Mobile Capital, Rent Shifting, and the Source of Inefficiencies

Hajime Takatsuka*

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Abstract

By using a monopolistic-competition model with rent shifting via mobile capital, I examine effects and equilibrium of import, export, and capital taxes of two countries, and then clarify the inefficiency source of policy equilibrium. Main results are twofold. First, the Nash equilibrium is inefficient, and a larger country imposes taxes to have a higher world price of its products, while a smaller country imposes taxes to have a higher local relative price of imported varieties. This is because countries producing fewer varieties can expand firm scale and shift rent to them more by raising (resp. lowering) local relative prices of imported (resp. exported) varieties. Second, although rent-shifting externalities appear in the model, global efficiency as well as Pareto efficiency can be achieved without terms-of-trade (TOT) manipulations. TOT motives induce protective behavior of trading partners by influencing the international income distribution.

Key words: Mobile Capital, Rent Shifting, Terms of Trade, Country Size, Inefficiencies.

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*Graduate School of Economics, Osaka Metropolitan University, Sugimoto 3-3-138, Sumiyoshi, Osaka 558-8585, Japan. E-mail:takatsuka.hajime@gmail.com

1 Introduction

"During his state visit to Britain, US President Donald Trump asserted that his imposition of tariffs has significantly boosted investment in the United States. He claimed that without these tariffs, the country would only be experiencing a fraction of its current investment levels." (September 18, 2025, The Economic Times)

As President Trump said, tariffs are now seen as a policy tool for attracting capital, which is expected to lead to an increase in number and scale of firms, and therefore employment. The present study focuses on the rent-shifting effect of trade taxes through attracting capital, and examines each country's tax behavior, the Nash equilibrium, and its Pareto and global efficiencies. Specifically, by using a two-country, two-factor, one-sector monopolistic-competition model, I examine characteristics and equilibrium of import, export, and capital taxes of two countries, and then clarify the inefficiency source of policy equilibrium. Thanks to the simplicity of the one-sector model, we can obtain fully analytical results even if the two countries are asymmetrically sized, and the determination of them can be represented diagrammatically.

The results are summarized as follows. First, capital taxes have both aspects of import and export taxes in the present model, and, hence, three tax instruments are redundant and any two of them are sufficient for each government to maximize welfare. Second, the Nash equilibrium is inefficient, and a larger country imposes taxes to have a higher world price of its products, while a smaller country imposes taxes to have a higher local relative price of imported varieties. This is because countries producing fewer varieties can expand firm scale and shift rent to them more by raising (resp. lowering) local relative prices of imported (resp. exported) varieties. Third, although rent-shifting externalities appear in the model, global efficiency as well as Pareto efficiency can be achieved without terms-of-trade (TOT) manipulations. TOT motives induce protective behavior of trading partners by influencing the international income distribution. This shows that the TOT motives are the source of the two inefficiencies.

Until now, theoretical studies on delocation effects and profit-shifting effects of trade policies in monopolistically competitive markets almost rely on models with multiple sectors and only one immobile factor; namely, labor (e.g., Venables, 1987; Ossa, 2011, 2012, 2014; Campolmi et al., 2014, 2025; Bagwell and Staiger, 2015). In that case, both delocation and profit-shifting effects of trade taxes are simple variations of production-efficiency effects (Helpman and Krugman, 1987, Ch.7). In a monopolistically competitive market, markups lead to under-allocation of labor to the industry. Thus, an increase in demand for goods of the industry by using trade policies leads to shifting labor to the industry and making production more efficient. Under this framework, hence, if domestic policies such as labor taxes/subsidies are introduced, labor allocation can be endogenously efficient and delocation and profit-shifting effects disappear (Campolmi et al., 2025); otherwise, labor allocation remains inefficient, so that both effects also remain.

Meanwhile, Bagwell and Staiger (2015) show that, even if delocation effects remain due to absence of domestic policies, efficient trade taxes can be implemented when each country has a complete set of trade-tax instruments while does not manipulate its TOT, and, thus, externalities via delocation effects are not necessarily addressed in trade agreements. More specifically, following Bagwell and Staiger (1999, 2002), they define *politically optimal* (PO) taxes as those taxes that would hypothetically be chosen by governments unilaterally if they did not value the pure international transfer associated with the TOT movements induced by their unilateral tax choices, and then show that the PO tax is efficient policy.[1] However, their discussion is around the second-best situation with domestic inefficiencies. Furthermore, their results are obtained under a quasi-linear preference without income effects, and their framework is unable to care the difference of Pareto and global efficiencies even without domestic inefficiencies.

In contrast, the present study employs a one-sector and two-factor (i.e., immobile labor and mobile capital) monopolistic-competition model with income effects, and features rent-shifting effects by fixing number of firms.[2] It is noteworthy that our rent-shifting effects are *not* production-efficiency effects since production has been already efficient due to the assumption of one sector. Rent-shifting effects are generated in our model because capital is mobile between countries and firm scale can be increased by means of tax policies at the expense of outflow of capital from passive countries.[3] Unlike one-factor cases, rent-shifting effects never disappear even with domestic policies, and, thus, trade taxes are always accompanied with externalities via rent shifting as well as TOT manipulations. In such a situation, how is the policy (Nash) equilibrium if a complete set of trade-tax instruments is available? How does the PO equilibirum without TOT manipulations allocate resources? Is it Pareto efficient and/or globally efficient? They still remain open questions.

The present paper is also related to studies on capital tax competition, since capital taxes/subsidies are well-known policy tools for attracting capital, and have been studied in the field. Espe-

[1] Although Bagwell and Staiger (2012) obtain similar results on profit-shifting effects, they do not rely on monopolistic-competition models but use a Cournot competition model à la Brander and Spencer (1985).

[2] Although there are multiple industries and only one factor, Ossa (2012, 2014) employ a similar model and derive similar effects by fixing number of firms, but he calls it profit-shifting effects, because positive pure (abnormal) profit is generated by their assumption of no fixed input. In contrast, pure (abnormal) profit is zero and operating profit is distributed to labor (fixed input) as wage in the present paper. Therefore, we do not use the term of profit-shifting effects but use the term of rent-shifting effects.

[3] Thus, such effects are not observed in models with one sector and *one factor* (Gros, 1987; Campolmi et al., 2025).

cially, Wilson (1991) discusses a fundamental difference between tax competition and tariff wars. Namely, larger countries impose higher import tariffs to improve their TOT more in tariff wars (e.g., Kennan and Riezman, 1988; Syropoulos, 2002), while smaller countries impose lower capital taxes to attract capital more in tax competition (Bucovetsuky, 1991; Wilson, 1991). However, aforementioned comments of President Trump suggest that import tariffs play a role similar to that of capital taxes. Similarly, if capital taxes can affect the terms of trade, then the difference between the two may not be as clear-cut as Wilson (1991) stated.

Recently, Takatsuka and Zeng (2022) show that smaller countries impose higher import tariffs and they can win tariff wars by using a model with mobile capital. However, they consider neither capital taxes nor export taxes, and the difference as well as the similarity between capital taxes and trade taxes is not explored.[4] In addition, they focus on what occurs in Nash equilibrium of import tariffs, and discuss neither efficiency of the equilibrium nor the source of (possible) inefficiencies. In the present paper, I generalize the model of Takatsuka and Zeng (2022) by introducing capital taxes and export taxes, so that their model can be treated as a missing-instruments case of the present paper. Furthermore, to derive a more general principle, this paper focuses on world prices and local relative prices, like Bagwell and Staiger (1999), rather than tax rates themselves. As a result, we can show that smaller countries do *not* necessarily impose higher tariffs, and also clarify the differences in fundamental mechanisms between the aforementioned classic studies on tax competition and tariff wars and the present model.

In the former classic models, strong pricing power of larger countries is essential. Larger countries tend to impose higher tariffs since they can lower world prices of imported goods more, and, thus, improve its TOT by doing so, while they tend to impose higher capital taxes since they can lower pre-tax capital rent more, and, thus, suppress increases in post-tax capital rent by doing so. In the present model featuring rent-shifting effects based on monopolistic competition, however, number of varieties produced in each country plays a key role. Smaller countries have a stronger incentive to raise local relative prices of imported varieties since they substitute many types of imported varieties supplied by larger countries with few domestic varieties, and, thus, they can increase firm scale and shift rent to them more. Similary, smaller countries have a stronger incentive to lower local relative prices of exported varieties since larger trading partners substitute many types of domestic varieties with few imported varieties, and, thus, smaller exporters can increase firm scale and shift rent to them more. Conversely, larger

[4] In Section 6 of Takatsuka and Zeng (2022), they consider capital *import* taxes, which are, however, quite different from capital taxes in the present paper. Capital import taxes are imposed to extract rent from *foreign* capital, while capital taxes in the present paper are imposed on *all* capital used for production, and, thus, they are equivalent to production taxes.

exporters relatively prioritize improving their TOT over shifting rent to them, so that they set higher world prices of their products.

The remainder of this paper is organized as follows. In Section 2, I describe the model and derive the welfare decomposition in a similar manner of Helpman and Krugman (1989, Ch.2) and Campolmi et al. (2012, 2025), where the rent-shifting effect can be expressed as well as usual TOT and consumption-wedge effects. In Section 3, I derive the conditions of Pareto-efficient resource allocation and a globally-efficient one. I also compare the globally-efficient allocation with the laissez-faire equilibrium. In Section 4, unilaterally optimal and Nash policies are derived, and show the ineffciency of Nash policies and country-size effects on Nash tax rates. As special cases, missing-instruments cases are also discussed. Sections 5 shows that a globally-efficient resource allocation can be achieved by PO policies, and also compares welfare at the globally-efficient allocation, the laissez-faire equilibrium, and the Nash equilibrium. Finally, Section 6 is the conclusion.

2 The Model

2.1 Setup

The economy consists of two countries (home H and foreign F) and one sector supplying the differentiated varieties. Denote the amounts of labor in H by $\theta \in (0,1)$ and assume the measure of the worldwide endowment to be one. Capital is evenly held by all workers, and the measure of the total amount of capital is also one. Assume that labor is immobile but capital is freely mobile across countries. We define country size as the ownership share of factors so the size of country H is θ.

The production sector consists of a continuum of product varieties and is characterized by increasing returns to scale (IRS) and monopolistic competition. Workers are assumed to hold the same preferences, which are described by a constant elasticity of substitution (CES) utility on the varieties:

$$U = \ln \left[\int_0^{n^w} c(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where n^w is the mass of varieties in the world, and $c(i)$ is the consumption of variety i. Parameter $\sigma > 1$ is the elasticity of substitution between any two varieties. The expenditures in H and F are, respectively,

$$E = \theta\,(w + r) + T, \quad E^* = (1-\theta)\,(w^* + r) + T^*, \tag{2}$$

where r is the capital rent, w and T (resp. w^* and T^*) are the wage and the tax revenue in H (resp. F). We let capital be the numéraire, so that $r = 1$. Note that asterisk ($*$) refers to variables in F throughout the paper.

We assume Samuelson's iceberg transportation costs. Specifically, $\tau \in [1, \infty)$ units of the variety must be shipped for one unit to arrive from one country to the other country. In the present paper, we consider three types of policy instruments; namely, a production (capital) tax/subsidy (τ_R) on firms' marginal costs, a tax/subsidy (τ_I) on imports, and a tax/subsidy (τ_X) on exports. Specifically, $\tau_m > 0$ indicates an *ad valorem* tax rate for $m \in \{R, I, X\}$, i.e., $\tau_m \in (0, 1)$ implies a subsidy and $\tau_m > 1$ implies a tax. The tax revenue is evenly distributed to all workers in the country.

For production, each firm needs a fixed input of one unit of labor and a marginal input of one unit of capital. Thus, production tax/subsidy (τ_R) is capital tax/subsidy in our model. The assumption that the fixed input is only labor and marginal input is only capital reflects the recent knowledge-based management and the mechanization of production. The total costs of producing x units of differentiated varieties in H and F are, respectively,

$$\mathrm{TC}(x) = w + x, \quad \mathrm{TC}^*(x) = w^* + x,$$

where note that the capital rent r is unity.

The varieties are supposed to be symmetric, so we can omit the variety name and simply use p to refer to the price of a variety in country H made in country H, p^* the price of a variety in country F made in country F, $\bar{p}$ the price of a variety in country H made in country F, and $\bar{p}^*$ the price of a variety in country F made in country H. The monopolistic competition framework of Dixit and Stiglitz (1977) implies that

$$p = \frac{\tau_R \sigma}{\sigma - 1}, \quad p^* = \frac{\tau_R^* \sigma}{\sigma - 1}, \quad \bar{p} = \tau \tau_I \tau_X^* p^*, \quad \bar{p}^* = \tau \tau_I^* \tau_X p. \tag{3}$$

From (1), the demands for each variety produced in H and consumed in H and F are

$$q = \frac{p^{-\sigma}}{P^{1-\sigma}} E, \quad \bar{q}^* = \frac{(\bar{p}^*)^{-\sigma}}{(P^*)^{1-\sigma}} E^*, \tag{4}$$

respectively, and the demands for each variety produced in F and consumed in F and H are

$$q^* = \frac{(p^*)^{-\sigma}}{(P^*)^{1-\sigma}} E^*, \quad \bar{q} = \frac{\bar{p}^{-\sigma}}{P^{1-\sigma}} E, \tag{5}$$

respectively. In the above expressions, P and P^* are the price indices in the two countries given by

$$P = \left[np^{1-\sigma} + n^*(\bar{p})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = p\left[n + n^*\left(\frac{\bar{p}}{p}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{6}$$
$$P^* = \left[n(\bar{p}^*)^{1-\sigma} + n^*(p^*)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = p^*\left[n\left(\frac{\bar{p}^*}{p^*}\right)^{1-\sigma} + n^*\right]^{\frac{1}{1-\sigma}},$$

where n and n^* are masses of firms in H and F, respectively. It is noteworthy that $\bar{p}/p$ and $\bar{p}^*/p^*$ are *local relative prices* of imported goods to domestic goods in H and F, respectively.

In equilibrium, output x of each variety in H is determined such that the gross profit just covers fixed costs:

$$(p - \tau_R)x = w \Leftrightarrow x = \frac{(\sigma-1)w}{\tau_R}. \tag{7}$$

A similar equation holds for F. Since each firm needs a fixed input of one unit of labor, we have

$$n = \theta,\ n^* = 1 - \theta. \tag{8}$$

and the amount of capital used in H is $\kappa = nx$. From these equations, we have

$$w = \frac{\tau_R x}{\sigma - 1} = \frac{\tau_R}{\sigma-1}\frac{\kappa}{\theta}, \quad w^* = \frac{\tau_R^* x^*}{\sigma-1} = \frac{\tau_R^*}{\sigma-1}\frac{1-\kappa}{1-\theta}. \tag{9}$$

The *world prices* of each variety produced in H and F are defined by

$$p^w \equiv \tau_X p = \frac{\tau_{RX}\sigma}{\sigma-1},\ p^{*w} \equiv \tau_X^* p^* = \frac{\tau_{RX}^*\sigma}{\sigma-1}, \tag{10}$$

respectively, where τ_{RX} and τ_{RX}^* are products of export tax rates and capital tax rates (i.e., $\tau_{RX} \equiv \tau_R\tau_X$ and $\tau_{RX}^* \equiv \tau_R^*\tau_X^*$) in H and F, respectively. Export tax rates and capital tax rates resemble each other in terms that they increase world prices of exported varieties. Meanwhile, the local relative prices can be expressed as

$$\frac{\bar{p}}{p} = \frac{\tau_I}{\tau_R}\tau_R^*\tau_X^*\tau = \tau_{RI}\tau_{RX}^*\tau,\ \frac{\bar{p}^*}{p^*} = \frac{\tau_I^*}{\tau_R^*}\tau_R\tau_X\tau = \tau_{RI}^*\tau_{RX}\tau, \tag{11}$$

where τ_{RI} and τ_{RI}^* are ratios of import tax rates to capital tax rates (i.e., $\tau_{RI} \equiv \tau_I/\tau_R$ and $\tau_{RI}^* \equiv \tau_I^*/\tau_R^*$) in H and F, respectively. Import tax rates and *inverted* capital tax rates resemble each other in terms that they increase local relative prices of imported goods to domestic goods. From (11), it is noteworthy that τ_{RX} and τ_{RX}^* also affect local relative prices $\bar{p}^*/p^*$ and $\bar{p}/p$, respectively.

In the following, we suppose that

$$p^{w} \geq r = 1 \Leftrightarrow \tau_{RX} \geq \frac{\sigma - 1}{\sigma}, \quad p^{*w} \geq r = 1 \Leftrightarrow \tau_{RX}^{*} \geq \frac{\sigma - 1}{\sigma}, \tag{12}$$

Inequalities (12) suggest that each country imposes taxes/subsidies such that world prices are greater than or equal to marginal costs ($r = 1$) of production to never lose by exporting varieties. This point will be discussed again in Section 2.3. As shown later, these inequalities always hold at equilibirum.

Tax revenue in H can be expressed by

$$T = \underbrace{(\bar{p} - \tau p^{*w}) n^{*} \overline{q}}_{\text{import-tax revenue}} + \underbrace{(\tau p^{w} - \tau p) n \overline{q}^{*}}_{\text{export-tax revenue}} + \underbrace{(\tau_{R} - 1) \kappa}_{\text{capital-tax revenue}} . \tag{13}$$

Substituting this equation into (2) and using (9), expenditure in H can be expressed as

$$E = \theta + (\bar{p} - \tau p^{*w})(1 - \theta)\overline{q} + (\tau p^{w} - \tau p)\theta \overline{q}^{*} + (p - 1)\,\kappa, \tag{14}$$

where the first term is capital income and the last term is the sum of wage income (i.e., gross profit) and capital-tax revenue. The second and third terms suggest that, other things being equal, a rise in the world price of varieties produced in a country increases the expenditure in the country while decreases the expenditure in the other country. This pure international transfer is the TOT effect.

2.2 Market equilibrium

Since $x = \kappa/\theta$ and $x^{*} = (1 - \kappa)/(1 - \theta)$, the market-clearing conditions for each variety produced in H and F are

$$\frac{\kappa}{\theta} = x = q + \tau \overline{q}^{*}, \; \frac{1 - \kappa}{1 - \theta} = x^{*} = q^{*} + \tau \overline{q}, \tag{15}$$

respectively. With (4) and (5), solving (14) and (15) for κ, E, and E^{*}, we have

$$\kappa = \frac{\theta^{2}(1 - \theta)\left[\left(\frac{\bar{p}}{p}\frac{\bar{p}^{*}}{p^{*}}\right)^{\sigma} - \tau^{2}\right] + \left[\theta\left(\frac{\bar{p}}{p}\right)^{\sigma} p^{w} + (1 - \theta) p^{*w} \tau\right] \theta \tau}{\mathcal{A}\left(p^{w}, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^{*}}{p^{*}}\right)}. \tag{16}$$

and

$$E = \frac{\theta \bar{p} \left[(p^w - 1 + \theta) \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right] \left[\theta \left(\frac{\bar{p}}{p} \right)^{\sigma - 1} + 1 - \theta \right]}{\mathcal{A} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)}, \tag{17}$$

$$E^* = \frac{(1-\theta) \bar{p}^* \left[(p^{*w} - \theta) \tau + \theta \left(\frac{\bar{p}}{p} \right)^\sigma \right] \left[(1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^{\sigma - 1} + \theta \right]}{\mathcal{A} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)}, \tag{18}$$

where

$$\begin{aligned} \mathcal{A} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right) \quad \equiv \quad & \theta(1-\theta) \left[\left(\frac{\bar{p}}{p} \frac{\bar{p}^*}{p^*} \right)^\sigma - \tau^2 \right] \\ & + \left[\theta^2 \left(\frac{\bar{p}}{p} \right)^\sigma p^w + (1-\theta)^2 \left(\frac{\bar{p}^*}{p^*} \right)^\sigma p^{*w} + \theta (1-\theta) (p^w + p^{*w}) \tau \right] \tau. \end{aligned}$$

Substituting (17) into (4) and (5), we have

$$q = \frac{\theta \left(\frac{\bar{p}}{p} \right)^\sigma \left[(p^w - 1 + \theta) \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right]}{\mathcal{A} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)} \equiv q \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right), \tag{19}$$

$$\bar{q} = \frac{\theta \left[(p^w - 1 + \theta) \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right]}{\mathcal{A} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)} \equiv \bar{q} \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right) = \left(\frac{\bar{p}}{p} \right)^{-\sigma} q, \tag{20}$$

and their counterpart q^* and $\bar{q}^*$ are also obtained in similar forms. Here, we assume that

$$\frac{\bar{p}}{p} \frac{\bar{p}^*}{p^*} = \tau_I \tau_X \tau_I^* \tau_X^* \tau^2 \geq \tau^2 \Leftrightarrow \tau_I \tau_X \tau_I^* \tau_X^* \geq 1 \tag{21}$$

for $\mathcal{A}$ and κ to be positive.[5] From (12) and (21), we know that E and E^* are positive. We also know that κ and demands (q, $\bar{q}, q^*$, and $\bar{q}^*$) are all functions of p^w, p^{*w}, $\bar{p}/p$, and $\bar{p}^*/p^*$ from (16), (19), and (20). By using (12) and (21), it can be easily shown that

$$\frac{\partial \kappa}{\partial \frac{\bar{p}}{p}} > 0, \ \frac{\partial \kappa}{\partial \frac{\bar{p}^*}{p^*}} < 0, \ \frac{\partial \kappa}{\partial p^w} > 0, \ \frac{\partial \kappa}{\partial p^{*w}} < 0.$$

The first two inequalities show that capital moves to a country if local relaive prices of its imported varieties are raised. This is because such a price change increases demand for domestically produced varieties, so that it is more profitable to produce in the country. Meanwhile, the last two inequalities show that capital moves to a country if the world price of domestically produced varieties are raised. As aforementioned, such a change of the wolrd price increases the

[5] As shown later, this inequality always holds at equilibirum.

expenditure (i.e., market size) in the country.[6]

Finally, from (6), we have

$$V=\ln\frac{E}{\theta P}=\ln\frac{\left(\frac{\bar{p}}{p}\right)^{\sigma}\left[\left(p^{w}-1+\theta\right)\tau+(1-\theta)\left(\frac{\bar{p}^{*}}{p^{*}}\right)^{\sigma}\right]\left[\theta+(1-\theta)\left(\frac{\bar{p}}{p}\right)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}{\mathcal{A}\left(p^{w},p^{*w},\frac{\bar{p}}{p},\frac{\bar{p}^{*}}{p^{*}}\right)},\tag{22}$$

$$V^{*}=\ln\frac{E^{*}}{(1-\theta)P^{*}}=\ln\frac{\left(\frac{\bar{p}^{*}}{p^{*}}\right)^{\sigma}\left[\left(p^{*w}-\theta\right)\tau+\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\right]\left[\theta\left(\frac{\bar{p}^{*}}{p^{*}}\right)^{1-\sigma}+1-\theta\right]^{\frac{\sigma}{\sigma-1}}}{\mathcal{A}\left(p^{w},p^{*w},\frac{\bar{p}}{p},\frac{\bar{p}^{*}}{p^{*}}\right)},\tag{23}$$

implying that V and V^* are also functions of p^w, p^{*w}, $\bar{p}/p$, and $\bar{p}^*/p^*$.

In summary, the following proposition holds:

Proposition 1 *(i) Amount of capital κ used in H, demands q, $\overline{q}, q^*$, and $\overline{q}^*$, and indirect utility V and V^* are all determined by the world prices (p^w and p^{*w}) and the local relaive prices ($\bar{p}/p$ and $\bar{p}^*/p^*$).*
(ii) κ increases in p^w and $\bar{p}/p$ while decreases in p^{*w} and $\bar{p}^*/p^*$.

Therefore, we immediately obtain the following result from the above proposition:

Corollary 1 *Amount of capital κ used in H and indirect utility V and V^* are all determined by τ_{RX}, τ^*_{RX}, τ_{RI}, and τ^*_{RI}.*

This shows that welfare-maximizing government of H controlls τ_{RX} and τ_{RI}, given their foreign counterparts τ^*_{RX} and τ^*_{RI}, implying that three tax instruments are redundunt and any two of them are sufficient for each government to maximize welfare. Thus, even if it is impossible to have a complete set of trade taxes (i.e., both import and export taxes) for each government due to some rules, it is harmless for maximizing welfare when domestic production taxes are available. In the real world, the use of export policy instruments is constrained by GATT/WTO, but such regulations are invalid in this economy.

Finally, as mentioned above, a rise in τ_{RX} positively affects both the world price p^w of home varieties and the local relative price $\bar{p}^*/p^*$ in F. Proposition 1 (ii) shows that the former effect works for acquiring capital as well as improving TOT, while the latter one works against acquiring capital. Meanwhile, a rise in τ_{RI} positively affects the local relative price $\bar{p}/p$ in H only. Proposition 1 (ii) shows that this effect works for acquiring capital. Similar relationships hold for τ^*_{RX} and τ^*_{RI} of country F.

[6]Effects of world prices and local relative prices on demands are shown in Appendix A.

2.3 Welfare decomposition

Following Helpman and Krugman (1989, Ch.2) and Campolmi et al. (2012, 2025), we decompose welfare changes by totally differentiating indirect utility. From (14), expenditure in H can be rewritten as

$$
\begin{aligned}
E &= \theta + (\bar{p} - \tau p^{*w})(1-\theta)\overline{q} + (\tau p^{w} - \tau p)\theta\overline{q}^{*} + (p-1)\,\kappa \\
&= \theta - \kappa + (1-\theta)\overline{pq} - (1-\theta)\tau p^{*w}\overline{q} + \theta\tau p^{w}\overline{q}^{*} - \theta p\tau\overline{q}^{*} + p\kappa \\
&= (\theta-\kappa) + \theta\tau p^{w}\overline{q}^{*} - (1-\theta)\tau p^{*w}\overline{q} + \left[p\theta\left(q + \tau\overline{q}^{*}\right) - \theta p\tau\overline{q}^{*} + (1-\theta)\overline{pq}\right] \\
&= \underbrace{\underbrace{(\theta-\kappa)}_{\text{capital balance}} + \underbrace{\theta\tau p^{w}\overline{q}^{*} - (1-\theta)\tau p^{*w}\overline{q}}_{\text{trade balance}}}_{0} + \underbrace{\left[\theta pq + (1-\theta)\overline{pq}\right]}_{\text{consumption spending}}, \qquad (24)
\end{aligned}
$$

where the third equality is from

$$\kappa = \theta x = \theta\left(q + \tau\overline{q}^{*}\right).$$

Totally differentiating (24), we have

$$
\begin{aligned}
dE &= -d\kappa + \theta\tau\overline{q}^{*}dp^{w} + \theta p^{w}d\left(\tau\overline{q}^{*}\right) - (1-\theta)\tau\overline{q}dp^{*w} - (1-\theta)\tau p^{*w}d\overline{q} + d\left[\theta pq + (1-\theta)\overline{pq}\right] \\
&= -\theta dx + \theta\tau\overline{q}^{*}dp^{w} + \theta p^{w}\left(dx - dq\right) - (1-\theta)\tau\overline{q}dp^{*w} - (1-\theta)\tau p^{*w}d\overline{q} + d\left[\theta pq + (1-\theta)\overline{pq}\right] \\
&= \left[\theta\tau\overline{q}^{*}dp^{w} - (1-\theta)\tau\overline{q}dp^{*w}\right] + \theta\left(p^{w}-1\right)dx + \left[\theta\left(p - p^{w}\right)dq + (1-\theta)\left(\bar{p} - \tau p^{*w}\right)d\overline{q}\right] \\
&\quad + \theta qdp + (1-\theta)\overline{q}d\bar{p}.
\end{aligned}
$$

From (6), it holds that

$$
\begin{aligned}
\frac{dP}{P} &= \theta P^{\sigma-1}p^{-\sigma}dp + (1-\theta)P^{\sigma-1}(\bar{p})^{-\sigma}d\bar{p} \\
&= \theta\frac{q}{E}dp + (1-\theta)\frac{\overline{q}}{E}d\bar{p}. \qquad (25)
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
dV &= \frac{dE}{E} - \frac{dP}{P} \\
&= \underbrace{\frac{\theta\tau\overline{q}^{*}dp^{w} - (1-\theta)\tau\overline{q}dp^{*w}}{E}}_{\text{(direct) TOT effect}} + \underbrace{\frac{\theta\left(p^{w}-1\right)dx}{E}}_{\text{rent-shifting effect}} \\
&\quad + \underbrace{\frac{\theta\left(p - p^{w}\right)dq + (1-\theta)\left(\bar{p} - \tau p^{*w}\right)d\overline{q}}{E}}_{\text{consumption-wedge effect}}. \qquad (26)
\end{aligned}
$$

As shown by existing studies (e.g., in Helpman and Krugman, 1989, Ch.2), the first and the third terms represent the *terms-of-trade* (TOT) and the *consumption-wedge effects*, respectively. The TOT effect indicates that an increase in the world price of exported varieties (p^w) raises welfare, although an increase in the world price of imported varieties (p^{*w}) reduces welfare. Meanwhile, the consumption-wedge effect means that an increase in the demestic demand for domestic varieties (q) raises welfare if their domestic price (p) is higher than their world price (p^w), while an increase in the demestic demand for foreign varieties ($\overline{q}$) raises welfare if their domestic price ($\bar{p}$) is higher than their world price (p^{*w}) multiplied by τ.

The second term in the above equation is similar to but different from those in existing studies. This term is based on the change of output (dx). In the cases of preveous studies, there are several industries and production factors are supposed to be immobile among countries (typically, labor), so that an increase in output of one sector means decreases in output of other sectors. Thus, Helpman and Krugman (1989, Ch.2, p.23) call such an effect "the gain from an improved production compositoin." In our case, there exists only one sector but two types of factors, i.e., immobile labor as fixed input and internationally mobile capital as variable input. Thus, an increase in domestic output implies that capital moves from F to H. The second term in the above equation shows that such capital inflow is beneficial if and only if the world price of varieties is higher than marginal production costs; i.e., the first inequality of (12) strictly holds.

As known by the process of obatining (26), such welfare gain is generated from an increase in export of varieties (i.e., $d\overline{q}^*$). When varieties produced in H are sold only in H, welfare gain does not occur because gross profit obtained by firms in H is just a transfer from consumers in the same country H. Meanwhile, when varieties produced in H are exported to F, welfare gain occurs since gross profit is obtained from consumers in country F. In that case, gross profit is shifted to the country with a rise in wage income and capital inflow from the country with a fall in wage income and capital outflow, and, thus, we call the effect represented by the second term the *rent-shifting effect*.

As discussed below (14), a rise in the world price of varieties produced in H increases the home expenditure. This change of the expenditure also affects the domestic output x as well as the domestic demand q and $\overline{q}$ in the model with income effect. This suggests that an improvement of the TOT in H influences the home welfare directly and indirectly. The first term of (26) represents the direct TOT effect, while indirect TOT effects occur via the second and third terms of (26).

When we classify possible effects of taxes, decomoposition (26) is useful. However, the change of welfare itself can be expressed more simply. Since the sum of capital balance and trade balance is zero in (24), we have

$$dV = \frac{d\left[\theta pq + (1-\theta)\overline{pq}\right]}{E} - \frac{dP}{P} = \frac{\theta p dq + (1-\theta)\bar{p}d\bar{q}}{E}, \tag{27}$$

where the last equality is from (25). As the equation indicates, the change of welfare in H is equal to the sum of changes in consumption valued at consumer prices in H over expenditure in the country.

3 Efficient Resource Allocation

3.1 Pareto-efficient allocation

First of all, we can easily show that the laissez-faire allocation is a Pareto-efficient one in the present one-sector model. Appendix B shows this fact by deriving the first-order conditions (FOCs) of Pareto-efficient allocation. In market equilibrium with taxes, the marginal rates of substitution of imported varieties to domestic ones in H and F are equal to the local relative prices (11), respectively. Therefore, from (56) in Appendix B, if

$$\tau_{RX}\tau^*_{RI} = 1, \;\; \tau^*_{RX}\tau_{RI} = 1, \tag{28}$$

the resource allocation is Pareto efficient. This is evidently because such tax rates are not distortive.

From (3) and (16) with $\tau_{RX} = \tau_{RI} = \tau^*_{RX} = \tau^*_{RI} = 1$, the laissez-faire allocation of capital is

$$\kappa = \theta + \frac{\sigma\theta(1-\theta)(2\theta-1)\phi(1-\phi)}{\theta(1-\theta)\left[\sigma(1+\phi^2)-(1-\phi^2)\right]+\left[1-2\theta(1-\theta)\right]\sigma\phi} \equiv \kappa_e, \tag{29}$$

where $\phi \equiv \tau^{1-\sigma} \in (0,1]$ is trade freeness. This suggests that

$$\kappa_e \gtreqless \theta \Leftrightarrow \theta \gtreqless \frac{1}{2},$$

i.e., a more-than-proportionate share of capital is employed in the larger country.

3.2 A globally-efficient allocation

We define the world welfare W as the simple sum of utilities across all individuals. Since varieties are supposed to be symmetric, we have

$$W = \theta \ln \left[n\left(c\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(\overline{c}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + (1-\theta)\ln \left[n\left(\overline{c}^*\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(c^*\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{30}$$

where c (resp. $\overline{c}$) denotes the consumption of a worker in H for a variety produced in H (resp. F) and c^* (resp. $\overline{c}^*$) denotes the consumption of a worker in F for a variety produced in F (resp. H). The social planner chooses c, $\overline{c}$, c^*, $\overline{c}^*$, and κ that maximize W, so that the Lagrangean is represented by

$$L = W + \lambda\left[\kappa - n\theta c - n\left(1-\theta\right)\tau\overline{c}^*\right] + \lambda^*\left[1-\kappa - n^*\left(1-\theta\right)c^* - n^*\theta\tau\overline{c}\right].$$

In this case, five endogenous variables $(c, \overline{c}, c^*, \overline{c}^*, \kappa)$ are determined by (56), (57), (58) in Appendix B, and

$$\begin{aligned} &\frac{c^*}{c} = \left(\frac{u}{u^*}\right)^{\sigma} \Leftrightarrow \\ &(c)^{\frac{1}{\sigma}}\left[n\left(c\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(\overline{c}\right)^{\frac{\sigma-1}{\sigma}} \right] = \left(c^*\right)^{\frac{1}{\sigma}}\left[n\left(\overline{c}^*\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(c^*\right)^{\frac{\sigma-1}{\sigma}} \right], \end{aligned} \tag{31}$$

which are obtained from $\partial L/\partial c = \partial L/\partial c^* = 0$ and $\lambda = \lambda^*$. Note that

$$\begin{aligned} u &\equiv n\left(c\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(\overline{c}\right)^{\frac{\sigma-1}{\sigma}}, \\ u^* &\equiv n\left(\overline{c}^*\right)^{\frac{\sigma-1}{\sigma}} + n^*\left(c^*\right)^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

Solving these equations, we have

$$c = \frac{1}{\theta + (1-\theta)\phi}, \ \overline{c} = \frac{\tau^{-\sigma}}{\theta + (1-\theta)\phi}, \tag{32}$$

$$c^* = \frac{1}{\theta\phi + 1 - \theta}, \ \overline{c}^* = \frac{\tau^{-\sigma}}{\theta\phi + 1 - \theta}, \tag{33}$$

and

$$\kappa = \theta + \frac{\theta(1-\theta)(2\theta-1)\phi(1-\phi)}{\theta(1-\theta)\left(1-\phi\right)^2 + \phi} \equiv \kappa_{FB} > \theta, \tag{34}$$

i.e., a more-than-proportionate share of capital is employed in the larger country again. Substituting (32) and (33) into utility function of each country, we also have

$$U = \frac{1}{\sigma - 1} \ln\left[\theta + (1-\theta)\phi\right] \equiv V_{FB}, \tag{35}$$
$$U^* = \frac{1}{\sigma - 1} \ln\left[\theta\phi + (1-\theta)\right] \equiv V_{FB}^*,$$

suggesting that the larger country has higher welfare (i.e., higher real income) in the globally-efficient (first-best) case.

3.2.1 Implementations

Let y and y^* to be per-capita income in H and F, respectively. Using demand functions of (4) and (5), we have

$$\frac{c^*}{c} = \frac{y^*}{y}\left(\frac{p}{p^*}\right)^{\sigma}\left(\frac{P^*}{P}\right)^{\sigma-1}.$$

Meanwhile, indirect utility functions of (22) and (23), we have

$$\left(\frac{u}{u^*}\right)^{\sigma} = \left(\frac{y}{y^*}\right)^{\sigma-1}\left(\frac{P^*}{P}\right)^{\sigma-1}.$$

From these two equations, (31) holds if and only if per-capita income measured by domestic variety in each country are equalized; namely, $y/p = y^*/p^*$. If Pareto efficiency holds, we have

$$\frac{y/p}{y^*/p^*} = \frac{\tau + \theta(\tau^\sigma - \tau)}{\tau + (1-\theta)(\tau^\sigma - \tau)} \frac{p^w\tau + (1-\theta)(\tau^\sigma - \tau)}{p^{*w}\tau + \theta(\tau^\sigma - \tau)} \tag{36}$$

from (17), (18), and (28). Furthermore, we know that the above equation increases in p^w, decreases in p^{*w}, and is equal to one for any τ, θ, and σ if and only if $p^w = p^{*w} = 1$. Thus, the globally-efficient allocation can be implemented in market equilibrium with (28) and $p^w = p^{*w} = 1$, which are expressed as

$$\tau_{RX} = \tau_{RX}^* = \frac{1}{\tau_{RI}} = \frac{1}{\tau_{RI}^*} = \frac{\sigma - 1}{\sigma}$$

by using (10).

In addition, when Pareto efficiency holds and $\theta > 1/2$, we have

$$\frac{\partial}{\partial p^w}\left(\left.\frac{y/p}{y^*/p^*}\right|_{p^{*w}=p^w}\right) > 0,$$

so that the laissez-faire equilibrium is *not* globally efficient since $y/p > y^*/p^*$ holds from $p^w = p^{*w} = \sigma/(\sigma-1) > 1$. In other words, per-capita income measured by domestic variety in each country is higher in larger countries in the laissez-faire equilibrium.

3.3 Comparison of laissez-faire allocation and the globally-efficient one

From (29) and (34), we have

$$\mathrm{sgn}\left(\kappa_e - \kappa_{FB}\right) = \mathrm{sgn}(2\theta - 1),$$

implying that

$$\kappa_e > \kappa_{FB} \ \ (> \theta) \Leftrightarrow \theta > \frac{1}{2}.$$

From (22) and (23), indirect utility at laissez-faire equilibrium in H and F are

$$V = V_{FB} + \ln\left\{1 + \frac{(2\theta-1)(1-\theta)\phi(1-\phi)}{\theta(1-\theta)\left[\sigma(1+\phi^2)-(1-\phi^2)\right] + \left[1-2\theta(1-\theta)\right]\sigma\phi}\right\} \equiv V_e, \quad (37)$$

$$V^* = V^*_{FB} + \ln\left\{1 - \frac{(2\theta-1)\theta\phi(1-\phi)}{\theta(1-\theta)\left[\sigma(1+\phi^2)-(1-\phi^2)\right] + \left[1-2\theta(1-\theta)\right]\sigma\phi}\right\} \equiv V^*_e, (38)$$

respectively. Using these equations, we can show that

$$V_e > V_{FB} > V^*_{FB} > V^*_e \Leftrightarrow \theta > \frac{1}{2}, \qquad (39)$$

where note that

$$V_e = V_{FB} = V^*_{FB} = V^*_e \Leftrightarrow \theta = \frac{1}{2}. \qquad (40)$$

In other words, welfare differential between two countries is larger under the laissez-faire equilibrium compared to the globally-efficient case.

The above results are summarized as the following proposition:

Proposition 2 *(i) Pareto-efficient allocation can be implemented in market equilibrium with* $\tau_{RX}\tau^*_{RI} = 1$ *and* $\tau^*_{RX}\tau_{RI} = 1$*, including the laissez-faire equilibrium.*
(ii)The globally-efficient allocation maximizing a utilitalian social welfare function (30) can be implemented in market equilibrium with $\tau_{RX} = \tau^*_{RX} = 1/\tau_{RI} = 1/\tau^*_{RI} = (\sigma-1)/\sigma$.
(iii) In the globally-efficient allocation, a more-than-proportionate share of capital should be employed in the larger country, although the share should be smaller than the laissez-faire one,

i.e., $\theta < \kappa_{FB} < \kappa_e$ *for* $\theta > 1/2$.

(iv) It holds that $V_e > V_{FB} > V_{FB}^* > V_e^*$ *for* $\theta > 1/2$.

4 Unilaterally Optimal and Nash Policies

4.1 Unilaterally optimal policies

We assume that the government in each country maximizes indirect utility of workers in its country by using tax instruments. Specifically, from Proposition 1 and Corollary 1, the FOCs of unilaterally optimal taxes in H are

$$\frac{\partial V}{\partial \frac{\bar{p}^*}{p^*}} \frac{\partial \frac{\bar{p}^*}{p^*}}{\partial \tau_{RX}} + \frac{\partial V}{\partial p^w} \frac{\partial p^w}{\partial \tau_{RX}} = 0 \text{ and } \frac{\partial V}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau_{RI}} = 0. \tag{41}$$

Similar conditions hold for F. It is noteworthy that τ_{RX} affects both $\bar{p}^*/p^*$ and p^w, while τ_{RI} affects $\bar{p}/p$ only.

4.1.1 Unilaterally optimal τ_{RX}

From (27), the effect of $\bar{p}^*/p^*$ on V can be expressed as

$$\begin{aligned} \frac{\partial V}{\partial \frac{\bar{p}^*}{p^*}} &= \frac{\theta p}{E} \frac{\partial q}{\partial \frac{\bar{p}^*}{p^*}} + \frac{(1-\theta)\bar{p}}{E} \frac{\partial \bar{q}}{\partial \frac{\bar{p}^*}{p^*}} \\ &= -\frac{(p^w - 1)\,\theta\,(1-\theta)^2 \left(\frac{\bar{p}^*}{p^*}\right)^{\sigma-1} \sigma\tau \left[(p^{*w} - \theta)\,\tau + \theta \left(\frac{\bar{p}}{p}\right)^{\sigma}\right] \left[\theta p \left(\frac{\bar{p}}{p}\right)^{\sigma} + (1-\theta)\,\bar{p}\right]}{E\mathcal{A}^2 \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*}\right)} \\ &\leq 0, \end{aligned} \tag{42}$$

where the second equality is from Appendix A and the last inequality is from (12). As long as $p^w > 1$, a rise in $\bar{p}^*/p^*$ lowers wage income in H via decreasing export from H to F and the rent-shifting effect from Proposition 1 (ii) and (26), and, hence, consumption q and $\bar{q}$ also decrease in H by income effects. Equation (42) shows that these consumption changes lower welfare.

In a similar way, the effect of p^w on V (direct and indirect TOT effects) can be expressed as

$$\begin{aligned} \frac{\partial V}{\partial p^w} &= \frac{\theta p}{E} \frac{\partial q}{\partial p^w} + \frac{(1-\theta)\bar{p}}{E} \frac{\partial \bar{q}}{\partial p^w} \\ &= \frac{\theta\,(1-\theta)\,\tau \left[(p^{*w} - \theta)\,\tau + \theta \left(\frac{\bar{p}}{p}\right)^{\sigma}\right] \left[\theta\tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\right] \left[\theta p \left(\frac{\bar{p}}{p}\right)^{\sigma} + (1-\theta)\,\bar{p}\right]}{E\mathcal{A}^2 \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*}\right)} \end{aligned}$$

$$> \quad 0, \tag{43}$$

where the second equality is from Appendix A and the last inequality is from (12) again. A rise in the world price of home products improves the home TOT, and increases q and $\overline{q}$ by income effect. Equation (43) shows that these consumption changes improve welfare.

Unilaterally optimal τ_{RX} is determined by the balance of a nonpositive effect of $\bar{p}^*/p^*$ on V and a positive effect of p^w on V. Thus, the FOC of unilaterally optimal τ_{RX} is

$$\begin{aligned} \frac{dV}{d\tau_{RX}} &= \underbrace{\frac{\partial V}{\partial p^w}\frac{\partial p^w}{\partial \tau_{RX}}}_{+} + \underbrace{\frac{\partial V}{\partial \frac{\bar{p}^*}{p^*}}\frac{\partial \frac{\bar{p}^*}{p^*}}{\partial \tau_{RX}}}_{-\text{ or } 0} = 0 \\ &\Leftrightarrow \left[\theta\tau + (1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\right] p^w - (p^w - 1)(1-\theta)\,\sigma\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} = 0, \end{aligned} \tag{44}$$

where the last equation is from (42) and (43). Therefore, if τ_{RX} is optimal, it holds that

$$p^w = \frac{(1-\theta)\sigma}{(1-\theta)(\sigma-1) - \left(\frac{\bar{p}^*}{p^*}\right)^{-\sigma}\theta\tau}, \tag{45}$$

which represents the relationship between p^w and $\bar{p}^*/p^*$, where it must hold that

$$\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} > \frac{\theta\tau}{(1-\theta)(\sigma-1)} \tag{46}$$

since $p^w > 0$. In other words, (45) determines the unilaterally optimal world price of varieties produced in H (p^w), given the local relarive price of the varieties in F ($\bar{p}^*/p^*$). We easily show that unilaterally optimal p^w decreases in $\bar{p}^*/p^*$, and

$$\lim_{\frac{\bar{p}^*}{p^*} \to \left[\frac{\theta\tau}{(\sigma-1)(1-\theta)}\right]^{\frac{1}{\sigma}}} p^w = \infty, \quad \lim_{\frac{\bar{p}^*}{p^*} \to \infty} p^w = \frac{\sigma}{\sigma - 1}.$$

In Figure 1, such a relationship between p^w and $\bar{p}^*/p^*$ is depicted as a downward sloping curve, called τ_{RX}. The world price p^w means the TOT of H, while the local relative price $\bar{p}^*/p^*$ can be interpreted as the "protectiveness" of market F, because domestic demand for varieties produced in F is larger and export from H is reduced if $\bar{p}^*/p^*$ is higher. The τ_{RX} curve indicates that country H raises (resp. lowers) the world price of its products when the protectiveness of market F is milder (resp. severer). Since raising the world price of home varieties ($p^w = \sigma\tau_{RX}/(\sigma-1)$) also strengthens protectiveness in the foreign country ($\bar{p}^*/p^* = \tau\tau_{RX}\tau^*_{RI}$) via τ_{RX}, there can be room for country H to raise p^w when $\bar{p}^*/p^*$ is lower.

From (45), we know that the τ_{RX} curve goes up as θ increases as shown by a dashed curve in Figure 2. This is because the negative rent-shifting effect of τ_{RX}, which is represented by the second term of (44), is relatively larger for smaller countries. Number of varieties produced in each country plays a key role. When the local relative price $\bar{p}^*/p^*$ is lowered, larger importing countries substitute a larger variety of domestic products with a smaller variety of imported products. Therefore, smaller exporters can increase firm scale by attracting capital and shift rent to them more. This suggests that the second term in (44) is relatively larger for smaller countries; i.e., smaller countries have a stronger incentive to lower $\bar{p}^*/p^*$. Conversely, improving their terms of trade (i.e., the first term of (44)) is more beneficial for larger countries even if they reduce wage income due to rent shifting.

In sum, the following lemma is obtained:

Lemma 1 *Country H unilaterally sets τ_{RX} so that $p^w > \sigma/(\sigma-1)$, $\partial p^w/\partial(\bar{p}^*/p^*) < 0$, and $\partial p^w/\partial\theta > 0$.*

4.1.2 Unilaterally optimal τ_{RI}

From (27), the effect of τ_{RI} on V can be expressed as

$$\frac{dV}{d\tau_{RI}} = \frac{\partial V}{\partial \frac{\bar{p}}{p}} \underbrace{\frac{\partial \frac{\bar{p}}{p}}{\partial \tau_{RI}}}_{+} = \frac{\theta p}{E} \underbrace{\frac{\partial q}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau_{RI}}}_{+} + \frac{(1-\theta)\bar{p}}{E} \underbrace{\frac{\partial \bar{q}}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau_{RI}}}_{-}. \tag{47}$$

A rise in the local relative price $\bar{p}/p$ in H decreases import ($\bar{q}$) while it increases consumption of domestically produced varieties (q) due to import substitution. Unilaterally optimal τ_{RI} is determined by the balance of these two effects. Thus, the FOC of unilaterally optimal τ_{RI} is

$$\begin{aligned} \frac{dV}{d\tau_{RI}} &= 0 \Leftrightarrow \theta p \frac{\partial q}{\partial \frac{\bar{p}}{p}} + (1-\theta)\bar{p}\frac{\partial \bar{q}}{\partial \frac{\bar{p}}{p}} = 0 \\ &\Leftrightarrow \frac{\bar{p}}{p} (= \tau_{RI}\tau^*_{RX}\tau) = -\frac{\theta \frac{\partial q}{\partial \frac{\bar{p}}{p}}}{(1-\theta)\frac{\partial \bar{q}}{\partial \frac{\bar{p}}{p}}}, \end{aligned} \tag{48}$$

suggesting that the unilaterally optimal τ_{RI} is determined so that the local relative price $\bar{p}/p$ in H is equal to the the marginal rate of substitution of increased consumption of domestic products for decreased imports when $\bar{p}/p$ rises. By using the results of Appendix A, we obtain

$$\frac{\bar{p}}{p} = \left[1 + \frac{\theta\,(p^{*w}-1)\,\tau + (1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}(p^{*w}-1)}{\theta p^w \tau + (1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}}\right]\tau. \tag{49}$$

Plugging (45) into (49), we have a relationship among $\bar{p}/p$, p^{*w}, and $\bar{p}^*/p^*$ by deleting p^w:

$$\frac{\bar{p}}{p} = \left[p^{*w} - \frac{(p^{*w}-1)\left(\frac{\bar{p}^*}{p^*}\right)^{-\sigma}\theta\tau}{(1-\theta)(\sigma-1)} \right] \tau.$$

This shows that the unilaterally optimal $\bar{p}/p$ increases in p^{*w} and $\bar{p}^*/p^*$, and it is equal to τ when $p^{*w} = 1$ for any $\bar{p}^*/p^*$.

A similar equation holds for country F as follows:

$$\frac{\bar{p}^*}{p^*} = \left[p^{w} - \frac{(p^{w}-1)\left(\frac{\bar{p}}{p}\right)^{-\sigma}(1-\theta)\tau}{\theta(\sigma-1)} \right] \tau. \tag{50}$$

For a given $\bar{p}/p$, the relationship between $\bar{p}^*/p^*$ and p^w is depicted as an upward sloping curve in Figure 1, called τ_{RI}^*. The τ_{RI}^* curve indicates that country F raises (resp. lowers) its protectiveness of the local market when the world price of H's varieties is higher (resp. lower). When $p^w = 1 = r$, firms in H are unable to shift rent by exporting their varieties because they are sold at marginal costs in F. In that case, country F does not have incentives to increase local demand for varieties produced there by protection. Eventually, the local relative price is set to be equal to the ratio of marginal costs, so that we have $\bar{p}^*/p^* = \tau$ when $p^w = 1$.

From (50) and the counterpart of (48), we also have

$$-\frac{\partial}{\partial\theta}\frac{(1-\theta)\frac{\partial q^*}{\partial \frac{\bar{p}^*}{p^*}}}{\theta\frac{\partial \bar{q}^*}{\partial \frac{\bar{p}^*}{p^*}}} = \frac{\partial}{\partial\theta}\frac{\bar{p}^*}{p^*} = \frac{(p^w-1)\left(\frac{\bar{p}}{p}\right)^{-\sigma}\tau^2}{\theta^2(\sigma-1)} \geq 0,$$

showing that the τ_{RI}^* curve rotates clockwise around $(\bar{p}^*/p^*, p^w) = (\tau, 1)$ when θ increases for a given $\bar{p}/p$ as shown by dashed line in Figure 2, where note that country F is smaller by a rise in θ. In other words, smaller countries have a stronger incentive to have a higher local relative price of imported varieties. Number of varieties produced in each country plays a key role again. As the positiveness of the left-hand side of the above equation shows, smaller countries can expand output for their workers more due to import substitution since they must substitute many types of imported varieties with a limited number of domestic varieties. This increases wage income more because of rent shifting.

Lemma 2 *Given a $\bar{p}/p$, country F unilaterally sets τ_{RI}^* so that $\bar{p}^*/p^* \geq \tau$, $\partial(\bar{p}^*/p^*)/\partial p^w > 0$, and $\partial(\bar{p}^*/p^*)/\partial\theta > 0$.*

4.2 Nash policies

As discussed above, the home world price p^w depends on the local relative price in F $\bar{p}^*/p^*$, which is expressed as a negative relationship (τ_{RX} curve) in Figure 1. Meanwhile, the local relative price in F $\bar{p}^*/p^*$ also depends on the home world price p^w which is expressed as a positive relationship (τ^*_{RI} curve) in Figure 1. Therefore, the intersection $(\bar{p}^*/p^*, p^w)$ of the two curves, i.e., point N, corresponds to a Nash equilibrium, while point E corresponds to the laissez-faire equilibrium. Similary, the intersection $(\bar{p}/p, p^{*w})$ of τ^*_{RX} curve and τ_{RI} curve, which are counterparts of the τ_{RX} curve and the τ^*_{RI} curve, also corresponds to the Nash equilibrium. However, it is noteworthy that the τ^*_{RI} (resp. τ_{RI}) curve is depicted given a $\bar{p}/p$ (resp. $\bar{p}^*/p^*$), and, hence, we have to examine whether such a Nash equilibrium uniquely exists.

Also, Lemmas 1 and 2 claim that the larger country imposes taxes to have a higher world price of their products while the smaller country imposes taxes to have a higher local relative price of imported goods. Although the shift from N to N' in Figure 2 shows a case that the findings also hold at a Nash equilibrium, this may not necessarily hold, since Lemmas 1 and 2 are true if the other country does not change its tax rates. For example, the world price p^w may decrease from N to N' in Figure 2 if the upward shift of τ_{RX} curve is small. The local relative price $\bar{p}^*/p^*$ may also decrease from N to N' in Figure 2 if $\bar{p}/p$, which is fixed in Lemma 2, decreases from N to N', since $\bar{p}^*/p^*$ increases in $\bar{p}/p$ from (50). Nevertheless, we can show that the findings in Lemmas 1 and 2 also hold at a Nash equilibrium.

A Nash equilibrium $(\tau_{RX}, \tau^*_{RX}, \tau_{RI}, \tau^*_{RI})$ corresponds to a root $(p^w, p^{*w}, \bar{p}/p, \bar{p}^*/p^*)$ of simultaneous equations (45), (50), and their counterparts. Examining these equations, Appendix C shows the following proposition:

Proposition 3 *(i) A unique Nash equilibrium exists.*
(ii) The Nash equilibrium is inefficient and it is characterized by $\tau_{RX} > \max\{1/\tau^*_{RI}, 1\}$ *and* $\tau^*_{RX} > \max\{1/\tau_{RI}, 1\}$.
(iii) At the Nash equilibrium, a larger country imposes taxes to have a higher world price of its products, while a smaller country imposes taxes to have a higher local relative price of imported varieties.

Proposition 3 (i)(ii) claims that a Nash equilibrium $(\tau_{RX}, \tau^*_{RX}, \tau_{RI}, \tau^*_{RI})$ uniquely exists as shown by point N in Figure 1 and its counterpart. Since $\bar{p}/p > \tau$ and $\bar{p}^*/p^* > \tau$, domestic products are excessively consumed in each country at the equilibrium. Meanwhile, Proposition 3 (iii) shows that country-size effects on world prices and local relative prices at the Nash equilibrium are consistent with Lemmas 1 and 2.

Since any two of three tax instruments are sufficient from Corollary 1, we consider a case that the use of export policy instruments is constrained, i.e., $\tau_X = \tau_X^* = 1$, to make it easier to understand Proposition 3 (iii). This result shows that if $\theta > 1/2$, it holds that

$$p^w > p^{*w} \Leftrightarrow \tau_R > \tau_R^*$$

and

$$\frac{\bar{p}}{p} < \frac{\bar{p}^*}{p^*} \Leftrightarrow (\tau_R^*)^2 \tau_I < (\tau_R)^2 \tau_I^*.$$

These inequalities show that capital tax rates are higher in the larger country, while import tax rates may be higher or lower there compared to the smaller country. This shows that the result of Takatsuka and Zeng (2022) that smaller countries impose higher import tariffs may not hold if capital taxes are available and $\tau_X = \tau_X^* = 1$.[7]

4.3 Missing instruments

If any two of three tax instruments are available, the aforementioned Nash equilibrium can be achieved. However, if only one tax instrument is available, the Nash equilibrium achieved will differ.

4.3.1 Only import/export taxes

The case that import taxes or export taxes are only available is a purely special case of the previous mulitple-tax case in terms that one of FOCs (41) is still valid, while the other one degenerates. Thus, it is also possible to explain the equilibrium using diagrams similar to those used before.

When import taxes are only available, we have $\tau_{RX} = 1$ and $\tau_{RI}^* = \tau_I^*$, so that the τ_{RX} curve degenerates to $p^w = \sigma/(\sigma - 1)$, while the τ_{RI}^* curve still has the same characteristics as the curve in Figure 1. Therefore, the Nash equilibrium in this case is expressed as point N in Figure 3, which is inefficient since $\bar{p}^*/p^* > \tau$. If the size of H (θ) increases, the τ_{RI}^* curve rotates clockwise around $(\bar{p}^*/p^*, p^w) = (\tau, 1)$ as in Figure 2, so that the Nash equilibirum shifts from N to N' in Figure 3. This suggests that τ_I^* increases since $\bar{p}^*/p^* = \tau_I^* \tau$ increases, while $\bar{p}/p$ and τ_I decrease. This is the mechanism behind the result of Takatsuka and Zeng (2022) that smaller countries impose higher import tariffs.[8]

[7] A similar result is obtained in the case with $\tau_R = \tau_R^* = 1$.

[8] Takatsuka and Zeng (2022) explain that the reason is that imposing tariffs affect factor prices so that re-

Meanwhile, if export taxes are only available, we have $\tau_{RX} = \tau_X$ and $\tau_{RI}^* = 1$, so that the τ_{RI}^* curve degenerates to

$$\frac{\bar{p}^*}{p^*}(= \tau_X \tau) = \frac{(\sigma - 1)\,\tau}{\sigma} p^w,$$

while the τ_{RX} curve still has the same characteristics as the curve in Figure 1. Therefore, the Nash equilibrium in this case is expressed as point N in Figure 4, which is inefficient since $\bar{p}^*/p^* > \tau$ again. Meanwhile, point E corresponds to the laissez-faire equilibrium, which is Pareto efficient. If the size of H (θ) increases, the τ_{RX} curve goes up as in Figure 2, so that the Nash equilibirum shifts from N to N' in Figure 4. This suggests that larger countries impose higher export taxes to have higher world prices of their products, leading to the relative local prices of imported varieties are higher in smaller countries.

4.3.2 Only capital taxes

When capital taxes are only available, we have

$$\frac{\bar{p}}{p} = \tau_{RX}^* \tau_{RI} \tau = \frac{\tau_R^*}{\tau_R}\tau,\ \frac{\bar{p}^*}{p^*} = \tau_{RX} \tau_{RI}^* \tau = \frac{\tau_R}{\tau_R^*}\tau,$$
$$p^w = \frac{\tau_{RX}\sigma}{\sigma - 1} = \frac{\tau_R \sigma}{\sigma - 1},\ p^{*w} = \frac{\tau_{RX}^*\sigma}{\sigma - 1} = \frac{\tau_R^*\sigma}{\sigma - 1}.$$

so that, unlike previous cases, tax rates τ_R in H affect all of p^w, $\bar{p}^*/p^*$, and $\bar{p}/p$, simultaneously. Also, tax rates τ_R^* in F affect all of p^{*w}, $\bar{p}/p$, and $\bar{p}^*/p^*$, simultaneously. Therefore, the present case is not a purely special case of the previous mulitple-tax one, and the FOC of unilaterally optimal capital taxes in H can be expressed as

$$\frac{dV}{d\tau_R} = \underbrace{\frac{\partial V}{\partial \frac{\bar{p}^*}{p^*}} \frac{\partial \frac{\bar{p}^*}{p^*}}{\partial \tau_R}}_{-\text{ or } 0} + \underbrace{\frac{\partial V}{\partial p^w} \frac{\partial p^w}{\partial \tau_R}}_{+} + \frac{\partial V}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau_R} = 0. \tag{51}$$

This shows that unilaterally optimal τ_R is determined by the balance of a nonpositive effect via $\bar{p}^*/p^*$ (the first term), a positive effect via p^w (the second term), and an effect via $\bar{p}/p$ (the last term), whose sign is ambiguous. A similar condition holds for F.

From the discussion of unilaterally optimal τ_{RX} (Lemma 1), larger countries impose a higher $\tau_{RX} = \tau_X \tau_R$ to raise their TOT even if they have a negative rent-shifting effect. Intuitively,

duce foreign demand, but such a negative effect of tariffs is relatively weak for small countries (Takatsuka and Zeng, 2022, pp.178-180). However, the explanation in this paper based on the role of variety number in import substituion is more persuasive and easier to understand.

hence, the sum of the first and the second terms of (51) is higher for larger countries. Meanwhile, from the discussion of unilaterally optimal τ_{RI} (Lemma 2), smaller countries impose a higher $\tau_{RI} = \tau_I/\tau_R$ to raise the local relative price of imported goods. Intuitively, hence, the third term of (51) is also higher for larger countries. Therefore, larger countries may impose higher capital taxes. In fact, Appendix D shows the following proposition:

Proposition 4 *When capital taxes/subsidies are only available, two countries impose capital subsidies* $\tau_R = \tau_R^* \in ((\sigma - 1)/\sigma, 1)$ *at* $\theta = 1/2$. *When* $\theta \neq 1/2$, *the larger (resp. smaller) country imposes a higher (resp. lower) capital tax rate and excessively consumes imported (resp. domestic) products.*

In the field of capital tax competition, Bucovetsuky (1991) and Wilson (1991) show that larger countries tend to impose higher capital taxes since they can lower pre-tax capital rent and, thus, suppress increases in post-tax capital rent by doing so. Proposition 4 demonstrates that a similar result holds in the present rent-shifting model. However, the mechanism is quite different. As aforementioned, number of varieties produced in each country is a key. Smaller countries have a stronger incentive to raise (lower) local relative prices of imported (exported) varieties, since smaller (larger) countries substitute many (few) types of imported varieties supplied by larger (smaller) countries with few (many) types of domestic varieties, and, thus, smaller countries can increase firm scale and shift rent to the countries more.

Domestic policies (including capital taxes/subsidies) have never been the subject of direct GATT/WTO negotiations because of respecting national sovereignty. Therefore, the present case may suggest an outcome of simple free trade agreements (FTAs), under which two countries commit themselves to abolish import and export taxes (i.e., $\tau_I = \tau_I^* = \tau_X = \tau_X^* = 1$) and they unilaterally determine domestic production taxes.

Proposition 4 shows that the simple FTA is inefficient when $\theta \neq 1/2$. Meanwhile, if $\theta = 1/2$, by using (36) and Proposition 2, we have

$$V_N < V_{FB} = V_S,\ V_N^* < V_{FB}^* = V_S^*,$$

where V_S and V_S^* are indirect utility under the simple FTA in H and F, respectively. From the continuity of each equilibrium with respec to θ, it holds that $V_N < V_S$ and $V_N^* < V_S^*$ near $\theta = 1/2$. In other words, around $\theta = 1/2$, welfare in both countries can be improved by the simple FTA.

5 Politically Optimal Policies and Inefficiency Sources

5.1 Politically optimal policies

Bagwell and Staiger (1999, 2002) define *politically optimal (PO) tariffs* as those tariffs that would hypothetically be chosen by governments unilaterally, if they did not value the world price movements (direct and indirect TOT effects) induced by their unilateral tariff choices; namely,

$$\frac{\partial V}{\partial p^w} \equiv 0 \text{ and } \frac{\partial V^*}{\partial p^{*w}} \equiv 0.$$

Applying this concept to the present model, we define *PO policies* as any combination of tax rates $(\tau_{RX}, \tau_{RI}, \tau^*_{RX}, \tau^*_{RI})$ that simultaneously satisfies

$$\frac{\partial V}{\partial \frac{\bar{p}^*}{p^*}} \frac{\partial \frac{\bar{p}^*}{p^*}}{\partial \tau_{RX}} = \frac{\partial V^*}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau^*_{RX}} = 0, \tag{52}$$

$$\frac{\partial V}{\partial \frac{\bar{p}}{p}} \frac{\partial \frac{\bar{p}}{p}}{\partial \tau_{RI}} = \frac{\partial V^*}{\partial \frac{\bar{p}^*}{p^*}} \frac{\partial \frac{\bar{p}^*}{p^*}}{\partial \tau^*_{RI}} = 0. \tag{53}$$

Among the above conditions, (53) is the same as the second condition of Nash equilibrium (41) because τ_{RI} and τ^*_{RI} do not affect the world prices p^w and p^{*w}, respectively. Thus, τ^*_{RI} curve in Figure 1 still holds in the case of PO policies. However, (52) is different from the first condition of Nash equilibrium (41) since $\partial V/\partial p^w \equiv 0$ is supposed in PO policies.

Equation (42) and (52) show that, as long as $p^w > 1$, country H continues to decrease $\bar{p}^*/p^*$ (i.e., τ_{RX}) to attract capital (shift profit) from F to H. Thus, it eventually holds that $p^w \equiv \sigma\tau_{RX}/(\sigma - 1) = 1$, suggesting that $\tau_{RX} = (\sigma - 1)/\sigma$ regardless of F's policy choice. The dashed line in Figure 1 represents politically optimal p^w. Since similar results hold for F, we have

$$p^w = p^{*w} = 1 \Leftrightarrow \tau_{RX} = \tau^*_{RX} = \frac{\sigma - 1}{\sigma}$$

in politically optimal policies.

Furthermore, by using (49) and (50), (53) suggests that

$$\frac{\bar{p}^*}{p^*} = \frac{\bar{p}}{p} = \tau \Leftrightarrow \tau_{RX}\tau^*_{RI} = \tau^*_{RX}\tau_{RI} = 1 \Leftrightarrow \tau^*_{RI} = \tau_{RI} = \frac{\sigma}{\sigma - 1}$$

hold in PO policies. In other words, the politically optimal policies are determined at the intersection of the dashed line and τ^*_{RI} curve, i.e., P, in Figure 1. Comparing N and P, we know that the TOT manipulation increases both p^w and $\bar{p}^*/p^*$. Similar results hold for p^{*w} and $\bar{p}/p$. From Proposition 2, we have the following proposition:

Proposition 5 *PO policies implement the globally-efficient allocation maximizing a utilitalian social welfare function (30), which implies that the TOT motives impede the global efficiency as well as Pareto efficiency.*

Therefore, the TOT motives can be interpreted as a source of two inefficiencies. We start to discuss from point P in Figure 1, which can be achieved without TOT manipulations if policy instruments are sufficiently available. With TOT manipulations, each country raises the world price of each products. For example, given the initial $\bar{p}^*/p^* = \tau$, country H raises its world price p^w from point P to D, which is on τ_{RX} curve in Figure 1. Similary, given the initial $\bar{p}/p = \tau$, country F raises its world price p^{*w}, but the magnitude is smaller than that of p^w if F is smaller than H (i.e., $\theta > 1/2$) because smaller countries place more importance on the rent-shifting effect. Such changes of world prices influence international income distribution. At the initial point P, per-capita income measured by domestic variety in each country are equalized, resulting in the global efficiency as discussed in Section 3.2.1. However, from (36), we know that the gap of per-capita income measured by domestic variety is widen by $p^w > p^{*w} > 1$, and, thus, the global efficiency collapses.

At the next step, local relative prices are adjusted. Specifically, the rise in world prices induce protective behavior of trading partners since they are trying to prevent rent outflow by using import substitution. As a result, the local relative price $\bar{p}^*/p^*$ in country F rises while the world price p^w of H's products falls along τ_{RX} curve from point D to N in Figure 1. A similar change occurs in country H, so that $\bar{p}/p > \tau$ and $\bar{p}^*/p^* > \tau$ hold, and Pareto efficiency also collapses.

Following Bagwell and Staiger (1999, 2002), in a monopolistic competion model featuring delocation effects, Bagwell and Staiger (2015) define tariff changes that conform to *reciprocity* as those that bring about equal changes in the volume of each country's imports and exports when valued at existing world prices, and they show that such tariff changes contribute to enhancing efficiency. We have similar results in the present case, if we apply the principle of reciprocity considering domestic capital taxes as well as trade taxes.

Specifically, we define tax changes that conform to reciprocity as those satisfy

$$p_0^w \Delta M^* = p_0^{*w} \Delta M + \Delta\kappa, \tag{54}$$

where 0 (resp. 1) denotes variables before (resp. after) tax changes , $\Delta M^* \equiv n\tau\Delta\bar{q}^*$, $\Delta M \equiv$

$n^*\tau\Delta\overline{q}$, and $\Delta\kappa$ are the home country's changes in the volume of exports of goods, imports of goods, and imports of capital, respectively.

Considering that the source of inefficiency is only the TOT motive (Proposition 5), we expect that tax changes satisfying reciprocity are (partially) efficiency enhancing. In fact, we can show that (i) starting at the Nash equilibrium, the two countries must both gain from a small reduction of τ_{RX} and τ^*_{RX} satisfying reciprocity and a small reduction of τ_{RI} and τ^*_{RI}; (ii) once the political optimum is achieved, neither country can gain from a small change of τ_{RX} and τ^*_{RX} satisfying reciprocity and/or a small change of τ_{RI} and τ^*_{RI}.[9] However, it is noteworthy that, in contrast to the case of Bagwell and Staiger (2015), such tax changes should care about domestic capital taxes.

5.2 Welfare comparison

We turn to the welfare in each country. Compared to the globally-efficient allocation or the laissez-faire equilibrium, can the Nash equilibrium give higher welfare in a country?

In the symmetric case ($\theta = 1/2$), the answer is evident, i.e., no. In this case, we have $V_e = V_{FB} = V^*_{FB} = V^*_e$ from (40). In addition, from Proposition 3, we have

$$V_N < V_{FB} = V_e,$$

where V_N represents indirect utility in H under the Nash equilibrium and $V_N = V^*_N$ holds in the symmetric case. This inequality shows that the Nash equilibrium gives lower welfare in both countries, i.e., a prisoner's dilemma occurs. From (39) and the continuity of the equilibrium with respec to θ, it holds that

$$V_N < V_{FB} < V_e,\ V^*_N < V^*_e < V^*_{FB}$$

if $\theta > 1/2$ and $\theta \simeq 1/2$.

On the other extreme case, i.e., if $\theta \to 1$, Appendix E shows that

$$V_N > V_e > V_{FB} > V^*_{FB} > V^*_e > V^*_N.$$

In other words, at the Nash equilibrium, the larger country can enjoy higher welfare relative to the globally-efficient allocation and the laissez-faire equilibrium.

Summing up the above results, we have the following proposition:

[9] The proof is available upon request.

Proposition 6 *Suppose that H is the larger country (i.e., $\theta > 1/2$). Then,*
(i) when θ is around a half, $V_e > V_{FB} > V_N$ and $V_{FB}^* > V_e^* > V_N^*$;
(ii) when θ is large enough, $V_N > V_e > V_{FB} > V_{FB}^* > V_e^* > V_N^*$.

Under PO policies, home and foreign welfare are equal to V_{FB} and V_{FB}^*, respectively, from Proposition 5. Noting this fact, Proposition 6 is similar to the result in the standard perfectly competitive case (e.g., Bagwell and Staiger, 1999; 2016, Section 2.1). Namely, if two countries are sufficiently symmetric, PO policies generate greater-than-Nash welfares for both countries; otherwise, they can generate a lower-than-Nash welfare for a country (larger one). Therefore, in general, it is difficult to achieve the globally efficient allocation by any negotiations even with the reciprocity principle.

6 Conclusions

By using a two-country, two-factor, one-sector monopolistic-competition model, this paper examines characteristics and equilibrium of import, export, and capital taxes of two countries, and then clarify the inefficiency source of policy equilibrium. We have shown that the Nash equilibrium is inefficient, and a larger country imposes taxes to have a higher world price of its products, while a smaller country imposes taxes to have a higher local relative price of imported varieties. However, only without TOT manipulations global efficiency as well as Pareto efficiency can be achieved, because TOT motives induce protective behavior of trading partners by influencing the international income distribution.

As mentioned in Introduction, US President Trump recognizes that import tariffs are an effective measure to attract capital. The present paper implies that, if export and/or capital taxes are available, such recognitions do not necessarily lead the world in a bad direction. To attract capital, it is necessary to simultaneously lower the export (world) price of one's own goods, as this will alleviate the protective measures taken by other countries. The problem lies in the TOT motive of taxes, which, conversely to the above, requires raising export (world) prices, as this would strengthen the protective stance of the importing country. While the principle of reciprocity considering capital movement is partially efficiency enhancing, when there is a large disparity in scale between two countries, it is generally difficult to escape from an inefficient Nash equilibrium where a large country can achieve high welfare without any agreements.

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Appendix A: Effects of world and local relative prices on demand

From (17), and (18), we can easily show that

$$\frac{\partial \mathcal{A}}{\partial p^w} > 0, \ \frac{\partial \mathcal{A}}{\partial p^{*w}} > 0, \ \frac{\partial \mathcal{A}}{\partial \frac{\bar{p}}{p}} > 0, \ \frac{\partial \mathcal{A}}{\partial \frac{\bar{p}^*}{p^*}} > 0.$$

If $\bar{p}$ is fixed, we also have

$$\frac{\partial E}{\partial p^w} > 0, \ \frac{\partial E}{\partial p^{*w}} < 0, \ \frac{\partial E}{\partial \frac{\bar{p}^*}{p^*}} \leq 0, \tag{55}$$

where the first and the last inequalities are equivalent to (43) and (42). The equality of the last one holds if and only if $p^w = 1$ because the rent-shifting effect (as well as the consumption-wedge effect) vanishes at $p^w = 1$.

From (19) and (20), we have

$$\begin{aligned}
\frac{\partial \overline{q}}{\partial p^w} &= \frac{\theta (1-\theta) \tau \left[(p^{*w} - \theta) \tau + \theta \left(\frac{\bar{p}}{p} \right)^\sigma \right] \left[\theta \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right]}{\mathcal{A}^2 \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)} > 0, \\
\frac{\partial q}{\partial p^w} &= \left(\frac{\bar{p}}{p} \right)^\sigma \frac{\partial \overline{q}}{\partial p^w} > 0.
\end{aligned}$$

In a similar way, we can show that $\partial q / \partial p^{*w} < 0$ and $\partial \overline{q} / \partial p^{*w} < 0$.

Furthermore,

$$\begin{aligned}
\frac{\partial \overline{q}}{\partial \frac{\bar{p}^*}{p^*}} &= - \frac{(p^w - 1) \theta (1-\theta)^2 \sigma \tau \left(\frac{\bar{p}^*}{p^*} \right)^{\sigma - 1} \left[(p^{*w} - \theta) \tau + \theta \left(\frac{\bar{p}}{p} \right)^\sigma \right]}{\mathcal{A}^2 \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)} \leq 0 \\
\frac{\partial q}{\partial \frac{\bar{p}^*}{p^*}} &= \left(\frac{\bar{p}}{p} \right)^\sigma \frac{\partial \overline{q}}{\partial \frac{\bar{p}^*}{p^*}} \leq 0, \\
\frac{\partial \overline{q}}{\partial \frac{\bar{p}}{p}} &= - \frac{\theta^2 \sigma \left(\frac{\bar{p}}{p} \right)^{\sigma - 1} \left[\theta p^w \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right] \left[(p^w - 1 + \theta) \tau + (1-\theta) \left(\frac{\bar{p}}{p} \right)^\sigma \right]}{\mathcal{A}^2 \left(p^w, p^{*w}, \frac{\bar{p}}{p}, \frac{\bar{p}^*}{p^*} \right)} < 0, \\
\frac{\partial q}{\partial \frac{\bar{p}}{p}} &= - \frac{(1-\theta) \tau \left[\theta (p^w + p^{*w} - 1) \tau + (1-\theta) p^{*w} \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right]}{\theta \left[\theta p^w \tau + (1-\theta) \left(\frac{\bar{p}^*}{p^*} \right)^\sigma \right]} \frac{\partial \overline{q}}{\partial \frac{\bar{p}}{p}} > 0,
\end{aligned}$$

where the first and the second inequalities are directly from (55) and, hence, equalities hold if and only if $p^w = 1$ again. The third and the last inequalities suggest that raising $\bar{p}/p$ enables import substitution. Similar inequalities hold for q^* and $\overline{q}^*$.

Appendix B: Pareto-efficienct allocation

Let

$$\begin{aligned}
u &\equiv n (c)^{\frac{\sigma - 1}{\sigma}} + n^* (\overline{c})^{\frac{\sigma - 1}{\sigma}}, \\
u^* &\equiv n (\overline{c}^*)^{\frac{\sigma - 1}{\sigma}} + n^* (c^*)^{\frac{\sigma - 1}{\sigma}}.
\end{aligned}$$

Since varieties are supposed to be symmetric, a Pareto-efficient allocation maximizes the following Lagrangean:

$$\begin{aligned} L &= \ln u^{\frac{\sigma}{\sigma-1}} + \lambda\left[\kappa - n\theta c - n\left(1-\theta\right)\tau\overline{c}^{*}\right] \\ &\quad + \lambda^{*}\left[1-\kappa - n^{*}\left(1-\theta\right)c^{*} - n^{*}\theta\tau\overline{c}\right] + \mu\left(u^{*} - \overline{u}\right), \end{aligned}$$

where c (resp. $\overline{c}$) denotes the consumption of a worker in H for a variety produced in H (resp. F) and c^* (resp. $\overline{c}^*$) denotes the consumption of a worker in F for a variety produced in F (resp. H), $\overline{u}$ is a given level of utility of a worker in F, and λ, λ^*, and μ are Lagrange multipliers for corresponding constraints. Noting that n and n^* are fixed from (8). The FOCs are

$$\begin{aligned} &\frac{\partial L}{\partial c} = \frac{1}{u} n c^{\frac{-1}{\sigma}} - \lambda n\theta = 0 \Leftrightarrow c^{\frac{-1}{\sigma}} = \theta\lambda u, \\ &\frac{\partial L}{\partial \overline{c}} = \frac{1}{u} n^{*}\left(\overline{c}\right)^{\frac{-1}{\sigma}} - \lambda^{*} n^{*}\theta\tau = 0 \Leftrightarrow \left(\overline{c}\right)^{\frac{-1}{\sigma}} = \theta\tau\lambda^{*} u, \\ &\frac{\partial L}{\partial c^{*}} = \frac{\mu}{u^{*}} n^{*}\left(c^{*}\right)^{\frac{-1}{\sigma}} - \lambda^{*} n^{*}\left(1-\theta\right) = 0 \Leftrightarrow \left(c^{*}\right)^{\frac{-1}{\sigma}} = \frac{\left(1-\theta\right)\lambda^{*} u^{*}}{\mu}, \\ &\frac{\partial L}{\partial \overline{c}^{*}} = \frac{\mu}{u^{*}} n\left(\overline{c}^{*}\right)^{\frac{-1}{\sigma}} - \lambda n\left(1-\theta\right)\tau = 0 \Leftrightarrow \left(\overline{c}^{*}\right)^{\frac{-1}{\sigma}} = \frac{\left(1-\theta\right)\tau\lambda u^{*}}{\mu}, \\ &\frac{\partial L}{\partial \kappa} = \lambda - \lambda^{*} = 0. \end{aligned}$$

From these FOCs, five endogenous variables $(c, \overline{c}, c^*, \overline{c}^*, \kappa)$ in a Pareto-efficient allocation are determined by the following five equations:

$$\left(\frac{\overline{c}}{c}\right)^{-\frac{1}{\sigma}} = \tau, \quad \left(\frac{\overline{c}^{*}}{c^{*}}\right)^{-\frac{1}{\sigma}} = \tau, \tag{56}$$

$$\kappa = n\theta c + n\left(1-\theta\right)\tau\overline{c}^{*}, \tag{57}$$

$$1-\kappa = n^{*}\left(1-\theta\right)c^{*} + n^{*}\theta\tau\overline{c}, \tag{58}$$

$$\overline{u} = n\left(\overline{c}^{*}\right)^{\frac{\sigma-1}{\sigma}} + n^{*}\left(c^{*}\right)^{\frac{\sigma-1}{\sigma}}. \tag{59}$$

Equation (56) implies that the marginal rate of substitution is equal to the ratio of marginal costs. Thus, the above equations evidently hold in the laissez-faire equilibrium if $\overline{u}$ is the equilibrium level of utility of a worker in F. This suggests that the laissez-faire allocation is a Pareto-efficient one.

Appendix C: Proof of Proposition 3

(i) From (50) and the counterpart, we have

$$\left(\frac{\bar{p}}{p}\right)^{\sigma} = \frac{(1-\theta)\tau^2}{\theta(\sigma-1)} \frac{p^w - 1}{p^w\tau - \frac{\bar{p}^*}{p^*}}, \tag{60}$$

$$\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} = \frac{\theta\tau^2}{(1-\theta)(\sigma-1)} \frac{p^{*w} - 1}{p^{*w}\tau - \frac{\bar{p}}{p}}. \tag{61}$$

Plugging (45) and the counterpart into the above equations, we have

$$\left(\frac{\bar{p}}{p}\right)^{\sigma} = \frac{(1-\theta)\tau^2}{\theta(\sigma-1)} \frac{(1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} + \theta\tau}{\theta\tau\frac{\bar{p}^*}{p^*} - (1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\left[(\sigma-1)\frac{\bar{p}^*}{p^*} - \sigma\tau\right]}, \tag{62}$$

$$\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} = \frac{\theta\tau^2}{(1-\theta)(\sigma-1)} \frac{\theta\left(\frac{\bar{p}}{p}\right)^{\sigma} + (1-\theta)\tau}{(1-\theta)\tau\frac{\bar{p}}{p} - \theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\left[(\sigma-1)\frac{\bar{p}}{p} - \sigma\tau\right]}. \tag{63}$$

Noting (46) and the counterpart and solving the above equations for $(\bar{p}/p)^{\sigma}$ and $(\bar{p}^*/p^*)^{\sigma}$, we have

$$\left(\frac{\bar{p}}{p}\right)^{\sigma} = \frac{(1-\theta)\tau^2}{\theta} \frac{(\sigma-1)\frac{\bar{p}}{p} + \tau}{(\sigma-1)^2\frac{\bar{p}^*}{p^*}\left(\frac{\bar{p}}{p} - \tau\right) - \sigma\tau^2},$$

$$\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} = \frac{\theta\tau^2}{1-\theta} \frac{(\sigma-1)\frac{\bar{p}^*}{p^*} + \tau}{(\sigma-1)^2\frac{\bar{p}}{p}\left(\frac{\bar{p}^*}{p^*} - \tau\right) - \sigma\tau^2}.$$

From these equations, $\bar{p}/p$ (resp. $\bar{p}^*/p^*$) can be expressed as a function of $\bar{p}^*/p^*$ (resp. $\bar{p}/p$) as follows:

$$\frac{\bar{p}}{p} = \frac{\sigma(1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} + \theta\left[(\sigma-1)\frac{\bar{p}^*}{p^*} + \tau\right]}{(1-\theta)(\sigma-1)^2\left(\frac{\bar{p}^*}{p^*} - \tau\right)}\left(\frac{\bar{p}^*}{p^*}\right)^{-\sigma}\tau^2 \equiv \rho\left(\frac{\bar{p}^*}{p^*}\right), \tag{64}$$

$$\frac{\bar{p}^*}{p^*} = \frac{\sigma\theta\left(\frac{\bar{p}}{p}\right)^{\sigma} + (1-\theta)\left[(\sigma-1)\frac{\bar{p}}{p} + \tau\right]}{\theta(\sigma-1)^2\left(\frac{\bar{p}}{p} - \tau\right)}\left(\frac{\bar{p}}{p}\right)^{-\sigma}\tau^2 \equiv \rho^*\left(\frac{\bar{p}}{p}\right). \tag{65}$$

They show that

$$\frac{d\rho}{d\frac{\bar{p}^*}{p^*}} < 0, \quad \lim_{\frac{\bar{p}^*}{p^*}\to\tau} \rho = \infty, \quad \lim_{\frac{\bar{p}^*}{p^*}\to\infty} \rho = 0,$$

$$\frac{d\rho^*}{d\frac{\bar{p}}{p}} < 0, \quad \lim_{\frac{\bar{p}}{p}\to\tau} \rho^* = \infty, \quad \lim_{\frac{\bar{p}}{p}\to\infty} \rho^* = 0.$$

The first inequality is from

$$\begin{aligned}
\frac{d\rho}{d\frac{\bar{p}^*}{p^*}} &= -\frac{(1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma+1} + \theta\left[(\sigma-1)\left(\frac{\bar{p}^*}{p^*}\right)^2 - (\sigma-3)\left(\frac{\bar{p}^*}{p^*}\right)\tau - \tau^2\right]}{(1-\theta)(\sigma-1)^2\left(\frac{\bar{p}^*}{p^*}-\tau\right)^2\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma+1}}\sigma\tau^2 \\
&< -\frac{(1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma+1} + \theta\tau^2}{(1-\theta)(\sigma-1)^2\left(\frac{\bar{p}^*}{p^*}-\tau\right)^2\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma+1}} < 0,
\end{aligned}$$

where we use $\bar{p}^*/p^* \geq \tau$, which is shown in Figure 1. In a similar way, the second inequality can be shown. Solid curves in Figure 5 show the graphs of ρ and ρ^*, and the intersection (i.e., point N) determines a unique root $(\bar{p}/p, \bar{p}^*/p^*)$, suggesting that $\bar{p}/p > \tau$ and $\bar{p}^*/p^* > \tau$. Since p^w and p^{*w} are also uniquely determined from (45) and its counterpart, a unique Nash equilibrium $(\tau_{RX}, \tau_{RX}^*, \tau_{RI}, \tau_{RI}^*,)$ exists.

(ii) As shown in Figure 1, at the Nash equilibrium (i.e., point N), it holds that

$$p^w > \frac{\sigma}{\sigma-1},\ \frac{\bar{p}^*}{p^*} > \tau,\ p^{*w} > \frac{\sigma}{\sigma-1},\ \frac{\bar{p}}{p} > \tau.$$

Thus, we immediately have

$$\tau_{RX} > 1,\ \tau_{RX} > \frac{1}{\tau_{RI}^*},\ \tau_{RX}^* > 1,\ \tau_{RX}^* > \frac{1}{\tau_{RI}},$$

leading to the inequalities in Proposition 3(ii). From Propositions 2(i), we know that the Nash equilibrium is inefficient.

(iii) From (64) and (65), we have

$$\frac{\partial\rho}{\partial\theta} > 0,\ \frac{\partial\rho^*}{\partial\theta} < 0. \tag{66}$$

This suggests that, as θ increase, solid curves of ρ and ρ^* in Figure 5 shift to dashed curves, respectively, and, thus, the Nash equilibrium also shifts from N to N', leading to the latter result of Proposition 3(iii).

Differentiating (62) and (63) with respet to θ and solving for $\partial(\bar{p}/p)/\partial\theta$ and $\partial(\bar{p}^*/p^*)/\partial\theta$ at $\theta = 1/2$, we have

$$\left.\frac{\partial\frac{\bar{p}^*}{p^*}}{\partial\theta}\right|_{\theta=1/2} = -\left.\frac{\partial\frac{\bar{p}}{p}}{\partial\theta}\right|_{\theta=1/2} = \frac{4\tau\left[(\sigma-1)\frac{\bar{p}}{p}+\tau\right]\frac{\bar{p}}{p}}{(\sigma-1)^2\left[\left(\frac{\bar{p}}{p}\right)^{\sigma+1} + \frac{\bar{p}}{p}\tau\right] + \sigma\tau^2} > 0,$$

which is consistent with the above result. Plugging these equations into derivatives of (45) and its counterpart with respet to θ, we have

$$\left.\frac{\partial p^w}{\partial \theta}\right|_{\theta=1/2} = -\left.\frac{\partial p^{*w}}{\partial \theta}\right|_{\theta=1/2} = \frac{4\tau\sigma\left(\sigma-1\right)\left(\frac{\bar{p}}{p}\right)^{\sigma+1}}{\left(\sigma-1\right)^2\left[\left(\frac{\bar{p}}{p}\right)^{\sigma+1}+\frac{\bar{p}}{p}\tau\right]+\sigma\tau^2}\frac{1}{\left(\sigma-1\right)\left(\frac{\bar{p}}{p}\right)^{\sigma}-\tau} > 0, \quad (67)$$

where the last inequality is from the counterpart of (46). Meanwhile, from (45), it holds that $p^w = p^{*w}$ if and only if

$$\left(\frac{\bar{p}}{p}\right)^{\sigma} = \frac{(1-\theta)^2}{\theta^2}\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma},$$

suggesting

$$\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma} = \frac{\theta\tau^2}{(1-\theta)(\sigma-1)}\frac{p^w-1}{p^w\tau-\frac{\bar{p}^*}{p^*}}$$

from (60). Comparing this equation with (61), it holds that $p^w = p^{*w}$ if and only if $\bar{p}/p = \bar{p}^*/p^*$, i.e., $\theta = 1/2$ from (66). With (67), it holds that $p^w > p^{*w}$ for any $\theta > 1/2$, leading to the former result of Proposition 3(iii). ■

Appendix D: Only Capital Taxes

From (42), (43), and (48), we have

$$\frac{dV}{d\tau_R} = \frac{\theta\left(1-\theta\right)\sigma\mathcal{B}\left(\tau_R,\tau_R^*;\theta\right)}{E\left(\sigma-1\right)\mathcal{A}^2\left(p^w,p^{*w},\frac{\bar{p}}{p},\frac{\bar{p}^*}{p^*}\right)},$$

where

$$\begin{aligned}\mathcal{B} \;=\;& \tau\left[\left(p^{*w}-\theta\right)\tau+\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\right]\left[\theta p\left(\frac{\bar{p}}{p}\right)^{\sigma}+\left(1-\theta\right)\bar{p}\right]\left[\theta\tau+\left(1-\theta\right)\left(\sigma-1\right)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\left(\frac{1}{\tau_R}-1\right)\right]\\ &-\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\sigma\left[\left(p^w-1+\theta\right)\tau+\left(1-\theta\right)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\right]\\ &\times\left\{\left[\theta(p^w+p^{*w}-1)\tau+\left(1-\theta\right)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}p^{*w}\right]\tau-\left[\theta p^w\tau+\left(1-\theta\right)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\right]\frac{\bar{p}}{p}\right\}. \qquad (68)\end{aligned}$$

Thus, the FOC of unilaterally optimal τ_R is $\mathcal{B} = 0$. In a similar manner, we know that the FOC of unilaterally optimal τ_R^* is $\mathcal{B}^* = 0$, where

$$\mathcal{B}^* \;=\; \tau\left[\left(p^w-1+\theta\right)\tau+\left(1-\theta\right)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\right]\left[\left(1-\theta\right)p^*\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}+\theta\bar{p}^*\right]$$

$$\times\left[(1-\theta)\,\tau+\theta\,(\sigma-1)\left(\frac{\bar{p}}{p}\right)^{\sigma}\left(\frac{1}{\tau_R^*}-1\right)\right]$$
$$-(1-\theta)\left(\frac{\bar{p}^*}{p^*}\right)^{\sigma}\sigma\left[(p^{*w}-\theta)\,\tau+\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\right]\times$$
$$\left\{\left[(1-\theta)\,(p^w+p^{*w}-1)\tau+\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}p^w\right]\tau-\left[(1-\theta)\,p^{*w}\tau+\theta\left(\frac{\bar{p}}{p}\right)^{\sigma}\right]\frac{\bar{p}^*}{p^*}\right\}. \quad (69)$$

In the case of $\theta = 1/2$, we evidently have $\tau_R = \tau_R^*$ and the Nash equilibrium is globally efficient from Proposition 2. In addition, from the above two FOCs, we have

$$\tau_R = \frac{2(\sigma-1)\tau^{\sigma}}{(2\sigma-1)\tau^{\sigma}-\tau} \in \left(\frac{\sigma-1}{\sigma}, 1\right). \quad (70)$$

Furthermore, applying the Implicit Function Theorem to (68) and (69) and using (70), we have

$$\left.\frac{d\tau_R}{d\theta}-\frac{d\tau_R^*}{d\theta}\right|_{\theta=\frac{1}{2}} = \frac{8\,(\sigma-1)\,(\tau^{\sigma}-\tau)}{(5\sigma-2)\tau+(6\sigma^2-7\sigma+2)\tau^{\sigma}} > 0. \quad (71)$$

Meanwhile, it holds that

$$\mathcal{B}-\mathcal{B}^*|_{\tau_R^*=\tau_R} = \frac{(2\theta-1)\sigma\tau\,(\tau^{\sigma}-\tau)\,[\theta\tau+(1-\theta)\tau^{\sigma}]\,[(1-\theta)\tau+\theta\tau^{\sigma}]\,\tau_R}{\sigma-1} = 0,$$

implying that, if $\tau_R^* = \tau_R$ holds, we have $\theta = 1/2$. In other words, $\tau_R^* = \tau_R$ never holds for any $\theta \neq 1/2$. With (70) and (71), if and only if $\theta > 1/2$, we have $\tau_R > \tau_R^*$ and, hence, the larger country has a higher capital tax rate. Since $\bar{p}/p < \tau$ and $\bar{p}^*/p^* > \tau$ at $\theta > 1/2$, the larger (resp. smaller) country excessively consumes imported (resp. domestic) products. ■

Appendix E: Proof of Proposition 6(ii)

From (35) and (37), we have

$$\lim_{\theta\to 1} V_{FB} = \lim_{\theta\to 1} V_e = 0,$$
$$\lim_{\theta\to 1}\frac{\partial V_{FB}}{\partial\theta} = \frac{1-\phi}{\sigma-1} > \frac{1-\phi}{\sigma\,(\sigma-1)} = \lim_{\theta\to 1}\frac{\partial V_e}{\partial\theta} > 0, \quad (72)$$

which are consistent with (39). Under the Nash equilibrium, we have

$$\lim_{\theta\to 1}\tau_{RX} = \infty, \quad \lim_{\theta\to 1}\tau_{RI} = 1,$$
$$\lim_{\theta\to 1}\tau_{RX}^* = 1, \quad \lim_{\theta\to 1}\tau_{RI}^* = \frac{\sigma-1}{\sigma}, \quad (73)$$

from (45), (49), and their counterpart in F.

Plugging (3) and (10) into (22), we have

$$\begin{aligned} V &= \ln \left[\theta\left(\tau_{RI}\right)^{1-\sigma}+(1-\theta)(\tau\tau^*_{RX})^{1-\sigma}\right]^{\frac{1}{\sigma-1}} \frac{\mathcal{C}(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI})}{\mathcal{D}(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI})} \\ &\equiv \widehat{V}\left(\theta,\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}\right). \end{aligned} \tag{74}$$

where

$$\begin{aligned} \mathcal{C}(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}) \equiv\ & \tau^*_{RX}\left(\tau_{RX}\right)^{\sigma-1}\left[(1-\theta)+\frac{\theta}{\phi}\left(\frac{\tau^*_{RX}}{\tau_{RI}}\right)^{\sigma-1}\right] \\ & \times\left\{\frac{(1-\theta)(\sigma-1)}{\phi}\left(\frac{\tau_{RX}}{\tau^*_{RI}}\right)^{\sigma}+\left[\sigma\tau_{RX}-(1-\theta)(\sigma-1)\right]\right\} \end{aligned}$$

and

$$\begin{aligned} \mathcal{D}(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}) \equiv\ & \theta(1-\theta)(\tau_{RX})^{\sigma-1}\left[\sigma\tau^*_{RX}-(\sigma-1)\right]+\theta\sigma(\tau_{RX})^{\sigma}\left[(1-\theta)+\frac{\theta}{\phi}\left(\frac{\tau^*_{RX}}{\tau_{RI}}\right)^{\sigma}\right] \\ & +\frac{(1-\theta)}{\phi}\left(\frac{\tau_{RX}}{\tau^*_{RI}}\right)^{\sigma}(\tau_{RX})^{\sigma-1}\left[(1-\theta)\sigma\tau^*_{RX}+\frac{\theta}{\phi}(\sigma-1)\left(\frac{\tau^*_{RX}}{\tau_{RI}}\right)^{\sigma}\right]. \end{aligned}$$

In a similar way, we obtain

$$\begin{aligned} V^* &= \ln \left[\theta\left(\tau\tau_{RX}\right)^{1-\sigma}+(1-\theta)\left(\tau^*_{RI}\right)^{1-\sigma}\right]^{\frac{1}{\sigma-1}} \frac{\mathcal{C}^*(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI})}{\mathcal{D}^*(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI})} \\ &\equiv \widehat{V}^*\left(\theta,\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}\right). \end{aligned} \tag{75}$$

where

$$\begin{aligned} \mathcal{C}^*(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}) =\ & \tau_{RX}\left(\tau^*_{RX}\right)^{\sigma-1}\left[\theta+\frac{1-\theta}{\phi}\left(\frac{\tau_{RX}}{\tau^*_{RI}}\right)^{\sigma-1}\right] \\ & \times\left\{\frac{\theta(\sigma-1)}{\phi}\left(\frac{\tau^*_{RX}}{\tau_{RI}}\right)^{\sigma}+\left[\sigma\tau^*_{RX}-\theta(\sigma-1)\right]\right\} \end{aligned}$$

and

$$\begin{aligned} \mathcal{D}^*(\tau_{RX},\tau_{RI},\tau^*_{RX},\tau^*_{RI}) \equiv\ & \theta(1-\theta)(\tau^*_{RX})^{\sigma-1}\left[\sigma\tau_{RX}-(\sigma-1)\right] \\ & +(1-\theta)\sigma\left(\tau^*_{RX}\right)^{\sigma}\left[\theta+\frac{1-\theta}{\phi}\left(\frac{\tau_{RX}}{\tau^*_{RI}}\right)^{\sigma}\right] \\ & +\frac{\theta}{\phi}\left(\frac{\tau^*_{RX}}{\tau_{RI}}\right)^{\sigma}(\tau^*_{RX})^{\sigma-1}\left[\theta\sigma\tau_{RX}+\frac{1-\theta}{\phi}(\sigma-1)\left(\frac{\tau_{RX}}{\tau^*_{RI}}\right)^{\sigma}\right]. \end{aligned}$$

Let $\tau_{RX}(\theta)$, $\tau_{RI}(\theta)$, $\tau^*_{RX}(\theta)$, and $\tau^*_{RI}(\theta)$ be the Nash-equilibrium tax rates. Then, V_N can be expressed as

$$V_N = \widehat{V}\left(\theta, \tau_{RX}(\theta), \tau_{RI}(\theta), \tau_{RX}^*(\theta), \tau_{RI}^*(\theta)\right).$$

The FOCs of τ_{RX} and τ_{RI} imply that

$$\frac{\partial \widehat{V}}{\partial \tau_{RX}} = \frac{\partial \widehat{V}}{\partial \tau_{RI}} = 0,$$

suggesting that

$$\frac{d\widehat{V}}{d\theta} = \frac{\partial \widehat{V}}{\partial \theta} + \frac{\partial \widehat{V}}{\partial \tau_{RX}^*}\frac{d\tau_{RX}^*}{d\theta} + \frac{\partial \widehat{V}}{\partial \tau_{RI}^*}\frac{d\tau_{RI}^*}{d\theta}.$$

In addition, from (74), we have

$$\lim_{\theta\to 1}\frac{\partial \widehat{V}}{\partial \tau_{RX}^*} = \lim_{\theta\to 1}\frac{\partial \widehat{V}}{\partial \tau_{RI}^*} = 0,$$

and, hence, it holds that

$$\begin{aligned}
\lim_{\theta\to 1}\frac{dV_N}{d\theta} &= \lim_{\theta\to 1}\frac{d\widehat{V}}{d\theta} = \lim_{\theta\to 1}\left[\frac{\partial \widehat{V}}{\partial \theta} + \frac{\partial \widehat{V}}{\partial \tau_{RX}^*}\frac{d\tau_{RX}^*}{d\theta} + \frac{\partial \widehat{V}}{\partial \tau_{IX}^*}\frac{d\tau_{IX}^*}{d\theta}\right] \\
&= \lim_{\theta\to 1}\frac{\partial \widehat{V}}{\partial \theta},
\end{aligned}$$

Using this fact, (74), (72), and (73), we have

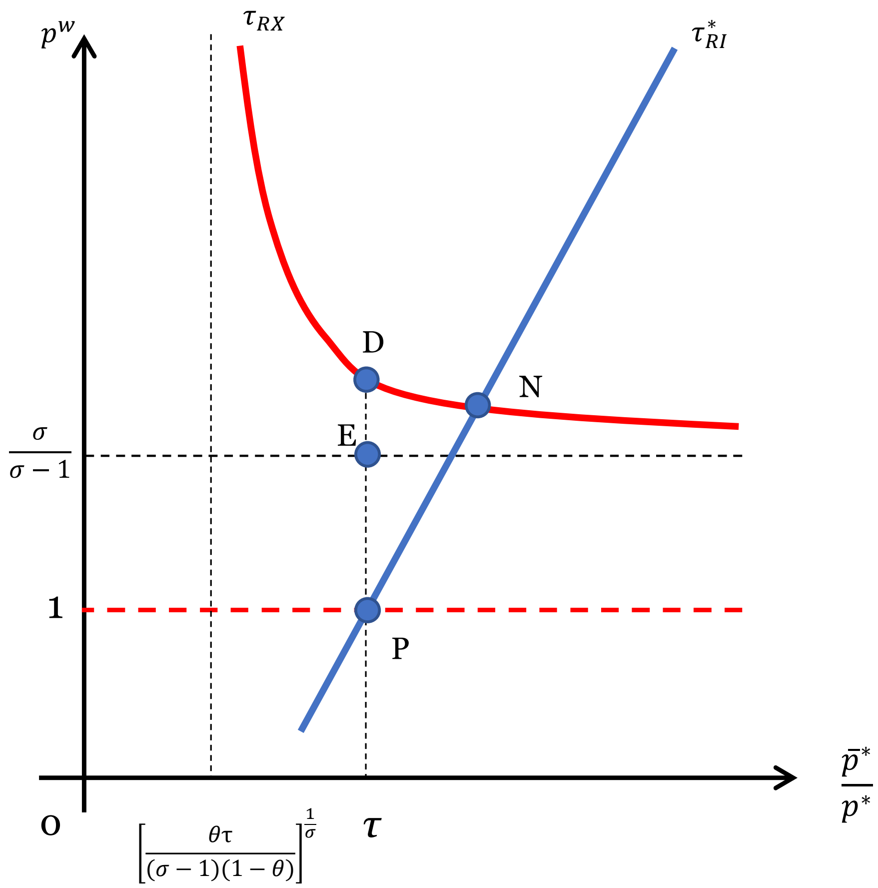
$$\begin{aligned}
\lim_{\theta\to 1}\frac{d\left(V_N - V_e\right)}{d\theta} &= \lim_{\theta\to 1}\frac{\partial \widehat{V}}{\partial \theta} - \lim_{\theta\to 1}\frac{dV_e}{d\theta} \\
&= \lim_{\theta\to 1}\frac{\left[1 - \sigma\left(\sigma - 2\right)\left(\tau_{RX} - 1\right) + \left(\sigma - 1 - \sigma\tau_{RX}\right)\phi\right]}{\sigma\left(\sigma - 1\right)\tau_{RX}} - \frac{1-\phi}{\sigma\left(\sigma - 1\right)} \\
&= -\frac{\sigma - 2 + \phi}{\sigma - 1} - \frac{1-\phi}{\sigma\left(\sigma - 1\right)} \\
&= -\frac{\sigma - 1 + \phi}{\sigma} < 0,
\end{aligned}$$

Equation (74) also shows that $\lim_{\theta\to 1} V_N = 0$, suggesting that $V_N > V_e$ near $\theta = 1$.

Finally, plugging (49) and (73) into (75), we have

$$\begin{aligned}
\lim_{\theta\to 1} V_N^* &= \lim_{\theta\to 1}\ln\frac{\sigma - 1 + \phi}{\sigma\tau\tau_{RX}} = -\infty \\
&< \ln\phi^{\frac{1}{\sigma-1}}\left(\frac{\sigma - 1 + \phi}{\sigma}\right) = \lim_{\theta\to 1} V_e^*,
\end{aligned}$$

where the last equality is from (38). ■



p^w
τ_{RX}
τ^*_{RI}
D
N
E
$\frac{\sigma}{\sigma-1}$
1
P
O
$\left[\frac{\theta\tau}{(\sigma-1)(1-\theta)}\right]^{\frac{1}{\sigma}}$
τ
$\frac{\bar{p}^*}{p^*}$

Figure 1

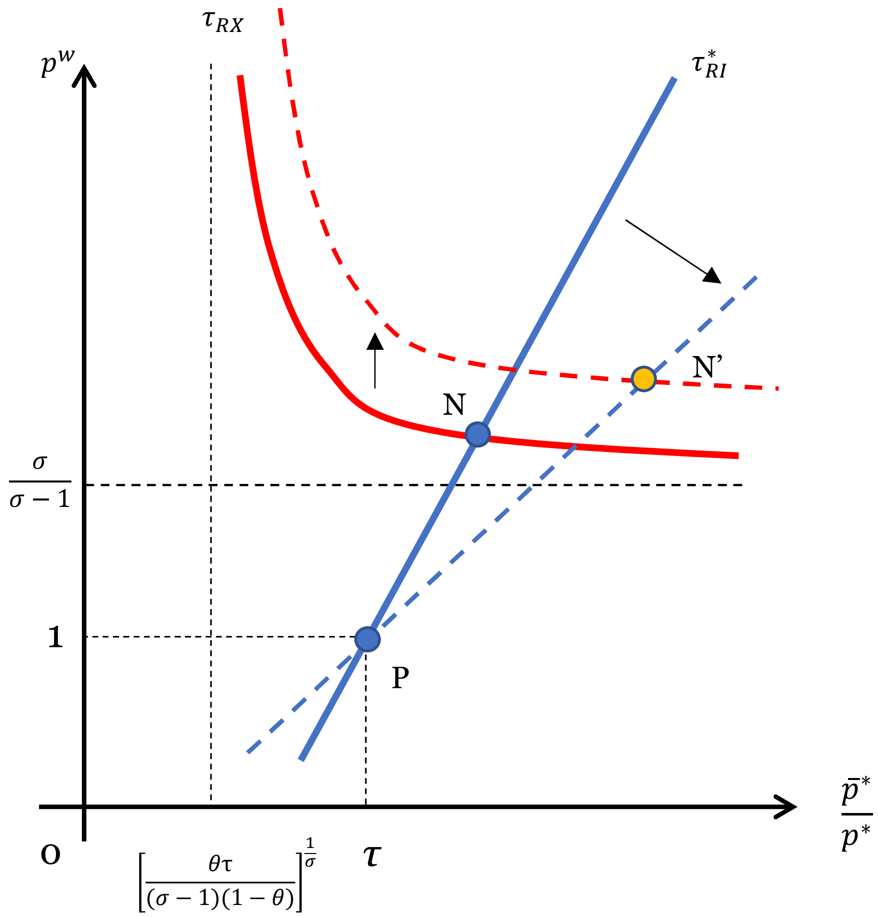

τ_{RX}
p^w
τ_{RI}^*
N'
N
$\frac{\sigma}{\sigma-1}$
1
P
$\frac{\bar{p}^*}{p^*}$
O
$\left[\frac{\theta\tau}{(\sigma-1)(1-\theta)}\right]^{\frac{1}{\sigma}}$
τ

Figure 2

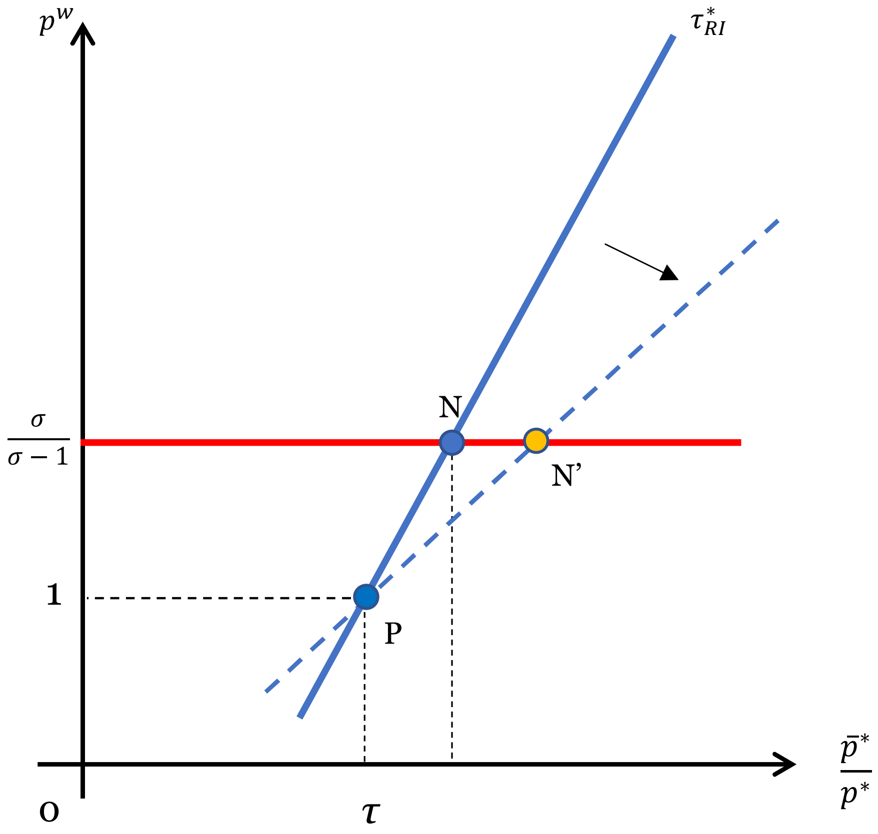
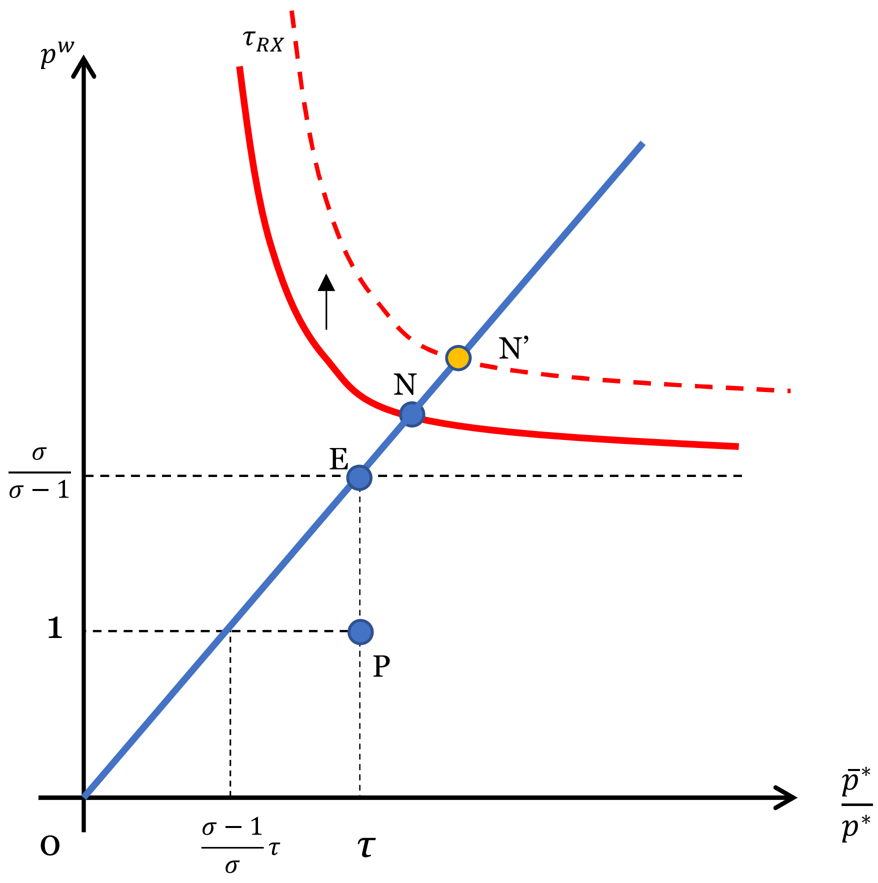


Figure 3



p^w
τ_{RX}
N'
N
E
$\frac{\sigma}{\sigma-1}$
1
P
O
$\frac{\sigma-1}{\sigma}\tau$
τ
$\frac{\bar{p}^*}{p^*}$

Figure 4

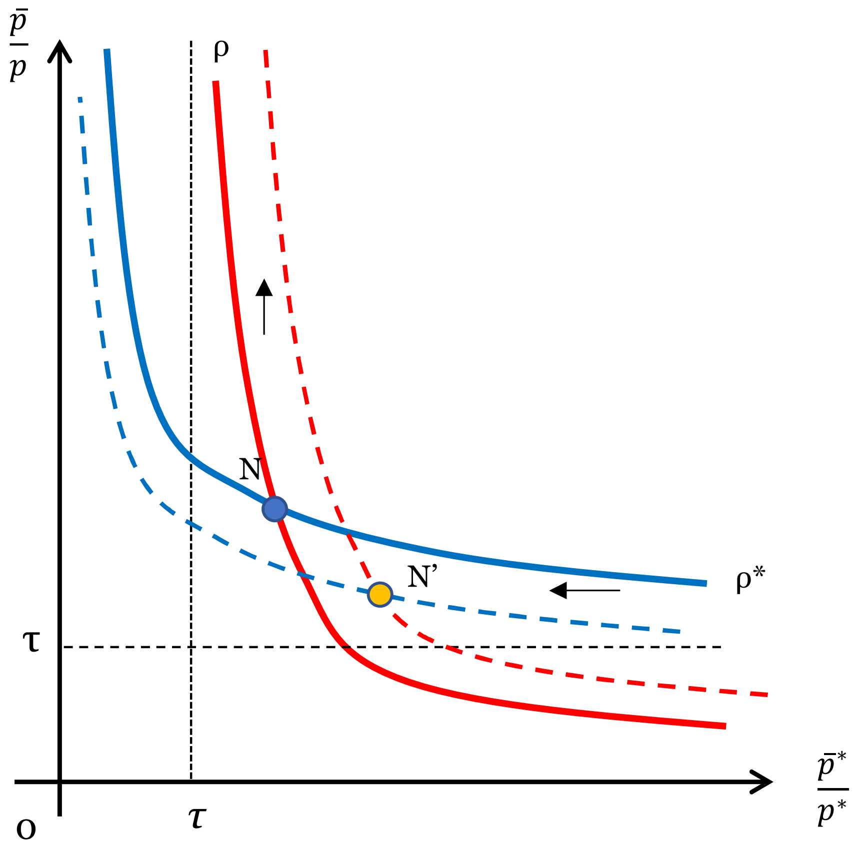


Figure 5