Spatial Herfindahl–Hirschman Index (sHHI): A New Density-Aware Market Competition Index Separating Inter-Type Rivalry from Intra-Type Cannibalisation

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Abstract: We introduce a new spatial competition index that blends the classical Herfindahl–Hirschman Index (HHI) with ideas from ecological competition. The metric is density-aware: it uses Voronoi catchments to assign population to the nearest unit (e.g., store or school) and summarises concentration at both the unit and type levels. We also extend the index to separate inter- and intra-type competition. As a proof of concept, we use simulations to test the behaviour of the index for different numbers of units and population-density schemes. The simulations show intuitive patterns: concentration rises when units are sited unevenly or when people are clustered; doing both amplifies the effect. Case studies from Warsaw—Żabka as a monopoly-like network, the Biedronka–Lidl duopoly, and public primary schools—are based on real data and serve as an empirical proof of concept. We also frame location choice as a practical optimisation problem for planners (minimise concentration) and managers (strengthen a focal type), showing how a density-aware approach changes the diagnosis and guides siting decisions. Results are robust across data and modelling choices and are reproducible with open data.

Keywords: spatial competition; density-aware index; Voronoi tessellation; spatial HHI (Herfindahl–Hirschman); inter- vs intra-type competition; location optimisation; urban services

JEL code: R12; R15; C63

Highlights:

- Density-aware Spatial HHI using Voronoi and population-weighted catchments.

- Decomposes inter-network rivalry vs intra-network cannibalisation.
- Validated on synthetic cities and Warsaw; robust to scale/resolution.
- Siting algorithm links the index to planning and retail decisions.
- Open, reproducible geocomputation (R/sf), ready for policy scenarios.

Introduction

Measuring market power without reference to space works reasonably well in industries where products and customers are mobile or markets are national in scope. It works much less well in urban services, where access is inherently local, customers are tied to neighbourhoods, and firms compete through where they place their units. Standard concentration metrics such as the Herfindahl–Hirschman Index (HHI) compress market structure into a single number by aggregating shares across firms, but they treat the economy as if it were placeless and populations as if they were uniformly smeared over a map (Hirschman, 1945; Herfindahl, 1950). In cities, neither assumption holds. People cluster; travel costs are asymmetric; small siting changes alter who wins a given street corner. An index that ignores spatial structure can therefore understate dominance in dense pockets and overstate rivalry in sparse ones. In short, if the question is "who serves whom, and where?", a density-aware, spatially explicit metric is needed.

The case for density is straightforward. A square kilometre in a high-rise district does not represent the same potential demand as a square kilometre of parkland. Two brands can operate the same number of units and yet capture very different effective market shares if one brand happens to cover a handful of dense blocks. Likewise, two cities with the same number of units per capita can exhibit sharply different competitive conditions because their people are distributed differently in space. A spatial index should, at minimum, (i) allocate potential demand to nearby units in a transparent way and (ii) weight territorial claims by people rather than land area. Only then does a concentration measure reflect the actual geometry of access.

On this point, ecology provides both language and tools. For decades, ecologists have treated space as a first-class determinant of interaction. Individuals occupy territories, compete more intensely with neighbours, and adjust survival to local crowding. A workhorse of this literature is the Voronoi (Dirichlet) tessellation, which partitions a habitat into zones of influence: each location is assigned to its nearest individual (Aurenhammer, 1991; Okabe et al., 2009). This simple geometric device turns qualitative proximity into a measurable map of inter- and intra-specific rivalry (Wiegand & Moloney, 2014). Translating that idea to urban markets is natural. Replace organisms with units (a store, a clinic, a school), and replace nutrients with population density (or age-specific need, income, etc.). The resulting catchments formalise "who is closest to whom" and let us compute demand-weighted shares that respect the city's uneven density field. The same ecological distinction between inter-specific (between species) and intra-specific (within species) competition also maps neatly to markets: rivalry between types (e.g., chains, operators, formats) versus crowding within a type (cannibalisation among a chain's own units).

Building on these insights, we propose a Spatial HHI that is explicitly density-aware. In our construction, population is assigned to the nearest unit using Voronoi catchments (with an optional gravity/Huff variant), producing unit-level shares that sum to one over the city (Huff, 1963; Okabe et al., 2009). Aggregating unit shares delivers type-level shares (e.g., by chain or operator). From these, we compute concentration at two layers: a unit-level spatial HHI that captures fragmentation across individual units, and a type-level spatial HHI that captures dominance across types. Because these two views can point in different directions—many small units can still belong to a dominant type—we introduce a nested measure that separates inter-type dominance from intra-type crowding. This nested index preserves interpretability: high inter-type values flag areas where one type dominates its rivals; high intra-type values flag places where a type mainly cannibalises itself.

We build a density-aware Spatial HHI by computing unit shares from Voronoi-assigned population (nearest-facility baseline, optional gravity/Huff), and study its behaviour in controlled simulations that vary siting (regular vs. random) and demand (uniform vs. clustered). We then extend the measure to a nested index that separates inter-type dominance from intra-type crowding, and operationalise siting as a tractable optimisation problem solved with a greedy-plus-swap heuristic. Finally, we demonstrate the approach on Warsaw case studies—Żabka, Biedronka vs. Lidl, and public primary schools—using open population grids and OSM facilities, and, where relevant, network distances (Aurenhammer, 1991; Huff, 1963; Church & ReVelle, 1974; Hakimi, 1964; Haklay & Weber, 2008; Boeing, 2017).

1. From Aspatial Indices to Spatial Ecology: Toward a Density-Based Competition Metric

The empirical study of competition has long relied on a triad of indicator families that, while conceptually distinct, share a common aspatial foundation. Structural measures—headed by the Herfindahl–Hirschman Index (HHI) and concentration ratios (C5, C10)—distil market power into aggregates of firm shares (McAuliffe, 2015). Behavioural measures rooted in price–cost margins—most famously the Lerner Index—translate the price–marginal-cost gap into an elasticity-based indicator of monopoly power (Shaffer & Spierdijk, 2017). A third, hybrid class embeds cost-margin information in aggregated concentration frameworks, linking mark-ups with market shares under standard input and demand conditions. These indicators are serviceable in antitrust and industry studies, yet their use is often sector-dependent: HHI is routine in telecommunications and airlines with high fixed costs and entry barriers (Affeldt et al., 2021), C5/C10 are common in fast-moving consumer goods, and Lerner-style metrics are applied in regulated sectors such as pharmaceuticals and banking (Soldatos, 2021). Still, all three families implicitly treat space as frictionless: firms are presumed to exert identical pressure on one another regardless of distance or the density that ties consumers to place.

The spatial blind spot is striking given the long tradition of location theory. From Hotelling's linear city (Hotelling, 1929), through Salop's circular variant (Salop, 1979) and related spatial models (Gabszewicz & Thisse, 1979), to contemporary spatial econometric and semiparametric competition games, theorists routinely embed distance, transport costs and neighbourhood effects in strategic interaction. Recent empirical work extends these insights with agent-based models and spatial simulations that accommodate non-uniform consumer distributions (Miura & Shiroishi, 2018; Ottino-Loffler et al., 2016). Yet the resulting metrics seldom migrate into practice. Regulators still clear mergers on national-level HHIs; retail chains choose sites with heuristics rather than transparent peer-pressure indices; and urban planners lack benchmarks to judge whether the mix of pharmacies, emergency wards or secondary schools in a district is socially optimal. As noted in recent reviews, traditional measures "often overlook the spatial dimension of accessibility and consumer distribution, particularly in dense urban settings," creating demand for geographically sensitive indicators (Zhang et al., 2020).

Understanding spatial competition at granular scales is increasingly important. Spatial interactions between consumers and firms, and the structure of urban agglomerations, shape location decisions and effective market power (Kopczewska, 2020, 2021). In urban planning, fine-grained analyses inform zoning, infrastructure and service provision, helping address spatial inequalities in access (Batty, 2013; Neutens et al., 2010). Crucially, the growing availability of fine-grained geospatial data—address-level points of interest, raster-based population grids and open road/building layers—enables density-sensitive competition metrics and spatial disaggregation beyond administrative containers. With advances in data science and geostatistics, analysts can now assign market power and competitive interaction to explicit territories and simulate their dynamics, expanding both resolution and policy relevance (van Leeuwen & Lijesen, 2012). This matters for three constituencies. Firms in saturated cities must balance cannibalisation against coverage; without a spatial index, location decisions rely on intuition or black-box models. Regulators face local monopolies that slip beneath national thresholds; a density-aware measure would reveal pockets of excessive power invisible to conventional HHIs. Public authorities require diagnostics that capture how population density and facility proximity jointly determine access. Early applications—e.g., Voronoi-based, population-weighted market shares for Warsaw supermarkets—demonstrate feasibility and policy relevance, showing how outlets in dense districts exert disproportionate influence and how spatially weighted concentration can diverge sharply from its non-spatial counterpart (Turolla, 2016).

Ecology offers both conceptual legitimacy and methodological guidance. Spatial competition has long been routine in ecology, where researchers map how plants compete for light or predators for territory. The workhorse is the Voronoi (Dirichlet) tessellation, which partitions space into mutually exclusive zones of influence: each location is assigned to its nearest seed (Aurenhammer, 1991). This geometry translates physical spacing into quantitative measures of neighbourhood interaction and crowding (Pielou, 1962; Pacala & Silander, 1985). Its simplicity has propelled it beyond ecology: network-Voronoi cells speed up k-nearest-neighbour queries (Kolahdouzan & Shahabi, 2004), Voronoi masks guide image segmentation (Cheddad et al., 2008), and tessellation-based perturbations support modern machine learning pipelines (Yildirim et al., 2021). The ecological precedent thus motivates a spatial competition index for firms: like organisms vying for resources, retailers and service providers vie for purchasing power embedded in residential density. A Voronoi-based approach allocates each consumer to the nearest facility and weights territorial claims by demographic mass, transforming qualitative insights of location theory into a quantitative analogue of territoriality.

Although scholars have proposed Voronoi- or Delaunay-based zones of influence and spatial concentration measures, innovations remain fragmented and case-specific. For instance, Delaunay-derived metrics in Queensland supermarkets reveal heterogeneous geographic competition across chains (Beare & Szakiel, 2009), while spatial diversion ratios and localised concentration indices can outperform national HHIs in retail mergers (Ellickson et al., 2020; Roig-Tierno et al., 2018). By fusing ecological theory (Tilman, 1988) with spatial econometrics, a unified, density-based spatial competition index can supply urban economists, planners and strategists with a demographically grounded indicator that supports store siting, antitrust scrutiny and equitable service provision (Birkin et al., 2002; Apparicio et al., 2008).

Finally, the ecological distinction between inter-specific and intra-specific competition provides a direct template for markets. Inter-specific (between-species) rivalry maps to inter-type competition (e.g., between chains, operators or formats), while intra-specific (within-species) rivalry maps to intra-type crowding—i.e., cannibalisation among a single chain's units. Any spatial index that respects territories and density should therefore separate these layers: diagnosing where a type dominates outsiders (inter-type) versus where it mainly steals from itself (intra-type). Embedding this separation in a single, interpretable framework is the key step from ecological inspiration to an operational tool for competition analysis.