Strategic Formula Apportionment with Endogenous Prices

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Abstract

Formula apportionment (FA) taxes a multinational enterprise (MNE) on a weighted average of its capital and sales shares across jurisdictions. Existing models of strategic FA competition typically hold prices fixed; this paper instead lets the MNE's prices respond continuously to the formula chosen. Three complementary specifications (an MNE-only differentiated-goods benchmark, a homogeneous-good Cournot benchmark with domestic firms, and a full differentiated-goods model with domestic firms) share a common structure: under equal statutory tax rates, the MNE's entire real allocation depends on the two countries' formulas only through their difference, the formula wedge. Common formulas neutralize the MNE's tax-planning incentive, but whether they are optimal splits sharply: tax-revenue and fiscal-welfare objectives favor common formulas, while MNE-profit and (in most specifications) consumer-surplus objectives favor maximally opposed formulas, since the wedge acts like an implicit output subsidy. Decentralized competition does not generally replicate the centralized optimum. Four extensions — joint competition in formulas and tax rates, free entry of MNEs, free entry of domestic firms, and a comparison with separate accounting — show the mechanism is robust and yield two sharper results: sales-only formulas implement the destination principle and make tax-rate competition exactly efficient, and free entry of domestic firms pins local consumer prices to constants independent of the formula.

1 Introduction

The last two decades have seen a marked rise in the participation of multi-regional firms (MRFs) in global economic activity, alongside growing capital mobility and foreign direct investment inflows. This has sharpened fiscal competition over corporate income taxes.[1] When an MRF operates in multiple regions, each regional authority may tax income generated within its borders, creating incentives for profit shifting across jurisdictions and consequences for regional tax revenue.

Measuring how much income is earned in each region raises a difficult conceptual and administrative problem. The European Union, for example, requires firms to maintain separate accounts by jurisdiction (Separate Accounting, SA); the U.S. and Canada instead use a system of Formula Apportionment (FA), under which a multi-regional firm's income is apportioned to a state or province by a weighted average of its shares of sales, property, and payroll located there.[2] Formally, if I denotes the set of states where the firm operates, the tax due to state $i \in I$ is $T^i = t^i \omega_i \Omega^i$, where t^i is state i's tax rate, Ω^i is the firm's taxable income as defined by state i's tax law, and ω_i is a weighted average of the firm's shares of property, sales, and payroll located in state i, with nonnegative weights summing to one.[3] States differ considerably in their choice of weights: the Multistate Tax Compact of 1967 recommended equal weighting of the three factors, but most states have since moved toward heavily or exclusively weighting sales. As of 2025, roughly three-quarters of states with a corporate income tax use a single-sales-factor formula; Table 1 reports the full cross-state detail.

[1]Devereux and Griffith (1998) emphasizes the importance of effective marginal tax rates on the locational decisions of foreign investors; DeMooij and Ederveen (2003) find that a 1 percentage point reduction in the host-country tax rate raises foreign direct investment inflows by 3.3 percent.

[2]Mintz and Smart (2004) describe the Canadian subnational system, where provinces apportion by the equally-weighted average of payroll and sales shares.

[3]Regional governments may also differ in how they define the tax base itself; the present paper considers one such difference, the deductible share of capital costs.

Table 1: State apportionment of corporate income — formulas for tax year 2025

State	Formula	State	Formula
AL	Sales	MT	Sales†
AK	3 Factor	NE	Sales
AZ	Sales / Double wtd Sales	NV	—
AR	Sales	NH	Sales
CA	Sales	NJ	Sales
CO	Sales	NM	3 Factor / Sales
CT	Sales	NY	Sales
DE	Sales	NC	Sales
FL	Double wtd Sales	ND	3 Factor / Sales
GA	Sales	OH	N/A (CAT)
HI	3 Factor	OK	3 Factor
ID	Sales	OR	Sales
IL	Sales	PA	Sales
IN	Sales	RI	Sales
IA	Sales	SC	Sales
KS	3 Factor	SD	—
KY	Sales	TN	Triple wtd Sales
LA	Sales	TX	Sales
ME	Sales	UT	Sales
MD	Sales	VT	Sales
MA	Sales†	VA	Double wtd Sales / Sales
MI	Sales	WA	—
MN	Sales	WV	Sales
MS	Sales / Other	WI	Sales
MO	Sales	WY	—
DC	Sales		

Note: Sales = single sales factor ($\alpha_{S,i} = 1$); Double (Triple) wtd Sales = three factors with sales double (triple) weighted; 3 Factor = sales, property, and payroll equally weighted. A slash separating two formulas indicates taxpayer option or formulas specified by state rules. "—": no state corporate income tax. Ohio levies a gross-receipts Commercial Activity Tax (CAT) in place of a corporate income tax, so no income-apportionment formula applies. †Mandatory single sales factor effective January 1, 2025. Source: Federation of Tax Administrators, "State Apportionment of Corporate Income" (tax year 2023, January 2023), updated for the 2025 Massachusetts and Montana single-sales-factor adoptions (Tax Foundation, "2025 State Tax Changes").

The European Union illustrates a different situation: a formulary system of the kind already used across U.S. states and Canadian provinces has repeatedly been proposed but never adopted. The 2011 Common Consolidated Corporate Tax Base (CCCTB) directive proposal would have apportioned a single EU-wide consolidated tax base among member states using a formula equally weighting sales, payroll, headcount, and assets, but its consolidation element drew opposition from several member states and the proposal was never adopted. The Commission relaunched it in 2016 as a two-step common-base/common-consolidated-base (CCTB/CCCTB) package, retaining the same three-factor apportionment logic (labor, assets, and destination-based sales, equally weighted), and, after that too stalled, withdrew both proposals in 2023 in favor of the current Business in Europe: Framework for Income Taxation (BEFIT) initiative, which again apportions a common EU tax base using equally weighted labor, tangible assets, and sales — initially through a transitional rule based on each group member's historical share of taxable income, with the permanent formulary method to follow.[4] Devereux and Loretz (2008) estimate that a compulsory EU-wide formula apportionment system would raise aggregate member-state revenue relative to separate accounting, but that the size and even the sign of country-level gains and losses depend sensitively on the formula weights chosen — precisely the strategic tension this paper's model formalizes. Unlike U.S. states, however, EU tax directives require unanimous Council approval, so a coalition of low-tax member states has so far been sufficient to block adoption at every attempt, leaving formulary apportionment a live but unresolved policy question rather than an established practice.

This apportionment method is straightforward to administer but creates complex incentive effects. Firms react to the formulas they face by reallocating property, sales, and employment across regions; because one region's formula choice affects the tax base — and hence residents — of other regions, formula choice becomes a strategic policy variable. If all regions adopt the same formula, exactly 100 percent of a firm's income is apportioned across states; non-uniform formulas can result in more or less than full apportionment.[5] A commonly cited prediction is a "race to the bottom" toward sales-heavy formulas, as regions compete to attract mobile capital (Anand and Sansing, 2000); Pinto (2007) instead shows that the generalized shift toward sales-heavy formulas can be rationalized as an attempt by regional governments to fund higher levels of local public goods.

This paper develops a theoretical framework that generalizes Pinto (2007) along several dimensions relevant to the FA debate: differentiated varieties with smooth (rather than corner) pricing responses to the formula, iceberg trade costs that constrains decisions concerning tax-motivated sourcing, and an explicit domestic sector taxed under separate accounting alongside the MNE. Two regional governments simultaneously choose the capital weight in their FA formula; the MNE then chooses prices and capital allocation, taking into account how prices alter both its sales share and its capital share in the apportionment formula. We characterize how consumer-oriented and tax-revenue-oriented governments choose formulas in the resulting decentralized game, and compare these outcomes to the centralized solution.

The paper's central contribution is to make prices — and hence the apportionment shares themselves — respond continuously to the chosen formula, rather than the corner-

[4]European Commission, COM(2011) 121 final; COM(2016) 683 final and COM(2016) 685 final; COM(2023) 532 final. BEFIT's transitional and permanent apportionment mechanisms are summarized in European Parliament, "BEFIT: Business in Europe – Framework for Income Taxation," Briefing PE 754.631 (2023).

[5]This need not hold exactly if the definition of taxable income differs across states.

solution sourcing regimes that arise under the constant-returns, exogenous-price benchmark of Pinto (2007). A substantial literature examines FA's incentive effects taking the formula as exogenously fixed (McLure, 1980, 1981, Gordon and Wilson, 1986, Mieszkowski and Zodrow, 1985, Pethig and Wagener, 2003, Kolmar and Wagener, 2004, Mintz and Smart, 2004); Anand and Sansing (2000) instead endogenizes formula choice, but the divergence in state incentives there is driven by an assumption of completely immobile inputs. We instead endogenize the formula in a setting where the MNE's own price and sourcing response is itself the source of strategic interaction across governments.

The remainder of the paper proceeds as follows. Section 2 lays out the shared model environment. Section 3 isolates the MNE-only benchmark with differentiated varieties. Section 4 reintroduces domestic firms in the tractable homogeneous-good Cournot limit. Section 5 reintroduces domestic firms in the full differentiated-goods model and presents our main results and the centralized formula problem. Section 6 summarizes four extensions — joint formula-and-tax-rate competition, endogenous entry of MNEs and of domestic firms, and a comparison with separate accounting. Section 7 concludes. Derivations, robustness checks, and extended results omitted from the main text for space are collected in the Appendix.

2 The Model

This section lays out the shared environment for the three specifications of the model: the MNE-only differentiated-goods benchmark of Section 3, the homogeneous-good Cournot benchmark of Section 4, and the full differentiated-goods model with domestic firms of Section 5.[6]

Table 2: The three model specifications compared

	Section 3 MNE-only	Section 4 Homogeneous Cournot	Section 5 Full differentiated
Domestic firms	Absent ($n_i = 0$)	Present, homogeneous	Present, differentiated (σ_D)
MNE's own routes	Differentiated (CES, η)	Perfect substitutes	Differentiated (CES, η)
MNE–domestic substitutability	—	Imperfect (CES, σ)	Imperfect (nested CES, σ)
Stage-two competition	Bertrand (prices)	Cournot (quantities)	Bertrand (prices)
Sourcing margin	Continuous, both routes active	Corner regimes (ρ_i; thresholds D_a^L, D_a^F)	Continuous, all four prices active

All three retain equal-statutory-rate wedge-only dependence on $D = \alpha_{K,a} - \alpha_{K,b}$ for the real allocation; country-specific MNE tax revenue and total local revenue retain a residual dependence on the common capital-weight level in every specification.

Two countries, a and b, each host consumers, a fixed mass n_i of symmetric domestic firms,

[6] The three specifications differ only in whether domestic firms are present and in the elasticity structure of demand and the nature of stage-two competition. The environment is summarized in Table 2.

and one affiliate of a multinational enterprise (MNE).[7] Capital is the sole factor of production and rents at the world rate $r > 0$; a unit of capital produces $A_D > 0$ units for a domestic firm and $A_M > 0$ units for the MNE.

Demand. Consumers in country $i \in \{a, b\}$ have quasi-linear utility

$$U_i = \nu_i + u_i(Q^i), \tag{1}$$

where ν_i is the numeraire and u_i is strictly increasing and strictly concave. Most of the analysis uses the log specification

$$U_i = \nu_i + A_i \ln Q^i, \qquad A_i > 0, \tag{2}$$

which is the $\chi \to 1$ limit of the more general isoelastic family

$$u_i(Q^i) = A_i \, \frac{(Q^i)^{1-\chi}}{1-\chi}, \qquad 0 < \chi < 1. \tag{3}$$

The homogeneous-good benchmark of Section 4 retains the isoelastic form applied to total market output Q_i.[8] The differentiated-sector quantity index Q^i is a nested CES aggregate,

$$Q^i = \left[\left(X_i^D\right)^{\frac{\sigma-1}{\sigma}} + \left(X_i^M\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \tag{4}$$

of a domestic composite X_i^D and an MNE composite X_i^M. The domestic composite aggregates n_i symmetric local varieties, each sold in quantity x_i^D by a representative domestic firm, with within-domestic elasticity $\sigma_D > 1$:

$$X_i^D = \left[n_i \left(x_i^D\right)^{\frac{\sigma_D-1}{\sigma_D}} \right]^{\frac{\sigma_D}{\sigma_D-1}} = n_i^{\frac{\sigma_D}{\sigma_D-1}} x_i^D. \tag{5}$$

The MNE sells two origin varieties in each destination market; $x_{\ell i}$ denotes the quantity produced in origin ℓ and delivered to market i. The MNE sub-index in country i is

$$X_i^M = \left[(x_{ii})^{\frac{\eta-1}{\eta}} + (x_{ji})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad \eta > \sigma, \qquad j \neq i, \tag{6}$$

[7] We initially assume that n_i is fixed; Section 6 relaxes this by allowing free entry and exit of domestic firms.

[8] Appendix A.1 verifies that maximizing consumer surplus is equivalent to minimizing the price index P^i under either specification, so the two are complementary rather than nested demand systems.

with $\eta > \sigma$ reflecting that the MNE's own origin varieties are closer substitutes for one another than for the domestic composite.

Solving the nested expenditure-minimization problem (Appendix A.1) yields the standard CES price indices and demand functions. For market a,

$$P_a^M = \left(p_{aa}^{1-\eta} + p_{ba}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{7}$$

$$P^a = \left[n_a^{\frac{1-\sigma}{1-\sigma_D}} (p_a^D)^{1-\sigma} + \left(P_a^M\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{8}$$

$$x_a^D = A_a (P^a)^{\sigma-1} (P_a^D)^{\sigma_D - \sigma} (p_a^D)^{-\sigma_D}, \tag{9}$$

$$x_{aa} = A_a (P^a)^{\sigma-1} (P_a^M)^{\eta-\sigma} p_{aa}^{-\eta}, \tag{10}$$

$$x_{ba} = A_a (P^a)^{\sigma-1} (P_a^M)^{\eta-\sigma} p_{ba}^{-\eta}, \tag{11}$$

where $P_a^D = n_a^{\frac{1}{1-\sigma_D}} p_a^D$ is the domestic price index, and market b is symmetric. Consumer surplus in country i is

$$CS_i = A_i \ln Q^i - P^i Q^i = A_i \ln\left(\frac{A_i}{P^i}\right) - A_i, \tag{12}$$

so, up to an additive constant, maximizing CS_i is equivalent to minimizing P^i.

2.1 Technology and Trade Costs

Domestic firms sell only in their local market; a domestic firm in country i selling x_i^D units demands capital $K_i^D = x_i^D / A_D$.

The MNE can produce in either country for either market. Local sales use one unit of capital per A_M units delivered. Exports face iceberg cost $\kappa > 1$, so delivering x_{ab} units from a to b requires $\kappa x_{ab}/A_M$ units of capital. The MNE's variable capital demands are therefore

$$k^{aa} = \frac{x_{aa}}{A_M}, \quad k^{ab} = \frac{\kappa x_{ab}}{A_M}, \quad k^{ba} = \frac{\kappa x_{ba}}{A_M}, \quad k^{bb} = \frac{x_{bb}}{A_M}. \tag{13}$$

We set fixed operating capital to zero throughout ($F_i = 0$), so the MNE's capital allocation is determined entirely by its output choices, and any realized quantity vector $(x_{aa}, x_{ab}, x_{ba}, x_{bb})$ implies a unique origin-destination capital allocation through the equations above.

2.2 Taxes

Domestic firms are taxed under separate accounting. If the statutory tax rate in country i is t_i and the deductible share of capital cost is $\delta_{D,i} \in [0, 1]$, the after-tax profit of a representative domestic firm is

$$\Pi_i^D = (1 - t_i)p_i^D x_i^D - (1 - t_i\delta_{D,i})rK_i^D. \tag{14}$$

The MNE is taxed under formula apportionment. Let

$$S_a = p_{aa}x_{aa} + p_{ba}x_{ba}, \qquad S_b = p_{ab}x_{ab} + p_{bb}x_{bb} \tag{15}$$

be MNE sales by destination market, and let $k^a = k^{aa} + k^{ab}$, $k^b = k^{ba} + k^{bb}$ be MNE capital employed by origin, with $k = k^a + k^b$. Country i assigns taxable income according to

$$\omega_i = \alpha_{K,i}\hat{\omega}_{K,i} + (1 - \alpha_{K,i})\hat{\omega}_{S,i}, \tag{16}$$

where $\hat{\omega}_{K,a} = k^a/k$ and $\hat{\omega}_{S,a} = S_a/(S_a + S_b)$ (country b uses the complementary shares).[9] The MNE tax base is

$$B = R - r\left(\delta_{M,a}\, k^a + \delta_{M,b}\, k^b\right), \qquad R = S_a + S_b, \tag{17}$$

where $\delta_{M,i} \in [0, 1]$ is the deductible share of MNE capital in country i; we assume $\delta_{M,i} = \delta_M$ for all i unless stated otherwise, so $B = R - \delta_M rk$. The MNE tax revenue collected by country i is $T_i^M = t_i\omega_i B$, and total MNE tax payments are

$$T^M = T_a^M + T_b^M \equiv \bar{\tau}B, \tag{18}$$

so MNE after-tax profits are

$$N = (1 - \bar{\tau})R - (1 - \delta_M\bar{\tau})rk. \tag{19}$$

Domestic-firm tax revenue is separate: for a representative domestic firm,

$$\tau_i^D = t_i\left(p_i^D x_i^D - \delta_{D,i}rK_i^D\right), \tag{20}$$

and aggregate domestic-firm tax revenue is $T_i^D = n_i\tau_i^D$. Total country tax revenue is $T_i = T_i^M + T_i^D$.

2.3 Timing

The game has three stages.

1. Governments simultaneously choose the capital weight $\alpha_{K,i} \in [0, 1]$ in their formula-apportionment system.

[9] To simplify, only the capital and sales weights are active, $\alpha_{W,i} = 0$ throughout.

2. The MNE and domestic firms observe the announced formulas and compete simultaneously; the implied capital allocations determine the formula shares. In the MNE-only and full differentiated-goods models (Sections 3 and 5), stage-two competition is in prices (Bertrand); in the homogeneous-good benchmark (Section 4), it is in quantities (Cournot).

3. Payoffs are realized.

3 The MNE-Only Benchmark: Differentiated Goods

We first isolate the case with no domestic firms: consumers, capital, iceberg trade costs, taxes, and timing are exactly as in Section 2 with $n_i = 0$, so the differentiated-sector quantity index Q^i collapses to the MNE origin-variety CES index alone. Setting $n_i = 0$ in Section 2's demand system gives the price index $P_i = (p_{ii}^{1-\eta} + p_{ji}^{1-\eta})^{1/(1-\eta)}$, outer demand $Q^i = (A_i/P_i)^{1/\chi}$ under the isoelastic specification (log limit $\chi \to 1$), and origin-variety demands $x_{\ell i} = A_i^{1/\chi} P_i^{\eta-1/\chi} p_{\ell i}^{-\eta}$; consumer surplus CS_i is decreasing in P_i. MNE sales S_a, S_b, capital k^a, k^b, the capital and sales shares $\hat{\omega}_{K,a}, \hat{\omega}_{S,a}$, the capital-sales wedge $\theta_a \equiv \hat{\omega}_{K,a} - \hat{\omega}_{S,a}$, the apportionment shares ω_a, ω_b, the tax base $B = R - \delta_M rk$, and MNE after-tax profit $N = (1-\bar{\tau})R - (1-\delta_M\bar{\tau})rk$ are exactly as defined in Section 2; under equal statutory rates $t_a = t_b = t$, the effective rate collapses to $\bar{\tau} = t(1 + D\theta_a)$, $D \equiv \alpha_{K,a} - \alpha_{K,b}$.

3.1 Stage-Two Equilibrium and the Formula Wedge

A stage-two equilibrium is a delivered-price vector $p = (p_{aa}, p_{ab}, p_{ba}, p_{bb})'$ maximizing MNE profit $N(p)$ given the formula pair. Because the formula pair enters the MNE's objective only through $\bar{\tau} = t(1 + D\theta_a(p))$, the separate levels $\alpha_{K,a}$ and $\alpha_{K,b}$ never enter the pricing problem directly — only their difference does.[10] Hence the entire MNE allocation (prices, quantities, sales, capital, the tax base, and MNE profit) is a function of the formula wedge D alone, $p^*(\alpha_{K,a}, \alpha_{K,b}) = p^*(D)$. The latter does not extend to tax revenue across countries: $T_i^M = t\omega_i B$ depends on the separate formula levels $\alpha_{K,i}$, so two formula pairs sharing the same D induce the same allocation but can split the tax base differently across countries.

The wedge-only structure delivers a global curvature result.

Proposition 1 (Global convexity of MNE profit in the formula wedge)**.** *Suppose equal statutory rates $t_a = t_b = t$; then MNE profit $N^*(D)$ is convex on $[-1, 1]$.*

[10](Appendix B derives the first-order condition and a closed-form solution under common formulas).

Under symmetric country fundamentals N^* attains its global minimum at $D = 0$ and rises with $|D|$ in either direction: the MNE weakly gains from any formula disagreement. By the envelope theorem, $dN^*/dD = -t\theta_a(p^*(D))B(p^*(D))$, which vanishes at $D = 0$ under symmetry (so that $\theta_a = 0$).

The wedge D also has a clean effect on the MNE's effective rate.

Proposition 2 (The formula wedge weakly lowers the MNE's effective tax rate)**.** *Let $t_a = t_b = t$ and fundamentals be symmetric. Then $D\theta_a^*(D) \le 0$ for every $D \in [-1, 1]$, equivalently $\bar{\tau}^*(D) \le t$, with equality at $D = 0$.*[11]

Aggregate MNE tax revenue does not inherit this clean structure. Since $T^{M,*} = \pi^*(D) - N^*(D)$ and $-N^*$ is concave by Proposition 1, revenue is concave in D if and only if pre-tax profit along the equilibrium path, $\pi^*(D)$, is concave, which is not guaranteed, because $p^*(D)$ maximizes the *after-tax* objective, not π.[12]

3.2 Policy Outcomes

Let country i's policy objective be $G_i = \lambda_C CS_i + \lambda_T T_i^M + \lambda_N N$, covering the consumer $(1, 0, 0)$, tax-revenue $(0, 1, 0)$, local fiscal welfare $(1, 1, 0)$, and MNE-friendly $(0, 0, 1)$ objectives. A decentralized formula equilibrium is a pair $(\alpha_{K,a}^*, \alpha_{K,b}^*)$ mutually best-responding in G_a, G_b.

Under equal statutory rates, the wedge-only reduction splits the payoffs into two classes. CS_i and N depend on the formula pair only through D, so the own capital weight is purely an instrument for moving the wedge D; own MNE tax revenue depends on the formula *level* as well, through ω_i. Three results follow:[13]

- **MNE-profit objective.** By Proposition 1, $N^*(D)$ is even and nondecreasing in $|D|$, so each government's best response is bang-bang at the extreme feasible wedge; the pure-strategy Nash equilibria are the maximally opposed pairs $(\alpha_{K,a}^*, \alpha_{K,b}^*) \in \{(1, 0), (0, 1)\}$.
- **Consumer-surplus objective.** If the MNE price index in the home market is monotonically decreasing in the own capital weight's contribution to the wedge (verified numerically), the best response is $BR_a(\alpha_{K,b}) = 1$ for every $\alpha_{K,b}$, and the unique equilibrium is the common pair $(1, 1)$.

[11] Proof in Appendix B.

[12] Appendix B derives an exact local curvature identity at $D = 0$ and a global result: under full capital-cost deductibility ($\delta_M = 1$), common formulas globally maximize aggregate MNE tax revenue. At $\delta_M = 0$, whether $D = 0$ is even a local maximum of revenue is shown numerically,not a theorem; the tax base can expand with the wedge, and in our baseline calibration it does, though the MNE's profit gain dominates so revenue still falls.

[13] Derivations in Appendix B.

- **Revenue and fiscal-welfare objectives.** At a symmetric diagonal point the mechanical and base-scale terms in the own-revenue derivative vanish (since $\theta_a = 0$ and $B'(0) = 0$ there), leaving a behavioral-share stationarity condition that pins down interior symmetric equilibria

$$\alpha_K^T = \frac{\hat{\omega}'_{S,a}(0)}{\hat{\omega}'_{S,a}(0) - \hat{\omega}'_{K,a}(0)}, \tag{21}$$

$$\alpha_K^F = \alpha_K^T + \frac{CS'_a(0)}{tB(0)\left[\hat{\omega}'_{S,a}(0) - \hat{\omega}'_{K,a}(0)\right]}, \tag{22}$$

for the pure tax-revenue and local-fiscal-welfare objectives respectively, both interior since capital leaves a country that taxes it more heavily on the formula ($\hat{\omega}'_{K,a}(0) < 0 < \hat{\omega}'_{S,a}(0)$) while consumers gain from a higher own capital weight ($CS'_a(0) > 0$, numerically verified). All three diagonal equilibria implement the identical common-formula allocation; the objective determines only *where* on the diagonal the game settles.

The centralized problem is one-dimensional: every centralized objective (aggregate revenue, aggregate consumer surplus, fiscal welfare, or world surplus including MNE profit) depends on the formula pair only through D.

When $\delta_M = 0$, an exact identity ties consumer surplus to MNE sales revenue, $CS_a + CS_b = \frac{\chi}{1-\chi} R^*(D)$: the consumer-surplus planner is exactly an MNE-revenue maximizer. Combined with Proposition 2, this produces a sharp split in centralized preferences: the revenue and fiscal-welfare planners prefer the common-formula diagonal (locally always, and globally under full deductibility), while the consumer-surplus and world-surplus planners prefer the maximally opposed formula pairs $D = \pm 1$. The formula wedge essentially acts like an output subsidy that lowers the MNE's effective tax rate and expands sales, which benefits consumers and the MNE by more than governments lose (in the numerical example).

3.3 Numerical Illustration

The baseline calibration is $A_a = A_b = 100$, $\chi = 0.5$, $\eta = 4$, $A_M = 1$, $\kappa = 1.15$, $F_a = F_b = 0$, $r = 1$, $\delta_M = 0$, $t_a = t_b = 0.30$.[14] Table 3 reports the exact decentralized and centralized formula choices for each objective.

[14] Appendix B reports the full price, quantity, and revenue tables and additional contour and best-response figures.

Table 3: Formula choices by objective, finite-elasticity MNE-only model

Objective	Decentralized eq.	Centralized optimum	$T_a^M + T_b^M$	$CS_a + CS_b$	N
Tax revenue	$(0.073, 0.073)$	diagonal ($D = 0$)	2485.260	8284.201	2899.470
Consumer surplus	$(1, 1)$	$(1, 0), (0, 1)$ ($D = \pm 1$)	$1406.633^\dagger$	8692.426	$3642.896^\dagger$
Local fiscal welfare	$(0.318, 0.318)$	diagonal ($D = 0$)	2485.260	8284.201	2899.470
MNE profit / world surplus	$(0, 1), (1, 0)$	$(1, 0), (0, 1)$ ($D = \pm 1$)	1406.633	8692.426	3642.896

†Values realized at the decentralized consumer-surplus equilibrium (coincide with the centralized optimum's allocation).

Figure 1 plots the four centralized objectives, together with MNE profit N, against the wedge D on $[-1, 1]$; by the one-dimensional reduction above, these profiles summarize the entire two-dimensional policy square. Aggregate revenue and fiscal welfare are single-peaked at the common formula; consumer surplus is locally flat at $D = 0$ and rises toward the corners; world surplus is non-concave, with a local maximum at $D = 0$ dominated by the corners; and MNE profit is globally convex with its minimum at $D = 0$, consistent with Proposition 1.

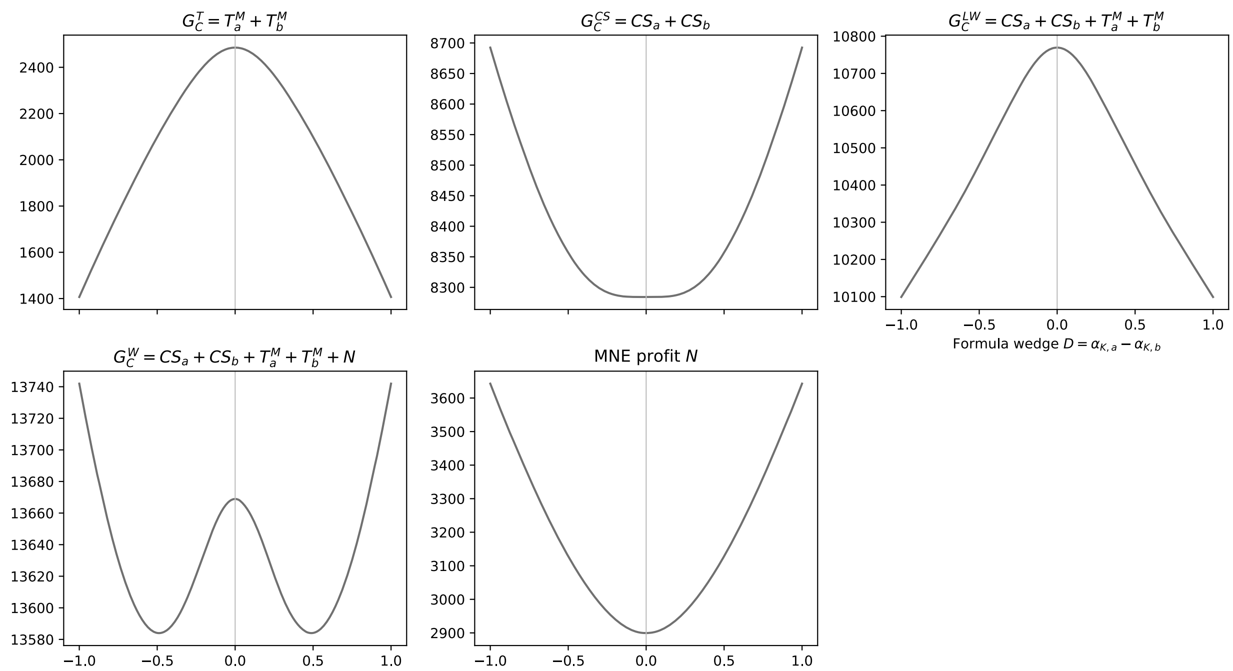

Figure 1: Finite elasticity, $\delta_M = 0$: the four centralized objectives and MNE profit N as functions of the formula wedge $D = \alpha_{K,a} - \alpha_{K,b}$. All curves are even in D with a stationary point at the common formula. Aggregate MNE tax revenue and local fiscal welfare peak at $D = 0$; aggregate consumer surplus is locally flat at $D = 0$ and maximal at the corners; world surplus is non-concave, with a local maximum at $D = 0$ dominated by the corners $D = \pm 1$; and MNE profit is globally convex with its minimum at $D = 0$ (Proposition 1).

4 Adding Domestic Firms: A Cournot Benchmark with Imperfect Substitution

This section reintroduces domestic firms, absent from the MNE-only benchmark of Section 3. To isolate the effect of formula apportionment on the sourcing and market-power margins, we adopt two simplifying assumptions: the MNE's own routes are perfect substitutes, and stage-two competition is in quantities (Cournot) rather than prices. Section 5 relaxes both assumptions, embedding domestic firms in the full differentiated-goods, Bertrand model.

The MNE's local and imported units are perfect substitutes within each destination: consumers care about $x_i = x_{ii} + x_{ji}$, not origin. This is new relative to the MNE-only benchmark, where the four routes were differentiated CES varieties with their own prices and quantities; here perfect substitutability lets the MNE's problem be stated as a choice of total quantity x_i and a local sourcing share ρ_i (route quantities x_{ii}, x_{ji} are then implied accounting objects), a decomposition specific to this section. The MNE good and the homogeneous domestic good are imperfect substitutes, so formula apportionment can relocate MNE production without changing the MNE product, while Cournot competition allocates expenditure between MNE and domestic producers. Two further structural results survive from the MNE-only benchmark: a scalar reduction of each market equilibrium conditional on the MNE's formula-adjusted shadow cost, and wedge-only real-allocation dependence under equal statutory rates.

4.1 Environment

Consumers in country i have quasi-linear utility $U_i = \nu_i + u_i(Q_i)$, $u_i(Q_i) = A_i Q_i^{1-\chi}/(1-\chi)$ (log limit at $\chi = 1$), over the CES quantity index

$$Q_i = \left[x_i^{\frac{\sigma-1}{\sigma}} + y_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad y_i \equiv \sum_{m=1}^{n_i} x_{mi}^D = n_i x_i^D, \qquad \sigma > 1, \tag{23}$$

of the MNE good x_i and the aggregate domestic good y_i (symmetric domestic firms).

Marginal utilities give inverse demands

$$p_i^M = A_i Q_i^{-\chi} \left(\frac{Q_i}{x_i} \right)^{1/\sigma}, \qquad p_i^D = A_i Q_i^{-\chi} \left(\frac{Q_i}{y_i} \right)^{1/\sigma}; \tag{24}$$

both MNE routes command p_i^M, but the MNE and domestic prices differ. Defining the MNE expenditure share

$$\lambda_i \equiv \frac{x_i^{(\sigma-1)/\sigma}}{x_i^{(\sigma-1)/\sigma} + y_i^{(\sigma-1)/\sigma}} \in (0,1), \tag{25}$$

CES homogeneity gives

$$p_i^M x_i = \lambda_i E_i, \qquad p_i^D y_i = (1-\lambda_i)E_i, \qquad E_i \equiv A_i Q_i^{1-\chi}, \tag{26}$$

and consumer surplus is

$$CS_i = \frac{\chi}{1-\chi} A_i Q_i^{1-\chi} \qquad (CS_i = A_i \ln Q_i - A_i \text{ at } \chi = 1). \tag{27}$$

The MNE chooses total quantity x_i and local sourcing share $\rho_i \in [0,1]$, so that route quantities are

$$x_{ii} = \rho_i x_i, \qquad x_{ji} = (1-\rho_i)x_i. \tag{28}$$

With iceberg cost $\kappa > 1$ and $\ell_i(\rho_i) \equiv \rho_i + \kappa(1-\rho_i)$, MNE capital by location of production is

$$k^a = \frac{\rho_a x_a + \kappa(1-\rho_b)x_b}{A_M}, \quad k^b = \frac{\kappa(1-\rho_a)x_a + \rho_b x_b}{A_M}, \quad k = k^a + k^b = \sum_i \frac{\ell_i(\rho_i)x_i}{A_M}; \tag{29}$$

a domestic firm uses x_i^D/A_D units of capital. MNE sales are $S_i = p_i^M x_i$ and $R = S_a + S_b$, and the formula shares are

$$\hat{\omega}_{K,i} = \frac{k^i}{k}, \quad \hat{\omega}_{S,i} = \frac{S_i}{R}, \quad \theta_i = \hat{\omega}_{K,i} - \hat{\omega}_{S,i}, \quad \omega_i = \hat{\omega}_{S,i} + \alpha_{K,i}\theta_i; \tag{30}$$

under equal rates, $\bar{\tau} = t(1 + D\theta_a)$ with $D = \alpha_{K,a} - \alpha_{K,b}$. MNE tax base and net profit are

$$B = R - \delta_M r k, \qquad N = (1-\bar{\tau})R - (1-\delta_M\bar{\tau})rk; \tag{31}$$

domestic firms remain under separate accounting,

$$\Pi_i^D = (1-t_i)p_i^D x_i^D - (1-t_i\delta_{D,i})\frac{r}{A_D}x_i^D, \qquad c_{D,i} \equiv \frac{1-t_i\delta_{D,i}}{1-t_i}\frac{r}{A_D}. \tag{32}$$

4.2 Cournot Equilibrium and the Scalar Reduction

The MNE and domestic firms choose quantities simultaneously, each internalizing its own price effect; the MNE also internalizes its tax base and formula shares, holding domestic quantities fixed in its Cournot derivative. Perceived marginal revenues are

$$MR_i^M = p_i^M \mu_i^M, \qquad \mu_i^M = 1 - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \chi\right)\lambda_i, \tag{33}$$

and

$$MR_i^D = p_i^D \mu_i^D, \qquad \mu_i^D = 1 + \frac{1}{n_i}\left[-\frac{1}{\sigma} + \left(\frac{1}{\sigma} - \chi\right)(1-\lambda_i)\right]; \tag{34}$$

perceived elasticities depend on CES shares and, for a domestic firm, on n_i. The domestic first-order condition is

$$p_i^D \mu_i^D = c_{D,i}; \tag{35}$$

the MNE's first-order condition defines a formula-adjusted shadow cost z_i,[15]

$$p_i^M \mu_i^M = z_i, \qquad z_i = \frac{(1-\delta_M\bar{\tau})r\ell_i/A_M + BtD\theta_{a,x_i}}{1-\bar{\tau}}. \tag{36}$$

Since CES inverse demands give $p_i^M/p_i^D = [(1-\lambda_i)/\lambda_i]^{1/(\sigma-1)}$, dividing the two first-order conditions collapses the four-variable market equilibrium to one scalar equation in λ_i:

$$\frac{z_i}{c_{D,i}} = \left(\frac{1-\lambda_i}{\lambda_i}\right)^{1/(\sigma-1)} \frac{\mu_i^M(\lambda_i)}{\mu_i^D(\lambda_i)}. \tag{37}$$

Proposition 3 (Conditional-equilibrium reduction)**.** *Fix sourcing and suppose $z_i, c_{D,i}, \mu_i^M, \mu_i^D > 0$. Every symmetric interior Cournot equilibrium corresponds to a solution $\lambda_i \in (0,1)$ of* (37)*; prices and quantities then follow uniquely. If the right-hand side is strictly decreasing in λ_i, the interior equilibrium is unique.*[16]

At $D = 0$, sourcing is local and $z_a = z_b = z_0 \equiv (1-\delta_M t)r/[(1-t)A_M]$; symmetric fundamentals then pin down a common share λ_0 from (37), and the equilibrium remains nondegenerate at $\chi = 1$ because each good faces a finite perceived elasticity (unlike the MNE-only benchmark's knife-edge, Section 3).

4.3 MNE Sourcing and the Formula Wedge

Holding quantities fixed, the sourcing share ρ_i changes capital costs and formula shares but not demand, so the KKT gradient is $G_{\rho_i} = (1-\delta_M\bar{\tau})r\frac{\kappa-1}{A_M}x_i - tDB\theta_{a,\rho_i}$.

Proposition 4 (Sourcing direction)**.** *If $D > 0$, then $\rho_b = 1$; if $D < 0$, then $\rho_a = 1$; at $D = 0$, $\rho_a = \rho_b = 1$ (proof in Appendix C).*

[15] Appendix C derives both first-order conditions.
[16] See Appendix C.

For $D > 0$, thresholds D_a^L (local-to-partial) and D_a^F (partial-to-full) partition $[0, 1]$ into three sourcing regimes: fully local, partial foreign sourcing, and fully imported, with country labels reversing for $D < 0$. The thresholds are implicit because quantities, the tax base, and share derivatives all respond through z_i and λ_i.[17]

Proposition 5 (Wedge-only real allocation). *Under equal rates, stage-two equilibrium is a function of D alone, $(x_i, x_i^D, Q_i, p_i^M, p_i^D, \lambda_i, \rho_i, k^i, N, B) = \mathcal{E}_i(D)$. Consumer surplus, domestic profits and taxes, aggregate MNE tax revenue, and aggregate real welfare depend on formulas only through D; country-specific MNE tax revenue and full local welfare also depend on $\alpha_{K,i}$.*

Under symmetric fundamentals, $x_a(D) = x_b(-D)$, $p_a^M(D) = p_b^M(-D)$, etc., so every symmetric aggregate objective is even in D ($D = 0$ is stationary when differentiable, though symmetry alone never establishes optimality).

4.4 Tax Revenue, Welfare, and the Common-Formula Benchmark

Local welfare is $W_i = CS_i + n_i\Pi_i^D + T_i^D + T_i^M$; cancelling domestic taxes against after-tax domestic profits gives $W_i = u_i(Q_i) - p_i^M x_i - \frac{r}{A_D} y_i + T_i^M$, and adding MNE net profit gives world welfare $W^{world} = \sum_i u_i(Q_i) - \frac{r}{A_D} \sum_i y_i - rk$.

Proposition 6 (Common formulas and sourcing efficiency). *At $D = 0$, the MNE sources locally, minimizing capital required for fixed (x_a, x_b). Under symmetric fundamentals, $D = 0$ is stationary for aggregate consumer surplus, domestic profit, MNE tax revenue, domestic welfare, and world welfare.*

The sourcing claim is global; stationarity follows from symmetry alone and does not establish global optimality, since a wedge also changes market power and consumption composition, not only trade costs.[18]

4.5 Numerical Illustration

The baseline calibration is $A_a = A_b = 120$, $n_a = n_b = 10$, $A_D = 1$, $A_M = 1.25$, $r = 1$, $\kappa = 1.25$, $t = 0.30$, $\delta_M = \delta_D = 0$, $\sigma = 4$, $\chi = 1$, solved on a grid of 201 formula wedges $D \in [-1, 1]$ [19] The MNE sources locally for $|D| \lesssim 0.28$, partially imports for $0.29 \lesssim |D| \lesssim 0.34$, and fully imports the tax-favored market's MNE good for $|D| \gtrsim 0.35$. Table 4 reports the equilibrium at five representative wedges.

[17] Appendix C gives the regime conditions and the interior-sourcing formula.

[18] Appendix C gives the full comparative-statics derivatives.

[19] Appendix C reports additional figures.

Table 4: Selected equilibria in the imperfect-substitutes Cournot benchmark.

D	x_a	x_a^D	Q_a	p_a^M	p_a^D	λ_a	ρ_a	k^a	k^b	N	B	$\mathcal{T}^M$
-1.000	15.921	5.561	86.378	2.120	1.551	0.281	1.000	32.462	0.000	30.098	72.687	10.127
-0.500	15.921	5.561	86.378	2.120	1.551	0.281	1.000	27.887	0.000	23.491	66.402	15.024
0.000	15.921	5.561	86.378	2.120	1.551	0.281	1.000	12.737	12.737	21.785	67.513	20.254
0.500	15.150	5.628	85.945	2.155	1.552	0.272	0.000	0.000	27.887	23.491	66.402	15.024
1.000	19.725	5.245	88.487	1.974	1.546	0.324	0.000	0.000	32.462	30.098	72.687	10.127

Notes: Country-b market variables follow by reflection. Parameters: $A_i = 120$, $n_i = 10$, $A_D = 1$, $A_M = 1.25$, $r = 1$, $\kappa = 1.25$, $t = 0.30$, $\delta_M = \delta_D = 0$, $\sigma = 4$, and $\chi = 1$.

Figure 2 plots MNE net profit, the taxable base, and aggregate MNE tax revenue $\mathcal{T}^M(D) = \bar{\tau}(D)B(D)$, all even in D. Net profit is minimized at the common formula and rises once opposed formulas induce tax-motivated foreign sourcing; the tax base is non-monotone; and aggregate MNE tax revenue is globally maximized at $D = 0$ in this calibration ($\mathcal{T}^M(0) = 20.254$ vs. 15.024 at $|D| = 0.5$ and 10.127 at $|D| = 1$).

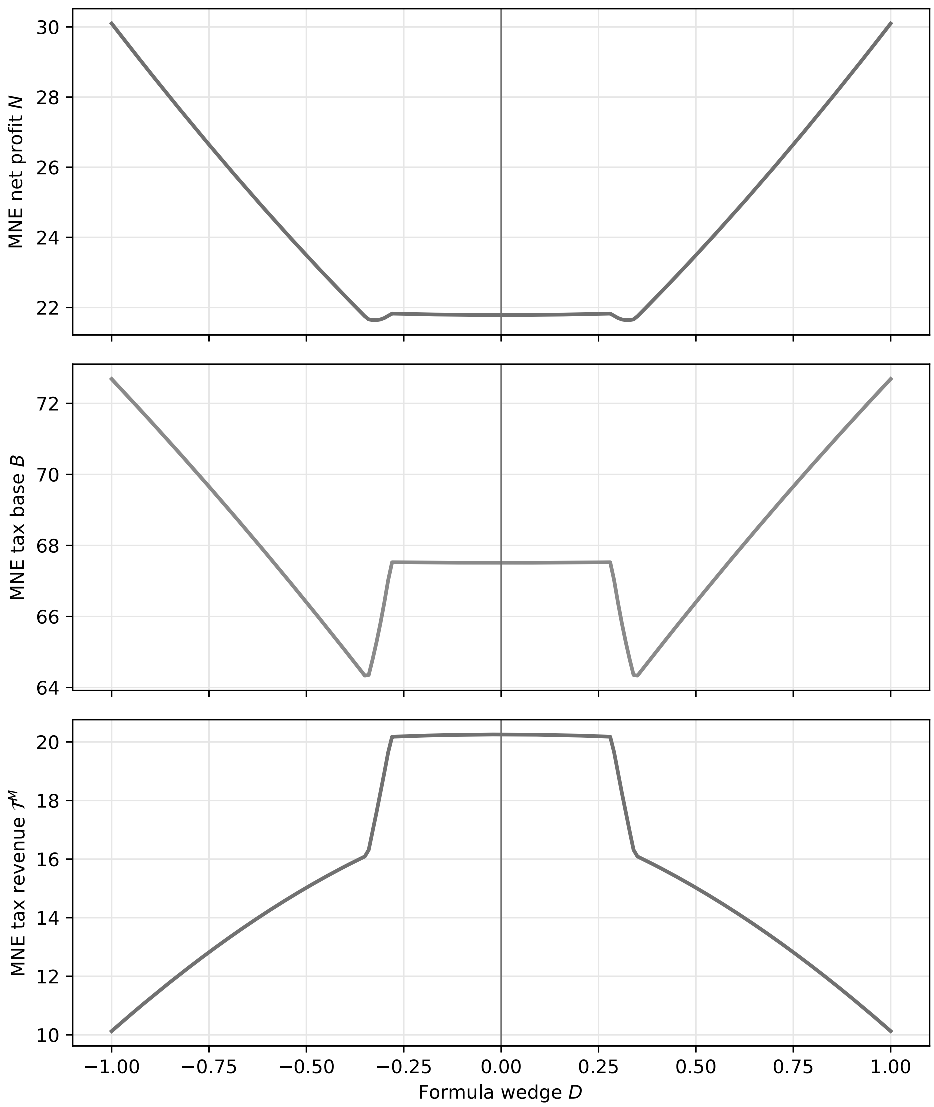

Figure 2: MNE after-tax net profit N, taxable base B, and aggregate MNE tax revenue $\mathcal{T}^M = \bar{\tau}B$ over the formula wedge.

4.6 Policy Games

Consider a game in which governments choose $(\alpha_{K,a}, \alpha_{K,b}) \in [0,1]^2$ guided by six payoff objectives: CS_i, $n_i\Pi_i^D$, T_i^D, $T_i^M = t\omega_i B$, $T_i = T_i^D + T_i^M$, N, and W_i. Proposition 5 shows: $CS_i, n_i\Pi_i^D, T_i^D, N$ depend on the formula system only through D; T_i^M, W_i depend on D and $\alpha_{K,i}$. This is because $\omega_i = \hat{\omega}_{S,i}(D) + \alpha_{K,i}\theta_i(D)$, so the $\alpha_{K,i}$-dependent terms cancel on summation for aggregate MNE tax revenue $\mathcal{T}^M(D) = tB(D)[1 + D\theta_a(D)]$ but not for the country split or W_i.

Two propositions (similar to those in the MNE-only benchmark), help us organize the game equilibria.[20]

Proposition 7 (Exact continuous best responses)**.** *Let $F_a(D), F_b(D)$ be any pair of country payoffs depending only on D, with $F_b(D) = F_a(-D)$ under symmetric fundamentals. Then $BR_a(\alpha_{K,b}) = \alpha_{K,b} + \arg\max_{D\in[-\alpha_{K,b},1-\alpha_{K,b}]} F_a(D)$, and symmetrically for b; a profile is a pure-strategy Nash equilibrium iff each action lies in the corresponding set.*

Proposition 8 (Sufficient conditions for a symmetric planner optimum)**.** *For any symmetric aggregate objective $\Phi(D) = \Phi(-D)$, $\Phi'(0) = 0$ whenever Φ is differentiable. The planner's choice is $D^* = 0$ if either Φ is strictly concave on $[-1, 1]$, or direct comparison shows $\Phi(0) \geq \Phi(D)$ for every D. Symmetry alone establishes stationarity, never global optimality.*

Restricting to $\chi = 1$, the local-sourcing regime's four-equation Cournot system exhibits an exact structural simplification.

Lemma 1 (Local decoupling at $D = 0$)**.** *At $D = 0$, the Jacobian of the MNE and domestic first-order conditions is block-diagonal across countries, and the first-order response $(dx_a/dD, dx_a^D/dD)|_0$ solves a single-country 2×2 linear system driven only by $tB_0\,\theta_{a,x_a}|_0$ (Appendix C).*

This generalizes the MNE-only benchmark's rent-margin logic: the formula wedge's only first-order channel into the market is through $\theta_{a,x_a}|_0$, with the response otherwise governed by country a's own market curvature. The 2×2 system's entries do not reduce to an elementary closed form for general σ.

Proposition 9 (Curvature at $D = 0$)**.** *At the baseline calibration, $N''(0) = 1.0715 > 0$ (a local minimum) and $\mathcal{T}^{M\prime\prime}(0) = -1.9639 < 0$ (a local maximum, a sign reversal from the corresponding perfect-substitutes calibration, where $\mathcal{T}^{M\prime\prime}(0) = +0.01387 > 0$). Neither sign is established in closed form.*

Both signs are local, evaluated within the local-sourcing regime around $D = 0$, and ultimately trace back to how the wedge moves the MNE's formula-adjusted shadow cost z_i in (36): dz_i/dD need not keep the same sign across the local, partial-import, and full-import sourcing regimes, so a curvature result signed at $D = 0$ does not extend automatically to a global statement over $D \in [-1, 1]$.

Table 5 reports the pure-strategy Nash equilibria and centralized optima for all six objectives at the baseline calibration.

[20]Proofs in Appendix C.

Table 5: Policy-game outcomes under the v2 (imperfect-substitutes) model, baseline calibration.

Objective	Nash equilibrium	Centralized D^C	Value at D^C	$\Phi''(0)$
CS_i	none (pure)	1.0000	832.9914	0.6978
$n_i\Pi_i^D$	(1,1)	-0.3430	9.8361	0.0579
T_i^M	(0,0)	0.0000	20.2539	-1.9642
T_i	(0,0)	0.0000	72.0000	-2.0573
W_i	(0,0)	-0.0000	911.6228	-1.3016
N	(0,1), (1,0)	1.0000	30.0978	1.0713

Notes: Parameters: $A_i = 120$, $n_i = 10$, $A_D = 1$, $A_M = 1.25$, $r = 1$, $\kappa = 1.25$, $t = 0.30$, $\delta_M = \delta_D = 0$, $\sigma = 4$, $\chi = 1$. $\Phi''(0)$ is the second derivative of the aggregate objective at $D = 0$ (PCHIP-interpolant finite difference; for N and $\mathcal{T}^M$ cross-checked against the exact computer-algebra values in Table 12).

The qualitative picture matches the MNE-only benchmark on every objective except one: CS_i has no pure Nash equilibrium (centralized optimum at a full-import corner); $n_i\Pi_i^D$'s Nash equilibrium is the weak corner $(1,1)$; T_i^M, T_i, and W_i share the sales-only Nash equilibrium $(0,0)$ with centralized optima at or near $D = 0$; and N's Nash equilibria are the maximally opposed corners, consistent with Proposition 9's $N''(0) > 0$. The one substantive difference is that the $\mathcal{T}^M$ game's centralized optimum is now a genuine local (and, at this calibration, global) maximum at $D = 0$, rather than the perfect-substitutes benchmark's calibration-dependent result in which a small formula asymmetry could raise revenue.

4.7 Main Findings

1. The MNE controls $(x_a, x_b, \rho_a, \rho_b)$; route quantities are implied accounting objects.

2. Conditional on z_i, the destination equilibrium reduces to the scalar share equation (37).

3. Unlike the MNE-only benchmark, where stage-two pricing is always interior (no sourcing corner), the MNE's sourcing choice ρ_i here is a genuine corner-solution problem: the KKT gradient G_{ρ_i} partitions $D \in [-1, 1]$ into local, partial-import, and full-import regimes, separated by the thresholds D_a^L, D_a^F. Within each regime, D reaches the formula-adjusted shadow cost z_i through a different combination of $\ell_i(\rho_i)$ and θ_{a,x_i} in (36), so dz_i/dD need not keep the same sign across regimes; results signed locally at $D = 0$, such as the curvature signs in Proposition 9, do not extend automatically to other regimes.

4. Equal rates preserve wedge-only dependence and reflection symmetry.

5. Imperfect substitution adds a composition margin: the market equilibrium's demand-system dependence is confined to the scalar share equation, but the policy game's local curvature (Proposition 9) is not reducible to the perfect-substitutes benchmark's closed form, and one sign (the curvature of aggregate MNE tax revenue at $D = 0$) reverses.

6. Every policy game's Nash-equilibrium and centralized-optimum structure reproduces the perfect-substitutes benchmark's qualitative pattern, verified numerically rather than proved analytically beyond Propositions 7–9 (Appendix C).

5 The Differentiated-Goods Model with Domestic Firms

We now reintroduce domestic firms into the finite-η differentiated-goods model of Section 3. The environment, tax structure, and timing are as in Section 2. The key departure from the Cournot benchmark of Section 4 is that consumers distinguish varieties by production origin: each origin-destination pair (i, j) carries a distinct delivered price p_{ij} and quantity x_{ij}, so the MNE controls the full price vector $(p_{aa}, p_{ab}, p_{ba}, p_{bb})$ rather than a pair of destination quantities and a scalar sourcing share ρ_i.

Fix a formula pair $(\alpha_{K,a}, \alpha_{K,b})$. A stage-two equilibrium is a collection of prices, quantities, capital demands, price indices, and apportionment/tax objects such that domestic firms and the MNE maximize after-tax profits taking the price indices and formula weights as given, demand and technology equations hold, and apportionment shares, tax payments, and the effective rate are computed from the resulting allocation. A stage-one equilibrium is a formula pair that is a mutual best response given the induced stage-two equilibria.

5.1 Domestic Firms and the MNE's Pricing Problem

Proposition 10. *For country $i \in \{a, b\}$, the equilibrium domestic price is*[21]

$$p_i^{D*} = \frac{\sigma_D}{\sigma_D - 1} \frac{r}{A_D} \frac{1 - t_i \delta_{D,i}}{1 - t_i}. \tag{38}$$

Domestic pricing is therefore independent of the MNE formula except through equilibrium demand; domestic capital is chosen residually, $K_i^{D*} = x_i^D(p_i^{D*}, P^i)/A_D$. The formula affects domestic firms only indirectly, by shifting the MNE prices that enter the market price index P^i and therefore domestic demand.

Given $(\alpha_{K,a}, \alpha_{K,b})$, the MNE chooses its four delivered prices to maximize $N = R - rk - \bar{\tau} B$,

[21] See Appendix D.

$B = R - \delta_M rk$. For any price p_{ij} the first-order condition is

$$\frac{\partial N}{\partial p_{ij}} = (1-\bar{\tau})\frac{\partial R}{\partial p_{ij}} - (1-\delta_M\bar{\tau})r\frac{\partial k}{\partial p_{ij}} - B\frac{\partial \bar{\tau}}{\partial p_{ij}} = 0, \tag{39}$$

a revenue effect, a capital-cost effect, and the formula-apportionment wedge (a price change moves the destination sales share and the origin capital share, which changes the effective tax rate). With within-market share $m_{ij} \equiv p_{ij}^{1-\eta}/(p_{ij}^{1-\eta} + p_{hj}^{1-\eta})$ ($h \neq i$ the other origin selling into j), upper-tier MNE share $\lambda_j \equiv (P_j^M/P^j)^{1-\sigma}$, and $c_j \equiv \eta - \sigma + (\sigma-1)\lambda_j \in (0,\eta)$, the CES demand system gives $\partial \ln x_{ij}/\partial \ln p_{ij} = -\eta + c_j m_{ij} < 0$ and $\partial \ln x_{hj}/\partial \ln p_{ij} = c_j m_{ij} > 0$: raising p_{ij} unambiguously lowers demand for that variety (the direct elasticity $-\eta$ always dominates the price-index feedback[22]) and raises demand for the rival origin variety in the same destination. Substituting these demand derivatives, the linear capital technology, and the derivative of $\bar{\tau} = t(1+D\theta_a)$ (under equal statutory rates, $D = \alpha_{K,a} - \alpha_{K,b}$, $\theta_a = \hat{\omega}_{K,a} - \hat{\omega}_{S,a}$) into (39) yields four fully expanded pricing equations, one per delivered price, that jointly determine $(p_{aa}, p_{ab}, p_{ba}, p_{bb})$.[23]

Remark 1 (Reflection identities for the four delivered prices)**.** *Under symmetric fundamentals ($t_a = t_b = t$, $A_a = A_b$, $n_a = n_b$, $F_a = F_b$), jointly relabeling the two countries and flipping the sign of the wedge leaves the four-equation pricing system unchanged, so along a continuously selected equilibrium branch $p_{aa}(D) = p_{bb}(-D)$, $p_{ab}(D) = p_{ba}(-D)$, and correspondingly for quantities. At $D = 0$ these collapse to $p_{aa} = p_{bb}$, $p_{ab} = p_{ba}$, recovering Proposition 11 below as the special case $D = -D = 0$.*[24]

Proposition 11 (Aligned-formula neutrality)**.** *Assume symmetric fundamentals ($t_a = t_b = t$, $A_a = A_b$, $n_a = n_b$, $F_a = F_b$) and $\alpha_{K,a} = \alpha_{K,b}$. Then a symmetric equilibrium exists with $p_{aa} = p_{bb}$, $p_{ab} = p_{ba}$, $k^a = k^b$, $S_a = S_b$, and $\bar{\tau} = t$.*

Proof. With aligned formulas, $\omega_a + \omega_b = \alpha_{K,a}(\hat{\omega}_{K,a} + \hat{\omega}_{K,b}) + (1-\alpha_{K,a})(\hat{\omega}_{S,a} + \hat{\omega}_{S,b}) = 1$ for any allocation (capital and sales shares each sum to one), so $\bar{\tau} = t$ independently of the MNE's pricing and capital choices, and $\partial\bar{\tau}/\partial p_{ij} = 0$ for every price: the FOC (39) collapses to $(1-t)\partial R/\partial p_{ij} - (1-\delta_M t)r\partial k/\partial p_{ij} = 0$, the undistorted pricing problem. Under symmetric fundamentals this problem is invariant to relabeling countries, so the symmetric price restriction $p_{aa} = p_{bb}$, $p_{ab} = p_{ba}$ is a fixed point of the four-equation system, delivering $k^a = k^b$, $S_a = S_b$, and $\hat{\omega}_{K,a} = \hat{\omega}_{S,a} = 1/2$. □

[22] Appendix D

[23] Appendix D.

[24] Appendix D.

Unlike the homogeneous-good benchmark, trade flows do not shut down: both MNE varieties remain active at aligned formulas because CES demand is strictly positive at finite relative prices, so the response to $\alpha_{K,a}$ and $\alpha_{K,b}$ is smooth throughout.

5.2 Local Comparative Statics and Payoff Responses

Writing $p = (p_{aa}, p_{ab}, p_{ba}, p_{bb})'$ and $F(p, \alpha_{K,i}; \alpha_{K,j}) = 0$ for the four first-order conditions, the implicit function theorem gives

$$\frac{dp^*}{d\alpha_{K,i}} = -\left(F_p\right)^{-1} F_{\alpha_{K,i}} \tag{40}$$

whenever the Jacobian F_p is nonsingular.

Proposition 12. *At an interior equilibrium, $\partial F_\ell/\partial\alpha_{K,i} = -t\theta_i(R_\ell - \delta_M r k_\ell) - tB\theta_{i,\ell}$ for each price equation ℓ, where $R_\ell \equiv \partial R/\partial p_\ell$, $k_\ell \equiv \partial k/\partial p_\ell$, $\theta_{i,\ell} \equiv \partial\theta_i/\partial p_\ell$. At a symmetric aligned-formula allocation ($\theta_i = 0$), this simplifies to $\partial F_\ell/\partial\alpha_{K,i} = -tB\theta_{i,\ell}$ and $dp^*/d\alpha_{K,i} = tB(F_p)^{-1}\theta_{i,p}$.*

Unlike the homogeneous-good benchmark (Lemma 1), the sign of $dp^*/d\alpha_{K,i}$ is not generally pinned down here: $\theta_{i,p}$ combines a capital-share derivative that is signable (a higher local-origin price lowers country i's capital share; a higher import price raises it) with a sales-share derivative that combines a direct revenue effect, an own-quantity effect, and a within-MNE substitution effect of ambiguous net sign, and $(F_p)^{-1}$ depends on the full four-equation Jacobian.

These price responses map into payoffs. Let country i maximize $\mathcal{G}_i = \beta_C CS_i + \beta_T T_i + \beta_D n_i \Pi_i^{D*} + \beta_N N^*$, $T_i = T_i^M + T_i^D$, nesting consumer-, revenue-, domestic-firm-, and profit-oriented governments (and, with $\beta_C = \beta_T = \beta_D = 1, \beta_N = 0$, the local-welfare benchmark used below). The analysis depend on four payoff derivatives:[25]

- $dCS_i/d\alpha_{K,i} = -A_i\, d\ln P^i/d\alpha_{K,i}$: consumers lose from a higher local MNE price index.
- $d(n_i\Pi_i^{D*})/d\alpha_{K,i} = (\sigma-1)n_i\Pi_i^{D*}\, d\ln P^i/d\alpha_{K,i}$: domestic firms gain from the same price index rising (softer MNE competition) the exact mirror of the consumer effect.
- $dT_i^M/d\alpha_{K,i} = t_i[\theta_i B + (B\omega_{i,p} + \omega_i B_p)\, dp^*/d\alpha_{K,i}]$: a mechanical apportionment term ($\theta_i B$, signable) plus a behavioral term through the induced price response (not signable in general); $dT_i^D/d\alpha_{K,i}$ moves only through the domestic price index, with no mechanical term.

[25] Appendix D.

- $dN^*/d\alpha_{K,i} = -t_i B\theta_i$ (envelope theorem): MNE profit falls in $\alpha_{K,i}$ exactly when country i's capital share exceeds its sales share.

The domestic-firm-only objective is the simplest case, $d\mathcal{G}_i/d\alpha_{K,i} = (\sigma-1)n_i\Pi_i^{D*}\, d\ln P^i/d\alpha_{K,i} > 0$: it is the mirror image of the consumer-only objective. The local-welfare objective trades off the consumer loss, the domestic-firm gain, and the fiscal effect, and is not signable in general.

As under equal statutory rates elsewhere in the paper, the feasible price set does not depend on the common component of $(\alpha_{K,a}, \alpha_{K,b})$, so the MNE profit function, the real allocation, consumer surplus, domestic-firm profits and taxes, and central welfare depend on the formula system only through the wedge D; country-specific MNE tax revenue $T_i^M = t_i\omega_i B$ and total local revenue T_i depend on $\alpha_{K,i}$. The iceberg cost κ governs how costly it is to exploit the wedge: a higher κ raises the capital cost of exported varieties (directly affecting θ_i) but also makes the MNE price exports higher and ship less, limiting the wedge's exploitability.

5.3 Numerical Results

The benchmark calibration is $\sigma = 4$, $\sigma_D = 5$, $\eta = 6$, $A_a = A_b = 120$, $n_a = n_b = 10$, $A_D = 1$, $A_M = 1.25$, $r = 1$, $\kappa = 1.15$, $t_a = t_b = 0.30$, $\delta_a = \delta_b = \delta_M = 0$, $F_a = F_b = 0$, solved on the grid $\alpha_{K,a}, \alpha_{K,b} \in \{0, 0.1, \ldots, 1\}$; these values keep both MNE affiliates active throughout, so formula changes tilt relative prices, sales, and capital continuously rather than generating the piecewise sourcing patterns of the homogeneous-good model. Table 6 reports the decentralized Nash equilibria and centralized maximizers for six objectives (continuous-optimization refinements where available; Appendix D reports the underlying line plots, contour surfaces, best-response curves, and mechanism figures for every row).

Table 6: Decentralized and centralized formula choices, finite-η differentiated-goods model

objective	decentralized	centralized
Tax revenue	(0.000, 0.000)	all common formulas
MNE tax revenue	(0.141, 0.141)	all common formulas
Consumer surplus	(1.00, 1.00)	opposed formulas
Domestic firms	(0.00, 0.00)	all common formulas
MNE profits	opposed formulas	opposed formulas
Local welfare	(0.107, 0.107)	near-flat plateau, $\alpha_{K,a} \in [0.00, 1.00]$, $\alpha_{K,b} \in [0.00, 1.00]$ (151 tied grid points)

The results can be summarized as follows:

- Tax and domestic-firm objectives select common (sales-heavy) formulas. The combined tax-revenue game has a unique decentralized equilibrium at $(0, 0)$; isolating MNE tax revenue alone from the domestic-firm base (T_i^D dominates T_i^M in magnitude but T_i^M is far more elastic to the formula, since it carries a direct mechanical apportionment term while T_i^D moves only through the domestic price index) instead produces a genuine interior equilibrium at $\alpha_K^{T^M} \approx 0.141$, which happens to lie exactly on the (aggregate-revenue-invariant) common-formula diagonal. The domestic-firm game similarly selects the sales-only corner $(0, 0)$, since common formulas remove the tax-planning wedge that would otherwise raise MNE prices and soften competition.

- Consumer-surplus and MNE-profit objectives select opposed formulas centrally but diverge decentrally. Decentralized consumer-surplus competition converges to the capital-heavy corner $(1, 1)$ (each government wants to tilt MNE pricing toward its own market), while the centralized consumer-surplus optimum is the maximally opposed pair: opposed formulas lower the MNE's effective tax rate, expand its scale, and lower the price index enough to raise total consumption even though surplus becomes asymmetric across countries. Once both governments compete for this effect the formulas end up aligned and the wedge disappears, so decentralization does *not* replicate the centralized outcome here. MNE-profit maximization is decentrally and centrally identical, at the maximally opposed corners $(0, 1)/(1, 0)$, lowering $\bar{\tau}$ from 0.30 (aligned) to about 0.143 (opposed).

- The decentralized local-welfare game has an interior equilibrium at $\approx (0.107, 0.107)$, strictly between the consumer- and domestic-firm-only corners.[26]

[26] Appendix D reports robustness checks along two additional dimensions: competition in the statutory tax rate rather than the formula weight, under isoelastic ($\chi < 1$) outer demand, and a joint game over both instruments simultaneously. The main finding is that aligned-formula neutrality and the qualitative pattern above survive away from log utility, though the revenue-maximizing tax rate falls sharply as demand becomes more price-elastic. Section 6.1 develops this joint tax-rate-and-formula game formally, with closed-form results.

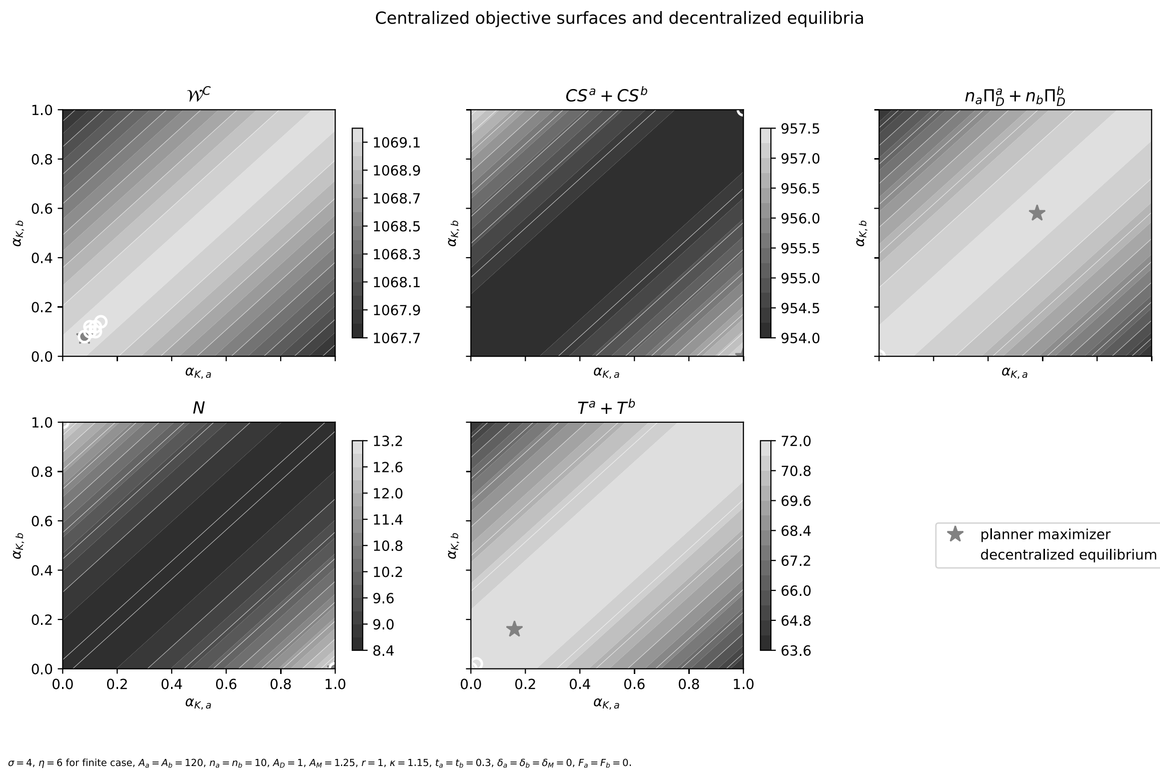

Figure 3: Centralized objective surfaces and decentralized equilibria. Common formulas are optimal for total welfare, domestic profits, and total taxes; opposed formulas maximize consumer surplus and MNE profits.

5.4 Centralized Solution

Suppose a planner chooses $(\alpha_{K,a}, \alpha_{K,b})$ to maximize world welfare with tax revenue rebated lump-sum. Since transfers do not affect resource feasibility, the objective reduces to

$$\mathcal{W}^C = CS_a + CS_b + \sum_{i=a,b} n_i \big(p_i^{D*} x_i^D - rK_i^D\big) + R - rk : \tag{41}$$

formula apportionment affects centralized welfare only through its distortion of prices, quantities, and capital, never through the transfer of tax revenue itself.

Proposition 13 (Diagonal invariance of the centralized problem)**.** *Assume $t_a = t_b = t$, $A_a = A_b$, $n_a = n_b$, and identical technologies. Then any common formula $\alpha_{K,a} = \alpha_{K,b} \in [0,1]$ solves the centralized problem* (41)*.*

Proof. Any common formula gives $\bar{\tau} = t$ and $\partial\bar{\tau}/\partial p_{ij} = 0$ for every price (as in Proposition 11), so the MNE FOCs reduce to the undistorted problem, independent of the common level. For $\alpha_{K,a} \neq \alpha_{K,b}$, $\bar{\tau}$ depends on the equilibrium allocation (equation (85) of Appendix D), so $\partial\bar{\tau}/\partial p_{ij} \neq 0$ generically and the FOCs carry the extra distortion term $-B\,\partial\bar{\tau}/\partial p_{ij}$; since taxes are pure transfers under lump-sum rebate, this distortion creates no offsetting welfare gain, only a reallocation away from the undistorted optimum. Hence any common formula weakly dominates any non-common pair. □

This is the differentiated-goods analogue of the baseline paper's centralized-revenue result, now extended to the full planner objective: once taxes are transfers, formula apportionment is purely distortionary, and common formulas eliminate the distortion regardless of the common level. Separating the planner objective into its components (Appendix D, Table 13) reproduces the pattern of Table 6: total welfare, domestic profits, and total taxes are each maximized by (a possibly different) common formula, while the sum of consumer surplus and total MNE profits are each maximized by opposed formulas.

5.5 Implications of the Assumptions

1. Finite CES substitutability ($\eta < \infty$, $\sigma < \infty$) is what turns the baseline's regime-switching sourcing problem into a smooth four-price problem; the $\eta = \infty$ perfect-substitutes limit instead turns the sourcing margin back into a Kuhn–Tucker problem.

2. Iceberg costs preserve an export margin in both directions throughout the policy space, since CES demand keeps every variety active at finite prices (unlike the corner sourcing regimes of the perfect-substitutes benchmark).

3. Fixed operating capital (set to zero here) would, if positive, make the capital factor more persistent and weaken the pricing response.

4. Imperfect deductibility amplifies the pricing wedge: a lower δ_M raises the effective marginal cost of MNE capital, and full deductibility ($\delta_M = 1$) mutes the direct capital-cost channel (though formulas can still distort prices through sales and capital shares).

5. The demand side is partial equilibrium: log outer utility fixes sectoral expenditure, isolating the apportionment mechanism but ruling out income feedback from rebated taxes or profit ownership.

6. The sharp common-formula centralized result (Proposition 13) uses symmetric fundamentals and equal statutory rates; under asymmetry the planner need not be indifferent among all common formulas, and the welfare-maximizing pair need not lie exactly on the diagonal (the result should be read as the benchmark symmetric case).

6 Extensions

This section develops four extensions to the baseline model of Sections 3–5: joint competition in formulas and tax rates, free entry of MNEs, free entry of domestic firms, and a comparison with separate accounting.

6.1 Joint FA and Tax-Rate Games

We initially assume that the formula system is held fixed at different wedges and governments compete in statutory tax rates (t_a, t_b), each maximizing its own MNE tax revenue $T_i^M = t_i \omega_i B$, with a centralized planner maximizing $T^M = T_a^M + T_b^M$. At an interior pricing optimum, $\partial p^*/\partial t_i = -[N_{pp}]^{-1} N_{pt_i}$, giving analytical best-response and centralized first-order conditions (Appendix F).

Proposition 14 (Centralized revenue-maximizing tax at the common formula)**.** *Let $\delta_M = 0$, $F_i = 0$, symmetric fundamentals, and a common formula ($D = 0$). Restricting to common tax rates $t_a = t_b = t$, aggregate MNE tax revenue is maximized at $t^C = \chi$, independently of A_i, A_M, κ, and η (proof in Appendix F: the common-formula closed form gives $B(t) \propto (1-t)^{(1-\chi)/\chi}$, so $\ln T^M = \ln t + \frac{1-\chi}{\chi} \ln(1-t) + const$ is strictly concave with peak at $t = \chi$).*

The comparison between decentralized and centralized rates turns on a fiscal externality, $\partial T_b^{M,*}/\partial t_a = t[B^* \partial \omega_b^*/\partial t_a + \frac{1}{2} \partial B^*/\partial t_a]$ at a symmetric point, combining a positive *share-shifting* term (a higher t_a pushes capital and sales toward b) and a negative *base* term (a higher t_a shrinks the common base); which dominates is not determined by theory, though it is resolved locally in the numerics below. One formula configuration makes the externality vanish exactly.

Proposition 15 (Sales-only formulas implement the destination principle)**.** *Let $\delta_M = 0$ and $\alpha_{K,a} = \alpha_{K,b} = 0$; demand scales may differ ($A_a \neq A_b$) and $F_i \geq 0$ is arbitrary. Then: (i) $T_i^M = t_i S_i$, a destination-based sales tax; (ii) the MNE's problem separates across destination markets, prices serving market i depending on t_i alone; (iii) $\partial T_j^M/\partial t_i = 0$, $j \neq i$ (i.e., the fiscal externality vanishes); (iv) each country's revenue-maximizing rate is $t_i = \chi$ regardless of the rival's rate (a dominant strategy), so the decentralized equilibrium coincides exactly with the centralized optimum.*

A sales-only, gross-revenue base makes each country's tax base sales to its own consumers, an object the MNE cannot relocate and the rival's tax cannot reach) shutting both channels of the externality simultaneously; $\delta_M = 0$ is essential (with $\delta_M > 0$ the base couples markets through total capital), but symmetry plays no role. Sales-only formulas are not, however, self-enforcing when formula weights are themselves strategic: at $(0,0)$ each country gains $tB\hat{\omega}'_{S,a}(0) > 0$ from tilting toward capital weight (Section 4's α_K^T logic), which is exactly what we explore next.

At isoelastic elasticities $\chi = 0.20$ and $\chi = 0.30$, Table 7 reports the tax-rate game holding the common capital weight at $\bar{\alpha}_K = 0.5$ and sweeping the wedge $D \in \{-1, 0, 1\}$, together with the aligned-formula cases $D = 0$ at $\alpha_{K,a} = \alpha_{K,b} \in \{0, \frac{1}{2}, 1\}$. By Proposition 14, the centralized rate at every $D = 0$ row is exactly $t^C = \chi$ (0.200 at $\chi = 0.20$, 0.300 at $\chi = 0.30$), independently of the common capital-weight level. The wedge cases satisfy the country-exchange reflection identity at both elasticities — e.g., at $\chi = 0.20$, $D = -1$ gives $(t_a, t_b) = (0.195, 0.135)$ and $D = 1$ gives the mirrored $(0.135, 0.195)$ — while the decentralized common rate at $D = 0$ falls as the common capital weight rises (from $t = 0.201$ at $\alpha_K = 0$ to $t = 0.145$ at $\alpha_K = 1$ for $\chi = 0.20$; from $t = 0.300$ to $t = 0.168$ for $\chi = 0.30$), even though the centralized planner holds its rate exactly fixed at χ throughout: the share-shifting externality of Proposition 14's discussion strengthens as the formula tilts toward capital. The $\alpha_K = 1$ decentralized rate is a non-converged best-response iteration endpoint rather than a verified fixed point (flagged †).

Table 7: MNE tax-rate game under isoelastic demand: wedge sweep at $\bar{\alpha}_K = 0.5$ and aligned-formula sweep at $D = 0$

		Nash equilibrium			Centralized		
χ	$(D, \bar{\alpha}_K)$	t_a	t_b	T^M	t_a	t_b	T^M
0.20	$(-1, 0.5)$	0.195	0.135	9.671	0.182	0.119	9.727
0.20	$(0, 0)$	0.201	0.201	13.560	0.200	0.200	13.560
0.20	$(0, 0.5)$	0.201	0.201	13.560	0.200	0.200	13.560
0.20	$(0, 1)^\dagger$	0.145	0.145	12.834	0.200	0.200	13.560
0.20	$(1, 0.5)$	0.135	0.195	9.671	0.119	0.182	9.727
0.30	$(-1, 0.5)$	0.287	0.155	7.758	0.275	0.130	7.801
0.30	$(0, 0)$	0.300	0.300	12.098	0.300	0.300	12.098
0.30	$(0, 0.5)$	0.274	0.274	12.030	0.300	0.300	12.098
0.30	$(0, 1)^\dagger$	0.168	0.168	10.152	0.300	0.300	12.098
0.30	$(1, 0.5)$	0.155	0.287	7.758	0.130	0.275	7.801

†: the decentralized best-response iteration did not converge to a symmetric pure-strategy fixed point (12 iterations, damping 0.5); the tabulated (t_a, t_b) is the iteration endpoint, not a verified equilibrium.

Letting each government choose *both* instruments jointly, $(t_i, \alpha_{K,i})$, closes most of the gap between the decentralized and centralized tax-only games at both elasticities: Table 8 reports $T^M = 13.289$ (Nash) against 13.560 (centralized) at $\chi = 0.20$ — about 98 percent efficient — and $T^M = 12.097$ against 12.098 at $\chi = 0.30$, essentially exact. Both governments load the formula onto the slow-moving sales share, muting the share-shifting channel, as in Proposition 15's logic; this is a second-order deviation from that proposition's exact efficiency, since sales-only formulas are not self-enforcing, but the resulting revenue cost is second order in the equilibrium tilt (Appendix D reports the full detail, including the $\chi = 0.35$ case).

Table 8: Joint tax-rate-and-formula game under isoelastic demand

	Decentralized Nash				Centralized			
χ	$\alpha_{K,a}$	$\alpha_{K,b}$	t	T^M	$\alpha_{K,a}$	$\alpha_{K,b}$	t	T^M
0.20	0.251	0.250	0.238	13.289	0.748	0.748	0.200	13.560
0.30	0.188	0.188	0.297	12.097	1.000	1.000	0.300	12.098

6.2 Endogenous Number of MNEs

This subsection replaces the single MNE of Section 3 with a continuum of measure $M \geq 0$ of symmetric MNE firms whose number is determined by free entry, adding a third demand tier (across firms, elasticity $\sigma_M > 1$) above the existing within-firm origin-variety tier ($\eta > \sigma_M$). Fixed affiliate capital $F_i > 0$ (which was assumed equal to zero up to now), is essential here to generate a nondegenerate entry margin.[27]

Each active firm solves the same pricing problem as the lone MNE with χ replaced by $1/\sigma_M$: under common taxes and formulas, the common markup is $\mu = 1/\sigma_M$ (the monopolistically-competitive CES markup, independent of M and of demand levels: entry and scale affect only quantities, but not markups). Unlike in the previous models, fixed capital breaks scale separability: because $P^i = M^{1/(1-\sigma_M)}p^i$, per-firm variable capital scales with $1/M$ while fixed capital does not, so the capital formula share depends on M and pricing, sourcing, and entry must be solved jointly.[28] Under equal statutory rates, the representative firm's problem is wedge-only in $D = \alpha_{K,a} - \alpha_{K,b}$, though the country split of tax revenue is not.

Table 9: MNE-entry formula games, baseline calibration

Objective	Centralized maximizers	Approx. continuous NE	Centralized value
T^M	all diagonal points, $D = 0$	$(0, 0)$	72.00
CS	$(0, 1)$ and $(1, 0)$	$(1, 1)$	1025.50
$W = CS + T^M$	all diagonal points, $D = 0$	$(0.521, 0.521)$	1081.62

We solve numerically the five-equation pricing-plus-entry system ($A_a = A_b = 120$, $A_M = 1.25$, $\kappa = 1.25$, $t = 0.30$, $\delta_M = 0$, $\eta = 6$, $\sigma_M = 4$, $F_a = F_b = 2$. The pattern in Table 9 closely

[27]With entry, log utility (used throughout) is well-behaved.
[28]See Appendix F.

mirrors the no-entry benchmark's qualitative results: the tax-revenue game decentralizes to the sales-only corner, and the centralized solution is the entire diagonal; the consumer-surplus game decentralizes to $(1, 1)$ even though the centralized planner prefers opposed formulas; and the welfare game balances these forces into the symmetric $\alpha_{K,i} = 0$ in the dectrnalized case, and the entire diagonal in the centralized case.

In the case of opposed formulas: consumer surplus increases (CS from 1009.62 to 1025.50), and entry rises only slightly (M^* from 10.50 to 10.60); aggregate MNE tax revenue declines (T^M from 72.00 to 53.57).

6.3 Endogenous Number of Domestic Firms

This subsection endogenizes the number of domestic firms n_i in the homogeneous-good Cournot benchmark of Section 4, via a Stage-2 free-entry decision: potential entrants pay an after-tax setup cost $\mathcal{F}_i = (1 - t_i\delta_{D,i})rF_i$ before Cournot competition. The central analytical result is a striking insulation property.

Lemma 2 (Price pinning)**.** *In any interior equilibrium with domestic firms active, total market output $\bar{Q}_i$ and market price $\bar{p}_i$ are pinned to constants determined entirely by local primitives $(A_i, c_{D,i}, t_i, \mathcal{F}_i)$, independent of MNE choices or the formula wedge D (proof in Appendix F: the zero-profit condition pins the domestic market share $\varphi_i(Q_i) = \sqrt{\mathcal{F}_i/[(1 - t_i)\chi A_i Q_i^{1-\chi}]}$, and substituting into the domestic FOC yields a single implicit equation in Q_i alone).*

Because domestic entry/exit exactly offsets any MNE expansion or contraction one-for-one, the MNE faces a linear residual demand at the pinned price: its sales $x_i(z_i)$ are linear and decreasing in its shadow cost z_i, and the number of active domestic firms is directly proportional to z_i: a rise in z_i (e.g., from a less favorable formula) crowds domestic firms in, while $z_i \to 0$ crowds them out entirely. Consumer surplus is therefore completely insulated from formula policy ($CS_i = A_i(\ln \bar{Q}_i - 1)$, constant), and the zero-profit condition cancels net domestic income from local welfare exactly, simplifying it to $W_i = CS_i + T_i^M + T_i^D$: governments care only about tax capture, not consumer prices or domestic profits.

Under country symmetry, with country b fixed at $\alpha_{K,b} = 0$, welfare simplifies to $W_b - CS_0 = tA$ (constant) and $W_a - CS_0 = tA + tD\theta_a R$, strictly below tA once D exceeds the local-sourcing threshold D_a^L (Section 4's sourcing-regime logic, rederived here with entry).[29]

Proposition 16 (Nash equilibrium set)**.** *The pure-strategy Nash equilibria when governments maximize W_i are $\mathcal{N} = [0, D_a^L] \times [0, D_a^L]$: any pair of small weights is an equilibrium.*

[29]Proofs of both of the propositions belwo are in Appendix F.

Proposition 17 (Centralized optimum)**.** *Joint welfare is maximized on the diagonal band* $\mathcal{C} = \{(\alpha_{K,a}, \alpha_{K,b}) : |\alpha_{K,a} - \alpha_{K,b}| \leq D_a^L\}$ *.*

Since $\mathcal{N} \subset \mathcal{C}$, every Nash equilibrium is Pareto-efficient, but the decentralized game restricts the viable policy set to the low-weight square within the (larger) efficient band.

6.4 FA vs. Separate Accounting

In this subsection we examine how the finite-elasticity model of Section 5 changes if the MNE is taxed under separate accounting (SA) with transfer pricing rather than formula apportionment, holding the real side of the model (demand, technology, domestic firms) fixed. Under SA, affiliate i's tax base is the income booked in country i using an internal transfer price q_{ij} on shipments to $j \neq i$; transfers cancel in the consolidated base ($B_a^{SA} + B_b^{SA} = R - \delta_M rk$, as under FA), but unlike FA each country taxes *source* income, not a formula share of the global base. Writing MNE profit as a sum of route margins μ_{ij} (Appendix F), the SA pricing FOC replaces FA's formula-share derivative $-B\,\partial\bar{\tau}/\partial p_{ij}$ with route-specific tax wedges depending on the destination rate, the origin rate, and (for exports) the transfer price.

If the arm's-length principle permits a range $q_{ij} \in [\underline{q}_{ij}, \overline{q}_{ij}]$, transfer pricing becomes a pure profit-shifting margin: since $\partial N^{SA}/\partial q_{ij} = (t_j - t_i)x_{ij}$ holding delivered prices fixed, the optimal transfer price sits at whichever endpoint moves income from the higher-tax to the lower-tax affiliate (any point in the range is optimal if $t_i = t_j$). With equal statutory rates, SA resembles FA's aligned-formula neutrality in that there is no tax-planning incentive, though the two regimes still differ in *how* revenue is assigned across countries (source accounting versus formula shares).

With unequal rates ($\Delta \equiv t_b - t_a > 0$), the comparison sharpens. Under FA the effective rate decomposes as $\bar{\tau}^{FA} = t_a(1 + D\theta_a) + \Delta\omega_b$: a rate-differential term plus the formula-mismatch term, so a common formula ($D = 0$) no longer neutralizes tax planning when $\Delta \neq 0$ (the MNE wants to move formula share away from the high-tax country).

Under SA, D is not a policy variable: the transfer price simply moves to $q_{ab}^* = \overline{q}$, $q_{ba}^* = \underline{q}$ (shifting income from high-tax b to low-tax a). Numerically ($t_a = 0.25$, $t_b = 0.35$), SA gives slightly higher MNE net profit than FA at $D = 0$, but sufficiently large FA formula mismatches let the MNE do better under FA than the bounded SA transfer-pricing margin allows, by changing real sourcing patterns rather than only book income.[30]

Finally, in a mixed regime with one FA country and one SA country, the MNE faces both wedges simultaneously, and the single FA country's weight $\alpha_{K,i}$ still matters even when $t_i = t_j$: by the envelope theorem, $dV^{mix}/d\alpha_{K,i} = -tB\theta_i$, exactly the FA-only envelope result

[30] Appendix F reports the full payoff-component figures.

of Section 3, so equal statutory rates do *not* neutralize the FA country's formula choice in a mixed regime the way they do in either full-FA (aligned formulas) or full-SA. Numerically this produces genuine non-monotonicity: MNE net profit falls then rises in $\alpha_{K,a} \in [0, 1]$ as the sign of θ_a flips, and aggregate MNE tax revenue is hump-shaped, so aggregate welfare $W = CS + \Pi^D + T^D + T^M$ is non-monotone as well. [31]

7 Conclusions

This paper develops a theoretical framework for strategic formula apportionment in which multinational prices, and hence the sales and capital shares that enter the tax formula itself, respond continuously to the formula chosen, rather than jumping across the corner sourcing regimes of the constant-returns, exogenous-price benchmark this paper generalizes.

We consider three complementary specifications: an MNE-only differentiated-goods benchmark, a homogeneous-good Cournot benchmark with domestic firms reintroduced, and the full differentiated-goods model with domestic firms. They all describe a similar pattern of results:

1. Under equal statutory tax rates, the entire real MNE allocation (prices, quantities, sourcing, the tax base, and MNE profit) depends on the formula pair only through the wedge $D = \alpha_{K,a} - \alpha_{K,b}$; only the distribution of MNE tax revenue across countries depends on the specific formula chosen by the countries. This holds in every specification and is what makes the policy games tractable.

2. Common (aligned) formulas neutralize the MNE's tax-planning wedge and are always a stationary point of every symmetric objective; whether they are a local or global optimum depends on the specific objective under consideration. Tax-revenue and local-fiscal-welfare objectives are maximized at (or very near) common formulas throughout; MNE-profit and, in most specifications, aggregate consumer-surplus objectives are instead maximized by maximally opposed formulas, since the formula wedge acts like an implicit output subsidy that lowers the MNE's effective tax rate and expands scale.

3. Decentralized competition does not generally replicate the centralized optimum. Even where both regimes agree on the qualitative direction (common vs. opposed), the decentralized equilibrium level typically differs from the centralized one — sometimes only in the split of a fixed pie (Section 3's revenue and fiscal equilibria), sometimes in a way that changes real allocations. Consumer-surplus competition is the sharpest case:

[31] Appendix F.

decentralized governments converge to *common*, capital-heavy formulas even though a planner would choose *opposed* formulas, because each government's unilateral incentive to tilt pricing toward its own market is self-defeating once both governments compete on the same margin.

4. The homogeneous-good and differentiated-goods models share the same wedge-only structure and qualitative policy-game pattern, but one sign genuinely reverses between them: aggregate MNE tax revenue is locally concave at the common formula in the CES imperfect-substitutes Cournot model; the opposite result is foundin the perfect-substitutes benchmark. Both signs are numerically dtermined, not closed-form theorems.

5. The extensions in Section 6 are not merely robustness checks. Sales-only formulas implement the destination principle exactly and make decentralized tax-rate competition the same as the centralized tax rates (Proposition 15). Free entry of domestic firms delivers an even sharper insulation result: local consumer prices are pinned to constants independent of the MNE's allocation or the formula wedge (Lemma 2), so every NE of the resulting formula game is also a centralized solution, even though it spans only a strict subset of the efficient set. And comparing formula apportionment with separate accounting shows that the two systems are not simply different accounting conventions for the same tax base: aligned FA formulas neutralize a distortion that separate accounting handles differently (through bounded transfer-pricing incentives), and a mixed regime with only one FA country reintroduces the FA formula-choice margin even when statutory tax rates are equal.

These results provide a concrete, theoretically grounded map of when and why formula apportionment distorts real economic allocations, and of exactly which of the baseline model's conclusions survive replacing exogenous prices and corner sourcing regimes with a genuinely price-responsive MNE.

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