carbon tax rate is given by the government specific political policy.The maximization of utility with respects η yields

$$\frac{\partial U}{\partial \eta} = -\alpha(\sigma-1)\lambda + \beta[1+(\sigma-1)-\lambda\eta(\sigma-1)]\eta^{-1} = 0 \tag{40}$$

and

$$\eta^{*} = \frac{\sigma}{(\sigma-1)}\frac{(1-\alpha)}{\lambda} \text{ with } \frac{(1-\alpha)}{\lambda} < \frac{(\sigma-1)}{\sigma}. \tag{41}$$

Thus, based on the (41), it is shown that the optimal distribution share η^{*} is independent of the tax rate.[5] Since the η^{*} must be less than one, thus the relative parameters must be constrained by $\frac{(1-\alpha)}{\lambda} < \frac{(\sigma-1)}{\sigma}$, and we have $\frac{\partial\eta*}{\partial\alpha} < 0$, and $\frac{\partial\eta^{*}}{\partial\lambda} < 0$, and also it is shown that

$$\frac{\partial\eta^{*}}{\partial\sigma} = -\frac{1}{(1-\alpha)^{2}}\frac{(1-\alpha)}{\lambda} < 0. \tag{42}$$

[5]Around the optimal distribution share, the second order condition is negative as shown in the Appendix, it guarantees the local maximum property.

Therefore, under the given parameter set is satisfied by $\frac{(1-\alpha)}{\lambda} < \frac{(\sigma-1)}{\sigma}$,[6] we have

Proposition 2 *When the carbon tax rate t is given, the optimal distribution share η^* is decreasing either in α, λ ,or σ.*

This proposition implies that when the government levy the tax rate on the carbon emissions is determined by the consideration of a specific political policy, then the optimal distribution share of the taxation revenue η^* on the finance of local public goods will decrease if (i), the consumption of manufacturing goods is relatively more important than that of local public good for the households; (ii) the elasticity between the manufacturing goods is greater; (iii) the share of carbon emissions in the production of manufacturing goods is higher. The economic implication is intuitive.

4.3 The determination of both the tax rate and the share of distribution simultaneously

Finally, we move to examine the scheme that if the government simultaneously determine the optimal tax rate t and optimal allocation ratio η of taxation revenue to finance the local public good, we can obtain that[7]

$$t^{**} = \frac{\gamma}{(\alpha)^{\alpha}(1-\alpha)^{\beta}} \frac{(\sigma-1)}{\sigma}. \tag{43}$$

and

$$\eta^{**} = \frac{\sigma}{(\sigma-1)} \frac{(1-\alpha)}{\lambda} \text{ with } \frac{(1-\alpha)}{\lambda} < \frac{(\sigma-1)}{\sigma}. \tag{44}$$

Since both the tax rate t^{**} and the allocation share η^{**} must be less than one, thus the parameter sets (α, β, γ, λ, σ) must be bounded by

[6]The sufficient condition for this parameter set would be $1 < \alpha + \lambda$.

[7]Around the optimal tax rate and distribution share, the second order condition is satisfied as shown in the Appendix, it makes share the local maximum exists.

$$Max\ (\frac{(1-\alpha)}{\lambda}, \frac{\gamma}{(\alpha)^{\alpha}(1-\alpha)^{\beta}}) < \frac{(\sigma-1)}{\sigma}. \tag{45}$$

Under this constrain condition, we have

Proposition 3 *(i), the optimal rate t^{**} is increasing in the negative externality γ and the elasticity σ, and decreasing in β;*

(ii), the optimal distribution share η^{**} is decreasing either in α, λ ,or σ.

Consider the government has the authority to decide the optimal tax rate t^{**} and allocation ratio η^{**} of taxation revenue to finance the local public good, simultaneously, then it is shown that the optimal tax rate t^{**} is independent of λ. It implies that the negative production technology induce by the carbon emission would be endogenously determined in the decision of both the optimal tax rate t^{**} and allocation ratio η^{**}.

5 Concluding Remarks and Extensions

In this study, we consider there emerges carbon pollution emissions when the manufacturing firms produce the goods, which in turn induce the negative externality on the household in neighborhood community, especially on the residents' health. And in order to compensate the households' negative externality, the government needs either to provide some clean-air infrastructures or green facilities such as parks to households in terms of a local public goods which has a positive externality to benefit residents and then smooth the negative externality from carbon emissions, or subsidy the households as an income transfer, thus the local government have the authority to levy the carbon pollution tax on the firms and to determine how to efficiently employ the pollution taxation revenue with respects to households's welfare. In the literature, the government either equally redistributes the taxation revenue to households as income transfers, or employs the whole taxation revenue to finance the provision of the local public goods. In this study, we complement the literature to consider the other alternative mechanism, the government doesn't bounded by the previous two extreme alternatives, it can employ a part of the taxation revenue to finance the local public goods, and uses the other parts to redistribute to the households. We examine and analyze the optimal allocation schemes of using the carbon taxation revenue with respects to the household welfare within a theoretical framework. And we discuss three alternative schemes of determination the optimal taxation rate and the optimal employment the carbon taxation revenue sourcing by the carbon emissions, respectively.

First, if the share of taxation revenue to finance the local public goods is exogenously given by a specific political policy consideration, the optimization tax rate is increasing in pollution externality, but decreasing in either the consumption weight of the aggregate manufacturing good or local public good. Second, we determine the optimal distribution share of the taxation revenue to finance the local public goods when the carbon tax rate is given by the government specific political policy consideration. It is shown that the optimal distribution share is decreasing in the weight of the consumption of manufacturing goods, or the elasticity between the manufacturing goods is greater; also it is decreasing in the share of carbon emissions in the production process. Finally, we discuss the more general scheme when both the share and tax rate endogenous determining by the government simultaneously. It documents that the optimal rate is independent of the share of carbon emissions in the production process, and is increasing in the negative externality of pollution emission and the elasticity of manufacturing good, and is decreasing in the weight of the local good consumption; and the optimal distribution share is decreasing either in the weight of consumption manufacturing good, the share of carbon emissions in the production process,or the elasticity of manufacturing good.

In an extension to our baseline setting, it would be interesting to move this study extending to an open economy. Since under the current globalization economy, most countries are seriously concerning the effects of carbon pollution emission on the residents' health as well as for global warming, and especially for the extreme climate change, thus most countries in the world always try to reduce the carbon emissions across the industries, and then the international trade among the goods, and across the industries as well as the countries would impose more serious regulation policies on the carbon emissions for the firms' production. Therefore, each country must adopts the more appropriate environmental policy as well as pollution taxation mechanism to smooth or reduce the carbon emissions to match all these request regulations and agreements for their trade partners. And in turn the optimal decision on the pollution taxation and the employment the tax revenue between/among the countries would be more sophisticated, it remains to the future study (Spector 2001).

Appendix

From (34), the indirect utility function of household is given by

$$U = \frac{L}{1+(1-\lambda)(\sigma-1)}\left\{[1+(1-\lambda\eta)(\sigma-1)]^{\alpha}[\eta\lambda(\sigma-1)]^{\beta}w - \gamma\lambda(\sigma-1)\left(\frac{w}{t}\right)\right\}, \quad \text{(A1)}$$

and from (35), the wage rate is given by

$$w = \left[\frac{\sigma}{(\sigma-1)}\frac{1}{[1+(1-\lambda)(\sigma-1)]}\frac{L}{f}\right]^{\frac{1}{(1-\lambda)(\sigma-1)}}\left(\frac{\phi_d}{\Phi}\right)^{\frac{1}{(1-\lambda)}}(t)^{\frac{-\lambda}{(1-\lambda)}}. \quad \text{(A2)}$$

1, The allocation share of taxation revenue η is given

The FOCis given by

$$\frac{\partial U}{\partial t} = -K\frac{\lambda}{1-\lambda}\left[[1+(1-\lambda\eta)(\sigma-1)]^{\alpha}[\eta\lambda(\sigma-1)]^{\beta}(t)^{-\frac{1}{1-\lambda}} - \gamma(\sigma-1)(t)^{\frac{-2+\lambda}{(1-\lambda)}}\right] = 0, \quad \text{(A3)}$$

where

$$K \equiv \frac{L}{1+(1-\lambda)(\sigma-1)}\left[\frac{\sigma}{\sigma-1}\frac{1}{[1+(1-\lambda)(\sigma-1)]}\frac{L}{f}\right]^{\frac{1}{(1-\lambda)(\sigma-1)}}\left(\frac{\phi_d}{\Phi}\right)^{\frac{1}{1-\lambda}} > 0. \quad \text{(A4)}$$

where $\Phi \equiv \lambda^{-\lambda}(1-\alpha)^{\lambda-1}$. The FOC yields

$$t = \frac{\gamma(\sigma-1)}{[1+(1-\lambda\eta)(\sigma-1)]^{\alpha}[\eta\lambda(\sigma-1)]^{\beta}} \equiv t^{*} > 0. \quad \text{(A5)}$$

The SOC is satisfied because

$$\left.\frac{\partial^2 U}{\partial t^2}\right|_{t=t^*} = -\frac{K\lambda}{1-\lambda}\gamma(\sigma-1)(t^{*})^{\frac{-3+2\lambda}{(1-\lambda)}} < 0. \quad \text{(A6)}$$

2, The tax rate t is given

The FOC is

$$\frac{\partial U}{\partial \eta} = -Kt^{-\frac{1}{1-\lambda}} \left[1 + (1-\lambda\eta)(\sigma-1)\right]^{\alpha} \left[\eta\lambda(\sigma-1)\right]^{\beta} \left[-\frac{\alpha\lambda(\sigma-1)}{1+(1-\lambda\eta)(\sigma-1)} + \frac{\beta}{\eta}\right] = 0, \tag{A7}$$

which leads to

$$\eta = \frac{\beta\sigma}{\lambda(\sigma-1)(\alpha+\beta)} \equiv \eta^{*}, \text{ where } \frac{\beta}{\lambda(\alpha+\beta)} < \frac{(\sigma-1)}{\sigma}, \text{ since } \eta < 1. \tag{A8}$$

When $\beta = 1-\alpha$, which corresponds to the case you used,

$$\eta^{*} = \frac{\sigma}{(\sigma-1)}\frac{(1-\alpha)}{\lambda}, \text{ with } \frac{(1-\alpha)}{\lambda} < \frac{(\sigma-1)}{\sigma}. \tag{A9}$$

The SOC is

$$\left.\frac{\partial^2 U}{\partial \eta^2}\right|_{\eta=\eta^{*}} = -K\left[1+(1-\lambda\eta^{*})(\sigma-1)\right]^{\alpha}\left[\eta^{*}\lambda(\sigma-1)\right]^{\beta}(t)^{-\frac{\lambda}{1-\lambda}}\left[\alpha\frac{\lambda^2(\sigma-1)^2}{\left(1+(1-\lambda\eta^{*})(\sigma-1)\right)^2} + \frac{\beta}{(\eta^{*})^2}\right] < 0 \tag{A10}$$

Therefore, the SOC is satisfied.

3, The determination of both the tax rate and the share of distribution simultaneously

Since the η^{*} is independent of t, then substituting the η into the previous t, the FOCs of both η and t, are, respectively, given by:

$$t^{**} = \frac{\gamma(\sigma-1)}{\left(1+\left(1-\frac{\beta\sigma}{(\sigma-1)(\alpha+\beta)}\right)(\sigma-1)\right)^{\alpha}\left(\sigma\frac{\beta}{\alpha+\beta}\right)^{\beta}}. \tag{A11}$$

and

$$\eta^{**} = \eta^{*} = \frac{\beta\sigma}{\lambda\left(\sigma-1\right)\left(\alpha+\beta\right)} \text{ with } \frac{\beta}{\lambda\left(\alpha+\beta\right)} < \frac{\left(\sigma-1\right)}{\sigma}. \tag{A12}$$

Because

$$\left.\frac{\partial^2 U}{\partial\eta\partial t}\right|_{t=t^{**},\eta=\eta^{**}} = \left.\frac{\partial^2 U}{\partial t\partial\eta}\right|_{t=t^{**},\eta=\eta^{**}} = 0, \tag{A13}$$

the Hessian matrix is given by:

$$H \equiv \begin{vmatrix} \left.\frac{\partial^2 U}{\partial t^2}\right|_{t=t^{**},\eta=\eta^{**}} & 0 \\ 0 & \left.\frac{\partial^2 U}{\partial\eta^2}\right|_{t=t^{**},\eta=\eta^{**}} \end{vmatrix} > 0. \tag{A14}$$

Since

$$\left.\frac{\partial^2 U}{\partial t^2}\right|_{t=t^{**},\eta=\eta^{**}} < 0 \text{ and } \left.\frac{\partial^2 U}{\partial\eta^2}\right|_{t=t^{**},\eta=\eta^{**}} < 0 \tag{A15}$$

Apparently, the sufficient condition for a local maximum is satisfied.

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