Solar Power Diffusion in Urban Area:

Complexity and Structural Changes

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ABSTRACT

Decentralized generation of solar power has become one of the great energy alternatives for the world and it is in its first stages of dissemination as a new disruptive technology, with diverse effects on the economic development and the organization of the urban centres, generating structural changes that reach the functioning of the energy market, the urban dynamics and the household's behaviour who plays a simultaneous role as consumer and producer. Currently, there are few models of diffusion of decentralized solar generation using agent-based and most of them. Most of then use heterogeneous agent to generate macroeconomic time series only at country and region level. We designed a diffusion model to overcome the existing limitations by modelling a city of quasi-real topology that integrates several layers and add environmental and several micro and macroeconomic variables that are important for a systemic understanding of the problem. Relevant contributions include: a.) a model capable of dealing with the problem from the urban centre perspective; b.) spatially oriented diffusion that depends on the topological distribution of residences in an urban centre; c.) a real time-line with hourly solar radiation incidence, consumption and production in the short-run, synchronized with monthly households decision and accountability going up to 10 years of projection. d.) integration between the physical network (IEEE 33 standard) and

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economic decisions; e.) endogenous price setting considering different rules and market regulation framework; f.) cash flow of system operators and households; and finally, g.) evaluation of public policy, such as subsidies, minimum prices, tax and investments.

Key-words: Technological Diffusion, Solar Power, Urban Economics, Complex Systems, Agent-Based Model

1 Introduction

The technological advance in the generation of solar power through the use of photovoltaic panels (PV) has come to a threshold that can initiate structural changes in the energetic matrix of countries and cities. The falling costs and increasing cell-generating capacities coupled with other innovations in consumer equipment, such as LED technologies, when combined, can lead society to enter in energy abundance regimes (Diamandis and Kotler, 2012), or at least, sustain high standards of energy use without pressuring environmental resources and additionally reducing CO2 emissions by replacing oil-based generating plants.

The diffusion of decentralized solar power generation is typically a complex system, due to its interactions with various dimensions of reality, including economic dynamics of the markets, stabilizing physical network with varying generating nodes, interactive behaviour of the consumers who can behave as producer at the same time they consume, decentralized decision in demand and supply side, organization of urban centres and the environment. Consumer/supplier behaviour deserve attention and one of the sources of complexity comes from microeconomic aspects through the families interaction in social networks that affect their investment decisions and hence speeding the diffusion process. Another source of complexity relay on the electrical network which now must incorporate thousands of new generating nodes whose creation depends on the decentralized decision from the households. Stabilize such network is a tremendously challenging for engineers. In the model, we use a recognized algorithm to stabilize a physical network when new nodes are created and at the same time, they produce a floating flux of energy depending on the hourly radiation in a specific latitude and longitude. The solar radiation and temperature in the model can easily be calibrated for any city in the globe.

The diffusion of decentralized solar power generation opens up opportunities for economic growth, brings virtuous structural changes that, in addition to changing energy matrix, can promote a better income distribution at a personal level and increase per capita income by allowing families to sell energy in the system. Moreover, the diffusion promotes environmentally sustainable structural changes, as well as the reallocation of labour to a greener productive sector.

To deal with such a complex system we have developed an agent-based model that integrates important microeconomic and micro-physical characteristics which is capable of devising different emerging patterns of economic and urban development that can help to understand some important parts of this puzzle.

The model, better described in section (3), focus on urban development and contains several components and layers that allow us to analyse some of the important aspects we discuss in this introduction. Model's future developments may incorporate other important aspects like the impact of PV diffusion in the labour market structure, real topology of an urban centre, environmental impacts, industrial and public illumination, changing in the habits, introduction of electric cars, to mention some. The current version of the model is based on a quasi-real urban topology of a town with 32 neighbourhoods of equal size which can support different population density, differentiated by per capita income, profile towards technology and social relationship. Families living in these neighbourhoods are exposed to monthly decisions to invest their own resources in solar power generation. In the current version, we suppose that all investment required to expand the physical network is done by the household. Another future version should consider that when the households invest in generation, the transmission companies must also invest to expand the infrastructure of the physical network to support new producers and stabilize their complex network.

One of the major theoretical contributions of this model is the algorithm developed to integrate a physical network based on the IEEE 33 standard (section 4) coupled with spatial dynamics of an urban area and the economic decisions made by the market players. The model thus can be used to analyse and predict various micro and macroeconomic variables and to evaluate public policies.

2 Solar Generation Models in the Literature: innovation diffusion, social and physical network

The starting point to analyse the adoption of photovoltaic technology is the first models of innovation diffusion such as Rogers (1962, 2003), Chow (1967), Bass (1969), who first introduced the idea that the diffusion process of durable goods works like a contagious movement where some “innovators” helps to ignite the dissemination, followed by “early adopters” individuals and others with more conservative behaviour until the product reaches the maturity stage[1].

Such theories and models, particularly Rogers, highlighted several mechanism and variables which may accelerate or slow down the diffusion such as social relationship or network among individuals, word-of-mouth, mass media, homo and heterophily and leadership opinion. This early literature has called attention to the prominent role of consumer’s behaviour in the diffusion process. The social network concerns about how the consumers interact in such models and complement other variables like price, profit seeking, quality, efficiency of the technology by the supply side.

More recently the social network literature has given importance to many other motives why individuals create social links with others producing networks with multiples layers which can affect the diffusion of a innovation. Such social networks can emerge from motives so diverse as assortative, relational and proximity mechanisms (Rivera et al., 2010) or cognitive salience, collocation or *propinquity* and frequency of interaction (Bahulkar et al., 2017), and many causes of homophily such as genre, race, social class, job position, family, friendship (McPherson et al., 2001). There is also an upsurging neo-epidemic literature devoted to explaining the innovation diffusion process such as Young (2009). But this networks and respective motives are described in contexts so different as the large populations using some digital communication platform (e.g Messenger) in planetary scale (Leskovec and Horvitz, 2007) or so strict as the relationship among students in a college (Bahulkar et al., 2017), or small artistic community in Europe (Basov, 2018). Notwithstanding, the transposition of such multi layer social networks to a context of diffusion innovation, in particular to the diffusion of PV technology, must be done with care, not all motives apply. In the model described in section (3) we back to this point while describing the behaviour of households toward the adoption of solar power generation. Important reasons which should be take into account

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[1] In fact, one of the first famous model of innovation diffusion date back Ryan and Gross (1943) who analyse the diffusion of hybrid seed corn among 259 farmers in 1928 in Iowa, USA. In this study, the authors detected that the curve of adopters had a logistic shape, which became a standard behaviour for almost later models of diffusion of innovation.

when modelling the behaviour of households regarding to the consumption of energy is given by Siebert et al. (2017).

More recently, agent-based model of diffusion has been used in different context like environmental innovation (Schwarz and Ernst, 2009), healthcare (Dunn and Gallego, 2010), alternative fuel vehicles (Zhang et al., 2011), biomass fuel (Günther et al., 2004), among others. A general model interacting consumers and firms was proposed by (Einloft et al., 2016). The diffusion of PV follows the same principle observed in hundreds of other cases, but assuming idiosyncratic elements which distinguish such technology[2].

One of the first studies dedicated to the determinants of PV technology diffusion was Cesta and Decker (1978). This research was written in the first seconds of the diffusion of PV when the technology was giving the early steps leaving the laboratories toward the market. The authors applied a questionnaire using the Delphi method of interview to determine the factors that may either inhibit or stimulate solar energy commercialization. The main factors nominated by the experts interviewed were product cost, lack of product knowledge, lack of governmental support, and public concern over the energy crisis. No social network was mentioned at that moment, maybe because the social network theory was not so diffused and developed as today.

Three main approaches to model diffusion of power generation technologies has been used since then to analyse and forecast each specific energy market[3]: a.) technical high-resolution models; b.) inter-temporal optimization models and c.) agent-based models. There are advantages and disadvantages in using each approach which was discussed by Wittmann (2008, p. 9-12) at length. For the same reason described there, we are using an agent-based approach. It allows for the heterogeneity among actors, includes different decision algorithms, admits strong interactions in multi-layers network, generates macro dynamics from micro behaviour, and accounts for distributed technologies and policy frameworks. We could add to this long list the easy of interacting the evolution and dynamics in time and spatial dimension in a unique framework.

In recent literature Zhao et al. (2011) and Palmer et al. (2015) has implemented ABMs (Agent-Based Models) to simulate the diffusion of photovoltaic systems[4]. Both authors model exclusively the diffusion of PV technology, which is the special interest for our proposals.

The model of Zhao et al. (2011) is a hybrid two-level simulation model, where the lower-level concerns the calculation of the payback period of an individual household based on hourly electricity generation and consumptions, tax and financial incentives, PV module price, and hourly electricity price. At the higher-level the heterogeneous households make a weekly decision about adopt or not the technology, going to 20 years forward. The household choice is influenced by four factors: payback period, household income, word-of-mouth and advertisement effect. The profile of a household varies by income and number of members. The model thus is calibrated for two residential areas in two different regions in Tucson, Arizona and New York, USA. The model also considers two types of financial incentives: investment tax credit and feed-in tariff which allows for simulating different public policy. The tax rebate is considered constant. The energy consumption by the households is computed hourly based on a basket of durables good and habits derived from real surveys. By the supply side, the amount of energy depends on the panel efficiency (a fixed parameter) and hourly solar radia-

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[2] For a comprehensive review of innovation diffusion models, encompassing different markets, products and approaches, see Kumar (2015).

[3] Including different kind of distributed technologies: micro-gas turbines, reciprocating engine co-generation, solar cells, solar thermal collectors, micro-hydro generation, stationary fuel cells, hydrogen, and eolic turbines.

[4] A survey on ABM models of innovation diffusion in different markets or products can be found in Kiesling et al. (2012).

tion, but no amount of energy produced by household is shown. In this model, the author just computes the number of adopters and no energy balance of all city is shown. Although the price varies hourly depending on the demand, the price, in fact, is exogenously fixed assuming an arbitrary decreasing level in the long-run.

The model developed by Palmer et al. (2015) departs from Zhao et al. (2011) extending the model in some new directions: the model is adapted to study the Italian PV market[5]; characterize the agents according to Rogers (1962, 2003) adopter categories; and a new procedure to calibrate the initial conditions. Like Zhao, the price of the energy is exogenous and fixed in the beginning of a simulation, and the households evaluate a linear and additive utility function with four weighted components: payback period, environmental benefits to invest in PV, income, social networks or communications. But differently, the amount of energy produced by households depends on four variables: available rooftop area for PV modules, the efficiency of the solar cells, the PV system efficiency, and the irradiation at standard conditions, which is assumed to equal to 1 kW/m^2. Another novelty introduced in the model is an equation to compute the costs to install a PV plant like administrative fees, maintenance and depreciation. One important feature of Palmer's model is that it is a regional model rather than urban. The population of Italy is distributed by the regions preserving the characteristics of each region and then the model is computed for all the country. An important feature introduced by Palmer is the type of house inhabited by a family. An agent represents a household living in a single or two-family house where a single one has a larger rooftop area. During the initialization routine, the agents are allocated in these houses according to the income level.

The agent-based aspects of both model are the same: a household adopts the new technology based in a number of adopters in the neighbourhood. While in Zhao's model assume a threshold number of 4 links for all, in Palmer's model the number of links varies across households and mostly occurs among agents belonging to the same socio-economic background. A common aspect of this models is that despite the fact they consider a city (Zhao et al) or regions (Palmer et al) where families live, the diffusion occurs only in the time dimension at an aggregated level.

Moving on, we build an ABM with some features presents in that models but expand then in many new directions. First, we take the urban space explicitly, by locating a household in a specific place in a lattice (urban area). The urban space has a topology and can be configured with a varying population density which affects the aggregated balance of power produced and consumed. The model expands the analyses from adoption/diffusion only to the spatial dynamics of urban areas. Can a city be self-sufficient and even export energy during some time? Second, we explicitly consider a physical layer of an electrical network. Each household is connected in a physical grid which requires investment by the power distribution companies to absorb new individual producers and stabilize the network. Instead of having a tree system like IEEE 33 bars alimented be a central plant, now we have to consider that each household can connect to a IEEE node as a new producer. Third, the model can be easily adapted for any city in the world adapting the urban topology, temperature, radiation, economic and social variables. Fourth, we explicitly account the economic and electrical variables following a bottom-up procedure, so that we can analyse micro and macro dynamics not only in time but also in urban space. Fifth, we use a endogenous mechanism to set the price to buy and sell energy considering many kinds of regulation. And finally, sixth, we use an more explicit mathematical notation to deal with time and space and make the dynamics more comprehensible and transparent. Others minor changes are being introduced. These new

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[5] Which include a system of incentives, price regulation, demography, the behaviour of households and radiation.

features and details will be explained in the section (3).

3 The Model

The objective of this work is to simulate the diffusion of residential photovoltaic panel in an artificial town using real data to calibrate temperature and radiation and economic and social proprierties of a population. Two network layers spatially oriented are used both to transmit electricity among residences and to connect the households with social links.

The physical network for transmission of energy is built using IEEE 33 bar standard. Based on that we can evaluate the impact of new suppliers or new nodes which potentially destabilize the network. At the intermediate level, the IEEE 33 bar system was used to simulate the power distribution, based on parameters to deal with the floating charge and generation behaviour. Thus we use the Newton-Raphson method to stabilize the network, to calculate the power flux and to evaluate the behaviour of the electrical quantities and energy losses on a daily horizon.

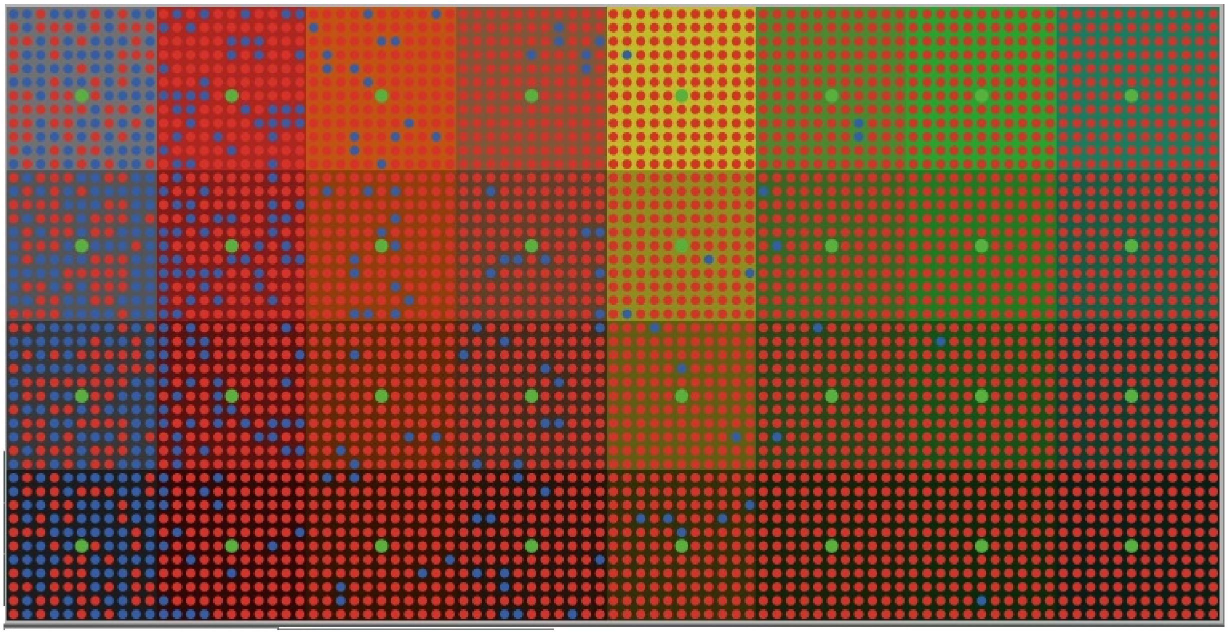
At micro-economic level, agent-based techniques were implemented to model the interactive behaviour of the households who use some heuristic to make a decision whether to adopt photovoltaic panel or not. The town is inhabited by a population of heterogeneous households, taking into account mainly, (1) the propensity to adopt new technologies (2) the level of household income, and (3) the social influence of communication with other agents and (4) the return on investment (ROI). To estimate the return on investment, the model considers the investment costs, the depreciation, the energy price at the moment of the decision, and calculates the expected net present value (NPV) for the period of 10 years. The timeline can easily be expanded for a longer period. The number of the years is a parameter that can be set. Some authors like Benedito (2009) and Jannuzzi et al. (2009) suggest that the lifespan of this technology is 20 years.

The propensity to use new technologies follows the division presented by Rogers and Shoemaker (1971), classifying them in five categories: innovators, early adopters, early majority, late majority and laggards. Different percentage of the population in each profile can be set as a parameter in the model. The income distribution standard among the households in the town replicate approximately the real standard observed in data but different Gini curves can be chosen at the beginning of a simulation. Finally, the social influence among households includes a world of mouth (WoM) mechanism which helps to accelerate or retard the photovoltaic diffusion (Britt, 1966; Buttle, 1998; Sweeney et al., 2012).

To deal with such a complex system we build a temporally and spatially oriented agent-based model. The spatial dimension represents the two-dimensional topology of a quasi-real urban centre with latitude and longitude orientation where the households live. The latitude and longitude of the city as a whole determine the temperature and solar radiation which, in the end, affects the energy production function. The radiation and temperature are equal in all the town but vary hourly along the year. So, we can distinguish the latitude and longitude of a household living in one city from the other living in any city on the globe.

Inside the city the urban area is modelled as coordinate point in an 88x44 lattice, where a househouse live. This city has, therefore, 3872 address. In the simulation ahead such space is constant, but the size of the city can be set at the beginning of a simulation. Inside this space, the households (residences) make a decision whether to adopt new technology becoming an energy supplier at the same time they remain consumers. New adopters, constitute new nodes in physical (electrical) network layer what may cause instability. Physical instability requires to run an optimization algorithm (in IEEE3 standard) which computes the necessity to expand physically the network by public or private investment by government or firms shifting the electrical network topology. In the current simulation, notwithstanding, the topology is constant and just receive new producer. The households occupy a place in the landscape, represented by a coordinate (x, y). For the sake of simplicity, all residence was calibrated to have the same size and there is a uniform rooftop area exposed to solar radiation. The area converted to solar energy is limited only by the income of the households[6]. The stylized urban area can be seen in figure (1). In this figure, the residences are represented by points, and the points with installed solar panels are marked as blue while non-adopters are red. The figure is a snapshot of one simulation exercise. This landscape evolves over time. The big square in the background are neighbours and the different colour means that the income varies from high (left oriented) to low (right oriented) level. There are richer and poorer neighbour which is an important economic variable that intervene in the process of diffusion.

Figure 1: Topology: Stylized Urban Centre



The green points represent the distribution sub-stations of a centralized generation system in an IEEE 33 standard. The links of such a network can be seen in figure (4). In the model, the centralized system produces a constant quantity of energy. When households start to produce their energy the energy budget of the city eventually becomes surplus and can be exported to other regions or sold to other types of consumer than residentials. In this version, no mechanism of exportation has been implemented. We consider the city as a closed system.

The timeline of the model reproduce the real-time and unify hourly production function in the short-run with monthly long-run dynamics. We use the subscript $h \in [1, 24]$ to denote hour and the subscript $t \in [0, 120]$ to denote month in a 10-year horizon. While the production and consumption function is computed hourly, due to the households daily cycle of life and habits, the investment function, which is responsible by diffusion dynamics, is computed once a month. The household budget also is updated once a month when the bills must to be paid and revenue received. The figure (2) shows the timeline in a comprehensive way. We assume a representative day inside the month and therefore, once we compute daily variable, we can add up to a monthly value just multiplying it by 30. This procedure helps to save computations cost and became a worth strategy when we need to run hundreds of simulation to explore the behaviour space of the parameters.

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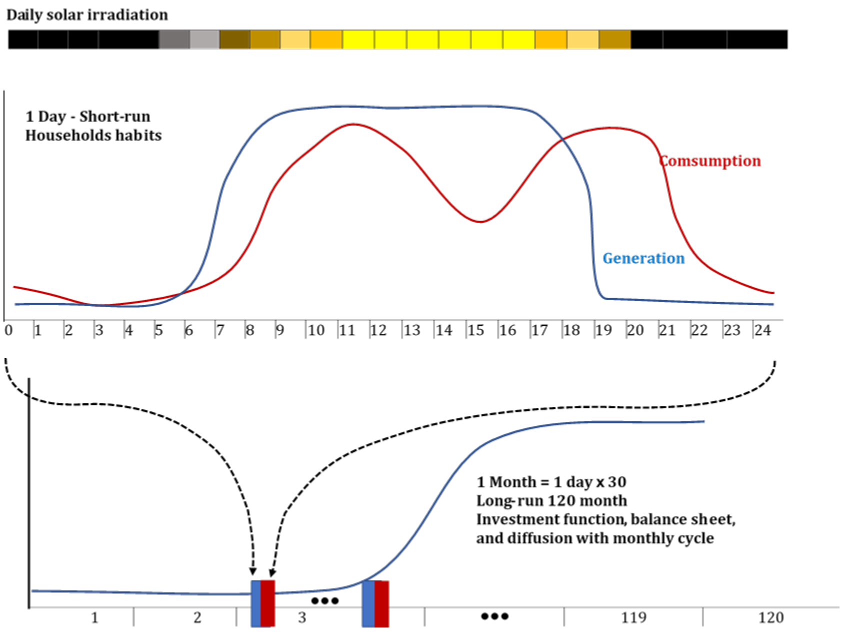
[6] In a new version of the model this features will be set to replicate more real feature of a town.

Figure 2: The Timeline of the Model



3.1 *Climatic variables*

The generation of solar energy in urban area depends on the temperature and solar irradiation. This varies across cities in the globe depending on the longitude and mainly the latitude. We calibrate the model with real hourly data of city Curitiba-Brazil, located at Lat 25° 25' 40" S and Long 49° 16' 23" W. To avoid short-run climate variation we compute the monthly average temperature and radiation incidence covering the period of jan/01/2012 to dec/31/2017, over a period of 5 years. The figure (3) shows the hourly radiation by day and month in the year 2017. We then use this monthly average as a representative day in the respective month and compute the temperature and radiation adding a random normal distribution shock:

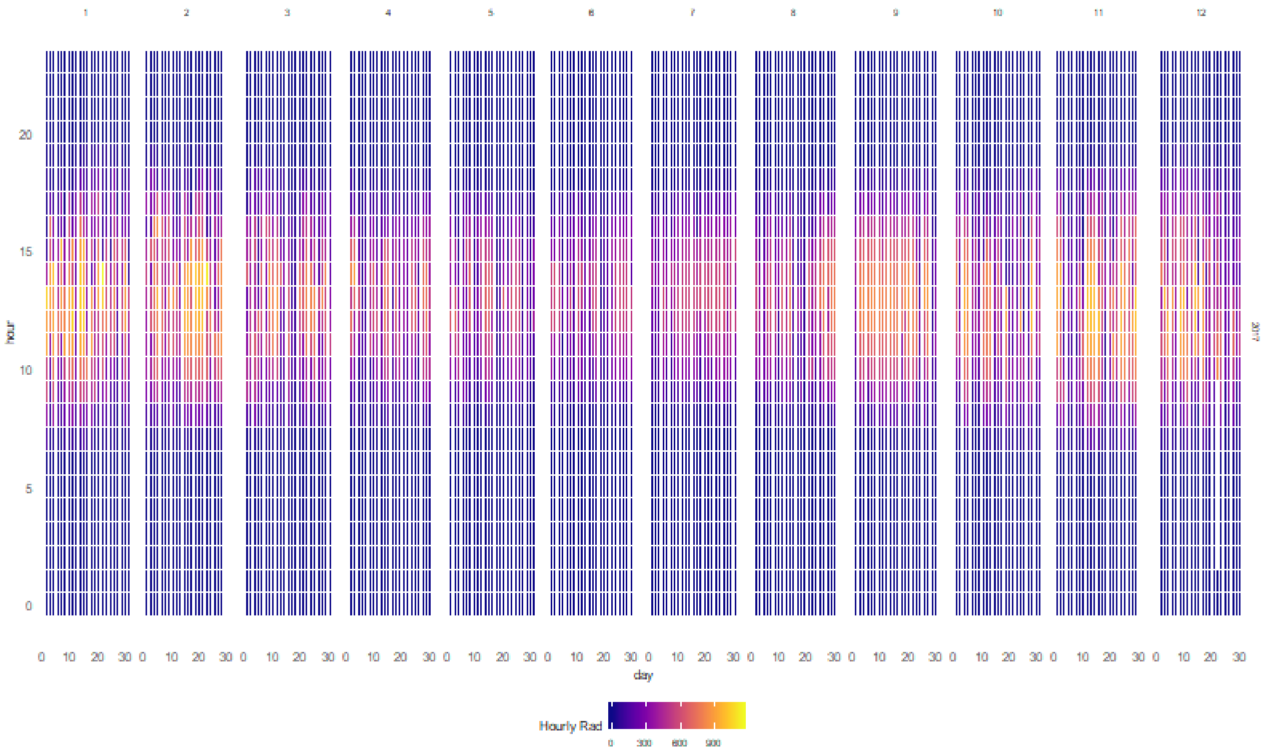
$$T_{h,t} = \bar{T}_{h,t} + e^1_{h,t} \qquad \text{where} \quad e^1_{h,t} \sim N(0, \sigma^{e1}_{h,t}) \tag{1}$$

and

$$R_{h,t} = \bar{R}_{h,t} + e^2_{h,t} \qquad \text{where} \quad e^2_{h,t} \sim N(0, \sigma^{e2}_{h,t}) \tag{2}$$

where $T_{h,t}$ is the temperature in the hour h in the month t, $\bar{T}_{h,t}$ is the 5 years average temperature in hour h in the month t, R_t is the solar radiation, $\bar{R}_{h,t}$ is the average and $e_{1,2,h,t}$ are random errors with normal distribution with $N \sim (0, \sigma^{1,2}_{h,t})$. Notice that T and R vary cyclically along the day and along the month in the year.

Figure 3: Hourly Radiation in Curitiba, Brazil in 2017



3.2 *The consumption of energy by the households*

All the residences in the city start consuming an amount of energy depending on your income level buying all energy in the market and producing nothing. Since the demand for energy can vary cyclically during the day, the demand (charge) by households is computed hourly and accumulated by taking the sum of the day, $C_i^{day} = \sum_{h=1}^{24} \mathcal{C}_{i,h}(Y_i)$. Considering this as a representative day in the month we can compute the charge monthly (time t) by multiplying this representative day by 30, according to the following equation:

$$C_{i,t} = 30 \times \sum_{h=1}^{24} \left[\mathcal{C}_{i,h,t}(Y_i) + e^3_{i,h,t}\right] \tag{3}$$

where $C_{i,t}$ is the monthly (t) consumption measured in kWh of a household i with income equal to Y_i, $\mathcal{C}_{i,h,t}(Y_{,i})$ is a implicit consumption function which depend of the household income its respective habit and $e^3_{i,h,t}$ is a random shock with a normal distribution. The household consumption habits vary according to income in such a way that the hourly (h) consumption pattern of a poor households is different of a rich one (see also figures 5 in section 5). We allow for small variation in income inside a neighbour in the urban space.

3.3 *Generation of solar energy by a residence: the production function*

The balance between consumed and generated energy by a household can be modelled by distinct ways depending on the regulation rules of the market, which may restrict the generation or not. A household can produce more kWh it consumes and freely sells the surplus in the market. Another option is to impose a restriction where a household sell an amount of energy limited by its consumption level, with some inter-temporal mechanism of compensation to correct daily unbalances. In this version of the model, we will assume this second case, based in actual Brazilian regulatory framework.

According to the actual rules of ANEEL[7], the value of the power produced and sold can only be deduced from the monthly consumption with a compensating mechanism to correct an eventually unbalance in the next 12 months. Such a mechanism means that $C_{i,t} = G_{i,t}$, and more precisely is:

$$\sum_{t=1}^{12} C_{i,t} = \sum_{t=1}^{12} 30 \times \sum_{h=1}^{24} G_{i,h,t} \tag{4}$$

where $\sum_{t=1}^{12} C_{i,t}$ is the annual consumption accumulated in 12 month (t) which must be equal to the power generated in the same period. The amount of power generated by a household $G_{i,t}$ will be explained later. But the compensation is made in monetary terms, not in kWh. so, we need to compute the equivalent monetary expression, given by the following equation:

$$\sum_{t=1}^{12} 30 \sum_{h=1}^{24} (C_{i,h,t} P^{C}_{i,h,t}) = \sum_{t=1}^{12} 30 \sum_{h=1}^{24} (G_{i,h,t} P^{G}_{i,h,t}) \tag{5}$$

The equation (5) can not be computed directly since the household does not know exactly its stream of consumption and production in 12 months. Its individual consumption is subject to a random shock (see equation 3) and additionally the temperature and radiation are subject to a similar behaviour. To overrun this, we take the expectation of equation (5), getting:

$$30E\left(\sum_{t=1}^{12}\sum_{h=1}^{24}(C_{i,h,t} P^{C}_{i,h,t})\right) = 30E\left(\sum_{t=1}^{12}\sum_{h=1}^{24}(G_{i,h,t} P^{G}_{i,h,t})\right) \tag{6}$$

or

$$\sum_{t=1}^{12}\sum_{h=1}^{24}[E(C_{i,h,t})E(P^{C}_{i,h,t})] = \sum_{t=1}^{12}\sum_{h=1}^{24}[E(G_{i,h,t})E(P^{G}_{i,h,t})] \tag{7}$$

Applying the expectational operator in the consumption, generation and respective prices (for price see 23 and 21 ahead) and substituting in (7) we get:

$$12\sum_{h=1}^{24} \bar{C}_{i,h} P^{C}_{i,h,t} = \sum_{t=1}^{12}\sum_{h=1}^{24}(E(G_{i,h,t}) P^{G}_{i,h,t}) \tag{8}$$

which is the balanced equation that warrant the annual consumption and production of a household. In the case of Brazilian economy this equation works like a restriction operating on the investment function. A household will never invest more than it consumes, by legal imposition. In the main model's algorithm this restriction can be deactivated, allowing for different framework rules.

The next step is to compute the amount of power to be generated by a household after it decides to adopt new technology $G_{i,t}$. The investment function $(K_{i,t})$ will be explained in the subsection 3.4. Given the physical amount of solar capital $K_{i,t}$ in kWh, the amount of power to be generated $(G_{i,t})$ depends on the technological capacity of the photovoltaic panel which, in turn, depends on the temperature and radiation:

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[7] Agência Nacional de Energia Elétrica [National Electrical Power Agency]- ANEEL, normative nº 482/2012.

$$G_{i,h,t} = K_i \left(\frac{R_{h,t} + e^2_{h,t}}{1000W} \right) [1 - 0,0045(T_{h,t} + e^3_{h,t} + 15)] \tag{9}$$

where the numeric constants reflect the state of art in photovoltaic cells technology and temperature and radiation are subject to a random normal shock with mean equal zero. We assume the technology is constant but in the case of an endogenously process of technological changing this fixed number could be replaced by some equation explaining the pace that the technology evolves.

Taking the expected value of equation (9) we get:

$$E(G_{i,h,t}) = K_{i,t} \left(\frac{R_{h,t}}{1000W} \right) [1 - 0,0045(T_{h,t} + 15)] \tag{10}$$

We can simplify this equation a bit by defining the variable $\Omega_{h,t}$

$$\Omega_{h,t} = \frac{R_t}{1000W} [1 - 0,0045(T_{h,t} + 15)] \tag{11}$$

and getting a shorter equation:

$$E(G_{t,i}) = K_i \Omega_{h,t} \tag{12}$$

Now we can use the equation (12) into (7) and produce:

$$12 \sum_{h=1}^{24} (C_{i,h} P^C_{i,h,t}) = \sum_{t=1}^{12} \sum_{h=1}^{24} (K_i \Omega_{h,t} P^G_{i,h,t}) \tag{13}$$

Since K_i is constant within the sum operation on h and t we can place it in the outside and then solve the equation (13) to K_i getting:

$$K_i = \frac{12 \sum_{h=1}^{24} (C_{i,t} P^C_{t,i})}{\sum_{t=1}^{12} \sum_{h=1}^{24} (\Omega_{h,t} P^G_{i,h,t})} \tag{14}$$

Equation (14) is the physical amount of capital in photovoltaic cells to be installed when a household decides to adopt this technology. The equation shows that the physical installed capacity of a household is a consequence of its *annual* expected value of consumption and production in monetary terms. To compute the installed capacity in monetary terms we use a linear function, which means we assume no economy of scale:

$$P^K_{i,t} = \rho_0 + \rho^K_1 K_{i,t} \tag{15}$$

where $P^K_{i,t}$ is the total investment faced by a household i in the month t, ρ_0 is the minimum value to be invested or a fixed cost and ρ^K_1 is the variable unitary cost of capital. In order to introduce a subsidy policy incising in the price of equipments which benefit the households, it is possible to add an exogenous parameter ϕ such that we get the final equation for the amount of investment to be done by a household to install a plant:

$$P^K_{i,t} = (1 - \phi)(\rho_0 + \rho^K_1 K_{i,t}) \tag{16}$$

3.4 The Investment by households and firms or government

In the model, we distinguish two types of investment: the one taken by households ($I^H_{i,t}$ when it decides to adopt solar generation, and the one taken by firms $I^N_{i,t}$ in infrastructure to expand the physical network. We simplify the supply side assuming only one firm which produces and *distribute* all energy. In the time $t = 0$ all energy distributed by the physical network and sold to the households comes from the traditional source. At each point of time ($t = month$) the households evaluate a utility function to decide whether to invest or not in the solar generation. In this version of the model, the amount of money invested is paid in cash[8].

At each month households take into account a series of factors internal and external to their family nucleus to make the decision to acquire the photovoltaic systems in their residences. The total probability is a linear combination of each determinant as follow:

$$\mathcal{P}(a)_{i,t} = \alpha_1 \mathcal{F}_1(ROI_{i,t}) + \alpha_2 \mathcal{F}_2(Links_{i,t}) + \alpha_3 \mathcal{F}_3(Y_i) + \alpha_4 \mathcal{F}_4(Profile_i) \tag{17}$$

In the equation (17), $\mathcal{P}(a)_{i,t}$ is the probability that each family i has to adopt (a) the photovoltaic system in the month t motivated by each of the four determinants. The parameters $\alpha_{1...4}$ represent the weight each decision-making factor has within the equation and the $\mathcal{F}_{1...4}$ are the implicit functions for each determinant of the investment decision (x): ROI, social links, income and the household profile toward technology. We must ensure that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \leq 1$.

The functions $\mathcal{F}_{1...4}$ are a cumulative probability distribution function. For all components of (17) we assume a logistic function:

$$\mathcal{F}(X)_{i,t} = \frac{2}{1 + e^{-\theta^x_0 X^{\theta^x_1}_{i,t}}} - 1 \tag{18}$$

where $\mathcal{F}(X)_{i,t} \in [0, 1]$, $X_{i,t}$ is the value of each one of the four determinants (x) and $\theta^x_{0,1}$ are parameters to control the turning point of the logistic function, again, for each determinant.

To evaluate whether a household will adopt new technology we compare the value of equation (17) with a random number uniformly distributed $U_{i,t} \in [0, 1]$. If the value computed in (17) is higher than $U_{i,t}$ the new technology will be adopted in time t:

$$f(n)_{i,t} = \begin{cases} 0 & \text{if } \mathcal{P}(a)_{i,t} \leq U_{i,t} \\ 1 & \text{if } \mathcal{P}(a)_{i,t} > U_{i,t} \end{cases} \tag{19}$$

Then, the capital to be created by household i in time t is given by the equation (14) subject to test (19):

$$K_{i,t} = f(n)_{i,t} K_i \tag{20}$$

3.5 Prices

The dynamics of price is not a trivial mechanism and can vary depending on how government regulate the market, going from a completely free price mechanism to a completely fixed one. Rules can be designed also to regulate consumer (P^C) and producer (P^G) prices and between these two extremes (completely free or fixed price), some specific mechanisms can be designed to attend different proposals.

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[8] In a future version, a credit mechanism will be implemented, where the government creates a specific line of credit with a grace period to amortize the loan, and complementary, a mechanism of compensation where the energy sold by a household can be used to repay the loan and the interest. In this case, only a few or even zero initial payment needs to be done in cash.

From de *production* side the price mechanism can first distinguish between two regimes: the regime type 1 where the selling price (P^G) is based on the cost of solar energy, and the regime 2 where the price is forced by rules to be equal to the price of consumption. These regimes can be represented be equation (21):

$$P_{i,t}^G = \begin{cases} \beta_1 + \beta_2 E_t^G & \textit{if price regime is type 1} \\ \beta_1 + \beta_2 P_{i,t}^C & \textit{if price regime is type 2} \end{cases} \tag{21}$$

where $\beta_{1,2}$ are parameters which make the price fixed or flexible, E_t^G is the unitary cost of generating solar energy in kWh and $P_{i,t}^C$ is the price that a households buy energy in the market. The Brazilian case, for instance, is the case of regulation regime 2 and $\beta_1 = 0$ and $\beta_2 = 1$, forcing the households sell energy by the same price they buy it, $P_{i,t}^G = P_{i,t}^C$. The German case is characterized by the regime 1 with $\beta_1 = \bar{P}$ and $\beta_2 = 0$, or in others terms, a case where government fixes the selling price. In the German case, the constant price helps strongly to stabilize the households expectation contributing so far to accelerate the diffusion of the solar energy. In the Brazilian case, the rule was designed also to eliminate the uncertainty but by anchoring the selling price at the consumption price. In this case the incertitude is not completely eliminated since the consumption price can vary by changing in the other source of energy, affecting the return of the investment.

From de *consumption* side others two distinct features are considered: a.) tax rate; b.) intra-day price variation to discourage consumption in rush hours ; c.) an income level differentiation due to social public police. In part "a" government can charge the households with a percentage of tax on the price. In the part "b", the consumption price can be fixed during all the day or can change each hour depending on the daily cycle to reduce the gap between the kWh supplied and demanded by households along the day. Finally in case "c" it is possible that the public policy distinguishes between poor and rich household and adopt a lower consumption price (P^C) if the income is below a threshold. These options are set at the beginning of a simulation and the rule is imposed on all households and remains the same until the end. This threefold price mechanism can be formalized by the following equation:

$$\hat{P}_{i,t}^C = \frac{1 + \boldsymbol{\tau}_2(h) - \boldsymbol{s}(Y_i)}{1 - \tau_1} E_t \tag{22}$$

where $\hat{P}_{i,t}^C$ is the consumer price before market adjustment (see equation 23 forward) to be paid by a household i in time t, τ_1 is the percentage of tax charged at the final price, $\boldsymbol{\tau}_2$ is a vector of 24 element which depends on the hour of the day. If all the h elements are $\boldsymbol{\tau}_{2,h} = 0$ the price paid by households remains constant during all day and if some h element $\boldsymbol{\tau}_{2,h} > 0$ the price during the respective hour will be higher to discourage high level of consumption during the energy rush time. $\boldsymbol{s}$ is a vector of length 5 to represent the subsidy rate which depends on the household income level (Y_i) divided into 5 classes, and E_t is the unitary cost of the energy incurred by the firms considering all sources of energy[9]. We assume that there is only another alternative source of energy than the solar so that the cost of the energy in time t is a weighted average between both, $E_t = \phi P^f + (1-\phi) P_{t-1}^G$ where the weight is the share of each source in aggregate level and P^f is the fixed cost of traditional source (hydroelectric, nuclear, etc) and P_{t-1}^G is the cost of the solar kWh.

We assume also that the price paid by households decrease as the number of adopters increase, following a standard supply curve with negative and linear inclination. Let H represent the fixed number of households living in the city and H_t^s the number of adopters. The final price can be adjusted by the market as suggested by the following equation:

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[9] Note that we distinguish the cost of solar energy generated by the households (E^G) of the average cost of the energy considering all sources (E_t).

$$P^{C}_{i,t} = \hat{P}^{C}_{0,i}\left[(1-\beta_0)\left(\frac{H^{s}_{year-1}}{H}\right)\right] \tag{23}$$

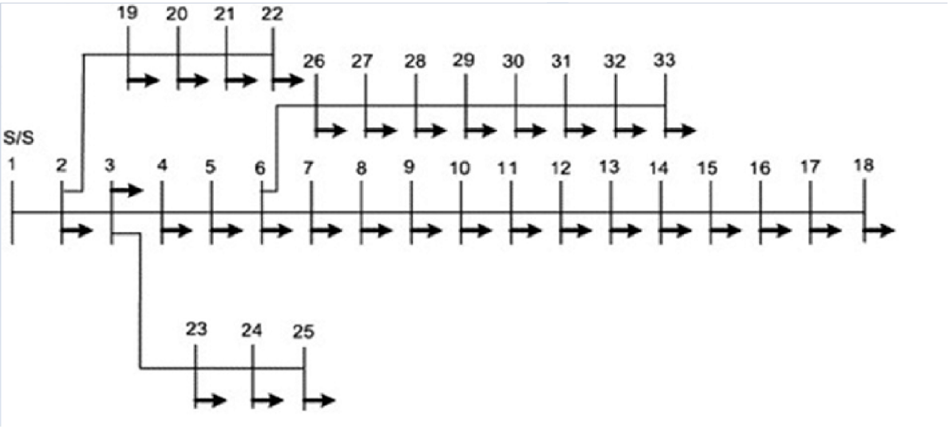
where $P^{C}_{i,t}$ is the *final* price of energy to be paid by a household *after* the market adjustment, $P^{C}_{i,0}$ is the price at the starting point and β_0 is a parameter that regulates how much the share of adopters contributes to decrease in the price. We assume that a contract to sell energy fixes prices for a year, so the equation (23) is computed once by year and not monthly.

4 Electrical Network

To simulate the power flow in networks, we use the IEEE 33 bus system. This system is commonly used to model distribution lines, as in the work of Arasteh et al. (2016). The slack bus bar (node 1) represents the source of energy that feeds the city externally. It is the reference bar, any variation of load or generation must be discounted in it. The other 32 bars are representations of the electrical transformers. Each is linked to a neighbourhood and the power flow calculation is performed relative to them. The electric transmission network is simplified compared to a real network, but it is useful to analyse the possible impacts on the hourly level in the system.

In this work to calculate the power flow, we used the Newton-Raphson method, which consists of estimating a group of variables based on the roots of a function and on some previously known parameters. For this, an initial approximation is chosen for this, after that, the equation of the tangent line (by means of derivatives) of the function at that point and the intersection of it with the axis of the abscissa, in order to find a better approximation to the root. By repeating the process, an iterative method is created to find the root of the function.

Figure 4: Diagram of IEEE 33 Bars



We use this network to distribute energy in the stylized city. Each bar in figure (4) corresponds to a neighbour (square) in figure (1). There are 32 bars and 32 neighbours which match perfectly. In figure (4) bar number 1 is the network input node. All the power entering by this node correspond to energy generated by the external source, for instance, hydroelectric, nuclear and others. At the beginning of a simulation, when no household produces energy, the totality of consumption will be attended by this node. When the solar power starts to diffuse in the city, the new amount of energy produced will compete with external source and at some point of the diffusion process the city became self-sustainable and even more, the city can become surplus so far.

In this version of the model, there is no industrial consumption of power. This is not essential to the dynamics of diffusion. It would be easy to place some industries in some neighbour and add some demand for street lighting. This could make the history more realistic but would not change dramatically the plot. The urban structural change can be pushed by household only, instigated by diverse public policies which can accelerate the capital deepening in small decentralized plants.

The decentralized generation of energy produces a remarkable structural change in urban development. The decentralized production system can transform urban development radically by making the urban area self-sustainable. How far and deep this structural change can go, will depend on how many variables and public policies are combined. This is a complex system with multiples solutions that only a simulation design can highlight. This is what will be done in the next section.

5 Simulations

In the following simulation, we explore a base-line simulation and discuss the results quickly. More robustness and sensibility analysis can be done than we did now. Notwithstanding, we take some real data to calibrate the model so that the results are sufficient to explore some appealing urban dynamics.

5.1 Base-line simulation

The main parameters are summarized in table (1). Some equations have random terms so we run 10 simulations with different random seeds and take the average to ensure a minimum of robustness in the results. All the trajectories for this 10 simulation are similar, so, we can use average values as a representative case. To better understand the dynamics of the system in behaviour space for some selected parameters we make a simple sensitivity analysis mapping the result space of selected variables for a range of such parameters. At the end, we explore some alternatives scenarios by particular calibration.

The model was calibrated to the city of Curitiba-Brazil, located at Lat 25° 25' 40" S and Long 49° 16' 23" W. This location determines the average radiation and temperature which at the end, contributes to the power generation of PV. In this version of the model, we use a stylized urban topology with 32 neighbourhoods with the same size and population density. A new version of the model is being prepared to deal with real urban topology[10]. The size of the stylized city is defined by the size of the grid. We use a grid with 88×44 resulting in 3872 spatial positions, where 32 are occupied by a distribution station (IEEE 32). At the end there are 3840 places where a household lives distributed uniformly among 32 neighbour. This a scale of a real city. Considering that 3 people live in a typical family or residence, this simulated city is comparable to a city with 3840×3=11,520 inhabitants. If the reader is interested in a higher city, for instance, with about 1.0 million inhabitants, just multiply this

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[10] Curitiba currently has an estimated population of 1,917,185 inhabitants in 2018 (IBGE), distributed in 76 neighbourhoods occupying an urban territory of 435 km^2 with 319,4 km^2 in urban space. Considering the urban area only, the population density is 6,010 inhabitants by km^2. Considering the average size of a family of 2.97 (IBGE, 2008) we have currently 645.516 families estimated what means in average 8,494 families or households by neighbourhood. In the stylized urban area, we use a grid with 32 neighbourhood of equal size where live 132 households. Thus, the scale of stylized urban are is about 132:8,494 or 1:64. One household in the model represents, in average, 64 households, in the case of Curitiba, Brazil.

results by 100 to have an approximated result, apart from some scale effects which we do not address right now.

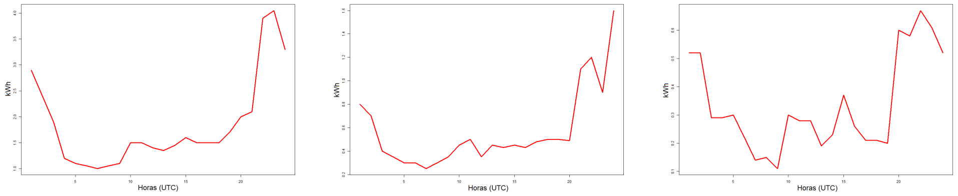
Table 1: Parameters of base-line simulation

Description	Param	Equation	Value
Size of City	Grid		88×44
Number of Households	H		3840
Number of Neighbours	N		32
Probability weight ROI	α_1	17	0.175
Probability weight Links	α_2	17	0.400
Probability weight Income	α_3	17	0.150
Probability weight Profile	α_4	17	0.150
Income distribution			4x4x16x4x4
Social network: distance	$\gamma_{1,2}$		0.40, 3.0
Logist distr. function	$\gamma_{3,4,5}$	17	15,2,0.04
Price system (regulation)			common
Subsidy	s_i	23	0
Household comsumption	β_0	23	0.6995
Household production	$\beta_1, 2$	21	0, 0.99
P^C, P^G difference	τ_1	22	0
Household production	τ_2	22	appendix 2
Tax rate	τ_3	22	0.7295
Radiation	R	22	appendix 3
Temperature	T	22	appendix 3
Probability function	θ_0	19	6.0
Probability function	θ_1	19	5.0

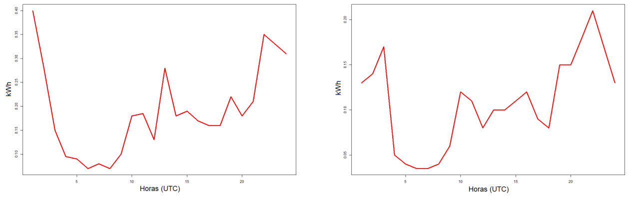
As explained in equation (3) the hourly consumption of power depends on the income level and the pattern vary across the households. The figure (5) below shows the empirical hourly pattern of each income class assumed in this simulation, which correspond to the habit of households in Curitiba, Brazil according to Francisquini (2006). As the reader can notice, the habits vary considerably between class. In the model's algorithm, these data are stored in hourly matrix which is scanned by the function $\mathcal{C}_{i,h}(Y_i)$ (see equation 3).

Figure 5: Hourly household consumption pattern by income class (kWh)

(a) class A **(b)** class B **(c)** class C



(d) class D **(e)** class E



The relationship between income and consumption is explained in the table (2) below.

Table 2: Household income and monthly consumption^2

Income bracket	**Class**	**Min-Max kWh**^{1}	**KW/month**
$\geq$ 18.740	A	1.000 - 4.050	$\geq$ 500
9.370-18.740	B	0.250 - 1.600	300-500
3.748-9.370	C	0.110 - 0.670	200-300
1.874-3.748	D	0.007 - 0.400	100-200
0-1.874	E	0.035 - 0.210	$\leq$ 100

(1) Source: Francisquini (2006).

(2) Income bracket represents the Brazilian income distribution and consumption standard. The amount of kWh consumed vary along the day between the *min* and *max* which is different for each income level. The number of households inside each class is a parameter which can be set at the beginning of a simulation. Inside each class, the households are distributed according to a normal distribution.

The figures below show the result of a simulation taking the base-line parameters above. Each time step in the simulation represents 1 hour of a day. We assume the day is a representative day in a month so that we jump from one day to a month by multiplying 1d × 30 = 1month, saving thus considerable computational costs. The horizon for diffusion is 10 year or 120 months. Taking all together we have 24h×12month×10year = 2880 time steps.

The figure (6) shows the cumulative number of households who adopted solar power. With the base-line calibration, about 23% of households in the city will adopt solar power after 2880 steps time or 10 years. The figure (7) shows the cumulative amount of monetary resources devoted to adoption by the households. At the end of 10 years this population of 3840 households invested approximately $35 million. Almost the totality was made by class A and a minor share by class B. As can be seen in the table (1) we used 4×4×16×4×4 income distribution in the sequence ABCDE. This resembles a symmetric distribution of population where the middle class (C) occupy 16 neighbours and A,B,D,E 4 each one. In each neighbour live 132 families. Considering this we can say that we have 132×4=480 households in class A, B, D, E and 120×16=1.920 in class C. Taking class A and B together this 960 families invested almost $35 million or $35.458 by households, a value not far from the reality for the price of the equipment today.

Figure 6: Adoption

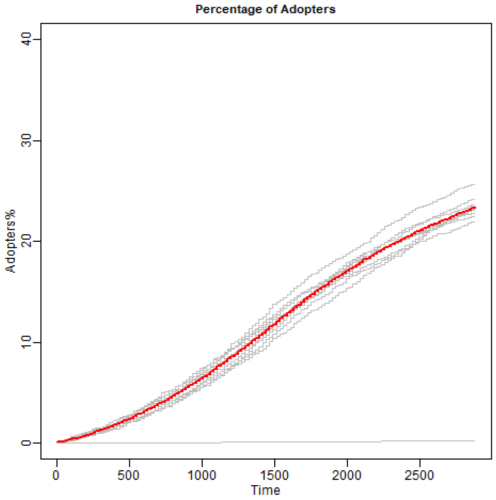
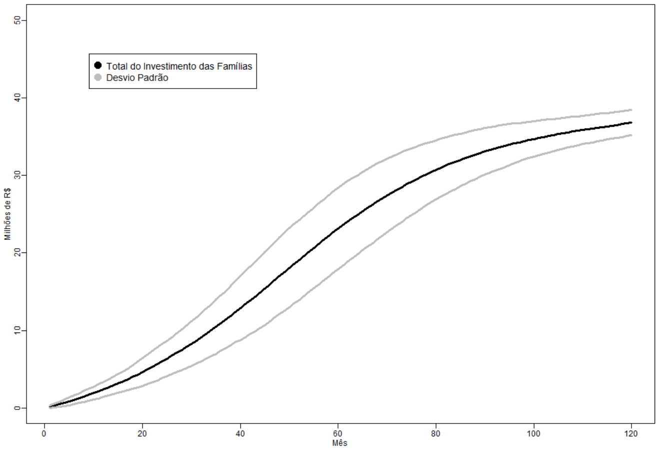


Figure 7: Generation Capacity by Income Class



Even more, personal income distribution plays an important rule in the diffusion. In figure (8) we can see that the income class A is responsible for almost all adoption followed by class B. The other three classes contribute nothing to the diffusion. The main cause for this is not only the income distribution by self, but also the mechanism adopted in the Brazilian case by imposing a restriction such that the equation (8). As a poor household consume few amount of energy it doesn't have economic incentive, since the the small size of the plant won't produce a positive cash flow to compensate the investment. In this case, only a purposeful public policy would help this families. The current Brazilian system is an income concentrator system. High concentrated economies will be faced with a strong restriction and some compensatory public policy should be designed. Since this base-line takes into account the current regulatory framework, prices and cost, the diffusion of innovation does not reach the middle class. This *middle-class barrier* is an important result revealed by the model. If a household is able to invest now and harvest income benefit in the future, the diffusion of solar energy could increase the income inequality in a country with one of worst income distribution in the world, as it is the case in Curitiba, Brazil as a whole.

Figure 8: Generation Capacity by Income Class

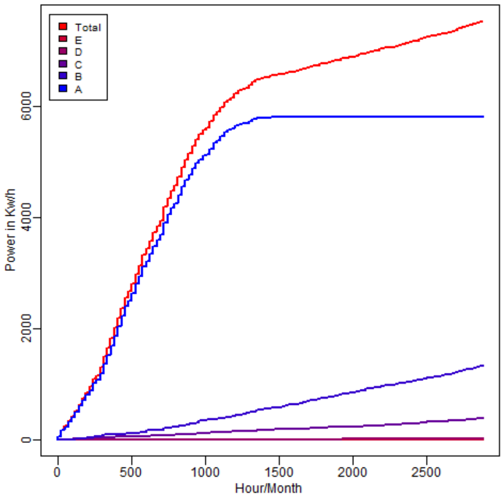
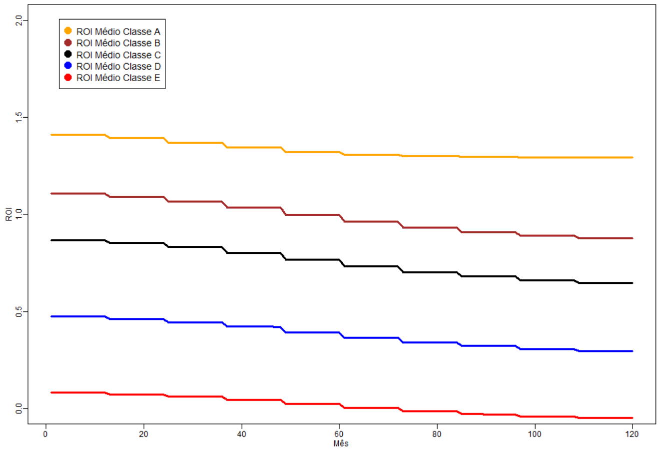
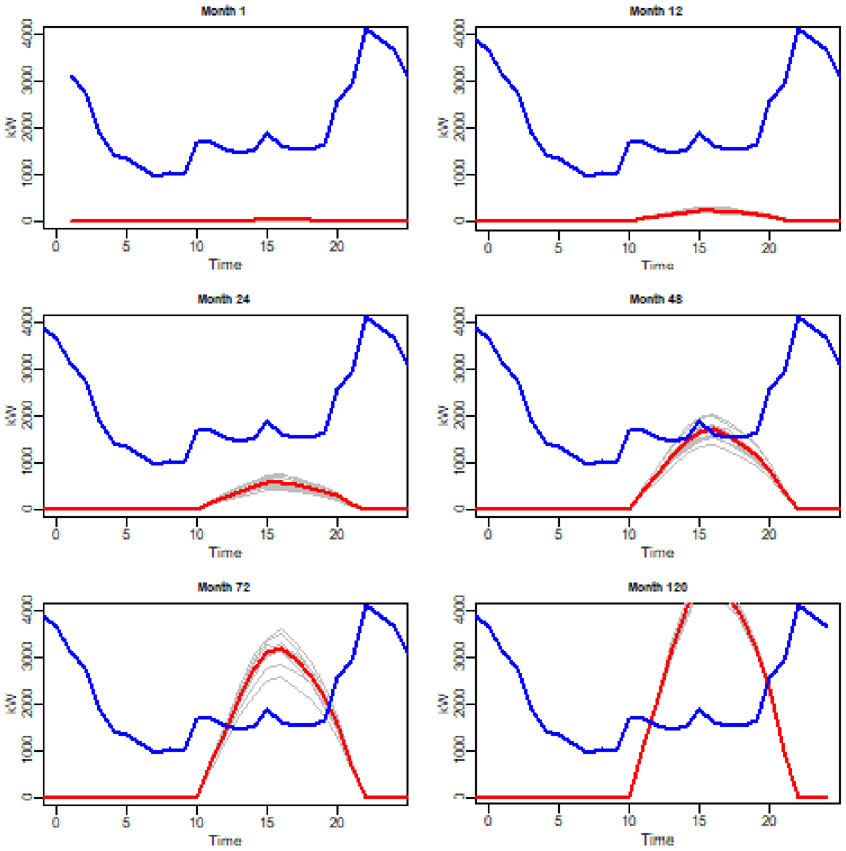


Figure 9: ROI by Income Class



Another important aspect to analyse is the intraday dynamics. As we explained in section (3) the temperature and radiation change during the day and along the year causing unbalance between the amount of power generated and consumed. This is challenging for engineers since it is extremely difficult to stabilize such electrical network. Ideally, the amount of power entering and leaving the network by multiples nodes (here 3840 potential nodes or households) needs to be balanced. Beyond that, there is intra-day balance of power. The habits of the inhabitants differ from the time of radiation, as we show in figure (2). This unbalance can be seen in figure (10). Even in month 120, after 23% of residences have adopted solar panels, the surplus is concentrated in some range of hours. The city will only be surplus from 10:00 to 20:00. The daily surplus is the area between curve red(generation) and blue (residential consumption). The important result here is that an increasing city-surplus begins to be generated after 48 month or 4th year. As time goes on, the surplus continues to grow. This is the excess energy that can be sold for other uses or exported others regions or stored to be consumed in other moments. This excess can be overtaken by using batteries, technology does not addressed in this version. What the model shows is the amount of power surplus which can be stored in such batteries. Knowing this, it would not difficult to compute how many batteries should be demanded by households, given some storage capacity of a cell.

Figure 10: Total energy generated hourly in one day



The figure (11) and (12) depicts the total of energy generated by this city, yet considering the baseline calibration. The first figure shows the power balance during all period. The green line represents the power measured in kWh and the time corresponds to 2880 time steps (24h×12month×10year). Remember that 1 month is a representative day ×30 so we decrease the necessity of computation by a factor of 30. The figure shows the difference between consumption and generation. The positive value above the red straight line means that the generation exceeds the consumption. At the beginning, the balance is negative because all the energy in this city needs to be supplied by a centralized system importing energy from abroad. Gradually, the city starts to produce its residential energy and remains surplus most of the time, except at night, when solar panels stop to work. The high variance of the kWh is due to the hourly frequency of the data. The rush time of generation and consumption do not match perfectly, even when the diffusion reach 23% of the residences as shown if the previous figure. Finally, the figure (12) shows the total installed capacity by kWh, measured in the rush hour.

Figure 11: Solar Power Consumed and Generated - kWh

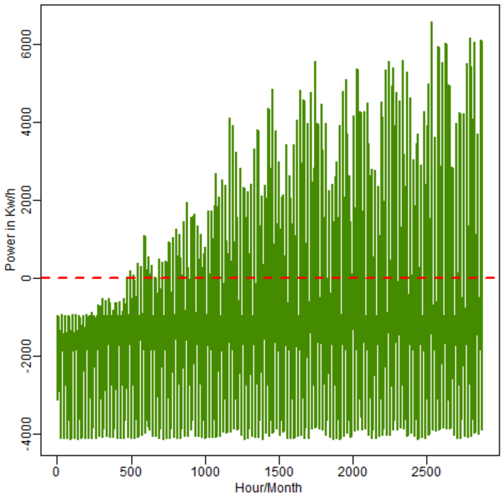
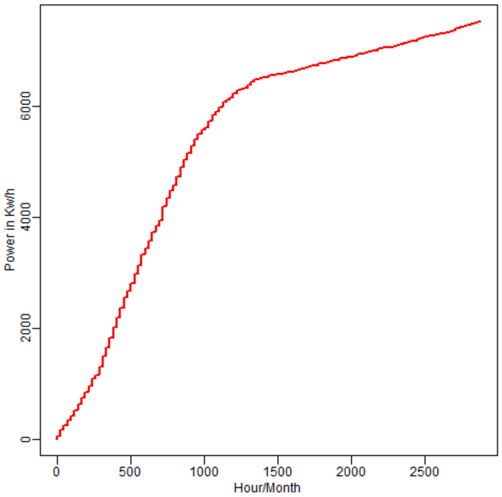


Figure 12: Diffusion of Solar Power - Installed Capacity in kWh



5.2 Alternatives Sceneries

Given the base-line simulation, the question one could ask is how the adoption or diffusion would behave under the shifting of some parameters. This counter-factual exercise sheds light on possible public policies and regulatory framework that could be carried out. To test this we suggest to analyse complementary alternatives shifting: a.) tax rate, b.) interest rate; c.) subsidy; d.) social network or number of links by household. Of course, the last one is a sociological variable which is not under the absolute control of a policy maker but can be stimulated by advertising or some kind of advantage to local collective adoption. For instance, join residences in a block to make a collective decision.

a.) Tax rate

The effect of decreasing the tax rate is shown in figures (13) and (14). The figure (13) shows the number of adopter in each step time, from 1 to 120 month for a range of tax rate, varying from 0 to 45%. The figure (14) shows the percentage of new adopters compared with base-line simulations at the end of time, in month 120. Each point in this figure is a simulation at different tax rate. The incentive to adoption by decreasing tax rate is slight exponential and seems not to have an effect below 10%, starting to increase after this threshold point.

Figure 13: Decreasing Tax Rate

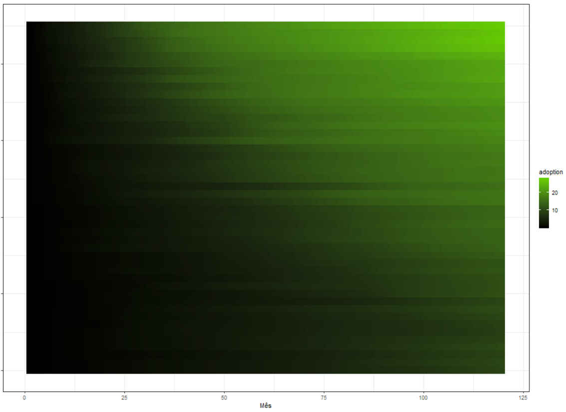
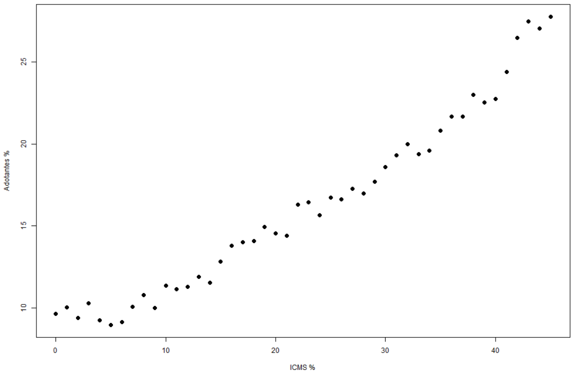


Figure 14: Diffusion of Solar Power - Installed Capacity in kWh



b.) Interest rate

The effect of decreasing interest rate is shown in figures (15) and (16). The interest rate has an impact on the ROI which, in its turn, reduces payback and increases the likely of adoption, according to equation (17)[11]. The figure (15) shows the number of adopter in each step time, from 1 to 120 month for a range of interest rate, varying from 0 to 15%aa. The figure (16) shows the percentage of new adopters compared with base-line simulations at the end of time, in month 120. Each point in this figure is a simulation at different tax rate. The incentive to adoption by decreasing interest rate is slight exponential and have a continuous effect, increasing the percentage of adopters by up to 40%.

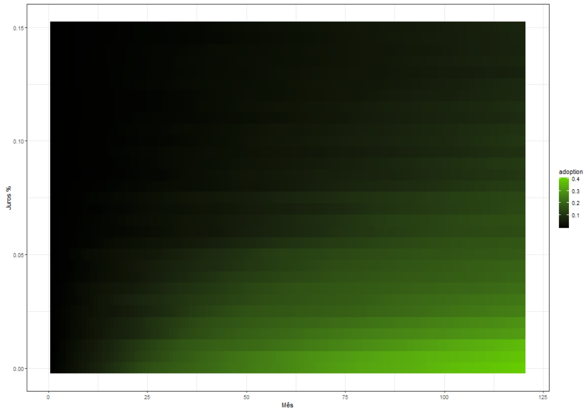


Figure 15: Decreasing Interest Rate

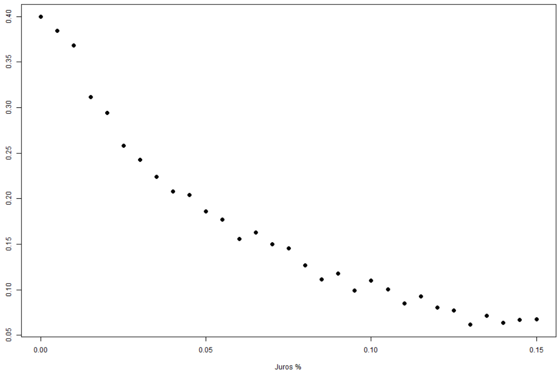


Figure 16: Diffusion of Solar Power - Installed Capacity in kWh

c.) Subsidy

The effect of decreasing interest rate is shown in figures (17) and (18). The subsidy policy can be implemented by price system. In this case, the government guarantees the energy price sold by households is a percentage above the unitary cost price. The subsidy has an impact on the ROI and at the end increases the likely of adoption. This is quite obvious but the models to helps to compute the magnitude of a such efect. The figure (17) shows the number of adopter in each step time, from 1 to 120 month for a range of subsidy, varying from 0 to 40%. The figure (18) shows the percentage of new adopters compared with base-line simulations at the end of time, in month 120. Each point in this figure is a simulation at different subsidy rate. The incentive to adoption by increasing subsidy is slight exponential and have a continuous effect, increasing the percentage of adopters by up to 40%.

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[11] The equation for ROI computation was omitted in the text in this draft.

Figure 17: Increasing Subsidy

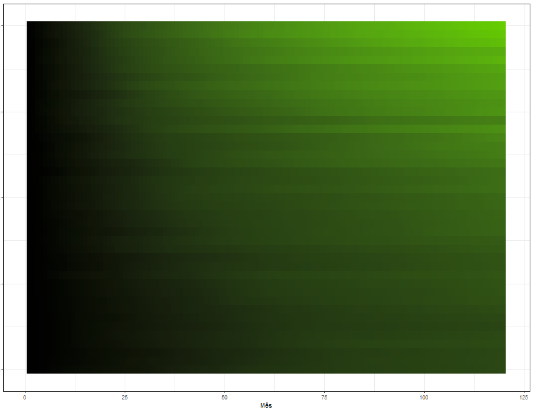
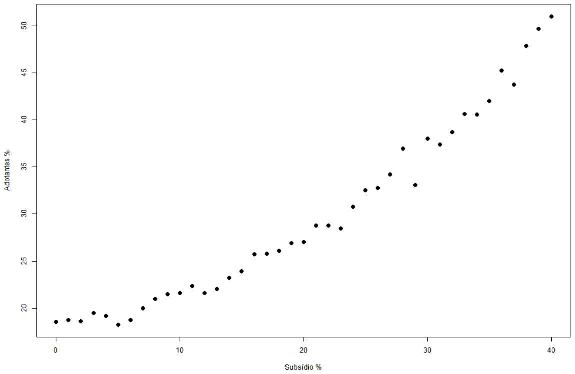


Figure 18: Diffusion-Installed Capacity in kWh



d.) Social Network or number of links

Finally, the last case is the sensitivity of diffusion to the number of social links a household has. Different from the other cases above, we test the model using just three values for the number of links: 4, 6 and 8. The effect of the increasing number of links is shown in figures (19) and (20). The increase in the number of links can be obtained in practice by using advertisement or some kind of stimulus for the collective decision, for instance, by stimulating household in a block adopt the PV panel at the same time. More about the strategies of advertisement and stimulus for word-of-mouth can be found in Rogers (1962, 2003); Buttle (1998); Sweeney et al. (2012). The figure (19) shows the percentage of adopter in each step time, from 1 to 120 month for for each number of links. The figure (20) shows the total of monetary resources invested in PV panels in each case. The increase of number of links has non-linear effects, what can be seen comparing the displace of blue line to each other. The hourly balance of power charge and generation is shown in figure (**??**), where the surplus during the sunny-day.

Figure 19: Adoption by Links

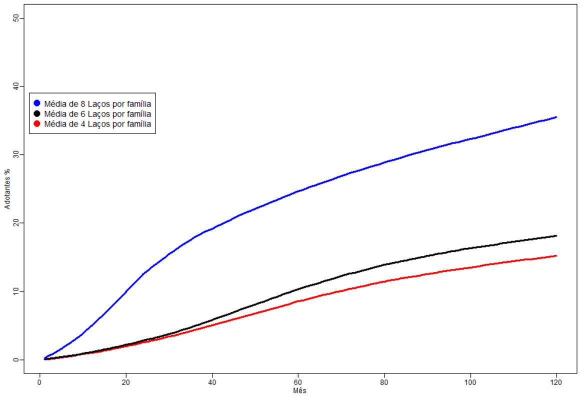


Figure 20: Installed Capacity in kWh

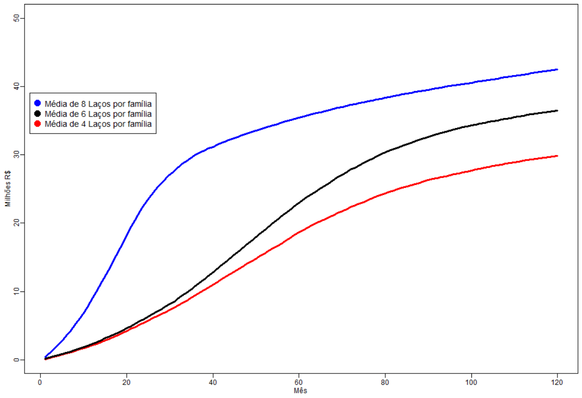
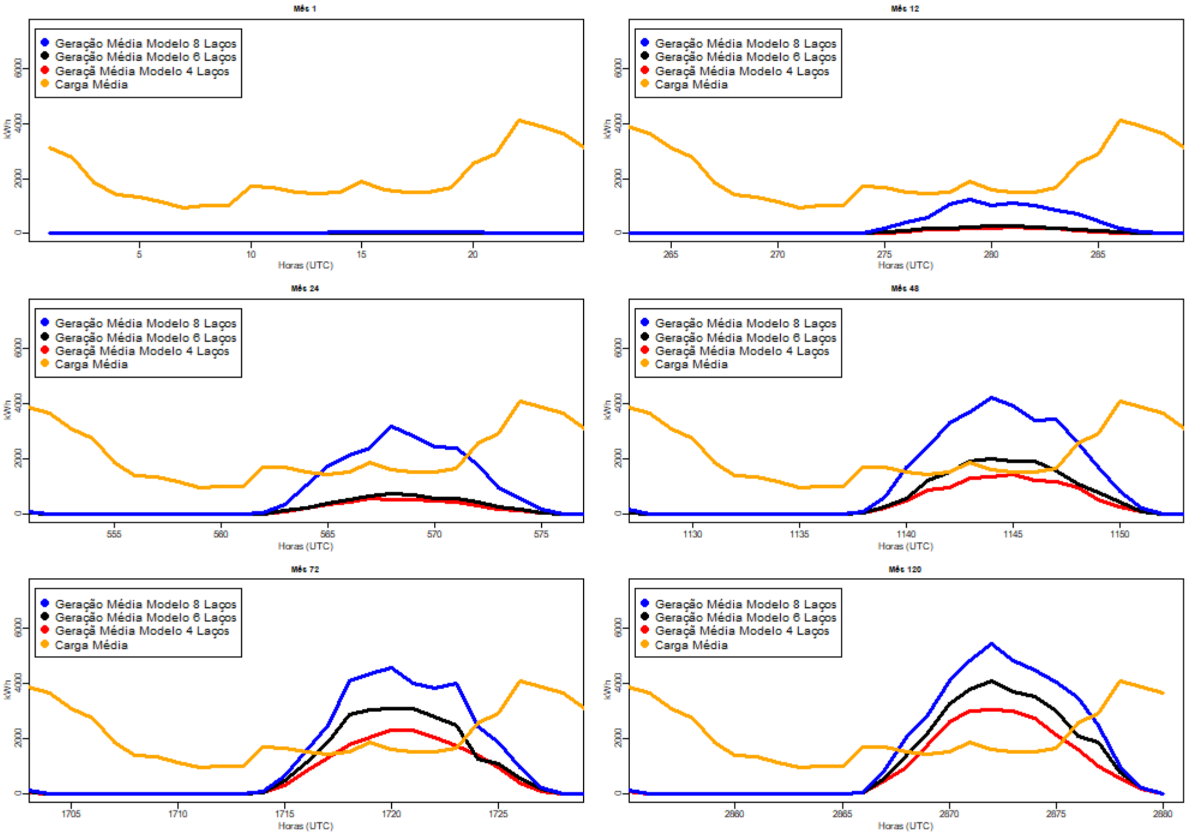


Figure 21: Hourly balance of energy in selected months - in kWh



6 Conclusion

In this work, we used an agent-based modelling framework to analyse and evaluate both the diffusion of photovoltaic systems, the impacts on the electric system, the impact within an urban area, the budget of households, considering real parameters and variables such as income distribution, panels capabilities, temperature and radiation in a specific Latitude and Longitude, price, tax, subsidy, interest rate, social habits and network. This model deals with some degree of complexity due to the social relationship between households and physical networks with a varying number of nodes supplying an unstable amount of energy. A higher degree of complexity arises from the overlap of these two networks layer. The proposed hybrid (physical plus economic level) model made it possible to simulate complex details of people's responses to incentives based on various factors, as well as a number of public policy scenarios. Based on the results generated from the news experiment, governments and organizations can establish appropriate incentives to encourage the adoption of photovoltaic systems.

From the public policies point of view, the results found could guide the concessionaires/government to encourage adoption of right incentives to solve the growing demand for energy in urban areas, by designing an appropriating framework considering the effects of tax and interest rate, subsidy and network relationship.

As a final comment, we would like to reinforce that this is a work in progress and the results present at this stage of research are limited. We do not explore all the flexibility of the model by doing some kind of sensitivity analysis or tracing comparatives scenarios. We do not discuss a particular case, although we have calibrated the model base on a Brazilian city including radiation, temperature, income distribution, prices and market regulation. Despite this, the model can be easily adapted for comparative studies between cities and even countries with different rules. This is a first version of a more comprehensible model and some important addition is being planned for the next version: a.) use a real topology for urban space which includes real population density by neighbourhood, size of each neighbourhood; b.) explore the multiple price mechanisms; c.) allow a household to sell the excessing energy[12], d.) assume heterogeneity in the rooftop area of each household; e.) include technological progress which may a more efficient rate of conversion of solar rays into energy; f.) explicit the expansion of the physical network, g.) explicit the total investment done by firms and households separately; h.) include environmental and labour market impacts, i.) include industrial demand and public illumination and j.) allow for growing urban economies especially vertical or density growth.

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[12] Remember that we impose a restriction to household sell an amount of energy no higher than the amount it consume.

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7 Appendix

7.1 *The temperature of the photovoltaic panel*

The photovoltaic panel operates in a temperature higher than the ambient so $T^{mod}_{h,t}$ can be decomposed in parts:

$$T^{mod}_{h,t} = T^{amb}_{h,t} + T^{op}_{h,t} \tag{24}$$

T^{op}_{t} is effective temperature that a photovoltaic panel operates. The manufacturers assumes that this temperature is approximately 40°C above the ambient, so in the equation (9) ambient temperature considered

$$G_{i,h,t} = K_i \left(\frac{R_{h,t}}{1000W} \right) [1 - 0,0045(T^{amb}_{h,t} + 40 - 25)] \tag{25}$$

or

$$G_{i,h,t} = K_i \left(\frac{R_{h,t}}{1000W} \right) [1 - 0,0045(T^{amb}_{h,t} + 15)] \tag{26}$$

as it appears in the main text.