Optimal Population Distribution When Regional Economies, Disasters, and Utility Levels are Intercorrelated: Applying Portfolio Theory

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Abstract

In Japan, where a fifth of the world's large (≥magnitude 6) earthquakes occur, public and business sectors are often decentralized geographically in order to diversify the "risk." [1] In Yonemoto (2025), the author has characterized the optimal level of diversification applying the traditional portfolio theory. This study extends it theoretically by introducing the correlation in utility levels and empirically by using Bulgarian data.

There are many advanced preceding studies that have explored risk mitigations, such as the ones summarized in Brebbia (1998-), Okuyama and Chang (2004), Tatano and Takagi (2005), Jorissen and Stallen (2013), and Hagiwara and Asahi (2016), mostly given the population distribution or the system of cities. In contrast, this study derives the long-term or "ideal" distribution taking both the risk and the economic growth into account[2].

For long-term optimality, the classical portfolio theory, which has been developed since the seminal work of Markowitz (1959), can be the first step as it takes both fluctuation (disaster damages) and return (economic growth) into account. This paper applies the portfolio theory, based on Markowitz (1959), Adachi and Saito (1992), and Hibiki and Tanabe (2005), straightforwardly to the allocation of the population.

[1] For example, National Resilience Promotion Office, Cabinet Secretariat, Japan (2018) states "excessive population concentration status should be alleviated in the places that have high risk of occurrence of earthquakes and effective measures of boosting development of an autonomous, decentralized and coordinated nation should be considered and conducted in order to promote risk diversification" in its Fundamental Plan for National Resilience.

[2] Some researches have considered the locations under disaster risk but from different point of views. For example, Tatano et al. (2004) has analyzed locations within a city.

1. The Basic Model

Similarly to Yonemoto (2025), the following simple (linear) mean-variance model of Portfolio Theory is employed to allocate the population. Referring to Hibiki and Tanabe (2005), formulate the maximization problem as:

$$\max_{\boldsymbol{w}} \boldsymbol{r}^T\boldsymbol{w} - \gamma\boldsymbol{w}^T\boldsymbol{V}\boldsymbol{w} \quad \text{s.t.} \quad \mathbf{1}^T\mathbf{w} = 1,$$
$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_I \end{bmatrix}, \quad \boldsymbol{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_I \end{bmatrix}, \qquad \boldsymbol{V} = \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1I} \\ \vdots & \ddots & \vdots \\ \sigma_{I1} & \dots & \sigma_{II} \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \tag{1}$$

where w_i is the population share of region i, r_i is the expected return (economic growth rate) of i, σ_{ij} is the covariance between i and j (for $i = j$, the one represents the variance instead,) and γ is the coefficient of risk aversion, which is arbitrary chosen in this study, depending on the planner's objective. Then the corresponding Lagrangian is,

$$L = \boldsymbol{r}^T\boldsymbol{w} - \gamma\boldsymbol{w}^T\boldsymbol{V}\boldsymbol{w} + \lambda(1 - \mathbf{1}^T\mathbf{w}), \tag{2}$$

where λ is the Lagrangian multiplier. The first-order condition for maximization is,

$$\boldsymbol{r} - 2\gamma\boldsymbol{V}\boldsymbol{w} - \lambda\mathbf{1} = \mathbf{0},$$
$$\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{3}$$

Solving (3) for $\boldsymbol{w}$,

$$\boldsymbol{w} = \frac{1}{2\gamma}\boldsymbol{V}^{-1}(\boldsymbol{r} - \lambda\mathbf{1}), \tag{4}$$

Using the constraint in (1), λ is derived as follows:

$$\lambda = \frac{\mathbf{1}^T\boldsymbol{V}^{-1}\boldsymbol{r} - 2\gamma}{\mathbf{1}^T\boldsymbol{V}^{-1}\mathbf{1}}. \tag{5}$$

If there are non-negativity constraints on w_is,

$$w_i \geq 0 \quad \forall i, \tag{6}$$

The following Kuhn-Tucker conditions are added:

$$\frac{\partial L}{\partial w_i} \leq 0, \quad w_i \geq 0, \quad \text{and} \quad w_i\frac{\partial L}{\partial w_i} = 0. \tag{7}$$

In general, an algebraic solution (such as (4) in the case without non-negativity constraints) is not explicitly derived and the result is obtained numerically.

2. Externalities

This study extends the aforementioned model considering other types of externalities. For example, as Mui (1995) summarizes, since Veblen (1934) and Duesenberry (1949), various economic researches have been conducted assuming explicitly that the utility depends on some relative conditions or variables. It may be characterized by the comparison to the others, his/her state in the past or any other "reference" points. The related early works include Galbraith (1958), Sen (1966), Easterlin (1974) and Stigler and Becker (1977). The succeeding developments including the concepts of "altruism" and/or "envy" have been summarized by Kolm (1972), Abel (1990), Elster (1991), Carroll et al. (1997), Bowles (1998), Clark and Oswald (1998), and Dufwenberg et al. (2011). Yonemoto (2021a, b) has introduced state-dependency of utility into regional modelling. For example, one can consider the utility function of the following type:

$$u_{nt} = u\left(c_{nt}; c_{1t}, \cdots, c_{(n-1)t}, c_{(n+1)t}, \cdots, c_{Nt}\right), \tag{8}$$

where u_{nit} indicates the utility level of individual n in period t, which depends on the consumption levels of the other people (some located in other regions) $1 \cdots, n-1, n+1, \cdots, N$ and affected (lowered) by the disasters as they occur. It is often assumed that comparisons are made between one's own consumption and another's (e.g. $c_{nt} - c_{1t}$,) but for simplicity, this study assumes that people have an altruistic preference on the deviation of the consumption level of another person from his/her "normal" level:

$$\begin{aligned} &u_{nt} = u\left(c_{nt}; c_{1t}/c_{1t}^*, \cdots, c_{(n-1)t}/c_{(n-1)t}^*, c_{(n+1)t}/c_{(n+1)t}^*, \cdots, c_{Nt}/c_{Nt}^*\right), \\ &c_{nt}^* \equiv (1+r_n)c_{n(t-1)}^*, \end{aligned} \tag{9}$$

where c_{nt}^*s are the "normal" (expected) consumption levels. The (partial) derivative on each element is considered to be positive; that is, a person feels unhappy when being informed that another person's living standard is made lower than his/her "normal" level, due to a disaster for example (and happy when his/her situation gets improved.) Then, as a disaster negatively affects the utility levels (or the subjective returns) of the other regions, the corresponding rates are added to the variance-covariance matrix $\boldsymbol{V}$[3].

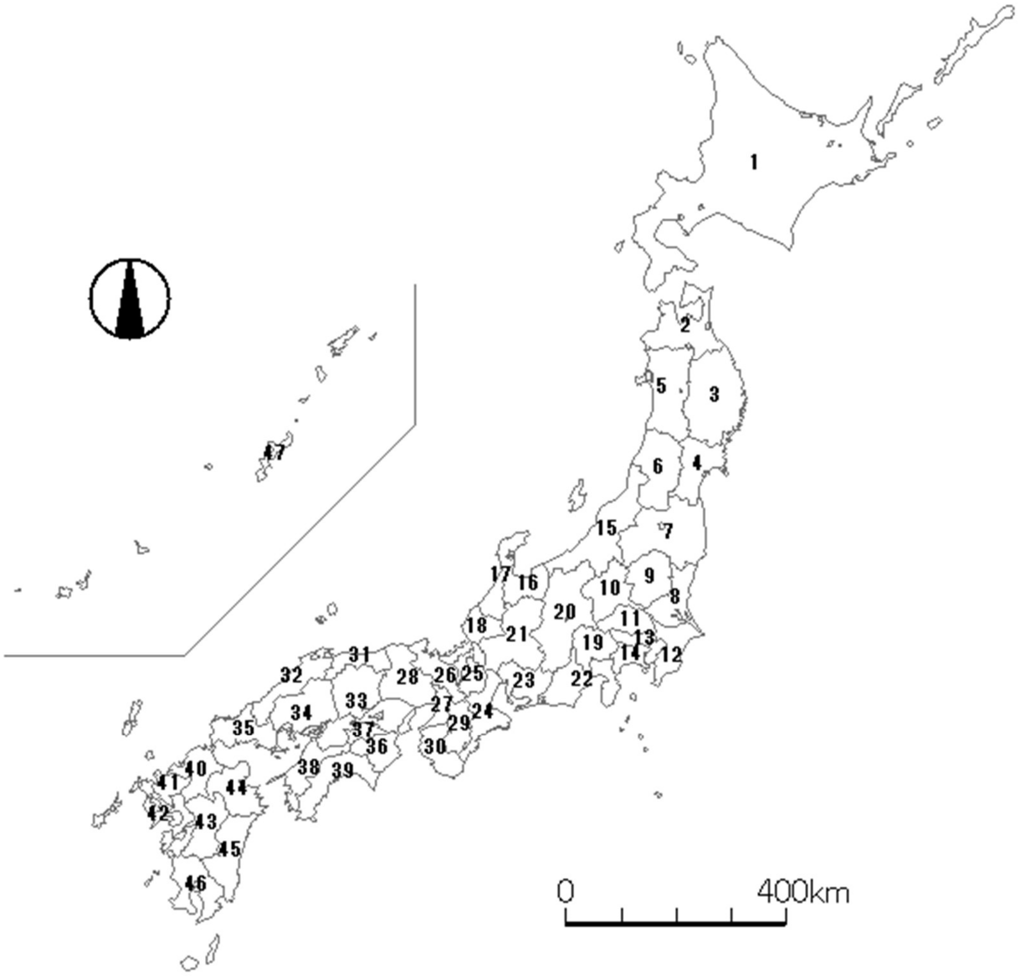
3. Japanese Case: Data and Methods

This study presents numerical examples using Japanese data (mostly on earthquake) and Bulgarian data (mostly on flooding.)

Historic Earthquake Data

Japanese earthquake data are based on Seismic Intensity Database, Japan Meteorological Agency. As the observation system started in 1919 and changed thoroughly after 1995, the Great Hanshin-Awaji Earthquake, this study uses data between them (1919-1995.) Collect all data whose seismic intensities are 4 or higher, and the highest one in each prefecture (see Figure 1 for their locations) is used to represent its effect. There have been 1,286 samples during this period including 16 samples whose intensity is 6 (the highest-realized scale before 1995.) Figure 2 depicts the epicenters of the last 10 years (1986-1995) of the data; it can be confirmed they are dispersed nationwide.

[3] This part of assumption can be made more complex and rigidly, but to keep the simple framework of the portfolio model, only $\boldsymbol{V}$ is modified here; the disaster does not affect the “normal” growths $\boldsymbol{r}$ but fluctuations $\boldsymbol{V}$.



1	Hokkaido	13	Tokyo	25	Shiga	37	Kagawa
2	Aomori	14	Kanagawa	26	Kyoto	38	Ehime
3	Iwate	15	Niigata	27	Osaka	39	Kochi
4	Miyagi	16	Toyama	28	Hyogo	40	Fukuoka
5	Akita	17	Ishikawa	29	Nara	41	Saga
6	Yamagata	18	Fukui	30	Wakayama	42	Nagasaki
7	Fukushima	19	Yamanashi	31	Tottori	43	Kumamoto
8	Ibaraki	20	Nagano	32	Shimane	44	Oita
9	Tochigi	21	Gifu	33	Okayama	45	Miyazaki
10	Gunma	22	Shizuoka	34	Hiroshima	46	Kagoshima
11	Saitama	23	Aichi	35	Yamaguchi	47	Okinawa
12	Chiba	24	Mie	36	Tokushima		

Figure 1: Japanese Prefectures

(Made by Author with MANDARA GIS software)

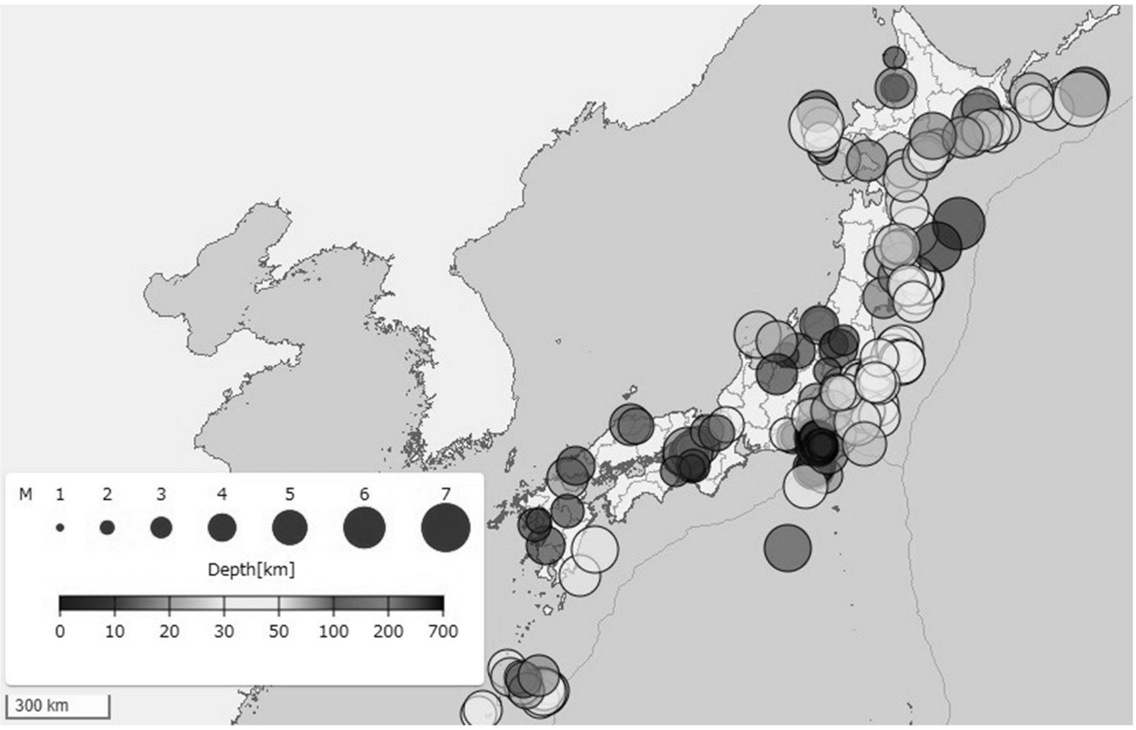


Figure 2: Example of Epicenters (≥ Intensity 4, Jan. 1986 - Dec. 1995)

Seismic Intensity Database, Japan Meteorological Agency

Converting Intensity to the Annual Rate of Damage

Since the seismic intensity is far from proportional to the degree of damage, first, it is converted to the acceleration (gal) by the following conventional equation[4]:

$$A = 10^{-047+\frac{SI}{2}}. \tag{10}$$

Then, by summing up for each year, annual A is obtained. It is converted to the rate of damage by using coefficient β estimated through the following regression (without a constant term):

$$\%DAMAGE_n = \beta A_n + e_n, \tag{11}$$

in which the original damage estimates (in monetary terms) are based on Cui et al. (2017) and Toyoda et al. (2020)[5].

[4] (10) is a part of the formulae used in "instrumental seismic intensity," Japan Meteorological Agency.

[5] The prefectural GDPs, which are used as numerators to calculate the "rates," are the ones of 2011,

No.	prefecture	avg growth	avg damage	No.	prefecture	avg growth	avg damage
1	Hokkaido	3.3%	4.5%	25	Shiga	5.0%	0.7%
2	Aomori	4.0%	2.2%	26	Kyoto	3.8%	1.1%
3	Iwate	4.1%	2.7%	27	Osaka	4.7%	0.5%
4	Miyagi	4.4%	1.9%	28	Hyogo	3.5%	2.1%
5	Akita	3.6%	0.8%	29	Nara	4.0%	0.7%
6	Yamagata	3.9%	0.9%	30	Wakayama	2.9%	0.9%
7	Fukushima	4.3%	3.4%	31	Tottori	3.5%	1.1%
8	Ibaraki	4.5%	3.3%	32	Shimane	3.6%	0.3%
9	Tochigi	4.6%	2.8%	33	Okayama	3.7%	0.6%
10	Gunma	4.2%	1.1%	34	Hiroshima	4.3%	0.9%
11	Saitama	5.8%	2.6%	35	Yamaguchi	3.5%	0.7%
12	Chiba	5.6%	3.1%	36	Tokushima	4.1%	0.6%
13	Tokyo	3.9%	7.0%	37	Kagawa	4.2%	0.6%
14	Kanagawa	5.2%	2.5%	38	Ehime	3.4%	0.6%
15	Niigata	3.9%	0.8%	39	Kochi	3.2%	0.7%
16	Toyama	3.9%	0.3%	40	Fukuoka	3.9%	0.2%
17	Ishikawa	4.2%	0.4%	41	Saga	3.7%	0.2%
18	Fukui	3.9%	1.0%	42	Nagasaki	3.5%	0.9%
19	Yamanashi	4.3%	2.6%	43	Kumamoto	3.8%	1.6%
20	Nagano	4.5%	3.2%	44	Oita	3.8%	0.8%
21	Gifu	3.9%	0.6%	45	Miyazaki	3.9%	1.3%
22	Shizuoka	4.2%	3.8%	46	Kagoshima	3.6%	1.8%
23	Aichi	4.7%	0.6%	47	Okinawa	4.6%	2.1%
24	Mie						

Table 1: Estimated Average Growth and Damage Rates[6]

Expected Return and Variance-Covariance Matrix

(Gross) returns are represented by the average growth rates of prefectural GDPs in 1955-2009[7]. Subtracting (11), 77 (years) × 47 (prefectures) matrix of returns is constructed and the variance-covariance (original $\boldsymbol{V}$ in (1)) is derived from it. And the row expected values of the matrix are $\boldsymbol{r}$s in (1). The average rates are presented in Table 1.

Prefectural Economic Accounts, Cabinet Office, Government of Japan. β has been estimated to be 0.043.

[6] High damage rate of Tokyo might be reflecting earthquake swarms around its territory of southern islands. Some geographically long or large prefectures, such as Tokyo, should be divided into several parts taking their particular characteristics into account (in future researches.)

[7] They are based on the time-series data of Prefectural Economic Accounts, Cabinet Office, Government of Japan. Because the data for 1975-1984 are not consistent with the ones for the other periods, they are simply extrapolated from 1985-1999 figures. Data for 1919-1954 are not officially released and thus not used. Note that annual growth rates are available but not used; the average rates are used for all periods in order to characterize the fluctuations due to the disasters but not by the business cycle.

Non-negativity Constraints

If w_is are derived to be non-negative without any constraints, those solutions are explicitly obtained from (4). However, if not, they need to be numerically calculated. This study uses Mathematica software (Wolfram 14.3) to obtain them.

Effcets of Altruism

Noting (9), for simplicity, the additional part of $\boldsymbol{V}$ is calculated as follows:

1. Obtain the data on the amount of donation during the disaster period. Dividing it by the total population of the damaged regions, express in terms of per capita recipient.
2. The same amount is divided by the total population of the unaffected regions and described in terms of per capita donor.
3. By the donation, the temporary disparity (due to the disaster) is expected to be mitigated by the sum of 1. and 2. because the temporary consumption/income of the recipient rises by 1. and that of the donor decreases by 2.
4. Dividing 3. by the amount of the actual damage (per capita), the rate of mitigation (by altruism) is calculated. The damage being perceived by the donors at least at this rate, noting (9), all the non-diagonal elements of $\boldsymbol{V}$ are raised by the rate.

When the Distance Matters

5. Moreover, if the distance from the damaged region to the donors' is taken into account, using regional donation data, the relationship between the rate of donation and the distance (presumably negative) is derived, and, 4. is modified accordingly.

4. Japanese Case: Parameters and the Results

First, the optimal allocation of the population is derived without altruism, using (1)-

(7)[8]. For $\gamma=0.5$, the resulting allocation of the population is 50.66% for No.27 (Osaka), 23.97% for No.23 (Aichi), 15.55% for No.40 (Fukuoka), 7.86% for No.17 (Ishikawa), and 1.96% No.25 (Shiga) (and almost zero for all the other prefectures.[9]) With low risk aversion, people are to be located in relatively growing regions with considerably fewer earthquakes. Note that, even then, the population needs to be somewhat dispersed.

For $\gamma=10$, the results are 49.62% for No.40 (Fukuoka), 14.35% for No.41 (Saga), 10.81% for No.17 (Ishikawa), 7.95% for No.27 (Osaka), 7.93% for No.16 (Toyama), 5.62% for No.32 (Shimane), 3.32% for No.23 (Aichi), 2.99% for No.47 (Okinawa), 0.09% for No.13 (Tokyo), and 0.03% for No.20 (Nagano). Now the population is more diversified mostly in the western Japan, the risk is taken more into account and even less-developing prefectures receive some parts.

Figure 3 summarizes the results of the above two cases.

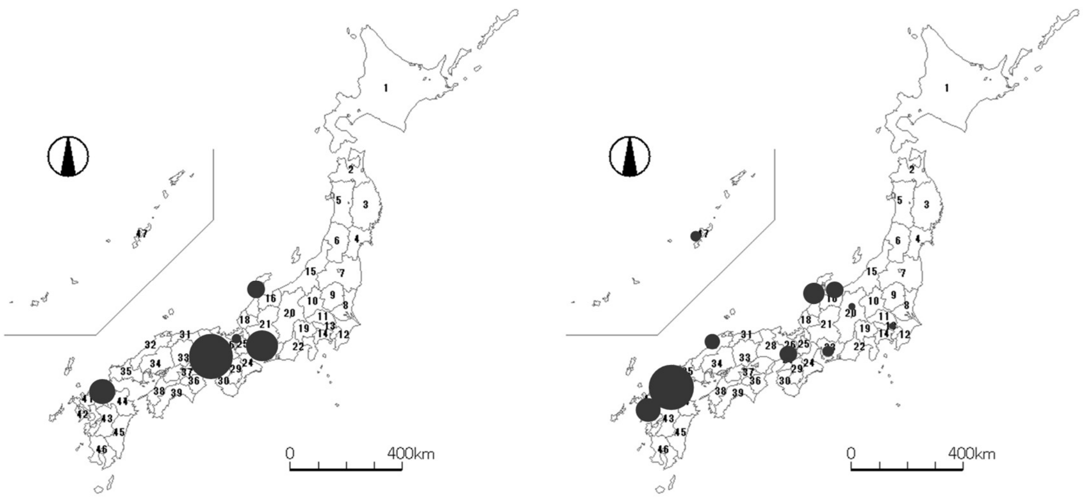


Figure 3: Japanese Results (Left: $\gamma=0.5$, Right: $\gamma=10$)

[8] The results are essentially similar to the ones derived in Yonemoto (2025). Some minor corrections and modifications have been made.

[9] Practically, it might be difficult to re-allocate the entire population in a short period according to this kind of criterion. However, at least it can be an indicator for new investments and developments.

For $\gamma = 20$, the number of the prefectures (with positive populations) increases more: 41.71% for No.40 (Fukuoka), 21.07% for No.41 (Saga), 10.81% for No.16 (Toyama), 9.98% for No.32 (Shimane), 8.48% for No.17 (Ishikawa), 5.41% for No.27 (Osaka), 1.71% for No.23 (Aichi), 0.31% for No.47 (Okinawa), 0.20% for No.5 (Akita), 0.15% for No.13 (Tokyo), 0.08% for No.20 (Nagano), 0.06% for No.6 (Yamagata), and 0.02% for No.10 (Gunma).

Altruism

Based on Japan Fundraising Association (2012), donation data on for 2011 Tohoku Earthquake are used to calculate the average donation 14,622 JPY / donor and as 76.4% of the population (mostly of the unaffected regions) donated, 11,171 JPY per capita. Converting to per capita recipient terms, 250,651 JPY / recipient, which counts for 10.29% of the actual damage / recipient (i.e. sufferer.) Adding this rate to all the non-diagonal elements of $\boldsymbol{V}$, for $\gamma = 10$, the resulting allocation is 67.25% for No.40 (Fukuoka), 11.29% for No.17 (Ishikawa), 7.95% for No.16 (Toyama), 7.46% for No.27 (Osaka), 3.46% for No.32 (Shimane), 7.67% for No.23 (Aichi), and 1.83% for No.41 (Saga).

Obviously, the populations are concentrated to the growing regions in comparison to the case without altruism. With altruism, the effect of a disaster is dispersed anyway so that physical diversification turns out to be less necessary; Ironically, more the sympathy on the sufferers, weaker the locality of an event and thus lower the effect of (locational) risk aversion.

In addition, based also on Japan Fundraising Association (2017), the relationship between the rate of donation and the distance has been estimated using 2016 Kumamoto Earthquake data [10]. Normalizing the rate in Kyushu, which includes the affected

[10] Because 2011 Tohoku Earthquake was rather a national event (disaster,) distance had little effect on the rate of people's donation. 2016 Kumamoto earthquake was more regional so that its data are used here. The

prefectures, to be 1 and 10.29% (calculated above) is multiplied by it. Using the modified $\boldsymbol{V}$, for $\gamma=10$, the results are 66.95% for No.40 (Fukuoka), 11.41% for No.17 (Ishikawa), 8.09% for No.16 (Toyama), 7.57% for No.27 (Osaka), 3.37% for No.32 (Shimane), 1.59% for No.41 (Saga), and 1.01% for No.23 (Aichi).

5. Bulgarian Case: Data and Methods

This study also conducts a similar analysis using Bulgarian disaster data. Disaster data are based on "crisis events" data of National Statistical Institute of Bulgaria, summarized by Vladev and Solakova (2025). Economic (including Consumer Price Index)

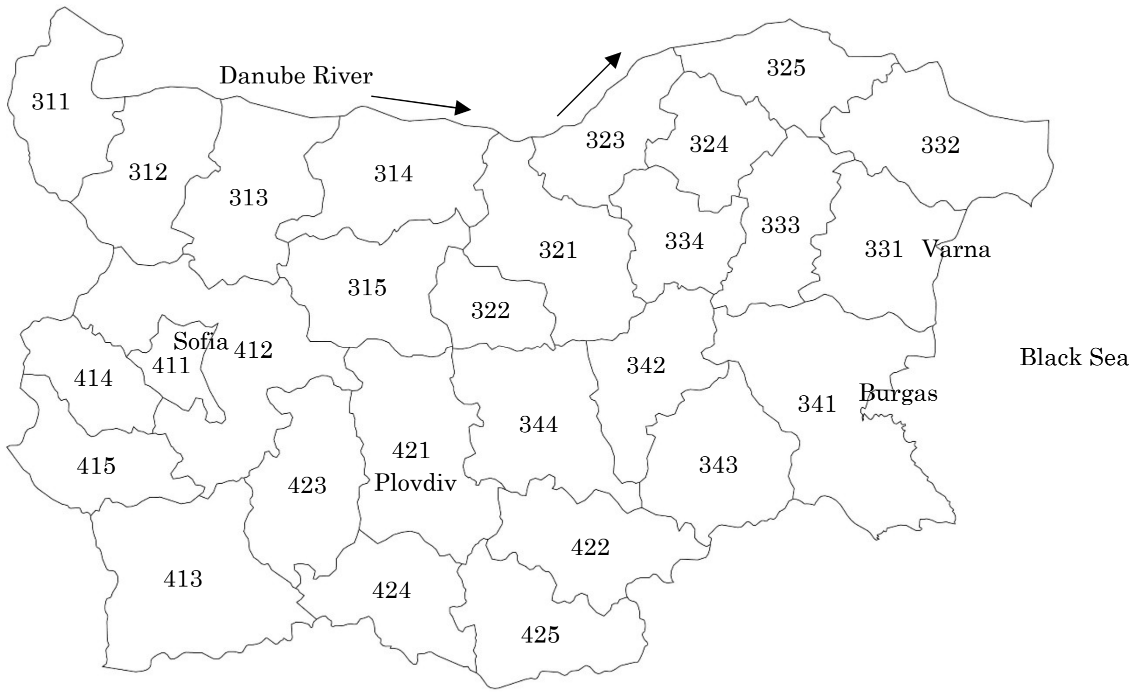


Figure 4: Bulgarian Oblasts

(Made from the original blank map by vepmaps.com)

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data are on Japanese 10 large regions such as Kyushu and Shikoku, the average of the distances from Kumamoto to prefectural governments of each region is calculated the donation rate is regressed on it. The coefficient is estimated to be −0.00008150 at 15% significance level.

oblast		flood damage (in 2025 price)											econ. growth
No.	name	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2010-2020
311	Vidin	0	207	379	0	751	0	0	0	0	0	0	1.3%
312	Montana	895	780	518	20	16,937	29	0	8	7,453	7,229	988	2.7%
313	Vratsa	0	359	0	6	39,300	968	0	0	0	0	0	3.1%
314	Pleven	1,786	1,235	27	0	2,107	3,951	18	2,168	0	0	370	2.8%
315	Lovech	747	6	6	0	251	0	0	0	0	1,352	0	2.3%
321	V. Tarnovo	364	476	2,770	1,184	36,292	3,142	201	4,834	8,074	2,463	2,133	2.3%
322	Gabrovo	226	122	918	126	10,392	0	4,290	0	0	0	108	3.1%
323	Ruse	414	1,353	204	243	2,025	6,329	4,382	950	429	950	2,253	3.5%
324	Razgrad	42	0	64	2,294	467	0	2	0	0	0	0	2.5%
325	Silistra	0	652	0	3,807	0	0	0	0	0	0	187	2.2%
331	Varna	4,249	2,050	1,857	2,055	57,553	5,408	7,166	1,007	16	450	972	3.2%
332	Dobrich	658	778	2	324	11,959	2,974	52	1,204	0	294	0	1.4%
333	Shumen	18	16	336	80	4,423	2,250	0	0	0	0	0	3.2%
334	Targovishte	86	3	0	0	0	0	0	875	368	0	0	2.5%
341	Burgas	801	218	582	20	24,675	207,498	1,240	177,589	3,541	3,487	1,439	0.6%
342	Sliven	1,994	1,121	0	0	94	2,035	270	53	192	1,102	310	2.2%
343	Yambol	1,966	0	0	0	423	0	0	0	0	0	0	2.2%
344	St. Zagora	419	0	1,275	0	11,655	2,570	0	0	0	118	0	2.5%
411	Sofia-capital	364	0	0	0	0	0	0	0	0	0	0	4.0%
412	Sofia	738	296,256	1,212	614	8,737	10,731	0	519	2,552	1,004	1	4.6%
413	Blagoevgrad	19,653	1,654	92	174	973	585	115	0	452	574	2,020	3.2%
414	Pernik	1,449	34	3	20	14	2	3	113	1	2,264	2,391	1.2%
415	Kyustendil	3,655	678	685	452	0	177	17	1,019	78	361	1	1.3%
421	Plovdiv	7,225	0	230	293	26,743	2,158	12,755	4,220	3,939	2,082	2,380	4.3%
422	Haskovo	444	674	6,550	0	6,456	343	260	0	0	1,630	1,284	2.8%
423	Pazardzhik	1,573	550	0	35	1,533	0	0	1,506	29	0	0	2.1%
424	Smolyan	13,149	12,650	12,161	9,497	5,186	14,097	7,919	1,609	5,957	3,028	5,406	2.0%
425	Kardzhali	266	349	1,779	1,698	1,486	3,686	8,353	6,368	8,481	1,683	1,034	6.2%

Table 2: Bulgarian Average Growth and Flood Damage (in million leva, BGN)[11]

data are from “macroeconomic data,” National Statistical Institute of Bulgaria.

Figure 4 indicates Bulgarian regions (oblasts) and Table 2, 3, and 4 are the (deflated) data on flood, landslide, and total (the sum of flood, landslide, fire, and earthquake) damages, respectively, used in the analysis.

Flood: Parameters and the Results

When only the flood damage is taken into account, as there has been little damage in Sofia city (the capita) during that period, to obtain non-zero rows and columns in $\boldsymbol{V}$, the data are merged with those of Sofia province (the surrounding region.) For γ =0.5 or 10

[11] The table in earlier version of this paper (extended abstract) submitted to this conference (ERSA 2026) had some errors in denomination. Please use this table (table 2) for reference.

oblast		landslide damage (in 2025 price)										
No.	name	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
311	Vidin	0	375	233	0	277	0	0	0	0	0	0
312	Montana	25	0	0	0	300	0	0	0	0	0	0
313	Vratsa	0	25	0	0	0	31	0	290	278	0	0
314	Pleven	0	271	0	0	0	0	0	39	0	0	0
315	Lovech	0	0	0	1,297	0	0	359	0	23	600	0
321	V. Tarnovo	279	3	1,444	0	260	320	96	455	0	0	0
322	Gabrovo	130	1	0	48	295	202	0	0	0	0	0
323	Ruse	0	0	0	200	12	248	11	0	868	1,829	0
324	Razgrad	0	0	0	0	0	0	0	0	0	0	0
325	Silistra	0	0	0	0	0	0	0	0	0	0	0
331	Varna	50	50	0	0	90	238	200	0	0	38	38
332	Dobrich	20	0	0	0	0	0	0	0	0	0	0
333	Shumen	0	0	0	0	130	0	0	0	0	0	0
334	Targovishte	7	0	240	0	0	0	0	0	0	0	0
341	Burgas	370	1,200	2,529	291,089	943	287	100	146	175	162	0
342	Sliven	2	811	0	0	0	539	72	0	0	1,400	420
343	Yambol	0	0	0	0	0	0	0	0	0	0	0
344	St. Zagora	0	0	0	0	0	0	0	0	0	0	0
411	Sofia-capital	214	267	0	0	0	0	0	0	0	0	0
412	Sofia	1,028	218,912	596	73	1	166	0	0	0	0	0
413	Blagoevgrad	13	0	105	120	0	1,096	835	0	0	0	0
414	Pernik	1	0	26	0	0	4	0	0	0	0	0
415	Kyustendil	0	51	0	0	499	72	274	145	5	1	35
421	Plovdiv	0	0	15	30	1,006	2,000	0	0	0	400	152,253
422	Haskovo	0	245	0	0	0	0	0	0	0	0	0
423	Pazardzhik	9	1,000	5,000	5	0	80	0	0	0	0	0
424	Smolyan	34	933	6,593	1,555	3,321	1,168	4,435	3,083	1,399	2,001	970
425	Kardzhali	0	646	603	42	2,157	3,560	3,250	3,562	3,500	1,670	1,280

Table 3: Landslide Damage (in million leva, BGN)

oblast		total disaster damage (in 2025 price)										
No.	name	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
311	Vidin	0	508	483	0	770	0	0	0	0	0	0
312	Montana	762	500	345	13	11,423	19	0	5	5,090	5,090	707
313	Vratsa	0	667	86	309	25,831	714	73	298	1,605	0	0
314	Pleven	1,107	1,063	18	0	1,384	2,592	18	1,489	59	54	265
315	Lovech	1,623	4	4	1,297	165	0	359	0	23	1,552	0
321	V. Tarnovo	510	340	3,323	789	24,094	2,391	241	3,666	5,514	1,737	1,527
322	Gabrovo	506	168	773	245	7,198	360	2,802	9	14	50	136
323	Ruse	255	868	135	390	1,342	4,400	2,863	631	1,161	2,498	1,613
324	Razgrad	26	0	542	1,828	607	311	255	200	200	0	0
325	Silistra	24	418	0	2,548	0	0	0	0	0	0	134
331	Varna	2,889	1,565	1,229	2,469	37,950	3,786	4,864	669	11	355	738
332	Dobrich	645	571	3	216	7,874	1,993	34	800	0	207	0
333	Shumen	11	10	222	53	3,035	1,476	0	1	0	0	0
334	Targovishte	60	2	252	0	0	0	0	581	251	0	0
341	Burgas	913	1,590	2,932	291,102	17,148	136,416	927	118,105	2,593	2,617	1,030
342	Sliven	1,231	1,530	0	0	62	1,874	248	35	131	2,258	682
343	Yambol	1,241	0	0	0	288	0	0	0	0	0	0
344	St. Zagora	317	0	986	0	7,654	1,771	125	2	0	83	0
411	Sofia-capital	573	281	5,424	0	0	0	0	0	0	0	0
412	Sofia	1,503	409,451	1,859	589	5,969	1,056	202	347	1,807	707	1
413	Blagoevgrad	12,124	1,282	179	239	639	1,868	940	0	309	409	1,624
414	Pernik	904	30	51,592	828	9	6	3	76	1	1,594	1,712
415	Kyustendil	2,249	542	452	341	500	1,103	285	822	67	255	36
421	Plovdiv	4,460	30	179	283	18,569	3,416	8,301	2,803	2,690	1,866	153,957
422	Haskovo	273	677	4,325	0	4,240	377	375	17	0	1,148	919
423	Pazardzhik	981	1,599	7,000	28	1,007	80	0	2,000	20	0	0
424	Smolyan	8,176	9,069	14,635	7,932	6,795	10,431	9,639	4,152	5,497	4,133	4,840
425	Kardzhali	164	870	1,778	1,173	3,133	6,398	8,756	7,792	9,292	2,855	2,020

Table 4: Total Disaster Damage (in million leva, BGN)

without altruism, the resulting allocation is 100% for No. 425 (Kardzhali), which is the recent fastest-growing region[12]. For $\gamma=100$, the results are 46.38% for No.425 (Kardzhali), 29.04% for No.421 (Plovdiv), 22.18% for No.411 and No.412 (Sofia, sum of the city and the province), and 2.40% for No.413 (Blagoevgrad.) Like Japanese cases, the population is dispersed as γ is raised sufficiently.

Landslide

Hereafter, because the (damage) values for Sofia city are positive, it is regarded as an independent region in the analyses. If only the landslide damage is considered, again, for $\gamma=0.5$ or 10 without altruism, the resulting allocation is 100% for No. 425 (Kardzhali). For $\gamma=100$, the allocation is 94.98% for No.425 (Kardzhali), 2.39% for No.421 (Plovdiv), 1.34% for No.412 (Sofia province), and 1.28% for No.341 (Burgas). To avoid the damage from landslide, Burgas, a port region, has relatively low growth but negative correlation with other regions, is also included.

Total

When the total (the sum of flood, landslide, fire, and earthquake) damages are taken into account, for $\gamma=0.5$ without altruism, the resulting allocation is 100% for No. 425 (Kardzhali). For $\gamma=10$, the allocation is 99.02% for No.425 (Kardzhali), and 0.98% for No.412 (Sofia province.) In the case $\gamma=100$, the results are 58.18% for No.411 (Sofia city), 34.03% for No.425 (Kardzhali), 6.63% for No.421 (Plovdiv), 0.86% for No.412 (Sofia province), and 0.31% for No.414 (Pernik) as illustrated in Figure 4 with darker circles;

[12] Kardzhali is a part Yuzhen Tsentralen, one of the fastest growing regions in EU. Kardzhali is not a "typical" province of Bulgaria: more than half of its population is Turkish origin, once prospered in lead and zinc industries, and experienced a large-scale structural transition in later periods. This study directly indicates the results without arguing or controlling for the special nature.

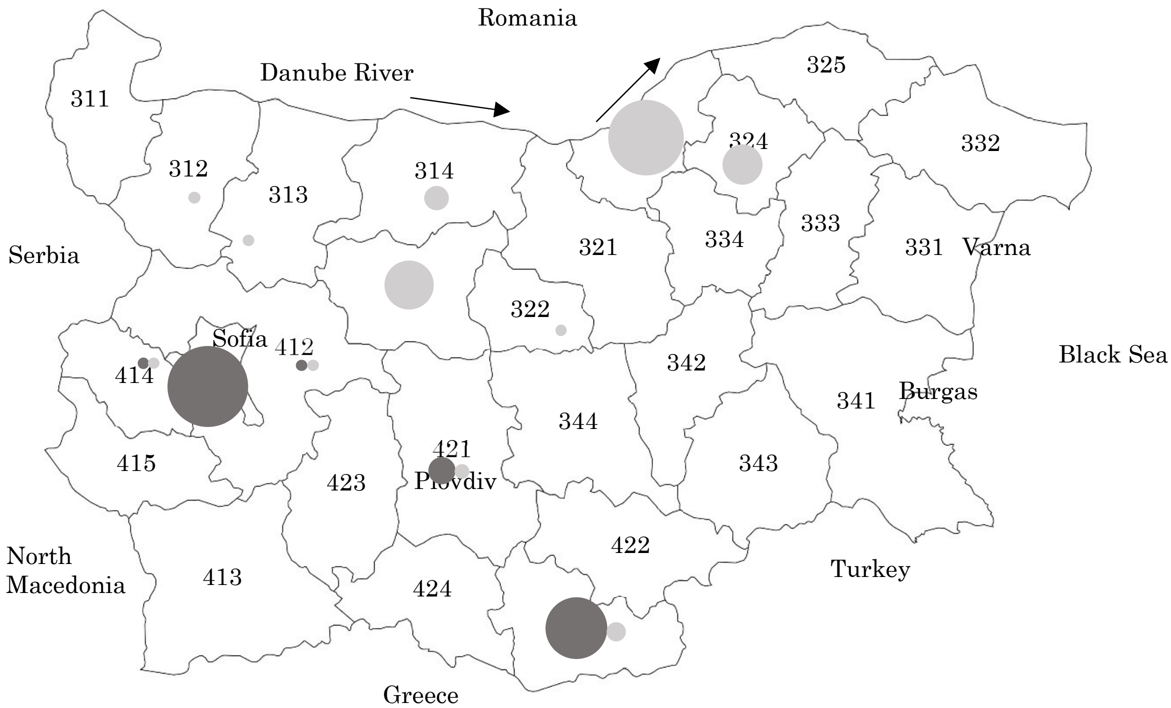

Figure 4: Some Bulgarian Results

(Total, Dark: γ=100, Light: γ=1,000 and +2% for No. 31- and 32-)

the capital is to have the largest share.

Altruism

For Bulgarian analysis, only the model without distance is used. Using Japanese rate (10.29%) and for γ=100 (total damages,) the allocation 67.08% for No.411 (Sofia province), 29.34% for No.425 (Kardzhali), 3.18% for No.421 (Plovdiv), and 0.40% for No.412 (Sofia province.) Similarly to Japanese results, concentration to the largest city becomes higher.

In Favor of the Regions along Danube

Because the regions along Danube River have been severely suffered by flood during the study period (particularly in 2014,) the results so far have shown no allocations to them. Seeking for more balanced results, the average growth rates of No.31- (Severozapaden) and No.32- (Severen tsentralen) are raised hypothetically by 2% (e.g. imagine the case in which some aids for flood-affected regions are made.) Then, the resulting allocation is 100% for No. 425 (Kardzhali) when $\gamma=0.5$ without altruism but when $\gamma=10$, it is 79.98% for No.425 (Kardzhali), 19.51% for No.323 (Ruse), and 0.51% for No.412 (Sofia province.) Ruse province, which is along Donau, is now included. When $\gamma=1{,}000$, it is 50.76% for No.323 (Ruse), 21.64% for No.315 (Lovech), 14.46% for No.324 (Razgrad), 5.32% for No.314 (Pleven), 3.31% for No.425 (Kardzhali), 2.01% for No.421 (Plovdiv), 1.35% for No.312 (Montana), 0.79% for No.414 (Pernik), and 0.35% for No.412 (Sofia province.) For lower digits, 0.004% for No.322 (Gabrovo), 0.002% for No.313 (Vratsa) are also counted. Those shares are indicated with lighter circles in Figure 4.

6. Concluding Remarks

This study has derived, applying classical portfolio theory and extending Yonemoto (2025), the "optimal" geographical distribution of population when the disaster risk varies and correlates among locations. The effect of reference-dependent preference (altruism, in particular) has been also taken into account.

Several analyses, based on Japanese and Bulgarian data, have been demonstrated. The results have indicated that, consistently with the theoretical part of Yonemoto (2025) in most cases, a few growing regions are chosen when γ (risk aversion) is low but more dispersed pattern is desirable when it is large. Altruism of the type modelled in this study tends to promote concentration.

Possible extensions of this study may include: use of more sophisticated portfolio model, more careful derivation of the estimated costs, and introduction of an economic model with detailed system of cities.

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