AGEING, HUMAN CAPITAL AND GROWTH:
A SPATIAL ANALYSIS

Alberto Bucci†   Pau Costa Capdevila‡

Rosella Nicolini‡

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ABSTRACT

We propose a theoretical framework that highlights the role of human capital in mitigating the negative effect of population ageing on long-run growth, in particular when a location is experiencing significant immigration inflows. Our theoretical results reinforce the idea that the investment in human capital is a key driver for economic growth. However, it also suggests that large inflows of migrants (in a location) can reduce per-capita investment in human capital, thereby negatively affecting that location growth rate. An empirical application of this theoretical framework is performed by referring to Spanish municipalities since this country has been experiencing a high rate of population ageing and immigration inflows since 2000. Econometric results endorse the theoretical outcomes.

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† ICEA (*International Centre for Economic Analysis*, Canada), and University of Milan. E-mail: alberto.bucci@unimi.it

‡ Departament d'Economia Aplicada – Univerisitat Autònoma de Barcelona

E-mail: pau.costa@uab.cat, rosella.nicolini@uab.cat

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1. Introduction

Population ageing is one of the most salient concerns for industrialized economies. The increase of the expectancy of life jointly with the drop of the fertility rate makes population shrink but, above all, reduced the size of the working-age population entailing important drawbacks for production and growth perspective. The economic literature explored the different options to understand the extent that production factors like human capital, immigration or technological change can overcome the potential recession effect generated by ageing and fertility drop (discussion in *Section 2*). However, the different contributions are not providing a unifying theoretical setting in which the potential effects of combining ageing with human capital formation and immigration are analyzed jointly with the purpose to sustain growth in ageing places.

This study aims at providing novel evidence in that direction. The principal novelty of this contribution is to elaborate a theoretical setting allowing for the long -term analysis of the economic implication in terms of growth perspective when a location (a general spatial unit that can be a region o a city, for instance) is selecting immigration and human capital formation as predominant strategies to contrast the reduction in growth perspective linked to the reduction of the stock of working-age population.

Our theoretical setting takes into consideration a one-sector growth model for a specific location with endogenous human capital accumulation, endogenous migration and endogenous fertility is suited for accommodating the *empty planet* (Jones, 2022) as long as mortality rate is high. Our framework of analysis identifies that as long as human capital accumulation may compensate for the population shrinking, in the very long run, the economic growth process of this specific location can even increase if the population growth rate is progressively declining. Nevertheless, if the selected location is a preferred destination for international migrants, then the total population of that location raises but the per-capita investment in human capital lowers (because of the canonical *dilution effect* of population) and, as a consequence, the economic growth reduces.

We also bring our theoretical outcomes to data by providing an empirical analysis referring to Spain. Literature emphasizes that the ageing process is not uniform across the territory but, rather, it is concentrated in rural and remote areas that are expected to suffer from lagging economies because of that (Daniele et al. 2019). Current empirical studies also emphasize the importance to tackle the question by exploiting granular data to be able to better control for the association between the international migration patterns and the ageing process (as, for instance, the Swedish case discussed in Backman and Karlsson, 2023).[1] Spain shares the same features of other countries being characterized by rural places suffering from an important ageing and depopulation process (Collante et al., 2014). The accessibility to data at municipal level and being able to discriminate them between urban and rural municipality provide the best conditions to test for the principal outcomes of our theoretical setting. We confirm that the human capital and immigration are productive factors that help to sustain economic growth in rural municipalities (that are the ones suffering from a sustained ageing process in Spain) but, in urban municipalities, the reverse happens. In addition, we are also able to provide quantitative evidence about the negative relationship between human capital and immigration. All these results call for further discussion about the effectiveness of policies (in the long-run) that are often implemented to attract immigrants in remote or depopulated rural areas to trigger local growth. Our setting suggests that this type of action can be effective in the short-run, but their implementation can harm the human capital cumulation in the long-run and, hence, turns to be negative for the foundation of the economic growth process.

[1] Shin (2026) also emphasizes that the use of granular data contribute to control for unobserved heterogeneity since regions or cities all share similar cultural environments, institutions, or policies.

This contribution is structured as follows. In the next section (Section 2) we discuss the most salient contributions in this research field, in Section 3 we develop our theoretical framework and in Section 4 we perform our empirical analysis. Finally, Section 5 concludes. Further material in included in the Appendix

2. Literature review

The role of human capital in economic literature is well established as one of the drivers for economic growth, but, recently, the constant ageing process of the population, above all in industrialized countries, fueled the debate on the effectiveness of this production factor. Becker (2002) proposes some elements for thinking about the consequences of an ageing society on the stock of human capital. The ageing process generates a clear effect: the loss of old and experienced workers (the retired persons) entails a loss in terms of productive capacity leading to a recession process if not controlled. A way to overcome this effect is by replacing the old workers with new worked and these new workers can contrast the loss of human capital of retired persons if they are endowed with a higher level of education.[2] Under this perspective, Autor et al. (2003) conclude that the upgrade of competences of new generation is likely since the technological progress change the nature of the labor tasks and call for more education.[3] However, Lindh (2004), by performing a quantitative exercise for a sample of European countries, concludes that, in an ageing society, human capital is still able to trigger the growth process not only by incorporating a technological change, but also importing labor. [4] This

[2] Another option to control for the shrinking of the labor force due to ageing is increasing the labor force participation of older workers. Focusing on the Korean case, Shin (2026) concludes that extending working life is insufficient and other policies need to be put in place to sustain growth in ageing economies and, in the Korean case, these policies are principally innovation, human capital formation and technology adoption.
[3] Chingri et al. (2026) contribute to this discussion by assessing the conditions for which a declining population growth rate can still maintain sustained per capita output growth when both human and automation capital increase in the production function.
[4] Acemoglu and Restepo (2017) conclude that ageing is not expected to entail consequences on per-capita growth because the reduction of labor supply can be compensated by technological

conclusion stems from the combination of several forces in place. On the one hand, the impact of human capital on growth is not constant across all the cohorts of the population. Castelló-Climent (2019) emphasizes that the degree of education of the middle-aged (40-49 years) cohort of workers has the largest impact on growth with respect of the educational level of the other group of ages. On the other hand, the ageing of the population does not only refer to the group of people that leave their job positions for retirement, but also to the reduced entry of new native workers because of the fertility drop of natives (Sobiech Pellegrini et al. 2024) and, hence, the need to count on incoming workers (basically immigrants) to preserve the productivity rate but a great uncertainty because the human capital endowment of immigrants varies a lot across countries for reasons ranging for the immigration policies to the available job openings (Grebeniyk et al. 2021).[5]

The distribution of available jobs adds another layer of complexity to the question: the spatial disparities. International migration flows are strongly dependent on demographic patterns and job opportunities. Mobility decisions for immigrants are driven by a clear cost-benefit analysis aimed at improving they standard of living in their home-countries starting by accessing to a better employment (Poutavaara, 2021) and job opportunities are usually unequally distributed across space and ageing is one of the components shaping this distribution. Prenzel (2021) discusses this outcome looking at the labor market in German regions. Population ageing influence the attractiveness of a region with respect to others. Ageing regions often host fewer natives in the middle-age life stage because they are attracted by places with more job opportunities. This dynamic is often linked to the dichotomy between urban and rural areas for which urban areas are likely to clusters more job opportunities and

improvements in the production process (for instance, robotization) and this change will allow not only to preserve the level of production but also increase it. Tovar Jalles (2026) endorses this conclusion with cross-country evidence for the period 1975-2023.

[5] Peri and Sparber (2019) provide a theoretical framework and empirical evidence to illustrate the consequences of the the trade-off between the tasks and abilities of immigrants versus the one of natives.

interregional mobility mostly follow a patter of displacement from the rural to the urban areas. Instead, international migration flows may follow different mobility patterns in accordance with the migrants' level of human capital and the existence of job openings matching their education that could also happen to cluster in rural or ageing regions. In this respect, the current literature is rather inconclusive about the effect of population ageing on growth perspectives. On the one hand, it has been assessed that there is a negative relationship between ageing and growth (Maestas et al. 2023) since the shrinking of the working-age population reduce the labor force and, as a consequence, reduce the level of production. On the other, there is another stream of literature that, instead, identify a positive relationship between ageing and growth. In this line, Prenzel and Iammarino (2021) and Nicolini and Roig (2024, 2025) conclude that the positive relationship between ageing and growth is linked to the presence of other two factors: immigration and human capital. As suggested in Nicolini and Roig (2024, 2025), international immigration is expected to find occupation in the destination countries either by replacing the retired persons in their job positions but also by providing additional serviced requested by the retired persons (above all healthcare or/and homecare services). In addition, Prenzel and Iammarino (2021) emphasize that in an ageing society, the investment in human capital con be perceived as a sustainable policy to, guaranteeing economic growth conditions when thinking of generating a positive impact on productivity (and hence, growth) via technology adoption or job training. However, in the literature there is scarce theoretical and quantitative evidence about the possible effects of combining ageing, migration and human capital as the factors leading to sustained growth. This study aims to provide novel evidence in this direction conditional on the uneven spatial distribution of the previous determinants and, hence, explicitly focusing on the dichotomy between rural and urban areas.

3. The theoretical setting

We build a one-sector growth model with: *(i)* Human capital investment, *(ii)* An endogenous *quality/quantity trade-off*, and *(iii)* An endogenous fertility rate (hence, population growth), in an attempt to assess whether a declining number of people (*e.g.*, workers) may be compensated by an increasing level of education (*e.g.*, human capital) in the population in order to sustain long-run economic growth. At this aim, consider a generic, representative spatial unit (a city or a region, for instance) i producing, in a perfectly competitive environment, a homogeneous consumption good $\left(Y_t\right)$ through a constant returns to scale aggregate production function that employs only human capital $\left(H_t\right)$:[6]

$$Y_t = AH_t, \qquad A > 0. \tag{1}$$

In Eq. (1), Y_t is local unit aggregate real GDP (the *numéraire*) and A is the marginal (and average) product of human capital. Since our focus is on the role of education, we omit any technological progress (*i.e.*, we assume that A is always constant), and think of human capital, H_t, as the number of workers, L_t, multiplied by the human capital of the typical worker, h_t, so that the quantity of workers, L_t, and their quality, h_t, are perfect substitutes in production:

$$Y_t = A\left(L_t h_t\right), \qquad H_t \equiv L_t h_t.$$[7]

Let $n_t \geq 0$ be a representative local birth rate at a generic time t, $d > 0$ its constant (and exogenous) death/mortality rate, and $m_t \gtreqless 0$ its (net) migration rate. Hence, at any time $t \geq 0$, the growth rate of the unit's population/workforce $\left(L_t\right)$ will be:

[6] For the sake of notational simplicity, unless necessary, we will drop the unit-subscript *i* in the rest of the analysis. This can be safely done in the light of our focus on a *'representative'* unit.
[7] We assume that the number of workers and the population size coincide (and are both equal to L). More formally, our operational hypothesis is that each individual in the population is educated ($h_t > 0$, $\forall t \geq 0$), and supplies inelastically one unit of (skilled) labor–services per unit of time.

$$\frac{\dot{L}_t}{L_t} = \left(n_t + m_t - d\right) \gtreqless 0, \qquad L\left(0\right) = L_0 > 0, \tag{2}$$

with $n_t L_t$, dL_t, and $m_t L_t \gtreqless 0$ representing the total number of local births, the total number of deaths in the same spatial unit, and the total (net) number of migrants flowing into/out of that location per unit of time, respectively. Notice that $m_t L_t$ can, in principle, be either positive or negative or else equal to zero.[8] The spatial unit's aggregate resource constraint reads as:

$$Y_t = AH_t = C_t + I_{H,t} \tag{3}$$

where $I_{H,t}$ is the local gross investment in human capital, and C_t is local aggregate consumption. The change in the local capital stock of human capital is given by:

$$\dot{H}_t = I_{H,t} - \delta_H H_t, \qquad H_t \equiv L_t h_t \tag{4}$$

where $0 \leq \delta_H \leq 1$ is the exogenous and constant depreciation rate of human capital. We assume that rearing *each single* child entails a *goods–cost* (in the form of expenditures to be borne once and for all) which is proportional to the per capita level of human capital $\left(h_t \equiv H_t / L_t\right)$ already attained in the spatial unit, *i.e.*:

$$\xi_t = bh_t, \qquad b > 0.^9 \tag{5}$$

Since $n_t L_t$ is the total number of local births per unit of time, $\xi_t n_t L_t$ will represent the aggregate amount of expenditures (in terms of goods) on child–rearing in the spatial unit at time *t*, and $\xi_t n_t = \left(bn_t\right) h_t$ will be the amount expended per capita. Overall, after taking into account also the goods–cost

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[8] In the paper we allow for mobility of people (migrations), but assume that the representative spatial unit has a balanced external position with respect to the trade of goods (and, eventually, assets) with the other spatial units in the world. We will come back later to the specification of the migration rate function, m_t, that we adopt in the current work.

[9] Following Barro and Sala-i-Martin (2004, pp. 412 and 413), we interpret ξ_t *"…as a large single outlay that represents the present value of expenditures for each child…"*, and assume that ξ_t increases with the per capita quantity of human capital.

related to fertility, the laws of motion of aggregate human capital in the spatial unit will finally read as:

$$\dot{H}_t = I_{H,t} - \delta_H H_t - \left(bn_t\right) H_t \tag{4'}$$

Notice that in Eq. (4′) the total amount of expenditures (in terms of goods) on child–rearing in the spatial unit *i* at time *t*, namely $\xi_t n_t L_t = \left(bn_t\right) H_t$, subtracts (goods–)resources to aggregate local human capital investment. In per capita terms, Eqs. (1), (3), (4′) can be recast, respectively, as:

$$y_t = Ah_t \tag{1'}$$

$$y_t = Ah_t = c_t + i_{h,t} \tag{3'}$$

$$\dot{h}_t = i_{h,t} - \delta_H h_t - \left(bn_t\right) h_t - \left(n_t + m_t - d\right) h_t, \qquad h\left(0\right) = h_0 > 0\,. \tag{4''}$$

where $y_t \equiv \dfrac{Y_t}{L_t}$, $c_t \equiv \dfrac{C_t}{L_t}$, and $i_{h,t} \equiv \dfrac{I_{H,t}}{L_t}$ represent per capita real GDP, per capita consumption, and per capita (gross) investment in human capital, respectively. To close the model, we need to specify a migration rate function, $m_t \equiv \dfrac{M_t}{L_t} \gtreqless 0$.

In this respect, we borrow from Barro and Sala-i-Martin (2004, pp. 385-386) the simplifying assumption that the current migration rate m_t responds solely (and positively) to the current wage rate of the representative spatial unit, w_t.[10] More formally, we assume that:

$$m_t = \underset{+}{m}\left(w_t\right) = \beta\left(w_t - \overline{\kappa}\right), \qquad \beta > 0, \qquad \overline{\kappa} \geq 0 \tag{6}$$

Given the production technology of the representative unit (Eq. 1), the current local wage rate is:

[10] In a more complete, but also more complicated model, the migration rate would respond positively to the difference between the present value of the domestic (*i.e.*, the representative unit's) wage and the present value of the wage that the migrant might obtain in other alternative units (*i.e.*, in the rest of the world). Here we try to keep the model as much simple as possible, with the ultimate objective of writing closed-form solutions that, hence, can be studied analytically.

$$w_t \equiv \frac{\partial Y_t}{\partial H_t} = A > 0,$$

Hence Eq. (6) boils down to:

$$m_t = m\left(w_t\right) = \beta\left(A - \bar{\kappa}\right) = m \gtreqless 0, \qquad \beta > 0, \qquad \bar{\kappa} \geq 0 \tag{7}$$

The parameter $\bar{\kappa} \geq 0$ is a proxy of the so-called *'push-factor of migration'*. Economically, it can be interpreted as some average of the domestic wage rates of the different origin countries of migrants and that each rational, forward-looking agent takes as given in the migrants' (representative) destination location. When $\bar{\kappa} > 0$, this parameter allows to have either positive, or negative, or else zero migration rates. Indeed, from Eq. (7) it is immediate to conclude that m will be: positive when $A > \bar{\kappa} > 0$; negative when $0 < A < \bar{\kappa}$; equal to zero when $A = \bar{\kappa} > 0$. On the other hand, if $\bar{\kappa} = 0$, then $m_t = A\beta = m > 0$, at any time $t \geq 0$. This means that we would *always* observe a positive and constant (net) migration rate in this case.[11] For the sake of generality, in what follows we will assume $\bar{\kappa} > 0$. We can now analyze the household's choices.

3.1 The Household's Problem

The representative spatial unit is populated by many structurally-identical households. Therefore, we may concentrate on the optimal behavior of a single (representative) infinitely-lived family with perfect foresight, and whose size coincides with that of the whole location $\left(L_t\right)$. Each member of the representative household is characterized by a *Millian-type* intertemporal utility function of the form (see Prettner, 2013, p. 818, Eq. 3):

[11] At this stage it is worth noting that in what follows we are going to characterize a competitive long-run equilibrium where $n > 0$ is constant, hence $\frac{\dot{L}_t}{L_t} = \left(n + m - d\right) \gtreqless 0$ is constant, too.

$$U \equiv \int_0^\infty u\left(c_t;n_t\right)\cdot e^{-\rho t}dt = \int_0^\infty \left[\ln\left(c_t\right)+\phi\ln\left(n_t\right)\right]\cdot e^{-\rho t}dt, \qquad 0<\phi<1, \qquad \rho\equiv\bar{\rho}+d>0 \tag{8}$$

where ϕ is the utility–weight that any single individual attaches to the number of children, n_t, and ρ is the overall discount rate. The pure time preference rate $\bar{\rho}>0$ is augmented by the mortality rate $d>0$ because individuals who face a risk of death are less likely to postpone consumption and fertility into the future. Yet, Eq. (8) is similar to Eq. (9.54) in Barro and Sala-i-Martin (2004), with two major differences:

- The total size of the representative household, L_t, does not enter the instantaneous utility function of any single member of the representative family, a specification that delivers a more tractable mathematical solution (Barro and Sala-i-Martin, 2004, p. 412; Jones, 2001);[12]

- Following, among others, Bucci and Prettner (2020), we adopt a logarithmic formulation for instantaneous utility, which serves the scope of making the problem more tractable analytically.[13]

Each individual chooses consumption $\left(c_t\right)$, the number of children $\left(n_t\right)$, and the (gross) investment in human capital $\left(i_{h,t}\right)$ so as to maximize (8), subject to (3′), the law of motion of h_t (4″) and L_t (2), and the migration rate function (7). Formally, the forward-looking, dynamic optimization problem that any representative agent living in a representative spatial unit *i* at time *t* seeks to solve is:

[12] This explains why we use a *'Millian-type'* intertemporal utility function. Jones (2022, p. 3505) also employs a specification for the flow utility of the type: $u\left(c_t;L_t\right)=\log c_t+\varepsilon\log L_t$, which includes L_t (the size of the representative family), rather than n_t, as we do in Eq. (8).

[13] See also Barro and Sala-i-Martin (2004, p. 415).

$$
\begin{aligned}
&\underset{\left\{c_t;\, n_t;\, i_{h,t};\, h_t;\, L_t\right\}_{t=0}^{t=+\infty}}{Max} U = \int_0^{\infty} \left[\ln\left(c_t\right) + \phi \ln\left(n_t\right)\right] \cdot e^{-\rho t} dt, \qquad \rho \equiv \bar{\rho} + d > 0 \\
&s.t.: \\
&y_t = Ah_t = c_t + i_{h,t} \\
&\dot{h}_t = i_{h,t} - \delta_H h_t - \left(bn_t\right)h_t - \left(n_t + m - d\right)h_t \qquad h\left(0\right) = h_0 > 0 \\
&m_t = \beta\left(A - \bar{\kappa}\right) = m \qquad \beta > 0, \quad \bar{\kappa} > 0 \\
&\dot{L}_t = \left(n_t + m - d\right)L_t \qquad L\left(0\right) = L_0 > 0
\end{aligned} \tag{9}
$$

The representative household living in the (potential) destination location takes as given the wage rate, w_t, hence the migration rate, m (that, in turn, will be always constant, in the light of the local aggregate production function that we are considering). However, the population growth rate, $\dot{L}_t / L_t = \left(n_t + m - d\right)$, will ultimately be endogenous, as n_t is endogenous.

3.2 Equilibrium Analysis

We now introduce the following definition.

DEFINITION (Yip and Zhang, 1997, p. 102): ***Competitive Balanced Growth Path (BGP) equilibrium***

A competitive BGP equilibrium is a competitive long-run equilibrium path along which all functions of time grow at constant (and, maybe, different) exponential rates, and n is constant.

We are now able to characterize the model's competitive BGP equilibrium.

***Proposition 1*:**

Along a competitive BGP equilibrium we observe that:

$$i_{h,t} = (A - \rho) \cdot h_t, \qquad \rho \equiv \overline{\rho} + d > 0, \qquad A > 0 \tag{10}$$

$$y_t = Ah_t \qquad\qquad c_t = \rho h_t \tag{11}$$

$$m = \beta\left(A - \overline{\kappa}\right) \gtreqless 0, \quad \beta > 0, \qquad\qquad \overline{\kappa} > 0 \tag{12}$$

$$n = \frac{\phi\left(\overline{\rho} + d\right)}{(1 + b)}, \qquad 0 < \phi < 1, \qquad b > 0, \qquad \overline{\rho} > 0 \qquad d > 0 \tag{13}$$

$$\frac{\dot{h}_t}{h_t} = A - \rho - \delta_H - (1 + b)n + d - m \tag{14}$$

$$\frac{\dot{L}_t}{L_t} = (n + m - d) = \frac{\phi\overline{\rho} + \beta(1 + b)\left(A - \overline{\kappa}\right) - (1 + b - \phi)d}{(1 + b)} \gtreqless 0 \tag{15}$$

$$\frac{\dot{i}_{h,t}}{i_{h,t}} = \frac{\dot{h}_t}{h_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} \equiv g = A(1 - \beta) - (1 + \phi)\overline{\rho} - \phi d - \delta_H + \beta\overline{\kappa} \tag{16}$$

Proof: See the *Online Appendix A*.

According to Eq. (13):

i. The equilibrium fertility rate, n, depends positively on the mortality rate, d;[14]
ii. A higher value of b increases the goods–cost of children and, accordingly, reduces n;

[14] This is rather standard in the endogenous fertility literature. As an example, see the simple, continuous-time, endogenous fertility model (without human capital, hence without any *quantity/quality trade-off*) by Barro and Sala-Martin, 2004, Section 9.2.2 (in particular, see their Eq. 9.59).

iii. A higher value of ϕ increases n by raising agents' preferences towards children;

iv. A higher value of $\bar{\rho}$ deters investment in human capital, h (Eq. 14), hence raises n via the endogenous quantity/quality trade-off (see *Proposition 2*).

Instead, according to Eq. (16):

i. A higher value of δ_H increases the cost of investing in human capital, and therefore reduces $\frac{\dot{h}_t}{h_t} \equiv g$;

ii. A higher value of ϕ increases n and therefore reduces the incentive to invest in children's human capital, hence economic growth, via the endogenous quantity/quality trade-off-channel (see *Proposition 2*);

iii. A higher value of $\bar{\rho}$ (or a higher value of d, or both), by raising the *'augmented'* subjective time discount rate, $\rho \equiv \bar{\rho} + d$, deters investment (in human capital), therefore reduces economic growth (g);

iv. A larger migration towards the spatial unit, $m = \beta\left(A - \bar{\kappa}\right)$, by raising the whole population growth rate (Eq. 15), lowers *ceteris paribus* investment in human capital per capita (Eq. 14) through the canonical dilution-effect of population, hence reduces economic growth.

The next table summarizes compactly these comparative-statics results:

$\uparrow \phi$	$\uparrow n$	$\downarrow g$
$\uparrow \bar{\rho}$	$\uparrow n$	$\downarrow g$
$\uparrow d$	$\uparrow n$	$\downarrow g$
$\uparrow b$	$\downarrow n$	
$\uparrow \delta_H$		$\downarrow g$

$\uparrow \bar{\kappa} > 0 \quad \Rightarrow \quad \downarrow m$		$\uparrow g$

Table 1: *The effects of changes in the model's parameters on the competitive BGP equilibrium values n and g*

The last row of Table 1 shows the negative correlation that, according to our model, does exist between the migration rate, m, and long-term economic growth, g. It is also evident from Table 1 that the model allows for the presence of an endogenous *quality/quantity trade-off* between fertility (n) and per capita human capital investment $\left(\dot{h}_t / h_t\right)$, as the next proposition demonstrates more formally.

***Proposition* 2:**
At a competitive BGP equilibrium, an endogenous quantity/quality trade-off between fertility and human capital investment does take place.

***Proof*:** To show the result, one should ultimately be able to demonstrate that:

$$\frac{\partial\left(\dot{h}_t / h_t\right)}{\partial n} < 0 .$$

In this regard, suppose there is an exogenous increase in the parameter that measures agents' preferences towards children, ϕ. This clearly raises n and, *ceteris paribus*, reduces $\dot{h}_t / h_t$ (see Eq. 14). ■

Assumption 1 introduces formal constraints on the relationship among (some of) the model's parameters.

Assumption 1:

$$A > \bar{\rho} + d > 0$$

$$A(1-\beta) > (1+\phi)\bar{\rho} + \phi d + \delta_H - \beta\bar{\kappa} .$$

Proposition 3

If Assumption 1 is satisfied, then at a competitive BGP equilibrium we observe:

$$\frac{\dot{i}_h}{h} > 0 \qquad\qquad \text{and} \qquad\qquad g > 0\,.$$

Variables (11) and (13) are always strictly positive, while $m = \beta\left(A - \overline{\kappa}\right) \gtreqless 0$.

Proof: See Eqs. (10)–(11)–(12)–(13) and (16). ■

Assumption 2:

$$\left(1+b\right)\left(A-\overline{\kappa}\right) > -\frac{\phi\overline{\rho}}{\beta}$$

Proposition 4

Under Assumption 2, at a competitive BGP equilibrium:

$\frac{\dot{L}_t}{L_t} > 0$	if $\quad 0 < d < \frac{\phi\overline{\rho} + \beta\left(1+b\right)\left(A-\overline{\kappa}\right)}{\left(1+b-\phi\right)}$
$\frac{\dot{L}_t}{L_t} = 0$	if $\quad d = \frac{\phi\overline{\rho} + \beta\left(1+b\right)\left(A-\overline{\kappa}\right)}{\left(1+b-\phi\right)} > 0$
$\frac{\dot{L}_t}{L_t} < 0$	if $\quad d > \frac{\phi\overline{\rho} + \beta\left(1+b\right)\left(A-\overline{\kappa}\right)}{\left(1+b-\phi\right)} > 0$

Table 2: *The sign of the endogenous rate of population growth at a competitive BGP equilibrium*

This suggests that in the model a negative population growth rate can be supported as a competitive BGP equilibrium outcome with endogenous fertility.

Proof: See Eq. (15). ■

Proposition 5

At a competitive BGP equilibrium there exists a negative correlation between the endogenous economic growth rate, g, and the endogenous population growth rate, $\dot{L}_t / L_t$. This means that in the very long run economic growth can even increase if the population growth rate were to be more negative.

Proof: After some (simple but tedious) algebra, the relation between g and $\dot{L}_t / L_t$ can be recast as:

$$g = A - \overline{\rho} - \delta_H + b\beta\left(A - \overline{\kappa}\right) - \left(1 + b\right)\left(\frac{\dot{L}_t}{L_t} + d\right), \qquad \frac{\dot{L}_t}{L_t} = \left(n + m - d\right).$$

Again, suppose there is an exogenous increase in the parameter that measures agents' preferences towards children, ϕ. This clearly raises the fertility rate, n, and, therefore, $\dot{L}_t / L_t$. In turn, with $b > 0$, the rise of $\dot{L}_t / L_t$ lowers g. ■

4. Empirical analysis

We perform our empirical analysis by exploiting Spanish data. Spain is one of the European countries with a sizable international immigration rate and an important ageing process in rural areas. Nicolini and Roig (2025) discussed the impact that the immigration inflow could entail in rural areas suffering from the ageing process: immigration impacts on economic growth of rural provinces experiencing ageing in the short run. In this contribution, we are producing additional evidence for analyzing the relationship among immigration-ageing-growth in the long run to test the results discussed in *Proposition 5*. To bring the theoretical outcome into data, we need to deal with the endogeneity issue: there is a clear double causality effect between growth and its factors of production (like human capital). In order to overcome this limitation, we are unable to implement an IV strategy, since we do not dispose of suitable data series at municipality level, but, rather, we can implement a convergence style estimation (e.g. Bisciari et al., 2020; Llorens i Jimeno and

Mestres Domènech, 2020; Maestas et al., 2023, Nicolini and Roig, 2024 and 2025 among others) to break down the double causality effect. Therefore, in our setting, we will consider the most recent growth rate -between 2015 and 2021- (at municipality level) and associate it to lagged covariates (set at year 2011).

As a consequence, in order to perform our empirical analysis, we consider data at municipality level (8132 municipalities) and we refer to two waves of Spanish Census (2011 and 2021), data on income per-capita (for years 2015-2021) are from the Atlas of Income Distribution (INE). We split all Spanish municipalities in three categories (urban, intermediate and rural) by applying the classification introduced by the European Union (2019/1130, 2 July 2019). According to that categorization, the Spanish territory is split into 1 km-grid squares, and the following categories are:

1)Mostly *rural municipality*: At least 50% of the population lives in rural grid cells;

2)*Intermediate municipality*: More than 50% but less than 80% of the population lives in urban agglomerations; and

3)Mostly *urban municipality*: At least 80% of the population lives in urban agglomerations.

According to the Spanish National Statistica Institute (INE), more than 5000 municipalities are classified as rural but they account for less than 20% of the total population. More precise descriptive statistics helps to figure out the structural demographic differences between rural and urban municipalities. We take into consideration all the municipalities that records at least one migration movement during our selected period.

Table 2: Descriptive statistics for urban municipalities
Source: INE, Calculus: Authors

Variables	Obs	Mean	SD	Min	Max
Avg income per capita growth (2015-2021)	248	0.0320	0.0071	0.0156	0.0684
Income per capita (2015)	248	11,240.6	2,533.6	6,521	23,861

Share working age immigrants (2011)	248	0.0799	0.0537	0.0112	0.3294
Share human capital (2011)	248	0.3087	0.1077	0.0670	0.6854
Share population older than 65 (2011)	248	0.1441	0.0411	0.0348	0.2464

Table 3: Descriptive statistics for rural municipalities

Source: INE, Calculus: Authors

Variables	Obs	Mean	SD	Min	Max
Avg income per capita growth (2015-2021)	5460	0.0372	0.0141	0.0674	0.3137
Income per capita (2015)	5460	10,087.5	2,129.1	3,281	25,555
Share working age immigrants (2011)	5460	0.0525	0.0568	0.0000	0.5467
Share human capital (2011)[15]	5460	0.1372	0.1235	0.0000	1.0000
Share population older than 65 (2011)	5460	0.3072	0.1127	0.0000	0.7885

Table 4: Descriptive statistics for intermediat municipalities

Source: INE , Calculus : Authors

Variables	Obs	Mean	SD	Min	Max
Avg income per capita growth (2015-2021)	1051	0.0361	0.0075	0.0109	0.0597
Income per capita (2015)	1051	9,788.2	2,156.6	5,935	20,010
Share working age immigrants (2011)	1051	0.0865	0.0716	0.0016	0.4504
Share human capital (2011)	1051	0.2272	0.0844	0.0000	0.6338
Share population older than 65 (2011)	1051	0.1567	0.045	0.0352	0.3681

[15] There are tiny municipalities (Cañizar, Azuelo, Lomas and others) whose total population is below 30 persons and official statistics records assess that all residents have -at least- a university degree.

The previous Tables 2-4 provide a clear picture about the structural differences between rural and urban areas by referring to economic and demographic figures. Rural municipalities record a higher percentage of people older than 65 years (doubling the rate for urban municipalities) as well as the share of working age immigrants, whereas the share of human capital (represented by the percentage of the population with a university degree) is lower (almost the half) then the one in urban municipalities. Finally, the (average)rate of growth of income per capita is comparable between rural and urban areas still being slightly higher in rural areas. The general picture that stems from these statistics emphasizes that rural areas are mostly offering low-skill job opportunities for immigrants mostly linked to tasks in the agricultural sectors or in service sector mostly in the healthcare or homecare associated with the large share of aged persons. Instead, urban areas are more attractive for high-skill workers, the share of immigrants is higher and their occupation can be both in skill position but also in low-skill positions (for instance, in services ranging from the financial sector to the hotels or restaurants).[16]

The differences in the demographic dynamics between urban and rural municipalities is well represented in the difference between fertility and death rate. In Table 5, we gather evidence about the population growth in urban and rural municipalities for the period 2011-2022 by distinguishing between natives and immigrants. Statistics are computed as the mean of net population changes across the municipalities belonging to one of the two categories (urban or rural municipalities). Up to 2020, the rural cities were experiencing a negative net growth rate (in terms of population changes) with respect to the urban ones. This trend was driven by a general positive fertility rate in the group of natives (in urban areas) and a negative one for foreign-born people.

[16] According to the report *Regional Information Centre for Western Europe - United Nations* (https://unric.org/en/migration-to-the-eu-facts-not-perceptions/), in 2022, in terms of occupations, EU and non-EU workers were more present in selected sectors: hotels and restaurants (9.1% no-EU and 4.2% for EU workers), construction (9.1% no-EU and 6.6% EU workers), and, domestic work (5.9% non-EU and 0.7% EU workers).

Table 5: Net population growth by spatial unit

Sources: INE, Calculus: authors.

		2011-2014			2014-2017			2017-2020			2020-2022		
	Obs	**Total**	**Native**	**Foreign**	**Total**	**Native**	**Foreign**	**Total**	**Native**	**Foreign**	**Total**	**Native**	**Foreign**
Urban	248	***0.002***	0.007	-0.037	***0.002***	0.005	-0.032	***0.008***	0.002	0.071	***0.000***	-0.001	0.012
		(0.012)	(0.011)	(0.036)	(0.008)	(0.008)	(0.037)	(0.008)	(0.007)	(0.034)	(0.010)	(0.009)	(0.036)
Rural	5460	***-0.019***	-0.016	-0.047	***-0.012***	-0.010	-0.038	***-0.007***	-0.010	0.043	***0.003***	0.001	0.049
		(0.071)	(0.070)	(0.242)	(0.107)	(0.105)	(0.233)	(0.098)	(0.096)	(0.250)	(0.034)	(0.031)	(0.304)

Note : Average growth value and standard deviation into brackets; Intermediate municipalities: 1087.

Instead, in recent times (post-Covid years), the situation is structurally different: the net population growth in rural municipalities is growing more than the one in the urban ones and this change of the trend is driven by the relatively high fertility rate of immigrants in rural municipalities. Therefore, the international migration is definitively responsible for the re-dynamization of urban municipalities.

As a consequence, first, we concentrate on the Spanish municipalities that were involved in (at least) an international migration movement in the period 2011-2021 in order to deal with a sample sharing the same key-feature and, hence, our sample shrinks to about 6500 observations.

Referring to the theoretical outcomes in Proposition 5, we first concentrate on the estimation of the relationship between migration and human capital growth (2011-2021) and income growth (2015-2021) by implementing a convergence-style estimations referring to the (working - age) immigration rate in 2011 (*immrate 2011*). The equations to be estimated are, for each municipality *m*:

$$Average\ Human\ capital\ growth_{m,\ (2011-2021)} = \alpha + \beta_1 Log\left(immrate_{m,\ 2011}\right) + \varepsilon_m, \quad (17)$$

$$Average\ Income\ growth_{m,\ (2015-2021)} = \alpha + \beta_1 Log\left(immrate_{m,\ 2011}\right) + \varepsilon_m, \quad (18)$$

The results are presented in Table 6:

Table 6: Estimations

Method: OLS

	Human capital growth (2011-2021)	**Average income growth (2015-2021)**
Log (Immigration rate$_{2011}$)	**-0.103***** (0.025)	**-0.065***** (0.025)
R-squared	0.022	0.014
Obs	6,416	6,514

Constant included; Standard errors into brackets; Errors: robust.

The estimation results presented in Table 6 confirms the negative relationship between human capital formation and immigration (for a sizable time period), as well as the negative one between immigration rate and income growth. Overall, the immigration rate seems not providing the right support on the long-term growth since it tempers the human capital formation that turns not being able to support economic growth. Put differently, the inclusion of immigrants in the labor force of incoming countries could be thought as a short-term solution to growth perspective of a location that could not last -in the long run- for its negative relationship with human capital formation.

However, the estimations results presented in Table 6 are the outcome for our overall sample of Spanish municipality. But it could be interesting to discriminate the effect that human capital and immigration may have on growth perspectives of rural and urban areas.

To this end, we propose an augmented version of the previous estimations to take into account this dimension and we estimate the following equation:

$$\boldsymbol{Average\ Income\ growth_{m,(2015-2021)} = \alpha + \beta_1 Log\big(Income_{m,2015}\big) + \beta_2 Log\big(Immrate_{m,2011}\big) + +\beta_3 Log\big(Share\ Human\ Capital_{m,2011}\big) + \beta_4 Log\big(Shareover65_{m,2011}\big) + \beta_5 RURAL + + \beta_6 Log\big(Immrate_{m,2011}\big) \cdot RURAL + \beta_7 Log\big(Share\ Human\ Capital_{m,2011}\big) \cdot RURAL + + \beta_8 Log\big(Shareover65_{m,2011}\big) \cdot RURAL + \beta_9 URBAN + \theta_{m,p} + \varepsilon_m,} \quad (19)$$

The previous equation to be estimated (still following the convergence-style approach) aims to disentangle the effect on of human capital, ageing and immigration on the growth rate (between 2015-2021) at municipal level by focuses on the role of these two factors in the rural municipalities. In this setting, we approximate the growth rate with the variation of the average level of income of each municipality (*Income*), since, it the available indicator at level of municipality for a sizable time-period. Then, we approximate the human capital as the percentage of citizens in a municipality m with at least a university degree in 2011 (*Share Human Capital),* the intensity of the ageing process as the percentage of citizens older that 65 years old in the municipality *m*, and the immigration rate as the percentage (over the total population) of persons born abroad whose their residence is in municipality *m* (*Immrate).* In addition, we introduce two dummies (*RURAL, URBAN*) to identify the municipalities that belong to each of the two groups as well as a dummy $\boldsymbol{\theta_{m,p}}$ to associate to each municipality the province (NUTS3) it belongs to and, finally, errors $\boldsymbol{\varepsilon_m}$ are treated either as robust.[1]

The results of the estimation of that equation are included in Table 7.
The total sample we are dealing with for that estimation is lower than the total number of municipalities in Spain because we do not dispose all information for the level on income of all of them for the period we are considering. The estimation is built a convergence estimation and our first result is to assess a slight convergence rate process between all municipalities in Spain in terms of income (the estimation of the coefficient $\boldsymbol{\beta_1}$ is negative).

[1] In our empirical exercise we privilege the robust to the clustered (by province) error correction because the three typologies of municipalities coexist in almost all provinces and, hence, we aim to avoid to create additional noise.

Table 7: Estimations

Method: Fixed effects

Dependent variable: Income growth $_{m,\ (2015\text{-}2021)}$

Variables	1	2	3	4
Log Income $_{(m,2015)}$	-0.0424*** (0.00324)	-0.0449*** (0.00369)	-0.0308*** (0.00156)	-0.0472*** (0.00386)
Log Share Human capital $_{(m,2011)}$		0.0102*** (0.00295)	0.0256*** (0.00420)	0.0377*** (0.00541)
Log Share Persons over 65 $_{(m,2011)}$	0.00749*** (0.00237)	0.0123*** (0.00259)	-0.00176 (0.00510)	-0.00261 (0.00565)
Log Share Immigrants $_{(m,2011)}$	-0.0343*** (0.00443)	-0.0351*** (0.0045)	-0.0230*** (0.00354)	-0.00441*** (0.00501)
Log Share Human capital $_{(m,2011)}$ **RURAL*			-0.0205*** (0.00389)	-0.0306*** (0.00450)
Log Share Human capital $_{(m,2011)}$ **URBAN*			-0.00294 (0.00604)	0.0174*** (0.00649)
Log Share Persons over 65 $_{(m,2011)}$ **RURAL*			0.0182** (0.00575)	0.0156** (0.00610)
Log Shae Persons over 65 $_{(m,2011)}$ **URBAN*			-0.0185* (0.0102)	-0.0270** (0.0114)
Log Share Immigrants $_{(m,2011)}$ **RURAL*			0.0110** (0.00502)	0.0112** (0.00492)
Log Share Immigrants $_{(m,2011)}$ **URBAN*			0.0104 (0.00794)	0.0049 (0.00822)
Dummy RURAL	NO	NO	YES	YES
Dummy URBAN	NO	NO	YES	YES
Province fixed effects	NO	NO	NO	YES
R-squared	0.302	0.305	0.229	0.312
Observations	6748	6743	6743	6743

Constant included; Standard errors into brackets; Errors: robust.

When referring to the overall sample, we retrieve the results already confirmed in the literature: the level of human capital contributes to the economic growth process, and, as a consequence, the ageing of the population and the inflows of immigrants with mostly a human capital endowment inferior to the one of natives are negatively associated with the growth perspective of the correspondent municipalities. However, our estimation emphasize that this result is principally associated with urban and intermediate municipalities. When referring to rural municipalities, human capital does not represent the fuel for the income growth, whereas population ageing and inflows of immigrants are. The dynamics that is likely to take place in rural municipalities is that the principal activities driving the growth process are mostly linked to the agricultural sector; hence the role of human capital is marginal in these production processes and the retirement of aged persons is mostly replaced by unskilled immigrants that guarantee the production standards and, in addition, the concentration of aged persons also for homecare or healthcare services that, once more, are likely to be provided by unskilled immigrants.

5. Conclusions (preliminary)

We provide a theoretical setting to assess the role of migration and human capital formation in fueling growth in places suffering from ageing population. Also, we are providing an original empirical exercise by referring to municipality data in Spain. Being able to handle information at so granular level allows for better controlling for territorial heterogeneity. One relevant conclusion of the theoretical setting, endorsed by the empirical analysis is the negative relationship between human capital and immigration in the long run. Hence, as discussed in the literature, ageing places can record positive growth rates in case the working aging population remain sizable or expand, but this trend is harming the human capital formation in the long run. Therefore, are policies attracting immigrants in the rural areas effective to guarantee the growth process in the long run ? They can have an impact in the short-run, but given the negative association between human capital and immigration, this is not expected to be a sustainable policy for ageing areas (mostly rural) in the long run. Could be mixed policies combining immigration and technology adoption be more sustainable in the long run ?

More complete and conclusive consideration could be put forward in case of performing empirical estimation exploiting longer time period. In addition, it could be valuable to explore causality between immigration and growth in rural areas by adopting and empirical setting as in Shin (2026) but, at time being, the lack of detailed data at municipal level prevent from being more conclusive in this dimension.

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APPENDICES

ONLINE APPENDIX A: *Proof of Proposition 1*

The *Hamiltonian function* (J_t) related to the intertemporal optimization problem (9) in the text reads as:

$$J_t = \left[\ln(c_t) + \phi \ln(n_t)\right] \cdot e^{-\rho t} + \lambda_t \left[i_{h,t} - \delta_H h_t - b n_t h_t - (n_t + m - d) h_t\right] + \\ \mu_t (n_t + m - d) L_t + \omega_t (A h_t - c_t - i_{h,t})$$

with $\rho \equiv \overline{\rho} + d > 0$, $0 < \phi < 1$, and $m_t = \beta(A - \overline{\kappa}) = m$. The co-state variables associated with the two state variables, h_t and L_t, are λ_t and μ_t, respectively, and ω_t is the Lagrange-multiplier associated to resource constraint (3′). The (necessary) first-order conditions read as:

$$\frac{\partial J_t}{\partial c_t} = 0 \quad \Leftrightarrow \quad \frac{e^{-\rho t}}{c_t} = \omega_t \tag{A.1}$$

$$\frac{\partial J_t}{\partial n_t} = 0 \quad \Leftrightarrow \quad \frac{\phi \cdot e^{-\rho t}}{n_t} + \lambda_t (-b h_t - h_t) + \mu_t L_t = 0 \tag{A.2}$$

$$\frac{\partial J_t}{\partial i_{h,t}} = 0 \quad \Leftrightarrow \quad \lambda_t = \omega_t \quad \Rightarrow \quad \frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{\omega}_t}{\omega_t} \tag{A.3}$$

$$\frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_t \quad \Leftrightarrow \quad \lambda_t (-\delta_H - b n_t - n_t - m + d) + \omega_t A = -\dot{\lambda}_t \tag{A.4}$$

$$\frac{\partial J_t}{\partial L_t} = -\dot{\mu}_t \quad \Leftrightarrow \quad \frac{\dot{\mu}_t}{\mu_t} = \underbrace{-(n_t + m - d)}_{-\left(\dot{L}_t / L_t\right)} \quad \Leftrightarrow \quad \frac{\dot{\mu}_t}{\mu_t} + \frac{\dot{L}_t}{L_t} = 0, \quad \forall t \geq 0 \tag{A.5}$$

along with the two transversality conditions:

$$\lim_{t\to+\infty} \lambda_t h_t = 0 \qquad \lim_{t\to+\infty} \mu_t L_t = 0$$

and the two initial conditions:

$$h(0) = h_0 > 0 \qquad L(0) = L_0 > 0 .$$

According to (A.5), $\mu_t L_t$ needs, at optimum, to be constant at any time $t \geq 0$. Using the transversality condition,

$$\lim_{t\to+\infty} \mu_t L_t = 0 ,$$

it follows immediately that it must be:[2]

$$\mu_t L_t = 0 , \qquad \forall t \geq 0 . \tag{A.6}$$

In the light of this result, Eq. (A.2) can be recast as:

$$\frac{\phi \cdot e^{-\rho t}}{n_t} = \lambda_t \left(1+b\right) h_t \tag{A.2′}$$

Along a competitive BGP equilibrium *n* is constant by definition. Take logs and derive with respect to time both sides of (A.2′) and get:

$$\rho = -\frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{h}_t}{h_t} \tag{A.7}$$

After combining (A.3) and (A.4), one immediately obtains:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \delta_H + \left(1+b\right) n + m - d - A \tag{A.8}$$

Use (A.8) and (4″) in the main text into (A.7) and immediately get:

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[2] Recall that the transversality conditions are themselves (necessary) optimality conditions of any dynamic optimization problem. By using (A.5), we observe that along a competitive BGP equilibrium $\dot{L}_t / L_t = \left(n_t + m - d\right)$ is constant by definition and the transversality condition implies: $\lim_{t\to+\infty} \mu_t L_t = \lim_{t\to+\infty} \mu_0 \cdot e^{-\left(\dot{L}_t / L_t\right)\cdot t} \cdot L_0 \cdot e^{\left(\dot{L}_t / L_t\right)\cdot t} = \lim_{t\to+\infty} \mu_0 L_0 = \mu_0 L_0 = 0$, where μ_0 and L_0 are the given initial conditions for μ_t and L_t , respectively. In other words, along a competitive BGP equilibrium, the transversality condition requires that $\mu_0 L_0 = 0$, or that $\mu_t L_t = 0$ at $t = 0$. However, since by (A.5) $\mu_t L_t$ needs to be constant at any time $t \geq 0$, it immediately follows that this constant must always (*i.e.*, $t \geq 0$) be zero along a competitive BGP.

$$i_{h,t} = \left(A - \rho\right) \cdot h_t \tag{A.9}$$

In words, (A.9) says that along a competitive BGP equilibrium the per capita (gross) investment in human capital, $i_{h,t}$, is always proportional to the per capita stock of human capital, h_t. The factor of proportionality is represented by the marginal (and average) productivity of human capital (A), net of the overall discount rate (ρ). Combination of (A.1), (A.3), and (A.8) ultimately delivers:

$$\frac{\dot{c}_t}{c_t} = A - \rho - \delta_H - \left(1 + b\right)n + d - m \tag{A.10}$$

where $m = \beta\left(A - \bar{\kappa}\right)$. Clearly, the growth rate of per capita consumption is constant along a competitive BGP equilibrium.

From the law of motion of h_t (Eq. 4″ in the text), together with (A.9), the growth rate of per capita human capital is:

$$\frac{\dot{h}_t}{h_t} = A - \rho - \delta_H - \left(1 + b\right)n + d - m = \frac{\dot{c}_t}{c_t} \equiv g, \qquad \rho \equiv \bar{\rho} + d > 0 \tag{A.11}$$

Using (1′) in the text and (A.11):

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{h}_t}{h_t} = \frac{\dot{c}_t}{c_t} = A - \rho - \delta_H - \left(1 + b\right)n + d - m \equiv g \tag{A.12}$$

Putting together (A.9) and (A.12):

$$\frac{\dot{i}_{h,t}}{i_{h,t}} = \frac{\dot{h}_t}{h_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{y}_t}{y_t} = A - \rho - \delta_H - \left(1 + b\right)n + d - m \equiv g \tag{A.13}$$

To obtain an expression for $\frac{c_t}{h_t}$, combine (3′) in the text with (A.9):

$$\frac{c_t}{h_t} = \rho \tag{A.14}$$

In order to obtain the endogenous value of n along a competitive BGP equilibrium, first of all notice (from A.1 and A.3) that:

$$\frac{e^{-\rho t}}{c_t} \cdot h_t = \lambda_t \cdot h_t, \tag{A.15}$$

Use now (A.15) and (A.14) into (A.2′), and finally obtain:

$$n = \frac{\phi\rho}{\left(1+b\right)}, \qquad \rho \equiv \overline{\rho} + d > 0 \tag{A.16}$$

Therefore, from (A.13) the common growth rate of the spatial unit at a competitive BGP equilibrium will ultimately be equal to:

$$\frac{\dot{i}_{h,t}}{i_{h,t}} = \frac{\dot{h}_t}{h_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} \equiv g = A\left(1-\beta\right) - \left(1+\phi\right)\overline{\rho} - \phi d - \delta_H + \beta\overline{\kappa} \tag{A.17}$$

Using (2) in the main text, (A.16), and the fact that $\rho \equiv \overline{\rho} + d > 0$, at a competitive BGP equilibrium the local population growth rate is ultimately equal to:

$$\frac{\dot{L}_t}{L_t} = \frac{\phi\overline{\rho} + \left(1+b\right)m - \left(1+b-\phi\right)d}{\left(1+b\right)} \tag{A.18}$$

where $m = \beta\left(A - \overline{\kappa}\right)$, $\beta > 0$, $\overline{\kappa} > 0$. ■