Innovation through intra and inter-regional interaction in economic geography*

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Abstract

We explore the mechanisms through which knowledge spillovers influence the spatial distribution of economic activities in a two-region economic geography model with vertical innovations. The chance of innovation depends on the *related variety*, i.e., the importance of interaction between researchers within the same region rather than across different regions. If related variety is high, knowledge spillovers are more localized and a higher economic integration leads to progressive agglomeration. If related variety is low, economic activities may re-disperse after an initial phase of agglomeration as integration gradually increases, because firms relocating to the smaller region leverage the concentrated knowledge base of the larger region to enhance their chance of innovation. The transition between spatial patterns may exhibit very diverse qualitative properties depending on the particular level of related variety.

Keywords: Innovation; inter-regional spillovers; economic geography; re-dispersion;
JEL codes: R10, R12, R23.

*We are thankful to Steven Bond-Smith, Sofia B. S. D. Castro, João Correia da Silva, Pascal Mossay, Pietro Peretto, Anna Rubinchik and Jorge Saraiva for very useful comments and suggestions. We would also like to thank the participants at the Fourth International Workshop "Market Studies and Spatial Economics", Université Libre de Bruxelles, ECARES, at the 8th Euro-African Conference on Finance and Economics / Mediterranean Workshop in Economic Theory, Faculty of Economics, University of Porto, and at the 7th Geography of Innovation Conference, University of Manchester. Funding Information: Japan Society for the Promotion of Science Grant/Award Number: 21K04299; Fundação para a Ciência e Tecnologia UIDB/04105/2020, UIDB/00731/2020 and PTDC/EGE-ECO/30080/2017. Part of this research was developed while José M. Gaspar was a researcher at the Research Centre in Management and Economics, Católica Porto Business School, Universidade Católica Portuguesa, through the grant CEECIND/02741/2017.

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1 Introduction

Geographical economics emphasizes the role of endogenous forces in shaping lasting and sizable economic agglomerations in the modern economy. However, in its aim to explain the spatial distribution of economic activities, there has been a narrow focus on pecuniary externalities through trade linkages,[1] with a few exceptions such as the notable works of Martin and Ottaviano (1999) and Martin and Ottaviano (2001). But even in the latter, the geography that drives innovation only goes through market interactions. However, in order to understand the processes of agglomeration and (de)-industrialization, it is crucial to develop theories that explore the interaction among multiple spatial linkages. We aim to fill this gap by explaining how the intra-regional and inter-regional interaction between researchers impacts knowledge creation and affects the spatial distribution of agents. Moreover, we study how the weight of such interactions interplays with economic integration to understand the evolution of the space economy as trade barriers decrease.

We combine the typical pecuniary externalities in geographical economics (Krugman, 1991b; Fujita et al., 1999; Baldwin et al., 2003) with the spatial diffusion of knowledge spawned from intra-regional and inter-regional interactions to infer about the circular causality between migration and knowledge flows. In the present work, production of knowledge affects the firms' capacity to innovate, which in turn allows the production of higher quality manufactured varieties in a region. The chance of successful innovations depends on the spatial distribution of mobile agents in the economy. Therefore, it is assumed that regional knowledge levels transfer imperfectly between regions, depending on the *related variety* (Frenken et al., 2007), i.e., the relative importance of interaction between agents within the same region rather than between different regions – which depends on several factors such as cognitive proximity, cultural factors, diversity of skills and abilities, among others. We assume further that the increasing complexity of each variety is offset by the available regional quality levels (cf. Section 3.3) generated from knowledge spillovers. Our modeling strategy is such that indirect utility differentials, which govern the migration of mobile agents between regions, are determined solely by trade linkages and by the spatial dimension of

[1] Reviews on the literature of geographical economics, or "new economic geography" models, are provided by e.g. Fujita et al. (1999), Baldwin et al. (2003), Fujita and Thisse (2013), Behrens and Robert-Nicoud (2011) and Gaspar (2018).

regional interaction (cf. Section 3.4). We thus avoid the explicit use of dynamics for the innovation process. This confers great analytical tractability and allows us to focus on spatial outcomes as a result of pecuniary factors and the *economic geography* of knowledge spillovers (Bond-Smith, 2021).

The level of related variety has been shown in the literature to vary with the degree of novelty in the innovation process (Mascarini et al., 2023). Accordingly, we explore different qualitative scenarios and provide a rich gallery of possibilities that depend on the level of related variety and how it affects spatial outcomes as economies become more integrated.

If related variety is low, then an increase in economic integration from a very low level initially fosters agglomeration in a single region. However, above a certain threshold, more integration leads to more symmetric spatial outcomes, because firms find it worthwhile to relocate to the smaller region in order to benefit from the sizeable pool of agents in the larger region, which increases the chance of innovation and expected profits in the smaller region. Therefore, when related variety is low (but not too low),[2] our model accounts for a (complete) re-dispersion of economic activities after an initial phase of agglomeration. That is, we are able to uncover a bell-shaped relation between economic integration and spatial development (Fujita and Thisse, 2013). In this case, knowledge spillovers constitute a local dispersion force that becomes relatively stronger as economic integration brings about the withering of agglomeration forces due to increasing returns to scale in manufacturing. But the process of (de)-industrialization is far from trivial; depending on the specific level of related variety, the relationship between economic integration and spatial imbalances occurs with very different qualitative properties, not yet described in the literature.[3] In fact, we show that increases in related variety are linked to more pronounced agglomerations in the industrialization process, and to more sudden (discontinuous) jumps towards dispersed outcomes, particularly for intermediate levels of economic integration.

By contrast, when related variety is high, knowledge spillovers become more localized and generate an additional agglomeration force. Re-dispersion becomes altogether impossible because within-region interaction is too important for innovation to make any deviation to a smaller region

[2] For exceedingly low values of related variety, the symmetric dispersion equilibrium is the unique stable equilibrium in the entire range of economic integration.

[3] To the best of our knowledge.

worthwhile. In this case, a higher trade integration increases the net agglomeration forces and promotes agglomeration, in line with the findings of Martin and Ottaviano (2001) and Martin and Ottaviano (1999). Moreover, an increase in the level of related variety benefits agglomeration and leads to greater spatial inequality.

We find that the complete re-dispersion of economic activities requires related variety to be low in the largest region, irrespective of the particular functional form for the chance of innovation. This captures the idea of a congestive effect in the production of knowledge, which makes innovation less likely in more congested regions. Thus, the existence of a re-dispersion phase hinges on the spatial dispersive or agglomerative nature of technological spillovers in the largest region.

The rest of the paper is organized as follows. Section 2 discusses some related literature. Section 3 introduces the spatial economic model and describes its short-run general equilibrium. Section 4 deals with the existence and stability of long-run equilibria. Section 5 studies the relationship between economic integration and spatial outcomes. In Section 6, we provide some comparative statics and a more general form for the firms' success of innovation. Finally, Section 7 is left for discussion and concluding remarks.

2 Literature Review

The formation of significant economic clusters is intricately linked to agglomeration economies, as the concentration of economic agents in urban centers yields a range of positive effects, both pecuniary and non-pecuniary (Duranton and Puga, 2004, 2015). The spatial economy can thus be seen as the result of trade-offs between such scale economies and the transportation costs incurred by the movement of goods, people, and information (Proost and Thisse, 2019).

However, most theories of endogenous agglmeration do not address how knowledge externalities operate between locations because they deliberately focus on trade linkages as *the* mode of inter-location interaction to investigate the role of pecuniary externalities, as well as to ensure tractability (Fujita and Mori, 2005). Such a narrow focus enables researchers to design a micro-founded model based on the firms' perspective using modern tools of economic theory. Nonetheless, it is true that further development in geographical economics requires modeling the creation and transfer of knowledge to infer how it affects the location of economic activities (Fujita and Thisse, 2013). In particular, the role of K-linkages has become increasingly relevant in the economic geography literature. Building upon pioneering works such as Berliant and Fujita (2008, 2009, 2012), one should hope that a new comprehensive economic geography theory fully integrates the linkage effects among consumers and producers and K-linkages in space. According to Fujita (2007), geography is an essential feature of knowledge creation and diffusion. For instance, people residing in the same region interact more frequently and thus contribute to develop the same, regional set of cultural ideas. However, while each region tends to develop its unique culture, the economy as a whole evolves according to the synergy that results from the interaction across different regions (i.e., different cultures). That is, according to Duranton and Puga (2001), knowledge creation and location are inter-dependent. Berliant and Fujita (2012) developed a model of spatial knowledge interactions and showed that higher cultural diversity, albeit hindering communication, promotes the productivity of knowledge creation. This corroborates the empirical findings of Ottaviano and Peri (2006, 2008). Ottaviano and Prarolo (2009) show how improvements in the communication between different cultures fosters the creation of multicultural cities in which cultural diversity promotes productivity. This happens because better communication allows different communities to interact and benefit from productive externalities without risking losing their cultural identities.

Therefore, combining the typical pecuniary externalities in geographical economics with the spatial diffusion of knowledge spawned from intra-regional and inter-regional interactions alike is important if we want to infer about an eventual circular causality between migration and the circulation of knowledge. In other words, geographical economics may shed light on the importance of knowledge exchanged between different regions through trade networks compared to "internally" generated knowledge.

Reinforcing the importance of heterogeneity in knowledge, it is crucial to discern about the *relatedness* of variety. This relatedness measures the cognitive proximity and distance between sectors that allows for a higher intensity of knowledge spillovers. According to Frenken et al. (2007), a higher *related variety* increases the inter-sectoral knowledge spillovers between sectors that are technologically related. This potentially adds a new dimension to the role of heterogeneity and location in the creation and diffusion of knowledge. Tavassoli and Carbonara (2014) have tested the role of knowledge intensity and variety using regional data for Sweden and found evidence that different types of related variety have an important weight in innovation. The level of related variety depends on the degree of novelty in the innovation process. A low related variety is typically associated with technological breakthroughs in innovation, while a high related variety is more important for incremental innovations (Castaldi et al., 2015; Innocenti et al., 2022). This confirms the relevance of the spatial determinants of innovation and knowledge creation.

In this paper, knowledge creation and diffusion amounts to vertical innovations in the manufacturing sector, which in turn draws from the literature on endogenous growth. Particularly in Schumpeterian growth theory, innovations that affect the quality of produced goods or a firm's cost efficiency are usually driven by stochastic processes, because the production of knowledge involves some sort of uncertainty. Therefore, we may think of quality as a proxy for a given firm's stock of knowledge. In the growth models developed e.g. by Aghion and Howitt (1990, 1998), Young (1998), Peretto (1998), Howitt (1999), or more recently Dinopoulos and Segerstrom (2010), innovations occur with a probability that depends on factors such as the amount of the firm's research effort, the common pool of public knowledge available to all firms, and the individual firm's quality level. Introducing geography and worker mobility in these frameworks allows the innovation success to also depend on the magnitude of regional interaction through the exchange of ideas between workers and producers alike among regions. If cognitive proximity is more important for innovation, then related variety is high and knowledge spillovers become more localized, thus promoting agglomeration (Baldwin and Martin, 2004). If interaction with foreign agents is more important, related variety is low and is likely to induce the dispersion of economic activities.

We follow the literature of *Quantitative Spatial Economics* (Redding and Rossi-Hansberg, 2017; Behrens and Murata, 2021; Kleinman et al., 2023) and purposefully abstract from modeling growth or the explicit use of dynamics in the innovation sector, thus ensuring analytical tractability. However, knowledge transfers *imperfectly* between regions, in accordance with the findings in the seminal work of Audretsch and Feldman (1996). Nonetheless, our modeling of the innovation sector renders the model scale-neutral. As such, its spatial outcomes are driven by the spatial nature of knowledge spillovers (and by its interplay with pecuniary externalities), rather than by implicit assumptions about scale returns in the innovation sector.[4]

[4]For a comprehensive discussion on how these implicit assumptions regarding returns to scale generate mistaken conclusions in geographical economics, we refer the reader to Bond-Smith (2021).

3 The model

The following is an analytically solvable footloose entrepreneur model (Baldwin et al., 2003). The economy is comprised of two regions indexed by $i = \{1, 2\}$, two kinds of labour, two productive sectors and one R&D sector. There is a unit mass of (skilled) inter-regionally mobile agents (which we dub scientists henceforth) and a mass $l \equiv \lambda > 0$ of (unskilled) immobile workers (just workers, for short) which are assumed to be evenly distributed across both regions, i.e., $l_i = \frac{\lambda}{2}$ $(i = 1, 2)$. The amount of scientists in region 1 is given by $z_1 \equiv z \in [0, 1]$ and fully describes the spatial distribution of agents in the economy.

3.1 Demand

The utility function of a consumer located in region i is given by:

$$u_i = \mu \ln \left(\frac{M_i}{\mu} \right) + B_i, \quad \mu > 0 \tag{1}$$

where B_i is the numéraire good produced under perfect competition and constant returns to scale. This good is produced one-for-one using L workers and its price is set to unity as is the wage paid to workers. The quality-augmented CES composite M_i is given by:

$$M_i = \left[\sum_{j=1}^{2} \int_{s \in S} \left(\delta_{ij}^{m(s)} d_{ij}(m, s) \right)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $d_{ij}(m, s)$ is the demand for manufactures in region i produced in region j for a given variety $s \in S$ of quality $m \in \{1, ..., k\}$, k being the highest quality rung achieved for any variety, S is the mass of varieties in region i and σ is the elasticity of substitution between any two varieties. The parameter $\delta > 1$ indexes the step size of quality improvements in region i after a successful innovation and k is the leading quality grade for any given variety s. Since $\delta^{m(s)}$ is increasing in m, the utility in (2) reflects the fact that consumers have a preference for higher quality (Dinopoulos and Segerstrom, 2006). Each consumer purchases only the good with the lowest quality adjusted price, $p_i(s)/\delta_i^m$. If any two goods have the same quality adjusted price, we assume that consumers will only buy the highest quality good (Dinopoulos and Segerstorm 2006; 2010; Davis and Şener, 2012). The firm responsible for each quality improvement for a variety s retains a monopoly right to produce that variety at the highest quality possible, i.e., k. Therefore, if the quality rungs $m = 1, ..., k$ have been reached, the *kth* innovator is the sole source of the good of variety s with the quality level δ^k (Barro and Sala-i Martin, 2004).

Since individual incomes depend on the distribution of labour activities, we have $y_i = 1$ for the workers, and $y_i = w_i$, which is the the compensation paid to the scientists that engage in research. Therefore, the regional income is given by:

$$Y_i = \frac{\lambda}{2} + w_i z_i.$$

The individual budget constraint is given by $B_i + P_i M_i = y_i + \bar{B}$, where $\bar{B}$ is the initial endowment of the numéraire and P_i is the regional price index. Since total expenditure on manufactures must equal P_i times the quantity of composite M_i, agents maximize (1) subject to the following budget constraint:

$$B_i + \sum_{j=1}^{2} \int_{s \in S} p_{ij}(s) d_{ij}(s) ds = y_i + \bar{B},$$

which yields the following optimal individual demands:

$$d_{ij}(s) = \mu \frac{a_i(s) p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}}, \; B_i = y_i + \bar{B} - \mu, \; M_i = \mu P_i^{-1}, \tag{3}$$

where $a_i(s) = \delta_i^{m(s)(\sigma-1)}$ is just an alternative measure of a variety s's quality in region i and P_i is the quality adjusted price index given by:

$$P_i = \left[\sum_{j=1}^{2} \int_{s \in S} a_{ij}(s) p_{ij}(s)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}. \tag{4}$$

We assume that $\bar{B} > \mu$ in order to assure that both types of goods are consumed. From (1) and (3), we obtain the indirect utility:

$$v_i = y_i - \mu \ln P_i - \mu + \bar{B}. \tag{5}$$

3.2 Manufacturing firms

For each firm, there is a variable input requirement of β workers. A manufacturing firm in region i thus faces the following cost:

$$C_i(q_i(s)) = \beta q_i(s), \tag{6}$$

where q_i is total production by a firm in region i.

Trade of manufactures between regions is burdened by transportation costs of the iceberg type. Let the iceberg costs $\tau_{ij} \in (1, +\infty)$ denote the number of units that must be shipped at region i for each unit that is delivered at region j. We have $\tau_{ij} = \tau_j \in (1, \infty)$ for $i \neq j$ and $\tau_{ij} = 1$ otherwise. The quantity produced by a firm in region i is thus given by:

$$q_i(s) = \sum_{j=1}^{2} \tau_{ij} d_{ij} \left(1 + z_j\right).$$

The profit of a manufacturing firm with the leading grade k of variety s in region i is given by:

$$\begin{aligned}\tilde{\pi}_i \equiv \pi_i(s \equiv s_k) &= \sum_{j=1}^{2} p_{ij}(s_k) d_{ij}(s_k) \left(\frac{\lambda}{2} + z_j\right) - \beta q_i(s_k) \\ &= \sum_{j=1}^{2} \left(p_{ij}(s_k) - \tau_{ij}\beta\right) d_{ij}(s_k) \left(\frac{\lambda}{2} + z_j\right),\end{aligned} \tag{7}$$

where s_k denotes the highest quality of a given variety s. Given (7) and the optimal individual demand in (3), the firm's profit maximizing price is the usual mark-up over marginal cost:

$$p_{ij} = \frac{\sigma}{\sigma - 1} \tau_{ij} \beta, \tag{8}$$

which does not depend on the quality of the firm's variety s. Under (4), the regional quality adjusted price index in (4) becomes:

$$P_i = \frac{\beta\sigma}{\sigma - 1} \left(\sum_{j=1}^{2} \phi_{ij} A_j n_j\right)^{\frac{1}{1-\sigma}}, \tag{9}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1)$ is the *freeness of trade* and n_i is the number of manufacturing firms (and manufactured varieties), and:

$$A_i = \sum_{s=1}^{n_i} a_i(s),$$

is the aggregate quality index for region i. The latter also constitutes a measure of the aggregate regional knowledge level.

Consider now that the aggregate knowledge level in one region is a public good such that it becomes readily available to the other region. Then $A_i = \max\{A_1, A_2\}$ so that both regions are able to reach the highest aggregate knowledge level available. This leads to the following assumption.

Assumption 1. $A_1 = A_2 = A$.

The rationale behind Assumption 1 is basically to eliminate first-nature advantages between regions (i.e., regional asymmetries). This not only makes the analysis much simpler but also allows us to focus on conveying the main message of the paper, which is the effects of regional interaction on the spatial distribution of economic activities. Under Assumption 1, the regional price index becomes simply:

$$P_i = \frac{\beta\sigma}{\sigma - 1}\left(A\sum_{j=1}^{2}\phi_{ij}n_j\right)^{\frac{1}{1-\sigma}}. \tag{10}$$

The higher the aggregate quality, the lower the cost of living in region i.

3.3 R&D sector

The modelling of the R&D sector builds on the literature of Schumpeterian growth theory (Aghion and Howitt, 1998). We assume that there is free entry in the R&D sector for each variety s. Innovation involves uncertainty. In order to innovate, a firm decides *ex-ante* to employ α scientists in the R&D sector. In doing so, a firm producing variety s at region i reaches a leading quality grade k with instantaneous probability:

$$\Phi_i(s) = \min\left\{\frac{bz_i + (1-b)z_j}{a_i(s)}\gamma A, 1\right\}, \tag{11}$$

where $b \in (0, 1)$ is the *weight of intra-regional interaction in the chance of innovation* and defines the importance of the exchange of ideas between researchers alike among regions, and $\gamma > 0$ is an efficiency parameter. We are assuming that the lowest quality grade possible is $\underline{a} > 1$ so that $a_i(s) > 1$, for any s. It is also reasonable to assume that the firm's research success is greater the higher the level of aggregate knowledge in region i, $A_i = A$, available to all firms alike. Finally, we assume that the innovation rate is decreasing in the complexity of each product, as measured by its quality level $a_i(s)$ (Li, 2003; Dinopoulos and Segerstrom, 2010).

It is worthwhile explaining how the underlying specification governing the innovation process depends on the magnitude of the interactions (or lack of them) between different "sets of ideas". Analogously to the interpretation of Berliant and Fujita (2012), we implicitly assume that each region holds its own set of ideas (or culture). Therefore, production of knowledge (which amounts to innovation), depends on the amount of "within region" interaction among scientists, but also on the interaction with scientists hailing from a different region. As such, we can say that b is sort of a measure of the *related variety* of knowledge (Frenken et al., 2007).

A higher related variety means that innovation benefits more from a regional common pool of ideas, that is, from the interaction between researchers and workers in the same region. Specifically, when $b \in (1/2, 1)$ – related variety is *high* –, we have that $\Phi_i(s)$ is increasing in z so that innovation in region 1 is more likely the more researchers live there. In this case, knowledge spillovers are more localized and generate an agglomeration force. Conversely, when $b \in (0, \frac{1}{2})$, we say that related variety is *low*, and innovation benefits from more researchers living in the other region. Knowledge spillovers induce a dispersion force in this case.

We may then look at the term $\gamma A \left[bz_i + (1-b)z_j\right]$ as the spatially-weighted average of global knowledge observed by each firm. We thus introduce a spatial mechanism of spillovers such that knowledge transfers imperfectly between researchers in different locations (Bond-Smith, 2021). In Section 6.2, we discuss a general case for the success of innovation that preserves this spatial nature of knowledge spillovers. Nonetheless, the specification in (11) is suitable because it provides detailed and interesting insights, without sacrificing analytical tractability.

When a manufacturing firm in region i, that produces variety s, innovates, it has the probability $\Phi_i(s)$ of reaching the leading quality grade k of its variety. A firm that successfully reaches the grade quality k becomes the quality leader and charges the monopolistic competitive price, collecting profits $\tilde{\pi}_i \equiv \pi_i(s_k)$. Lower quality products are considered obsolete. Firms who are unable to attain the leading quality grade face *creative destruction* and are priced out of the market (Aghion and Howitt, 1990). In what follows, the short-run general equilibrium will be comprised solely of firms that produce the highest quality possible of their variety $s \equiv s_k \in S$.

3.4 Short-run equilibrium

Given the research intensity α, each firm faces the following expected profit:

$$E\left[\pi_i(s)\right] = \Phi_i(s)\tilde{\pi}_i - \left[1 - \Phi_i\left(s\right)\right]0 - \alpha w_i,$$

where w_i is the nominal wage paid to scientists in region i.

Labour market clearing implies that the number of varieties (and hence firms) is given by $S = z_i/\alpha$, from where the price index in (10) becomes:

$$P_i = \frac{\beta\sigma}{\sigma - 1}\left(\frac{A}{\alpha}\sum_{j=1}^{2}\phi_{ij}z_j\right)^{\frac{1}{1-\sigma}}. \tag{12}$$

Given free-entry in the R&D sector, in equilibrium expected profits are driven down to zero, which yields the following condition:

$$w_i = \frac{\Phi_i(s)\tilde{\pi}_i}{\alpha}.$$

Using (3), (8), this becomes:

$$w_i = \frac{\mu a_i}{\alpha\sigma}\Phi_i\sum_{j=1}^{2}\left(\frac{p_{ij}}{P_i}\right)^{1-\sigma}\left(\frac{\lambda}{2} + z_j\right). \tag{13}$$

Replacing (8), (11) and (12) in (13) yields:

$$w_i = \frac{\mu\gamma}{\sigma}\left[bz_i + (1-b)z_j\right]\left(\frac{\frac{\lambda}{2} + z_i}{z_i + \phi z_j} + \phi\frac{\frac{\lambda}{2} + z_j}{\phi z_i + z_j}\right). \tag{14}$$

Finally, using (14) and (5), we get the indirect utility in region i:

$$v_i = \frac{\mu\gamma}{\sigma}\left[bz_i + (1-b)z_j\right]\left(\frac{\frac{\lambda}{2} + z_i}{z_i + \phi z_j} + \phi\frac{\frac{\lambda}{2} + z_j}{\phi z_i + z_j}\right) + \frac{\mu}{\sigma - 1}\ln\left[z_i + \phi z_j\right] + \eta, \tag{15}$$

where $\eta \equiv -\mu\left(\frac{\beta\sigma}{\sigma-1}\right) + \frac{\mu}{\sigma-1}\ln\left(\frac{A}{\alpha}\right) - \mu + \bar{B}$ is a constant.

Notice how our assumptions on the innovation rate in (11) imply that regional knowledge, $a(i)$ and A, does not affect indirect utility. This means that we can conveniently avoid modelling the dynamics of innovation. As a result, the spatial outcomes of our model are solely determined by pecuniary factors and by the spatial dimension of knowledge creation and diffusion, captured by the term $bz_i + (1-b)z_j$, as proposed by Bond-Smith (2021).

4 Long-run equilibria

Scientists are free to migrate between regions. In doing so, they choose the region that offers them the highest indirect utility. The long-run spatial distribution thus depends on the utility differential:

$$\Delta v(z) = v_1(z) - v_2(z). \tag{16}$$

We follow Castro et al. (2021) in the characterization of equilibria and their stability. There are two kinds of long-run equilibria which should be dealt with separately.

1. *Agglomeration* of all scientists in a single region $z^* = \{0, 1\}$ is an equilibrium if and only if $\Delta v(1) \geq 0$, or, equivalently, $\Delta v(0) \leq 0$.

2. *Dispersion* of scientists $z^* \in (0, 1)$ is an equilibrium if and only if $\Delta v(z^*) = 0$. If $z^* = \frac{1}{2}$ it corresponds to *symmetric dispersion.* Otherwise, it is called *asymmetric*.

Equilibria are stable if, after a perturbation such that $z = z^* \pm \epsilon$, with $\epsilon > 0$ small enough, the utility differential $\Delta v(z)$ becomes such that agents go back to their place of origin, i.e., $z = z^*$. A sufficient condition for stability of agglomeration is $\Delta v(1) > 0$ (or $\Delta v(0) < 0$). A sufficient condition for the stability of dispersion is that $\Delta v'(z^*) < 0$. When equilibria are *regular* (resp. $\Delta v(1) \neq 0$ and $\Delta v'(z^*) \neq 0$), these conditions are also necessary.[5]

4.1 Existence and multiplicity

Our first result regards the multiplicity of long-run equilibria. Given symmetry across regions, we focus on the case whereby region 1 is either the same size or is larger than region 2, i.e., $z \in \left[\frac{1}{2}, 1\right]$.

[5]In models that are well-behaved, the non-existence of irregular long-run equilibria holds in a full measure subset of a suitably defined parameter space.

Symmetric dispersion $z^* = \frac{1}{2}$ is called an *invariant pattern*, because it is a long-run equilibrium for the entire parameter range (Aizawa et al., 2020). Next, we have the following result regarding possible equilibria for $z \in \left(\frac{1}{2}, 1\right]$.

Proposition 1. *There are at most two equilibria for* $z \in \left(\frac{1}{2}, 1\right]$.

Proof. See Appendix A. □

We can be more precise regarding the existence of dispersion equilibria with the following result.

Proposition 2. *A dispersion equilibrium* $z \equiv z^* \in \left(\frac{1}{2}, 1\right]$ *exists if* $b \in \left(\max\left\{0, \tilde{b}\right\}, \hat{b}\right)$, *where:*

$$\tilde{b} \equiv \frac{\gamma(\sigma-1)(2z-1)\left[(z-1)z\left(\phi^2-1\right)+\phi^2\right]+\sigma\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi)}\right]}{\gamma(\sigma-1)(2z-1)(\phi+1)\left[2(z-1)z(\phi-1)+\phi\right]},$$

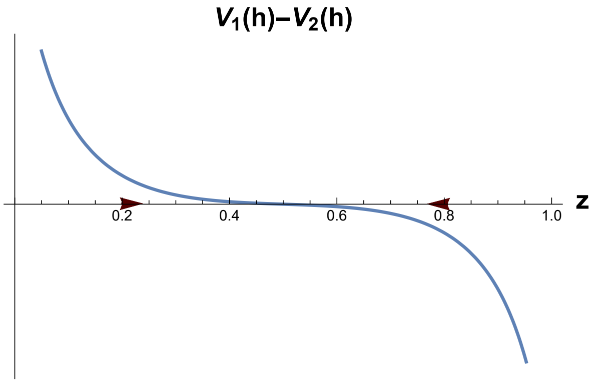
and

$$\hat{b} \equiv \frac{1+\phi^2}{(1+\phi)^2}.$$

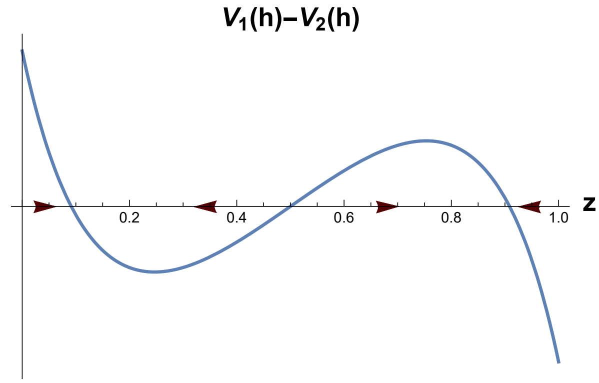
Proof. See Appendix A. □

Figure 1 shows one scenario with five qualitatively different cases, with varying freeness of trade ϕ, for $b \in (0, \frac{1}{2})$, regarding existence of the model's long-run spatial distribution, which exhaust all mathematical possibilities for the parameter values $(\lambda, \gamma, \sigma, b) = (2, 1, 5, 0.342)$. As a prelude to the forthcoming Section, Figure 1 also numerically depicts the local stability of each equilibrium, which is to be analysed analytically in greater detail in Section 3.2.

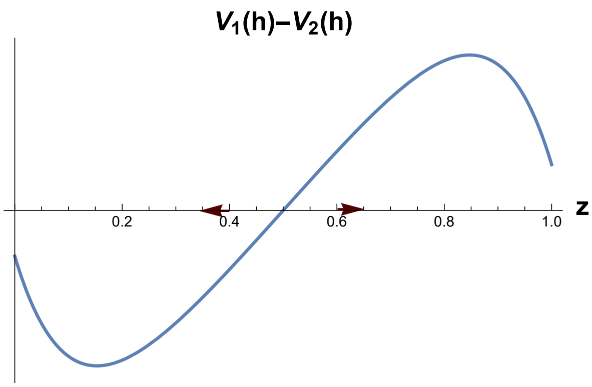
In Figure 1a, only symmetric dispersion exists and is stable for a very small ϕ. For a higher trade freeness we have one stable asymmetric dispersion for $z \in \left(\frac{1}{2}, 1\right]$ as portrayed in Figure 1b. For a greater ϕ, Figure 1c shows that the asymmetric dispersion equilibrium disappears and symmetric dispersion becomes unstable, whereas agglomeration becomes stable. For an even greater ϕ, Figure 1d illustrates an example of two long-run dispersion equilibria z^* for $z \in \left(\frac{1}{2}, 1\right]$ and symmetric dispersion $z = \frac{1}{2}$, whereby we can observe that both symmetric dispersion and the more agglomerated dispersion equilibrium are locally stable, whereas the less agglomerated equilibrium is unstable. The economy re-disperses and agglomeration does not exist in this particular case. However, as the trade freeness increases further, symmetric dispersion remains stable and all other equilibria disappear.



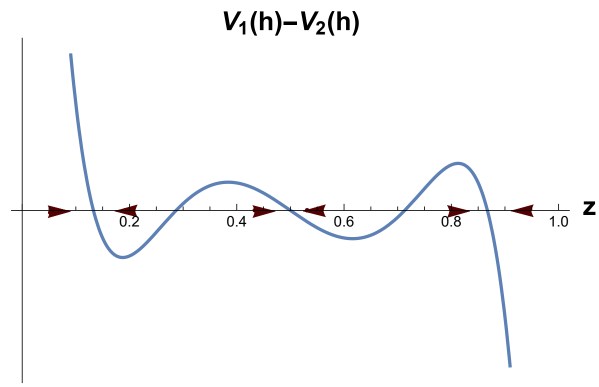
(a) Stable symmetric dispersion: $\phi = 0.1$.



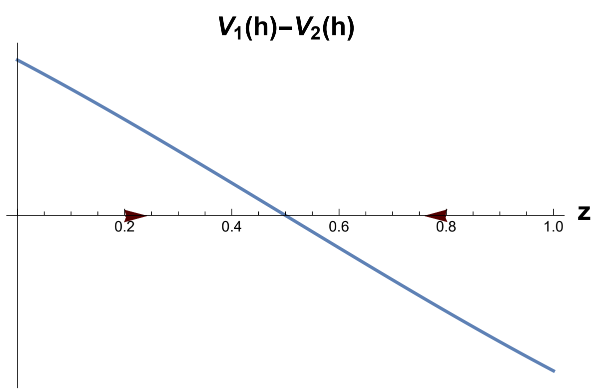
(b) Stable asymmetric dispersion: $\phi = 0.3$.



(c) Stable agglomeration: $\phi = 0.38$.



(d) Stable asymmetric dispersion: $\phi = 0.4$.



(e) Stable symmetric dispersion: $\phi = 0.8$.

Figure 1 – Long-run equilibria and their stability as ϕ increases.

When related variety is low, $b \in (0, \frac{1}{2})$, the model accounts for a "bell-shaped" relationship between economic integration and spatial imbalances (Fujita and Thisse, 2013), whereby firms are initially dispersed, then start to agglomerate in a single region as the trade freeness increases, but then find it worthwhile to relocate to the peripheral regions in order to benefit from higher expected profits due to the sizeable pool of scientists in the core which increases the chance of innovation in the periphery. In other words, when related variety is low, knowledge spillovers constitute a local dispersion force whose strength becomes relatively higher as economic integration increases and leads to the vanishing of agglomeration forces.

The case of high related variety, $b \in (\frac{1}{2}, 1)$, is much less diversified and can be accounted for resorting to a subset of the pictures from Figure 1. The history as economic integration increases is as follows. For a very low trade freeness, symmetric dispersion is the only stable equilibrium as in Figure 1a. For an intermediate value of ϕ, one asymmetric dispersion equilibrium arises which is the only stable one and becomes more asymmetric as ϕ increases further. This is akin to the picture in Figure 1b. Finally, the asymmetric dispersion equilibrium gives rise to stable full agglomeration in one single region once ϕ becomes very high. This is illustrated in Figure 1c. In other words, when intra-regional interaction is relatively more important, knowledge spillovers are more localized, and thus constitute an additional agglomeration force.

In the forthcoming Sections, we will analytically and numerically study in greater detail the local stability of the spatial distributions and the qualitative change in the model's structure as economic integration increases.

4.2 Stability

4.2.1 Agglomeration

Regarding agglomeration, using (15) in (16), we have that it is stable if:

$$\mathcal{S} \equiv \frac{\gamma \left[(b-1)(\lambda+2)\phi^2 + 2b(\lambda+1)\phi + (b-1)\lambda\right]}{2\sigma\phi} - \frac{\ln \phi}{\sigma - 1} > 0.$$

The second term is positive. Hence, agglomeration is always stable if the first term is also positive:

$$b > b_s \equiv \frac{(\lambda+2)\phi^2 + \lambda}{(\phi+1)\left[(\lambda+2)\phi + \lambda\right]}.$$

It is easy to check that $b_s < \frac{1}{2}$ if $\phi \in \left(\frac{\lambda}{\lambda+2}, 1\right)$, which means that, if $\phi \in \left(\frac{\lambda}{\lambda+2}, 1\right)$ and $b > \frac{1}{2}$, agglomeration is stable. In any case, we can conclude that agglomeration is always stable if related variety is extremely high.

Let us now define as *sustain point* (Fujita et al., 1999), a value of ϕ such that $\mathcal{S}(\phi) = 0$. We have the following result relating the freeness of trade and the relatedness of variety.

Proposition 3. *If $b < \frac{1}{2}$, there exist at most two sustain points, ϕ_{s1} and ϕ_{s2}, and agglomeration is unstable for $\phi \in (0, \phi_{1s}) \cup (\phi_{2s}, 1)$ and stable for $\phi \in (\phi_{1s}, \phi_{2s})$. If $b > \frac{1}{2}$, there exists a unique sustain point ϕ_{s1} and agglomeration is unstable for $\phi \in (0, \phi_{1s})$ and stable if $\phi \in (\phi_{1s}, 1)$.*

Proof. See Appendix A. □

The result in Proposition 3 suggests that an intermediate level of economic integration favours agglomeration only if the interaction with foreign scientists is relatively more important for the chance of successful innovation, i.e. if related variety is not too high. By contrast, if the within region interaction of scientists is more important, agglomeration is only possible when the freeness of trade is high enough.

4.2.2 Symmetric dispersion

Regarding symmetric dispersion $z^* = \frac{1}{2}$, using (15) in (16) we can say that it is stable if:

$$\mathcal{B} \equiv \gamma(\sigma - 1)\left[2b(\lambda+1)(\phi+1)^2 - (2\lambda+3)\phi^2 - 2\lambda - 1\right] + 2\sigma\left(1 - \phi^2\right) < 0. \qquad (17)$$

In fact, it is always unstable if the first term is positive, i.e. if:

$$b > \bar{b} \equiv \frac{(2\lambda+3)\phi^2 + 2\lambda + 1}{2(\lambda+1)(\phi+1)^2}.$$

This means that if related variety is prohibitively high, symmetric dispersion is surely unstable.

We can observe that $\mathcal{B}$ in (17) is a second degree polynomial in ϕ with at most two zeros, i.e., *break points* ϕ_{b1} and ϕ_{b2}, with $\phi_{b1} < \phi_{b2}$ and has a negative leading coefficient. Therefore, if both break points exist, we have that symmetric dispersion is stable for $\phi \in (0, \phi_{b1}) \cup (\phi_{b2}, 1)$ and unstable for $\phi \in (\phi_{b1}, \phi_{b2})$. The expressions for the break points, along with the conditions for their existence, are provided in Appendix A.4. We have the following result.

Proposition 4. *For $b > \frac{1}{2}$, symmetric dispersion is stable for $\phi \in (0, \phi_{b1})$ and unstable for $\phi \in (\phi_{b1}, 1)$, provided that b is not too high (otherwise it is always unstable). If $b < \frac{1}{2}$, symmetric dispersion is stable for $\phi \in (0, \phi_{b1}) \cup (\phi_{b2}, 1)$ and unstable for $\phi \in (\phi_{b1}, \phi_{b2})$, provided that b is not too low (otherwise it is always stable).*

Proof. See Appendix A.4. □

This means that our model accounts for the possibility of initial agglomeration as trade integration increases from a low level, and (complete) re-dispersion once trade integration becomes high enough.

4.2.3 Asymmetric dispersion

Although we cannot find an explicit stability condition for any asymmetric dispersion equilibrium $z^* \in \left(\frac{1}{2}, 1\right)$, we can use equation (21) that solves the equilibrium condition $\Delta v = 0$ given implicitly by $\lambda = \lambda^*(z)$ in the proof of Proposition 2 (Appendix A.2).[6] Then the stability condition of an asymmetric dispersion equilibrium is given by:

$$\frac{d\Delta v}{dz}(z^*)\bigg|_{\lambda=\lambda^*(z)} < 0.$$

Specifically, using (15) and differentiating (16) with respect to z, and evaluating at (21), we get that an asymmetric equilibrium $z^* \in \left(\frac{1}{2}, 1\right)$ is stable if $\lambda^*(z) > 0$ and:

$$\begin{aligned}\mathcal{G} \equiv &(2z-1)\left(\phi^2-1\right)\left[(2b-1)\gamma(\sigma-1)(1-2z)^2-\sigma\right]+ \\ &+\sigma\left[2z^2(\phi-1)^2-2z(\phi-1)^2+\phi^2+1\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]<0.\end{aligned} \tag{18}$$

We have the following result.

Proposition 5. *If $b < \frac{1}{2}$ an asymmetric equilibrium $z^* \in \left(\frac{1}{2}, 1\right)$ is stable for a high enough related variety. If $b > \frac{1}{2}$, an asymmetric equilibrium is always stable when it exists.*

[6] The same approach was adopted e.g. by Gaspar et al. (2018) and (Gaspar et al., 2021).

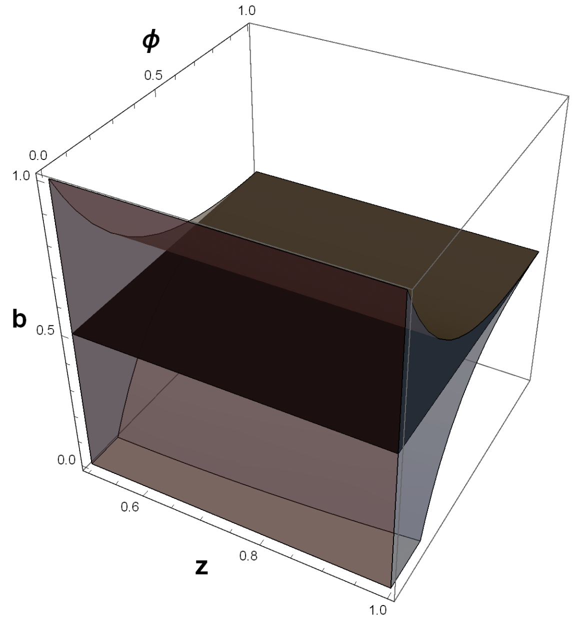


Figure 2 – Stability of asymmetric dispersion. The less opaque surface corresponds to $\mathcal{G} < 0$ and $\lambda^*(z) > 0$ in (z, ϕ, b)–space for $\sigma = 8$ and $\gamma = 1$. The black plane corresponds to $b = \frac{1}{2}$.

Proof. See Appendix A. □

Figure 2 illustrates the Proposition by setting $\sigma = 8$ and $\gamma = 1$ and plotting the surface corresponding to $\{(z, \phi, b) : \mathcal{G} < 0 \cap \lambda^*(z) > 0\}$. For $b < \frac{1}{2}$, an asymmetric equilibrium may exist that is not stable, and a higher b favours its stability. If $b > \frac{1}{2}$, an asymmetric equilibrium is always stable when it exists, but its existence seems to be favoured by a lower b. In other words, an asymmetric equilibrium exists and is stable when b is close enough to $\frac{1}{2}$.

Regarding ϕ, a higher freeness of trade seems to disfavour the stability of asymmetric dispersion.

5 The impact of economic integration

It is common in geographical economics to study the qualitative change of the spatial economy as economic integration increases. Accordingly, we investigate the existence of bifurcations in our model and employ the freeness of trade, ϕ, as the bifurcation parameter. First, suppose a break point exists, i.e., symmetric dispersion interchanges stability for some value of the freeness of trade. We have the following result.

Lemma 1. *At a break point $\phi \in \{\phi_{b1}, \phi_{b2}\}$, the symmetric dispersion undergoes a pitchfork bifurcation, which may be subcritical, supercritical or degenerate.*

Proof. See Appendix A. □

This Lemma asserts that a curve of asymmetric equilibria branches from symmetric dispersion at the break point, if it exists. The stability of the branch depends on the criticality of the bifurcation. If the pitchfork is subcritical, the asymmetric equilibria are unstable. If it is supercritical, the asymmetric equilibria are stable.

We now look at some bifurcation diagrams. To provide a complete gallery, we depict 6 qualitatively different scenarios, keeping most parameter values constant (except for the sixth scenario) and varying b, thus placing emphasis on changes in the value of related variety. These scenarios exhaust all mathematical/numerical possibilities. This can be shown through the combination of the analysis performed in the previous sections with various simulations under a very wide range of parameter values.[7] The six scenarios analysed in this Section are as follows:

(i). $(\lambda, \gamma, \sigma, b) = (2, 0.9, 8, 0.33)$; (ii). $(\lambda, \gamma, \sigma, b) = (2, 0.9, 8, 0.338)$;

(iii). $(\lambda, \gamma, \sigma, b) = (2, 0.9, 8, 0.339)$; (iv). $(\lambda, \gamma, \sigma, b) = (2, 0.9, 8, 0.35)$;

(v). $(\lambda, \gamma, \sigma, b) = (2, 0.9, 8, 0.55)$; (vi). $(\lambda, \gamma, \sigma, b) = (4, 0.9, 8, 0.55)$;

For a prohibitively low related variety, scientists disperse evenly among the two regions, irrespective of the value of the freeness of trade. The economic intuition is simple: a lower related variety implies higher chance of successful innovation with more scientists living in the other region. Hence, the nominal wage is higher when the scientists are more evenly distributed.

[7] Additionally, see Figure 9, the transition between different types of stable equilibria is qualitatively similar if we fix the value of ϕ and employ b as the bifurcation parameter instead. In fact, the symmetric dispersion can be shown to also undergo a pitchfork bifurcation for a critical value of b.

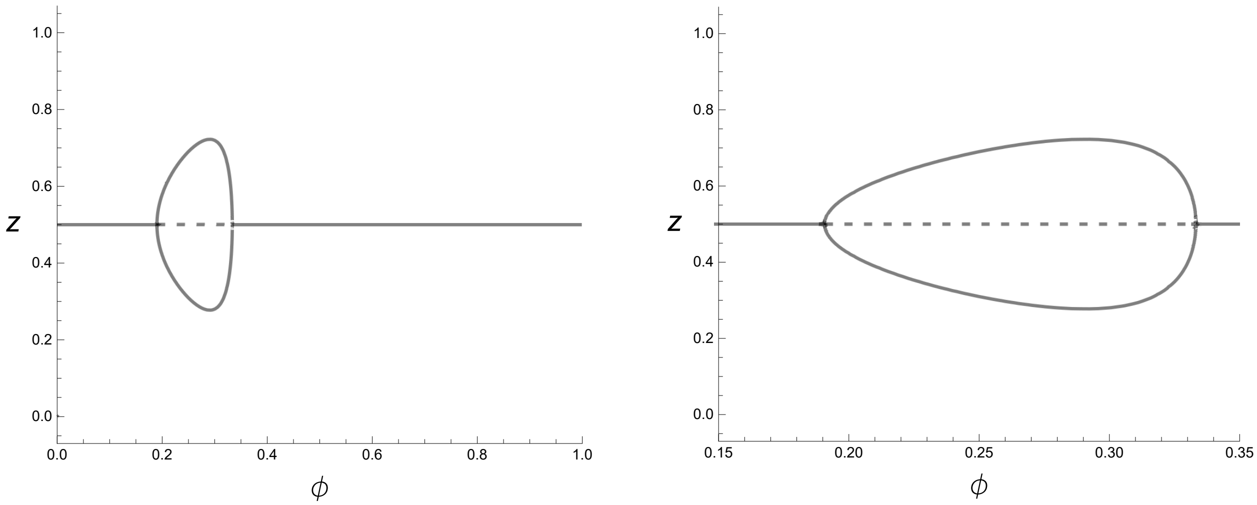


Figure 3 – Bifurcation diagram for scenario (i). Filled lines correspond to stable equilbria and dashed lines correspond to unstable equilibria.

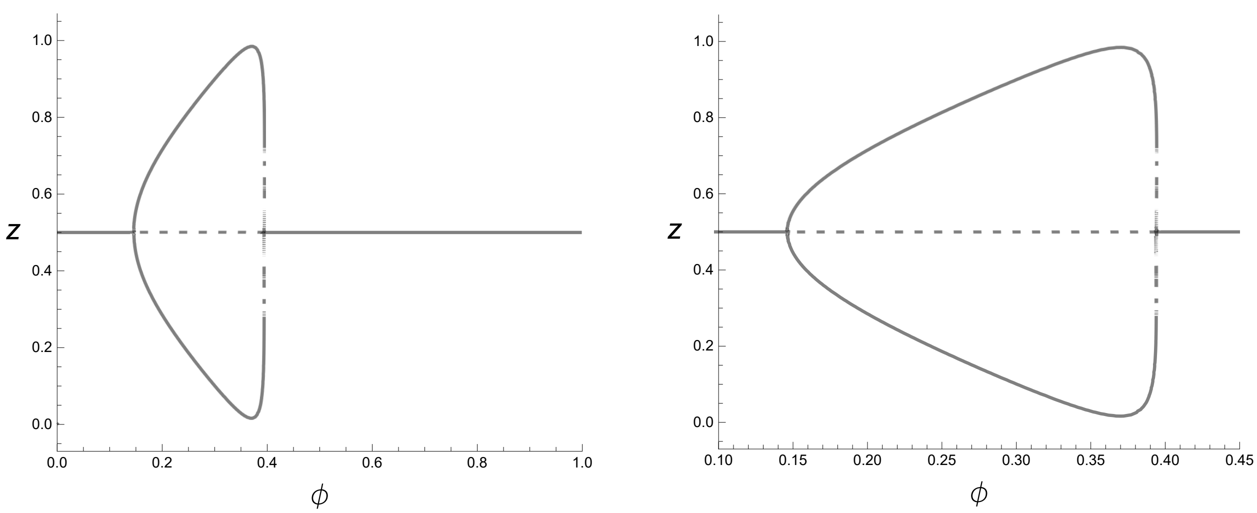


Figure 4 – Bifurcation diagram for scenario (ii). Filled lines correspond to stable equilbria and dashed lines correspond to unstable equilibria.

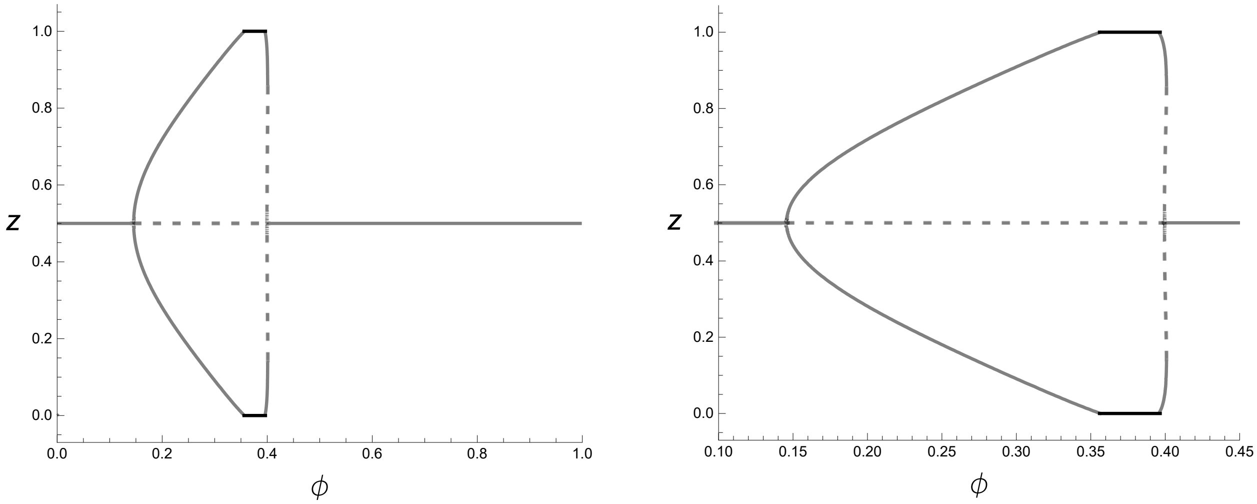


Figure 5 – Bifurcation diagram for scenario (iii). Filled lines correspond to stable equilibria, and dashed lines correspond to unstable equilibria.

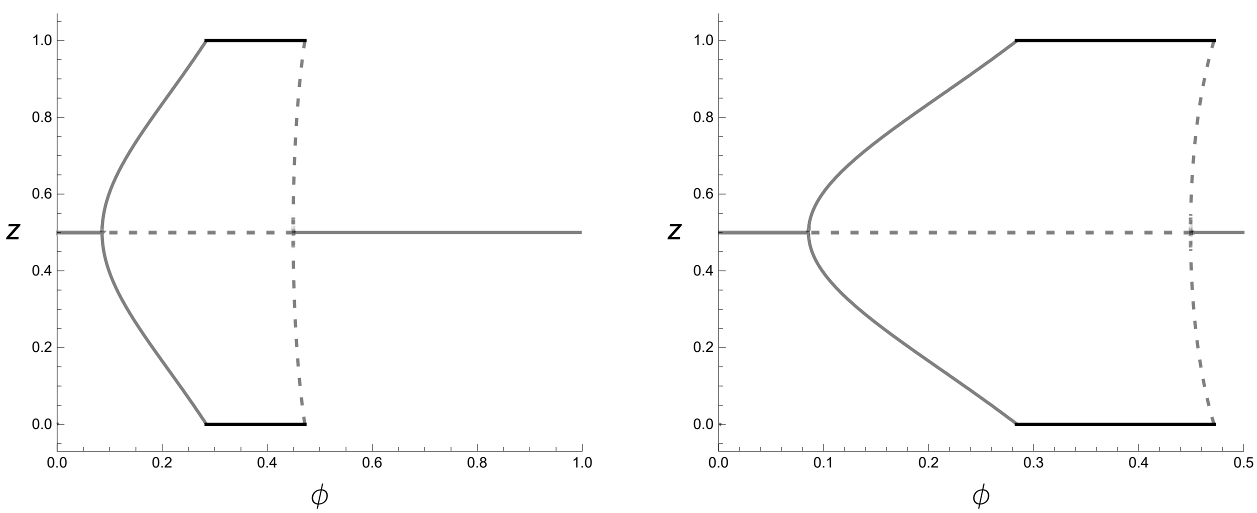


Figure 6 – Bifurcation diagram for scenario (iv). Filled lines correspond to stable equilbria and dashed lines correspond to unstable equilibria.

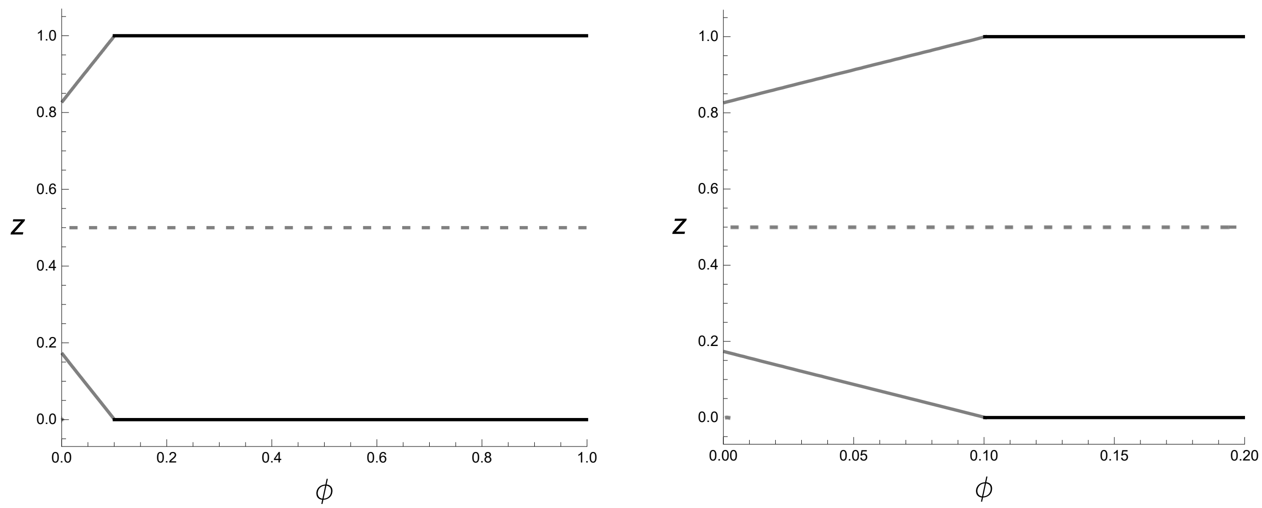


Figure 7 – Bifurcation diagram for scenario (v). Filled lines correspond to stable equilbria and dashed lines correspond to unstable equilibria.

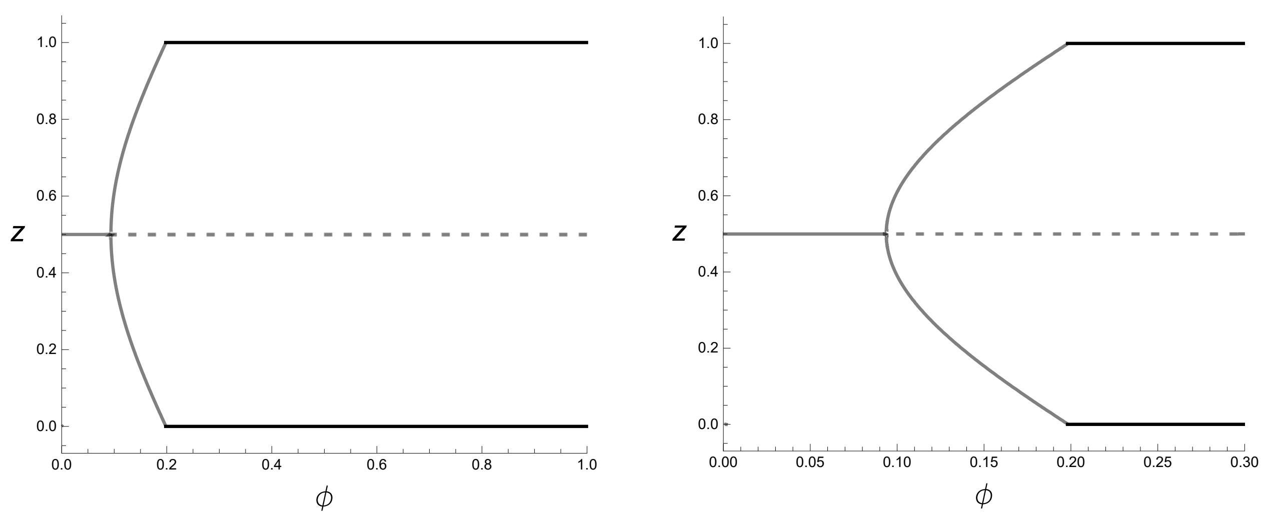


Figure 8 – Bifurcation diagram for $(\lambda, \gamma, \sigma, b) = (4, 0.9, 8, 0.55)$. Filled lines correspond to stable equilbria and dashed lines correspond to unstable equilibria.

In scenario (i), shown in Figure 3, related variety is such that within-region interaction is relatively less important ($b = 0.33$). For a low freeness of trade, symmetric dispersion is stable because firms wish to avoid the burden of a very costly transportation supplying to farmers from full agglomeration in a single region. As ϕ increases, the economy initially agglomerates, but then re-disperses as ϕ increases further. This re-dispersion process occurs because, for a very high economic integration, firms find it profitable to relocate to the less industrialized region in order to benefit from the pool of scientists in the more agglomerated region, which generates a higher chance of innovation and thus higher expected profits. Noteworthy, the turning point in the agglomeration process happens *before* industry reaches full agglomeration in a single region, as in Pflüger and Südekum (2008). However, contrary to the latter, our model does not predict full agglomeration in the entire parameter range of economic integration when related variety is low enough. Re-dispersion in scenario (i) is more akin to geographical economic models of vertical linkages between upstream and downstream firms by Krugman and Venables (1995); Venables (1996) and Puga (1999). However, in these models, re-dispersion is smooth altogether and occurs when workers are inter-regionally *immobile* and firms become too sensitive to regional cost differentials when economic integration is very high.

In scenario (ii), illustrated by Figure 4, related variety is just slightly higher, and the model still accommodates for re-dispersion. However, the re-dispersion process is not smooth – the economy suddenly jumps to symmetric dispersion from a fairly asymmetric equilibrium spatial distribution.

Scenario (iii) also just slightly increases related variety compared to the previous scenario (see Figure 5), and the story of spatial outcomes as economic integration increases is very similar, except that, in this case, full agglomeration is stable for a small range of intermediate values of ϕ, as predicted by Proposition 3. The parametrization here also corresponds to that illustrated in Figure 1.

The re-dispersion processes of scenarios (ii) and (iii) are uncommon in the literature of geographical economics; rather, such jumps occur in early models (Fujita et al., 1999; Baldwin et al., 2003) from the state of symmetric dispersion to catastrophic agglomeration (Behrens and Robert-Nicoud, 2011). The reverse discontinuous jump, i.e., from symmetric dispersion to partial agglomeration as trade costs steadily decrease, has been uncovered in the model by Pflüger and Tabuchi (2010), where *all* production factors, except land, which is used both for housing and production, are inter-regionally mobile. Their conclusions about spatial outcomes reveal a line-symmetry of scenario (iii): as integration increases, the economy jumps discontinuously from symmetric dispersion to partial agglomeration, and the ensuing re-dispersion is gradual and continuous.[8]

Figure 6 illustrates scenario (iv) and shows that the sudden re-dispersion process under a slightly higher b now happens from the state of full agglomeration directly to the state of symmetric dispersion. In both scenarios (iii) and (iv), the state of agglomeration is stable for intermediate values of economic integration, as in Robert-Nicoud (2008).

In scenario (v), for a sufficiently high related variety ($b > 1/2$), within-region interaction among scientists improves the chances of innovation enough such that the real wage becomes higher when they are either partially agglomerated in one region for low values of ϕ, or completely agglomerated in one region for a high enough ϕ. This is portrayed in Figure 7. Scenario (v) precludes the so-called "no black-hole condition" (Fujita et al., 1999), a condition that the constant elasticity of substitution σ must be high enough such that symmetric dispersion can be stable for low enough economic integration. As argued by Gaspar et al. (2018), this condition may be unwarranted if its exclusion allows for spatial outcomes other than ubiquitous agglomeration.

We can thus conclude that a higher related-variety is associated with a more pronounced agglomeration during the industrialization process, for intermediate values of economic integration, until eventually it becomes so high ($b > \frac{1}{2}$) that re-dispersion is no longer possible because within-region interaction among scientists is too important to make any deviation to a deindustrialized region worthwhile.

In scenario (vi) we illustrate the qualitative change in the spatial structure of the economy as ϕ increases for $b > 1/2$, but with a higher λ, since, with the parameter values of the previous scenario, agglomeration would be ubiquitously stable (and hence uninteresting) for higher values of b. In Figure 8, we can observe a supercritical pitchfork bifurcation, as in the model by Pflüger (2004), where there is no innovation. That is, for low levels of economic integration, symmetric dispersion is stable. As ϕ increases, one region smoothly becomes more and more industrialized en route to a full agglomeration whereby that region becomes a core.

[8] In our model, the assumption that unskilled workers are immobile is useful for tractability, as is the case of all footloose entrepreneur models (Baldwin et al., 2003). However, we make the reasonable conjecture that immobile labour generates an unnecessary dispersion force that changes the conclusions of our model compared to the case of a perfectly mobile workforce only in the sense of "reversed" stability as transport costs decrease, i.e. the line-symmetry of all scenarios (i)–(vi).

Besides the bifurcation at symmetric dispersion, in Figures 4 and 5 (scenarios (ii) and (iii)), a limit point $\phi \equiv \phi_l \in (\phi_{b2}, 1)$ is discernible at which two asymmetric equilibria, along a curve tangent to ϕ_l that lies to its left, collide and coalesce. This suggests that in both scenarios (i) and (ii) the model undergoes a saddle-node bifurcation at some asymmetric equilibrium $z^* \in \left(\frac{1}{2}, 1\right)$. This kind of bifurcation also appears in the two-region footloose entrepreneur model by Forslid and Ottaviano (2003) with heterogeneous agents analysed by Castro et al. (2021) and also in the Pflüger (2004) model extended to multiple regions by Gaspar et al. (2018). This kind of bifurcation seems to be associated with discontinuous jumps between some asymmetric equilibrium other than agglomeration and the symmetric dispersion once ϕ rises (falls) above (below) some threshold level.

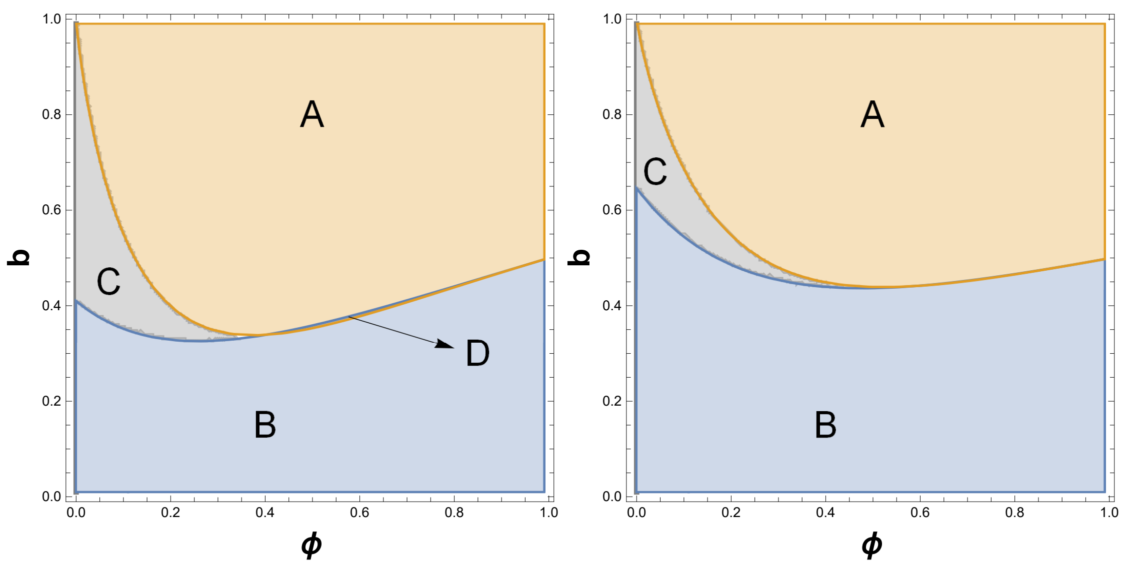


Figure 9 – Stability region for each equilibrium in (ϕ, b)-space. Region A denotes stable agglomeration, region B denotes stable symmetric dispersion, region C denotes stable asymmetric equilbria. In region D both agglomeration and symmetric dispersion are stable. Parameter values are $(\sigma, \gamma, \lambda) = (8, 0.9, 2)$, to the left, and $(\sigma, \gamma, \lambda) = (8, 0.9, 4)$, to the right.

It is clear that the transition along and between each type of stable equilibrium as ϕ increases depends on the level of related variety b. To conclude this Section, we portray the stability regions of the three types of equilibria (agglomeration, symmetric dispersion and asymmetric dispersion) in Figure 9, in the space of $(\phi, b) \in (0, 1) \times [0, 1]$. In region A, agglomeration is stable, which requires a high ϕ and a high b. In region B we have stable symmetric dispersion, which requires a low b and either a very low or a very high value of ϕ (as discussed before). Region C denotes stable asymmetric equilibria for low levels of ϕ and high enough levels of b. There is a thin region D where regions A and B overlap, indicating that both agglomeration and symmetric dispersion are simultaneously stable in that region. Finally, the region where both asymmetric dispersion and symmetric dispersion are stable is not visually discernible, which makes it a very unlikely outcome.

6 On the role of regional interaction

6.1 The impact of related variety: comparative statics

It is worthwhile investigating analytically how changes in the level of related variety affect the long-run spatial outcomes in the economy. Following Castro et al. (2021), we say that a change in related variety *favours agglomeration* if, due to the change: (a) symmetric dispersion may become unstable but not stable, (b) agglomeration may become stable but not unstable, and (c) asymmetric dispersion becomes more asymmetric.

Using (15) and (16), we have:

$$\frac{\partial \Delta v}{\partial b} = \frac{\gamma\mu(2z-1)(\phi+1)\left[\lambda - 4z^2 + \phi(\lambda + 4(z-1)z + 2) + 4z\right]}{2\sigma\left[z(\phi-1)+1\right]\left[z(1-\phi)+\phi\right]},$$

which is positive for $z \in \left(\frac{1}{2}, 1\right)$. We have the following result.

Lemma 2. *An increase in related variety favours agglomeration.*

Proof. See Proposition 9 of (Castro et al., 2021, p.197). □

The interpretation behind this result is straightforward: a higher b implies a higher chance of successful innovation in region i when more scientists live in region i. Hence, expected profits become higher, which leads to higher wages and, thus, a higher utility differential in region i. Therefore, a higher b makes stability of agglomeration (symmetric dispersion) more (less) likely for a given value of ϕ, and asymmetric dispersion becomes more asymmetric.

Suppose b is such that symmetric dispersion is the unique stable equilibrium for a low ϕ (Proposition 4), asymmetric dispersion is the unique stable equilibrium for intermediate values of ϕ (Proposition 5), and agglomeration is the unique stable equilibrium for a high ϕ (Proposition 3). Accordingly, the model undergoes a supercritical pitchfork bifurcation at symmetric dispersion.

As illustrated in Figure 10, an increase in b shifts the pitchfork bifurcation leftwards, provided that the set of stable equilibria remains unchanged.

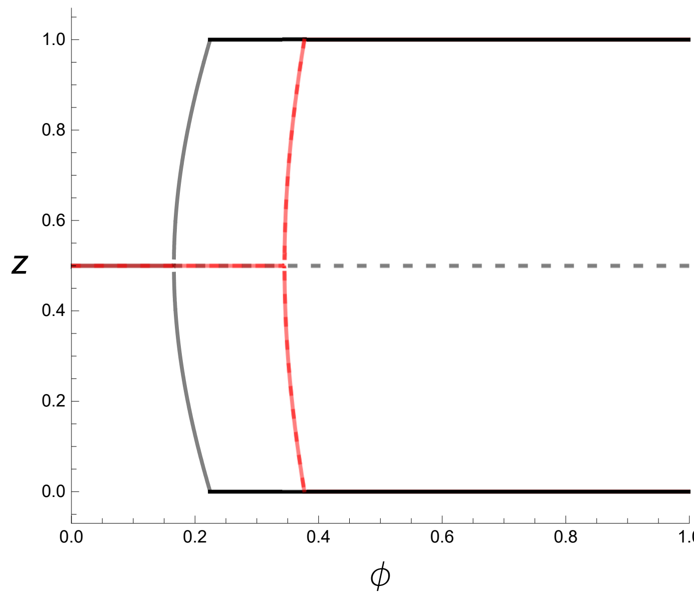


Figure 10 – Bifurcation diagrams: (i) to the left, in black, we set $b = 0.9$; (ii) to the right, in red, we set $b = 0.7$. Parameter values are $(\sigma, \gamma, \lambda) = (8, 1, 4)$.

6.2 Related variety and re-dispersion: a general case

Let us now consider a more general specification for the firms' success of innovation:

$$\Phi_i^G(s) = \min\left\{\frac{g_i(z_i, z_j; v)\gamma A}{a_1(s)}, 1\right\}, \tag{19}$$

with $g_1 \equiv g(z)$, $g_2 \equiv g(1-z)$, $g : [0,1] \mapsto [0,1]$ is a C^3 function of z, and $v \in \mathcal{V}$ is a vector of parameters which includes the related variety parameter $b \in (0,1)$. Additionally, assume that $g'(z) < 0$ if $b < 1/2$, and $g'(z) > 0$ if $b > 1/2$, for $z \in [\frac{1}{2}, 1]$. This reflects the fact that $g(z)$ constitutes a dispersion force in the larger region when related variety is low, and an agglomeration force in the larger region when related variety is high. This less restrictive assumption might be more reasonable in some contexts, e.g., if congestion is assumed to have an impact on knowledge creation (either positive or negative). Clearly, $\Phi_i(s)$ in (11) is a particular case of $\Phi_i^G(s)$ in (19). The indirect utility in region i is obtained by replacing the term $bz_i + (1-b)z_j$ in (15) with $g_i(z)$.

At symmetric dispersion, making use of the fact that $g_1'(\frac{1}{2}) = -g_2'(\frac{1}{2})$, we have at most two break points:

$$\phi_{b1}^G = \frac{\gamma(\lambda+1)(\sigma-1)\left[g'\left(\frac{1}{2}\right)+2g\left(\frac{1}{2}\right)\right]-\kappa}{2\left[\gamma g\left(\frac{1}{2}\right)(\lambda+2)(\sigma-1)+\sigma\right]-\gamma(\lambda+1)(\sigma-1)g'\left(\frac{1}{2}\right)},$$
$$\phi_{b2}^G = \frac{\gamma(\lambda+1)(\sigma-1)\left[g'\left(\frac{1}{2}\right)+2g\left(\frac{1}{2}\right)\right]+\kappa}{2\left[\gamma g\left(\frac{1}{2}\right)(\lambda+2)(\sigma-1)+\sigma\right]-\gamma(\lambda+1)(\sigma-1)g'\left(\frac{1}{2}\right)},$$

where

$$\kappa = 2\sqrt{2\gamma^2 g\left(\frac{1}{2}\right)(\lambda+1)^2(\sigma-1)^2 g'\left(\frac{1}{2}\right)+\left[\gamma g\left(\frac{1}{2}\right)(\sigma-1)+\sigma\right]^2}.$$

Since $g(\frac{1}{2}) > 0$, it can be shown that $\phi_{b2}^G \notin (0,1)$ if $g'(\frac{1}{2}) > 0$, i.e., if $b > 1/2$. Since symmetric dispersion is stable for $\phi \in (0, \phi_{b1}^G) \cup (\phi_{b2}^G)$, we can thus conclude that the complete re-dispersion of economic activities is not possible if related variety is high. This leads to the following result.

Proposition 6. *If intra-regional interaction is relatively more important in the larger region, then a complete re-dispersion of economic activities for high economic integration is not possible.*

In other words, the process of (complete) re-dispersion depends on the dispersive or agglomerative nature of knowledge spillovers at symmetric dispersion. When $b > \frac{1}{2}$, knowledge spillovers are localized and constitute an agglomeration force in the larger region, which becomes relatively stronger as trade barriers decrease. For a sufficiently high ϕ, re-dispersion is impossible because there is no incentive to relocate to the smaller region where innovation is less likely and expected profits are lower.

We take one step further by investigating what type of bifurcation the symmetric dispersion undergoes at some break point $\phi_b \in \left\{\phi_{b1}^G, \phi_{b2}^G\right\}$. We have the following derivatives:

$$\frac{\partial f}{\partial z}\left(\frac{1}{2};\phi_b\right) = 0;\ \frac{\partial^2 f}{\partial z^2}\left(\frac{1}{2};\phi_b\right) = 0;\ \frac{\partial f}{\partial \phi}\left(\frac{1}{2};\phi_b\right) = 0;$$

Furthermore, we have:

$$\frac{\partial^2 f}{\partial \phi \partial z}\left(\frac{1}{2};\phi_b\right) = \frac{8\mu}{(\phi+1)^3}\left\{\frac{\gamma g\left(\frac{1}{2}\right)\left[-2\lambda(\phi_b-1)-3\phi_b+1\right]}{\sigma}+\frac{\phi_b+1}{1-\sigma}\right\},$$

which is zero if and only if $g(\frac{1}{2}) = g_b$, with

$$g_b \equiv -\frac{\sigma(\phi_b + 1)}{\gamma(\sigma - 1)\left[2\lambda(\phi_b - 1) + 3\phi_b - 1\right]}.$$

If $g(\frac{1}{2}) \neq g_b$, then symmetric dispersion undergoes a pitchfork bifurcation (see Appendix A.6). In this case, its criticality is determined by the sign of the third-order derivative $\frac{\partial^3 f}{\partial z^3}(\frac{1}{2}; \phi_b)$. However, without additional details on the shape of $g(z)$ it is impossible to convey further information on these conditions and to relate them with b.

We conclude that, while the shift between distinct spatial outcomes as economic integration increases may display differences across different values of related variety, the presence of a (complete) re-dispersion phase should depend solely on the dispersive or agglomerative nature of knowledge spillovers.

7 Concluding remarks

We have analyzed a two-region economic geography model with vertical innovations. We looked at how the spatial creation and diffusion of knowledge and increasing returns in manufacturing interact to shape the spatial economy. Knowledge levels translate in a firms' capacity to innovate. In turn, the chance of successful innovations depends on the spatial distribution of mobile agents in the economy, i.e. on the intra- and inter-regional interaction between researchers. The weight of interaction between researchers within the same region rather than across different regions in the chance of innovation is a measure of the so-called *related variety* (Frenken et al., 2007). We thus introduce a spatial mechanism of knowledge spillovers conveying the idea that public knowledge transfers imperfectly across space, as argued by e.g. Krugman (1991a), or Audretsch and Feldman (2004), and supported by the empirical evidence of Audretsch and Feldman (1996).

We find that if related variety is low, the model accounts for (a complete) re-dispersion of economic activities after an initial stage of progressive agglomeration as trade integration increases from a low level. A lower related variety captures the idea that congestion dampens the production of knowledge, turning innovation into a dispersion force as one region becomes larger, since researchers have incentives to relocate to the smaller region and benefit from the sizable pool of agents living in the larger region. However, the relationship between economic integration and spatial imbalances is far from trivial, as we have shown a myriad of different qualitative possibilities regarding transitions between different stable states that depend on the particular level of related variety. This level determines the smoothness of the transition from either partial or full agglomeration to symmetric dispersion as trade integration reaches high enough levels.

On the other hand, if related variety is high, knowledge spillovers are more localized and generate an additional agglomeration force in the model. In this case, a higher economic integration leads to progressive agglomeration, as in Martin and Ottaviano (1999) and Martin and Ottaviano (2001). Re-dispersion for a high freeness of trade is precluded because there is no incentive to relocate to the smaller region where innovation is less likely and expected profits are lower. Further, we show that a higher related variety always favours agglomerated outcomes.

Finally, we reiterate that the modeling of the innovation sector in this paper purposefully abstracts from the dynamic processes that usually drive innovation and the creation and diffusion of knowledge. Such abstraction is useful because it allows the outcomes of the model to be entirely attributed to the spatial mechanism of knowledge spillovers (as proposed by Bond-Smith (2021)). However, we argue that this mechanism could be extended to models of Schumpeterian growth, where the explicit modelling of innovation dynamics would allow to infer about an eventual circular causality between regional growth and agglomeration patterns (Baldwin and Martin, 2004).

A Proofs

This appendix contains the more cumbersome formal proofs that support our main results.

A.1 Proof of proposition 1

Differentiating $\Delta v(z)$ in (16) yields:

$$\frac{d\Delta v}{dz}(z) = \frac{\mu P(z)}{2(\sigma-1)\sigma\left[z(\phi-1)+1\right]^2\left[z(1-\phi)+\phi\right]^2}, \tag{20}$$

where:

$$P(z) = a_1 z^4 + a_2 b z^3 - 2(1-\phi)a_3 z^2 + 2(1-\phi)a_4 z + a_5,$$

with:

$$\begin{aligned}
a_1 =& 4(1-2b)\gamma\mu(\sigma-1)(\phi-1)^3(\phi+1)\\
a_2 =& 8(2b-1)\gamma\mu(\sigma-1)(\phi-1)^3(\phi+1)\\
a_3 =& \gamma(\sigma-1)\left\{b(\phi+1)\left[(\lambda-2)\phi^2-\lambda+18\phi-4\right]-\phi\left[\lambda(\phi-1)\phi+\lambda+6\phi\right]+\lambda-8\phi+2\right\}\\
&+\sigma(\phi+1)(\phi-1)^2\\
a_4 =& \gamma(\sigma-1)\left\{\lambda(\phi-1)\left[b(\phi+1)^2-\phi^2-1\right]+2\phi\left[b(\phi+1)(\phi+5)-\phi(\phi+2)-3\right]\right\}\\
&+\sigma(\phi+1)(\phi-1)^2\\
a_5 =& \gamma(\sigma-1)\left\{\lambda\left(\phi^2+1\right)\left[b(\phi+1)^2-\phi^2-1\right]+2\phi\left[b\left(\phi^3+3\phi^2+\phi-1\right)-\phi\left(\phi^2+\phi+1\right)+1\right]\right\}\\
&-2\sigma\phi\left(\phi^2-1\right).
\end{aligned}$$

The denominator of (20) is positive, which means that the sign of $\frac{d\Delta v}{dz}(z)$ is given by the sign of $P(z)$, which is a fourth degree polynomial in z. Therefore, $\Delta v(z)$ has at most four turning points, and thus at most five equilibria for $z \in [0,1]$. We know that $z = \frac{1}{2}$ is an invariant pattern. By symmetry, we can establish that there exist at most two equilibria for $z > \frac{1}{2}$, which concludes the proof. □

A.2 Proof of Proposition 2

Proceeding as in Gaspar et al. (2018, 2021), the equilibrium condition $\Delta v(z) = 0$ yields:

$$\lambda \equiv \lambda^*(z) = -2\frac{b_1 b_2 + b_3 \ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]}{b_4}, \tag{21}$$

where:

$$\begin{aligned}
b_1 &= \gamma(\sigma-1)(2z-1) \\
b_2 &= \phi^2\left[2b(z-1)z + b - z^2 + z - 1\right] + (1-2b)(z-1)z + b\phi \\
b_3 &= \sigma\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right] \\
b_4 &= \gamma(\sigma-1)(2z-1)\left[b(\phi+1)^2 - \phi^2 - 1\right].
\end{aligned}$$

It is easy to note that $\lambda^*(z)$ has a vertical asymptote if and only if $b_4 = 0$, i.e., iff:

$$b = \hat{b} \equiv \frac{\phi^2+1}{(\phi+1)^2}.$$

For $z \in \left(\frac{1}{2}, 1\right]$, the log term of $\lambda^*(z)$ is negative, as is b_3. Next, we have $b_1 > 0$, and $b_2 > 0$ if:

$$b \geq \underline{b} \equiv \frac{(z-1)z\left(\phi^2-1\right)+\phi^2}{2(z-1)z\left(\phi^2-1\right)+\phi(\phi+1)},$$

where $0 < \underline{b} < \hat{b}$. Since $b_4 < 0$ only if $b < \hat{b}$, we have $\lambda^*(z) > 0$ if $b \in \left[\underline{b}, \hat{b}\right)$ and $\lambda^*(z) < 0$ if $b \in \left(\hat{b}, 1\right)$. For $b \in (0, \underline{b})$, we need further inspection.

We have that:

$$\frac{\partial\lambda^*}{\partial b}(z) = \frac{2(\phi+1)\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right]\left\{\gamma(\sigma-1)(2z-1)(\phi-1)+\sigma(\phi+1)\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]\right\}}{\gamma(\sigma-1)(2z-1)\left[-b(\phi+1)^2+\phi^2+1\right]^2},$$

which is positive for all $z \in \left(\frac{1}{2}, 1\right]$. The unique zero of $\lambda^*(z)$ in terms of b is given by:

$$b = \tilde{b} \equiv \frac{\gamma(\sigma-1)(2z-1)\left[(z-1)z\left(\phi^2-1\right)+\phi^2\right] - \sigma\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]}{\gamma(\sigma-1)(2z-1)(\phi+1)\left[2(z-1)z(\phi-1)+\phi\right]},$$

with $\tilde{b} < \underline{b}$ and $\lambda^*(z) > 0$ for $b \in (\tilde{b}, \hat{b})$. It is possible to show that $\tilde{b}$ is increasing in γ. Moreover, we have $\tilde{b} = 0$ if and only if:

$$\gamma = \gamma_c \equiv \frac{\sigma\left[z(1-\phi)-1\right]\left[z(1-\phi)+\phi\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]}{(\sigma-1)(2z-1)\left[(z-1)z\left(\phi^2-1\right)+\phi^2\right]} > 0.$$

This means that $\tilde{b} \geq 0$ if $\gamma \geq \gamma_c$ and $\tilde{b} < 0$ if $\gamma < \gamma_c$ and $\gamma_c \in (0, 1]$. Since $\gamma_c \in (0, +\infty)$, we have $\tilde{b} < 0$ if $\gamma_c > 1$. As a result, we have $\tilde{b} < 0$ if $\gamma \in (0, \min\{1, \gamma_c\})$ and $\tilde{b} \geq 0$ if $\gamma \in [\min\{1, \gamma_c\}, 1)$. Then $\lambda^*(z) > 0$ if $\gamma \in (0, \min\{1, \gamma_c\})$ and $b \in (0, \hat{b})$. Otherwise, we have $\lambda^*(z) > 0$ if $\gamma \in [\min\{1, \gamma_c\}, 1)$ and $b \in (\tilde{b}, \hat{b})$. Therefore, $\lambda^*(z)$ is positive for $b \in \left(\max\left\{0, \tilde{b}\right\}, \hat{b}\right)$ and negative for $b \in \left(0, \max\left\{0, \tilde{b}\right\}\right) \cup \left(\hat{b}, 1\right)$, where $\max\left\{0, \tilde{b}\right\}$ depends on γ_c and on the value of γ as described above.

Thus, we can assert that, if $b \in \left(\max\left\{0, \tilde{b}\right\}, \hat{b}\right)$, there exists a value of $\lambda > 0$ such that at least one (at most two) dispersion equilibrium $z \equiv z^* \in \left(\frac{1}{2}, 1\right]$ exists. This concludes the proof. □

A.3 Proof of Proposition 3

We have:

$$\lim_{\phi \to 0^+} \mathcal{S}(\phi) = -\infty \text{ and } \mathcal{S}(1) = \frac{\gamma(2b-1)(\lambda+1)}{\sigma}.$$

Therefore, $S(1) > 0$ if $b > \frac{1}{2}$ and we conclude that $\mathcal{S}(\phi)$ has at least one zero for $\phi \in (0, 1)$. Further, we have:

$$\frac{d\mathcal{S}}{d\phi}(\phi) = \frac{1}{2\phi^2}\left\{\frac{\gamma(b-1)\left[\lambda\left(\phi^2-1\right)+2\phi^2\right]}{\sigma} - \frac{2\phi}{\sigma-1}\right\},$$

whose sign depends on that of the second term, which is a second degree polynomial and thus has at most two zeros $\{\phi^-, \phi^+\}$, with $\phi^+ > \phi^-$. However, only ϕ^+ lies on the interval $\phi \in (0, 1)$:

$$\phi^+ = \frac{\sigma\left[\frac{1}{\sigma-1} - \sqrt{\frac{\gamma^2(b-1)^2\lambda(\lambda+2)}{\sigma^2} + \frac{1}{(\sigma-1)^2}}\right]}{\gamma(b-1)(\lambda+2)}.$$

Given that the leading coefficient of the polynomial is negative, we have that $\mathcal{S}(\phi)$ is increasing for $\phi \in (0, \phi^+)$ and decreasing for $\phi \in (\phi^+, 1)$. Thus, $\mathcal{S}(\phi)$ has at most two zeros for $\phi \in (0, 1)$, called sustain points ϕ_{s1} and ϕ_{s2} (with $\phi_{s1} < \phi_{s2}$).[9] If $b < \frac{1}{2}$, there exist at most two sustain points $\phi_{s1} \in (0,1)$ and $\phi_{s2} \in (0,1)$ and we have $\mathcal{S}(\phi) < 0$ for $\phi \in \{(0, \phi_{1s}) \cup (\phi_{2s}, 1)\}$ and $\mathcal{S}(\phi) > 0$ for $\phi \in (\phi_{1s}, \phi_{2s})$. If $b > \frac{1}{2}$, there exists one unique sustain point $\phi_{s1} \in (0,1)$ and we have $\mathcal{S}(\phi) < 0$ for $\phi \in (0, \phi_{1s})$ and $\mathcal{S}(\phi) > 0$ for $\phi \in (\phi_{1s}, 1)$, which concludes the proof. □

A.4 Symmetric dispersion

Using (17), the breakpoints are given by:

$$\begin{aligned}
\phi_{b1} &= \frac{\sqrt{\gamma^2(\sigma-1)^2\left[8b(\lambda+1)^2-4\lambda^2-8\lambda-3\right]+4\gamma\sigma(\sigma-1)+4\sigma^2}-2b\gamma(\lambda+1)(\sigma-1)}{\gamma(\sigma-1)\left[2b(\lambda+1)-2\lambda-3\right]-2\sigma} \\
\phi_{b2} &= -\frac{\sqrt{\gamma^2(\sigma-1)^2\left[8b(\lambda+1)^2-4\lambda^2-8\lambda-3\right]+4\gamma\sigma(\sigma-1)+4\sigma^2}+2b\gamma(\lambda+1)(\sigma-1)}{\gamma(\sigma-1)\left[2b(\lambda+1)-2\lambda-3\right]-2\sigma}.
\end{aligned} \tag{22}$$

The break point ϕ_{b1} lies in the interval $(0,1)$ if and only if:

(i). $\gamma \in \left(\frac{2\sigma}{(2\lambda+1)(\sigma-1)}, 1\right)$,

(ii). $b \in [b_1, b_2)$,

where:

$$\begin{aligned}
b_1 &= \frac{\left[\gamma(2\lambda+1)(\sigma-1)-2\sigma\right]\left[\gamma(2\lambda+3)(\sigma-1)+2\sigma\right]}{8\gamma^2(\lambda+1)^2(\sigma-1)^2}, \\
b_2 &= \frac{\gamma(2\lambda+1)(\sigma-1)-2\sigma}{2\gamma(\lambda+1)(\sigma-1)}.
\end{aligned}$$

We have that $b_1 \in \left(0, \frac{1}{2}\right)$ and $b_2 \in (b_1, 1)$. If, additionally, $b < \frac{1}{2}$, then we have also that $\phi_{b2} \in (0,1)$.[10] This means that the possibility of complete re-dispersion following agglomeration as ϕ increases requires that related variety is neither too high nor too low. However, if $b > \frac{1}{2}$, then ϕ_{b2} does not exist and we have a single break point ϕ_{b1} if conditions (i) and (ii) are satisfied.

[9] One of which is given by $\phi = 1$ if $b = \frac{1}{2}$.
[10] If $\gamma < \frac{2\sigma}{\lambda(\sigma-1)}$, then $b_2 \in \left(b_1, \frac{1}{2}\right)$ and the condition is trivially met by (ii). In this case, both break points exist.

A.5 Proof of Proposition 4

Taking the derivative of $\mathcal{G}$ in (18) with respect to b we get:

$$\frac{\partial \mathcal{G}}{\partial b} = -2\gamma(\sigma-1)(2z-1)^3\left(1-\phi^2\right) < 0, \;\; z \in \left(\frac{1}{2}, 1\right)$$

Next, solving $\mathcal{G}$ in (18) for b yields:

$$b = b_c \equiv \frac{(2z-1)\left(1-\phi^2\right)\left[\sigma+\gamma(\sigma-1)(1-2z)^2\right] - \sigma\left[2z^2(\phi-1)^2 - 2z(\phi-1)^2 + \phi^2 + 1\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]}{2\gamma(\sigma-1)(2z-1)^3\left(1-\phi^2\right)}. \tag{23}$$

As a result, we have $\mathcal{G} > 0$ for $b < b_c$ and $\mathcal{G} < 0$ for $b > b_c$. Next, we will prove that $b_c < 1/2$.

First, notice that $\lim_{\phi\to 1} b_c = \frac{1}{2}$. Next, we have:

$$\frac{\partial b_c}{\partial \phi} = \frac{\sigma \mathcal{N}}{2\gamma(\sigma-1)(2z-1)^3\left(\phi^2-1\right)^2\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right]},$$

where:

$$\begin{aligned}\mathcal{N} =& (2z-1)\left(\phi^2-1\right)\left[2z^2(\phi-1)^2 - 2z(\phi-1)^2 + \phi^2 + 1\right] - \\ &- 4\left[z(\phi-1)+1\right]^2\left[z(1-\phi)+\phi\right]^2 \ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right].\end{aligned}$$

The numerator of the derivative is negative. As for $\mathcal{N}$, observe that:

$$\begin{aligned}\frac{\partial \mathcal{N}}{\partial z} =& -4\sigma(2z-1)(\phi-1)^2 \times \\ &\times \left\{(2z-1)\left(1-\phi^2\right) + 2\left[z(\phi-1)+1\right]\left[z(\phi-1)-\phi\right]\ln\left[\frac{z(\phi-1)+1}{z(1-\phi)+\phi}\right]\right\}.\end{aligned}$$

The first term inside the curly brackets is positive and the second one is negative as is the log term. Therefore, we have $\frac{\partial \mathcal{N}}{\partial z} < 0$. Since $\mathcal{N}\left(z = \frac{1}{2}\right) = 0$ and given that $\mathcal{N}$ is continuous in z, we can conclude that $\mathcal{N} < 0$ for $z \in \left(\frac{1}{2}, 1\right)$. Thus, we have $\frac{\partial b_c}{\partial \phi} > 0$, which means that $b_c < \frac{1}{2}$. Thus, if $b > \frac{1}{2}$, we have $\mathcal{G} < 0$ for any value of λ such that $\lambda^*(z) > 0$. This concludes the proof.

A.6 Bifurcation at symmetric dispersion

We can get a better picture of the dynamic properties of the model by studying the type of local bifurcation that the symmetric equilibrium undergoes at some break-point $\phi = \phi_b$. After some tedious calculations, it is possible to show the following:

$$\frac{\partial f}{\partial z}\left(\frac{1}{2};\phi_b\right) = 0;\ \frac{\partial^2 f}{\partial z^2}\left(\frac{1}{2};\phi_b\right) = 0;\ \frac{\partial f}{\partial \phi}\left(\frac{1}{2};\phi_b\right) = 0;\ \frac{\partial^2 f}{\partial \phi \partial z}\left(\frac{1}{2};\phi_b\right) > 0,$$

where $\phi_b \in \{\phi_{b1}, \phi_{b2}\}$. According to Guckenheimer and Holmes (2002, pp. 150), the conditions above ensure that symmetric dispersion undergoes a pitchfork bifurcation at $\phi = \phi_b$. Further, we have

$$\frac{\partial^3 f}{\partial z^3}\left(\frac{1}{2};\phi_b\right) = -\frac{32\mu(1-\phi)}{(\sigma-1)\sigma(\phi+1)^4}\xi,$$

where

$$\xi = 3\gamma(\sigma-1)\left[b(\phi+1)^2-\phi^2-1\right]\left[\lambda(\phi-1)+2\phi\right]-\sigma(\phi-1)^2(\phi+1).$$

If $\xi > 0$, the derivative is negative, and thus the pitchfork is supercritical and a curve of stable asymmetric equilibria branches from symmetric dispersion to its right. If $\xi < 0$, the derivative is positive and hence the pitchfork is subcritical and a curve of unstable asymmetric equilibria branches from symmetric dispersion to its left. If $\xi = 0$ we say that the pitchfork is degenerate.

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