

# Years where decades happen:

## *A Comparative Static Analysis of Regional Welfare Disparities in Brazil*

Paulo Henrique Puça França\*    Tatiane Almeida de Menezes†

### Abstract

Since the late 1990s, Brazil has experienced some important economic developments such as a macroeconomic stabilization, the expansion of social programs and a sustained economic growth supported by favorable external factors. These transformations motivated extensive research on topics such as inequality and poverty, but there has been given less attention to welfare analysis per se. Since conventional metrics such as GDP, nominal income and expenditure often miss key dimensions of household well-being, this absence translates into a gap in the welfare economics literature.

This study contributes to this literature by constructing a spatially adjusted Welfare Index that addresses these limitations commonly associated with traditional metrics. We define a welfare-enhancing consumption bundle, apply equivalence scales to account for household composition and economies of scale and estimate a spatial price deflator using the weighted Country-Product Dummy (CPD) method, a hedonic regression approach widely used for regional Purchasing Power Parity adjustments to correct regional price differentials for different locations.

Using microdata from the Household Budget Surveys (POF) for the years 2002–2003, 2008–2009 and 2017–2018, we employ First and Second-Order Stochastic Dominance tests and Growth Incidence Curves (GIC) to analyze welfare dynamics across time, regions and income brackets. We find significant welfare dispersion across regions, highlighting how income disparities translate into welfare inequalities. In an intertemporal perspective, results indicate a first-order dominance relationship between periods, characterizing Pareto-improving growth that was strongly pro-poor. These dynamics exhibit high regional heterogeneity. To conclude, we perform some sensitivity analyses regarding different equivalence scales, deflation methods and consumption bundles. Through a robustness matrix and first-order dominance relations, we show that results are robust to different equivalence scales and deflation methods and are sensitive to the composition of the consumption bundle, which reinforces the importance of properly defining welfare indices to study welfare dynamics rigorously.

**Keywords:** Welfare economics; Stochastic dominance; Country-Product Dummy (CPD); Brazil.

**JEL Codes:** I31; R13; C43; D63.

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\*Masters student at PIMES/UFPE. E-mail: paulo.puca@ufpe.br

†Full Professor at the Economics Department, Federal University of Pernambuco (UFPE). Permanent member of PIMES, PPGGES and PPGIT. E-mail: tatiane.menezes@ufpe.br

# I Introduction

The discussion about welfare has been central to economics since its origins as a social science. Discussing how to allocate goods, distribute income and produce efficiently requires a prior discussion about welfare, more specifically about what makes each allocation more or less desirable than another alternative. The task of aggregating different individual preferences in a social welfare function is central to the theoretical and empirical effort to evaluate living conditions in a rigorous manner.

This paper constructs a Household Welfare Index for Brazil using household consumption data from the Household Budget Survey (POF) for the years of 2002-2003, 2008-2009 and 2017-2018. The index adjusts consumption for welfare relevance, household composition and regional price differentials. Using first-order stochastic dominance, second-order dominance and growth incidence curves, the paper examines whether welfare increased over time, whether gains were pro-poor and whether regional disparities declined. The Brazilian case is particularly relevant because the Brazilian economic literature has focused extensively on poverty and inequality, while paying less attention to welfare measurement itself. Although poverty and inequality measures are closely related to welfare, they do not necessarily capture changes in the full distribution of living standards, nor do they account for regional price differences or the composition of welfare-relevant consumption. The contribution of the paper is threefold: it constructs a welfare-enhancing consumption aggregate, adjusts household welfare for family size and composition and incorporates spatial price differences into the measurement of real living standards. Our results indicate that welfare increased throughout the welfare distribution between 2002-2003 and 2017-2018, with stronger gains among lower-welfare households. However, these national gains did not fully translate into regional convergence, as the North and Northeast regions continued to lag behind the South and Southeast. These results are robust across multiple specifications and sensitivity checks.

Different approaches have been taken in the study of welfare economics<sup>1</sup>, and in this paper we largely follow the approach proposed in [Deaton and Zaidi \(2002\)](#) for welfare analysis in *Guidelines*. This approach is based on the *money metric utility* framework which associates a household consumption bundle with a spatial price deflator, an essential adjustment in countries with large regional price disparities, such as Brazil. The money-metric approach remains relevant in contemporary welfare measurement as current papers employ this strategy to measure social welfare<sup>2</sup>

The study of welfare in the Brazilian context is especially relevant when considering an environment historically marked by inequality and poverty. In fact, 20th-century Brazil presents significant income disparities with a particularly pronounced regional dimension, as shown by [Azzoni \(2001\)](#), which demonstrates how the income distribution shares by region in Brazil remained practically unchanged over a period of almost 60 years between 1936 and 1994. Beyond the persistent interregional income distribution gap, the decomposition of inequality into within-regions and interregional inequality via Theil's inequality index allows us to observe that the regional aspect of inequality assumes an increasingly explanatory role during this period. This exposes not only high inequality but also spatially determined inequality.

Brazil's high inequality is a phenomenon that occurs in parallel with high poverty levels in the country, which is also marked by strong regional patterns. At the end of the 20th century, 49.3% and 56.7% of households in the North and Northeast, respectively, were below the poverty line.

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<sup>1</sup> See [Fleurbaey \(2009\)](#) for a full historical breakdown.

<sup>2</sup> See [Baqaee et al. \(2024\)](#) and [Jaravel and Lashkari \(2024\)](#).

The spatial concentration of poverty is particularly evident in the Northeast, which accounted for 48.4% of the poor and 57.9% of the indigent population, despite representing roughly one quarter of Brazil's population; this exposes inequality as a result of the chronic impoverishment of a significant portion of the population.

In contrast to the harsh scenario presented by the 20th century, there is a substantial body of literature on poverty and inequality that points to a significant decrease in these indicators since the second half of the 1990s, a phenomenon largely explained by the end of hyperinflation. [Silveira Neto and Menezes \(2010\)](#) presents data from the Household Budget Surveys (POF) for the years 1996-1997 and 2002-2003, showing a considerable drop in the Gini coefficient of per capita household consumption, a pattern also observed in other indices that seek to measure inequality. These results are corroborated by [Silveira-Neto and Azzoni \(2011\)](#), which shows how the faster growth of per capita income in relatively poorer regions at the turn of the century led to a consistent decrease in the  $FGT_\alpha$  family indices ( $P_0$ ,  $P_1$  and  $P_2$ ), mainly for the states of the North and Northeast regions. Also associated with this process, the expansion of social programs such as Bolsa Família and the Continuous Cash Benefit (BPC) also occupies a central position in understanding the transformation that occurred in Brazil during this period. [Silveira Neto and Azzoni \(2011\)](#) uses the decomposition of the Gini coefficient by income sources to identify the factors behind this change through the National Household Sample Survey (PNAD) microdata between 1995-2006 and finds that 24% of the decrease in social inequality during the period is attributable to these programs.

A decline in poverty or inequality does not necessarily reveal how the entire welfare distribution changed. It also does not indicate whether welfare gains were driven by real consumption improvements, changes in prices, household composition or regional differences in the cost of living. These traditional metrics fail to capture the particularities of various spending and consumption bundles, which vary significantly across regions. An analysis of this magnitude requires the construction of a proper Household Welfare Index.

The remainder of this paper is organized as follows. Section II explores the use of a money-metric utility function to measure household well-being and details the adjustment of expenditures for regional price disparities. Section III discusses the construction of a welfare-enhancing consumption bundle. Section IV presents the data. Sections V and VI discuss our main empirical results, detailing the dynamics of well-being in Brazil and conducting a sensitivity analysis. Finally, Section VII presents concluding remarks.

## II Money Metric Utility and True Cost-of-Living Index

Conceptually, the money metric utility arises as a method to empirically study welfare using a potent and settled approach within economics of the utility function. The main difficulty to appropriately deal with the consolidated concept of utility in an observational framework is that the utility is a unobservable and ordinal metric that does not allow for interpersonal comparisons. These limitations of an otherwise excellent notion motivated an effort to create a metric that somehow captures utility in a comparable basis across different individuals, and with [Samuelson \(1974\)](#) there was a formalization of this idea which consists of associating different levels of well-being with the income needed to achieve them. To this end, an expenditure function is constructed<sup>3</sup> whose arguments are vectors of prices and a given a vector of prices and a target utility level, the expenditure function gives the minimum expenditure required to

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<sup>3</sup> A function whose minimization is the dual problem to utility maximization, reflecting the minimum cost required to achieve a given level of utility.

attain that utility level. This utility would be able to represent the agent's well-being and make it comparable, since it can be monetarily measured when combined with the price vector.

Hence, the money metric utility consists of a monetary (and therefore comparable) measurement for different levels of welfare to different households across space and time. As a starting point, assuming that each household attempts to maximize its well-being through a consumption bundle defined in the nonnegative orthant  $\mathbb{R}^k$ , the problem of the household utility maximization that can be generally expressed as such:

$$v(\mathbf{p}, I) = \max_{\mathbf{c}} \{U(\mathbf{c}) : \mathbf{p} \cdot \mathbf{c} \leq I\} \quad (1)$$

That means that each household decides to allocate its income  $I$  via a consumption vector  $\mathbf{c}$  with an associated vector  $\mathbf{p}$  of prices in order to obtain a maximum utility. Given the indirect utility function  $v(\mathbf{p}, I)$ , it is possible to construct, by duality, an analogous problem through the expenditure function defined by:

$$e(\mathbf{p}, U) = \min_{\mathbf{c}} \{\mathbf{p} \cdot \mathbf{c} : U(\mathbf{c}) \geq U\} \quad (2)$$

Let  $\mathbf{c}$  express the optimal consumption bundle that solves the minimization problem for the household  $h$  as a function of a given utility level  $U$  and the price vector  $\mathbf{p}$  faced by each family, that implies that the money metric utility may be represented as:

$$u_m^h = \mathbf{c}(u^h, \mathbf{p}^0) \quad (3)$$

As suggested by [Deaton and Zaidi \(2002\)](#), in order to deal with the fact that the level of utility  $U^h$  is unobservable, a practical solution to theoretically work with the money metric utility with observable variables while avoiding making assumptions about the utility function specific properties is the usage of the first-order Taylor approximation, which represents an accurate approximation of the function in  $\mathbf{c}^*$ . Hence, we can rewrite the expression as follows:

$$\mathbf{c}(u^h, \mathbf{p}^0) \approx \mathbf{c}(\mathbf{p}^h, u^h) + \nabla_{\mathbf{p}} \mathbf{c}(\mathbf{p}^h, u^h) \cdot (\mathbf{p}^0 - \mathbf{p}^h) \quad (4)$$

Through Shephard's Lemma, it is possible to rewrite the expression above substituting the partial derivative as the hicksian demand to obtain:

$$u_m^h \approx \mathbf{c}(u^h, \mathbf{p}^h) + (\mathbf{p}^0 - \mathbf{p}^h) \cdot \mathbf{q}^h \quad (5)$$

$$\mathbf{c}(u^h, \mathbf{p}^h) + \mathbf{p}^0 \mathbf{q}^h - \mathbf{p}^h \mathbf{q}^h \quad (6)$$

Given that the term  $\mathbf{p}^h \mathbf{q}^h$  corresponds exactly to the bundle  $\mathbf{c}(u^h, \mathbf{p}^h)$ , we finally get:

$$u_m^h = \mathbf{p}^0 \mathbf{q}^h \quad (7)$$

Similarly to Samuelson's money metric utility,  $u_m^h$  expresses the household utility as a combination of vector of quantities consumed by each family given by  $\mathbf{q}^h$  and a general price vector given by  $\mathbf{p}^0$ . As in *Guidelines*, we rewrite the money metric utility by deflating the household consumption bundle by a price deflator that has the main goal of explicitly adjust different household expenditures to different price vectors that vary considerably across different regions, especially for developing countries such as Brazil. Therefore, it is essential to construct an approximation for the *True Cost-of-Living Index (True COLI)* that consists of a coherent representation of price indices and its spatial variation to properly adapt expenditure to local price levels.

The use of classical index numbers such as Laspeyres and Paasche (which is the deflator used in *Guidelines*) is insufficient to appropriately adjust household expenditure for a number of reasons. Firstly, those indices admit only bilateral comparability, alter their values for different base-locations and are intransitive<sup>4</sup>.

Superlative index numbers, such as Fisher and Törnqvist, improve upon Laspeyres and Paasche, but their exact cost-of-living interpretation depends on assumptions about preferences and functional forms as they represent ideal subcases for quadratic and homogeneous translogs preferences<sup>5</sup>. This limitation by itself is sufficient to argue against the use of those indices as they are unable to account for non-homothetic preferences.

A well-established approximation alternative for the estimation of the *true COLI* is the Country-Product Dummy (CPD), that originated in the context to overcome *missing data* problems by estimating through a hedonic regression missing prices for determined locations but soon began to be used in the context of estimating locational price differentials. In this sense, Rao (2005) argues that the CPD acts as a specific subcase of hedonic regressions where it is necessary to consider both the coverage differential in terms of products that are representative in consumption bundles and non-representative but also priced items; this adjustment occurs through the use of *expenditure shares*, thus forming the *weighted* CPD. This adaptation enables us to use the smallest conditional error in the minimization of weighted least squares problem giving more weight to observations with lower conditional variance as Silver (2002) reasons, which means in our case, goods that are more representative of household consumption.

Beyond base invariance and transitivity, fundamental characteristics for a spatial price differential estimator, the weighted CPD, being an econometric method, also provides the standard error for each locational price differential estimate, making the estimation more reliable within a range of bands and properly weights the price index by the economic relevance of each item, as argued by Diewert (2005). The treatment via *weighted least squares* (WLS) also softens potential problems of heteroscedasticity and heterogeneity of the consumption bundle, since the weighting occurs at the local consumption level without the need to construct a representative bundle.

To express the *weighted CPD*, let each good be indexed by  $i$ , each locality  $j$  and  $u$  be a measure of error, we can express hedonic prices as:

$$p_{ij} = \pi_i \eta_j u_{ij} \quad (8)$$

$$\ln(p_{ij}) = \ln(\pi_i) + \ln(\eta_j) + \ln(u_{ij}) \quad (9)$$

This allows us to estimate the CPD in its weighted version, which can be analytically represented as the following minimization:

$$WRSS = \sum_{j=1}^M \sum_{i=1}^N w_{ij} (\ln(p_{ij}) - \pi_1 D_1 - \pi_2 D_2 - \dots - \pi_n D_n - \eta_1 L_1^* - \eta_2 L_2^* - \dots - \eta_m L_m^*)^2 \quad (10)$$

In which  $w_{ij}$  represents the vector of weights formed by expenditure shares at a local level for each good in each location,  $D_i$  represents a dummy for each good and  $L_j$  represents a dummy for each locality. The minimization problem results in an estimation of the following type:

$$\ln(p_{ij}) = \sum \pi_i D_i + \sum \eta_j D_j + u_{ij} \quad (11)$$

<sup>4</sup> Moreover, Nakamura (2020) also analytically demonstrates that *Laspeyres* and *Paasche* are, respectively, the *upper* and *lower* bound of the *true COLI*.

<sup>5</sup> See Dumagan and Mount (1997).

Since this estimation occurs in a logarithmic scale, it is exactly the exponential term  $e^{\eta_j}$  that will measure locational price differential. Therefore, we can finally express our Household Welfare Index as nominal expenditure  $x_j^h$  adjusted by the regional price differential:

$$\text{Household Welfare Index}_j^h = \frac{x_j^h}{e^{\eta_j}} \quad (12)$$

### III Adjustments over the household consumption bundle

The task of constructing an aggregate consumption bundle capable of accurately reflecting living conditions for each household is vital for any welfare analysis. As a consequence, a series of questions emerge about the nature of expenditures that should be accounting as welfare-enhancing consumption to adequately measure not just household expenditure but its welfare per se.

Furthermore, specifics about family size and composition are not details, but are specially relevant as households experience scale economies in its consumption that may turn the sole use of the *per capita* measurement inherently biased towards larger families. Moreover, different family compositions with children, adults and elderly results in allocations that require different amounts spent for a similar level of well-being. In the following subsections, we will explore how to work through these particularities that influence our household welfare index.

#### III.A Welfare-enhancing consumption

Given the centrality of household consumption to our measure of welfare, it is essential to appropriately construct a welfare-enhancing consumption bundle, i.e., a consumption bundle built around goods and services that positively affect the current well-being of families. This enables us to observe current living standards, excluding expenditures that are better interpreted as taxes, transfers, investment or atypical outlays.

It should be evident that some outlays made by the household do not constitute welfare-enhancing consumption expenditures per se, such as taxes and fees, in such way that accounting those expenditures as positively affecting well-being could have the effect of overestimating true household welfare by computing any expenditures as welfare-enhancing.

Moreover, there is considerable nuance in how to properly account for different types of consumption within the household unit. Indeed, different accounting for specific kinds of consumption may bias the calculated welfare index by confounding real household well-being with atypical expenditures or undesirable spending, for instance.

To address these concerns, a great body of work has emerged within the economic literature with recommendations on how to construct a household aggregate consumption that emphasizes welfare-enhancing aspects accordingly. The adaptations carried out throughout this paper largely follow [Ravallion \(1994\)](#) criteria for the development of a household consumption aggregate:

1. **Comprehensiveness:** The consumption bundle to be considered should aim to encompass as many goods and services as possible consumed by the household in a given period of time;

2. Relevance: What truly affects the family's well-being is the consumption of goods, not the expenditure on them<sup>6</sup>;
3. Typical Consumption: The aggregate consumption bundle should aim to reflect items that are part of standard family consumption, excluding atypical expenses;
4. Valuation: Not all items consumed by the family are purchased in the market, but they should still be valued for consideration in the aggregate consumption bundle.

Many debates arise from those recommendations as the gap between what is feasible in a research design and what is theoretically desirable widens. As discussed throughout [Mancini and Vecchi \(2022\)](#), a great deal of subjectivity is inherently required in a welfare analysis as choices must be made along the way; what is required in the end is to evaluate how dependable the resulting conclusions are.

Each one of the four criteria is connected with trade-offs between empirical feasibility and credible results. The first criterion of comprehensiveness, for instance, proposes that the consumption bundle should include as much as possible of every good and service consumed, but metrics associated with the consumption of non-marketable goods are often irregular and unreliable in such a way that their inclusion may do more harm to the welfare index than make it more complete. The second criterion of relevance casts doubt on how to appropriately deal with durable goods as depreciation is an important factor in its consumption; the fourth criterion of valuation is not clear if the valuation should be made with market prices or production costs as the agent has not explicitly demonstrated their willingness to pay through revealed preferences. More important debates are intertwined with each one of these criteria that will not be extensively discussed in this paper. For a brief summary of the main dubious/controversial item, Table 1 presents the main items included or not and a justification<sup>7</sup>:

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<sup>6</sup> This distinction is less relevant for items of immediate consumption, such as food, but an essential particularity in the treatment of durable goods.

<sup>7</sup> Non-specified items were included in the welfare-enhancing bundle.

**Table 1** – Construction of the aggregate consumption bundle

| Item                                    | Included | Justification                                                                                                                                                   |
|-----------------------------------------|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Food items                              | Yes      | Their inclusion meets the comprehensiveness criterion.                                                                                                          |
| Public goods                            | No       | Excessive error associated with measuring the utility derived from public goods.                                                                                |
| Health                                  | Yes      | Following the recommendation of <a href="#">Mancini and Vecchi (2022)</a> , the expenditure elasticity of health is high enough to be considered <sup>8</sup> . |
| Education                               | Yes      | Its inclusion meets the comprehensiveness criterion.                                                                                                            |
| Durable goods                           | No       | The available data do not allow for a reliable estimation of the service flow or depreciation rate of each asset.                                               |
| Rent/Imputed rent                       | Yes      | Its inclusion meets the comprehensiveness criterion.                                                                                                            |
| Alcohol, tobacco, and narcotics         | No       | These expenditures do not represent welfare-enhancing expenses in the household context nor monotonic welfare properties.                                       |
| Taxes and fees                          | No       | Do not constitute welfare-enhancing expenditures but rather as income deductions.                                                                               |
| Ceremonies, parties, and leisure travel | No       | Excluded when they represent atypical or lumpy expenditures rather than regular household consumption.                                                          |
| Donations and gifts                     | No       | To avoid double counting of consumption.                                                                                                                        |

### III.B Family size and composition

Furthermore, it is important to notice that as the household includes shared household goods in the family spending, the growth of the number of people within the household implies a degree of economies of scale that arise from the dilution of these fixed costs among more members in the family unit.

More than just a theoretical point of contention, there is a great body of literature that suggests that not including these aspects in poverty and welfare analysis may bias the empirical measurement of any forthcoming index<sup>9</sup>. Although those shortcomings are well-known in the welfare literature, relevant works still argue in favor of the use of a *per capita* measurement for its merits in being simpler and more transparent while also making the cross-country comparisons easier to make.

In the context of measuring welfare in a single country, the merits of using a *per capita* measurement are less clear and its drawbacks are more serious. [Newhouse et al. \(2017\)](#) estimated child poverty with different parameters and found that the use of different parameters leads to estimates up to 3 p.p. larger in identifying poor children and [Salmeron-Gomez et al. \(2023\)](#)

<sup>9</sup> [Deaton and Muellbauer \(1986\)](#) and [Deaton and Paxson \(1998\)](#) discuss this topic in more detail.

observe that the number of children that live in poverty is significantly larger for households with a great number of children, which prompts the question of how the economies of scale may justify the heterogeneity of the estimation.

Different equivalence scales can be used to adjust household consumption for family size and composition. For its simplicity, we adhere to the common equation given by:

$$ES = (A + \alpha C)^\theta \quad (13)$$

Where  $\alpha$  represents a parameter of consumption equivalence between children (people aged under 14) and adults and  $\theta$  represents a parameter of economies of scale. For this present work,  $\alpha$  was established as  $\alpha = .55$  as this is the median value estimated for the Brazilian case in [Vaz and Vaz \(2007\)](#). For the parameter of economies of scale  $\theta$ , a value was established between the value recommended for poor and for rich countries by [Deaton and Zaidi \(2002\)](#) as .8 representing a compromise between the two since Brazil is an upper-middle-income country and there is absence of a consensual national parameter.

## IV Data

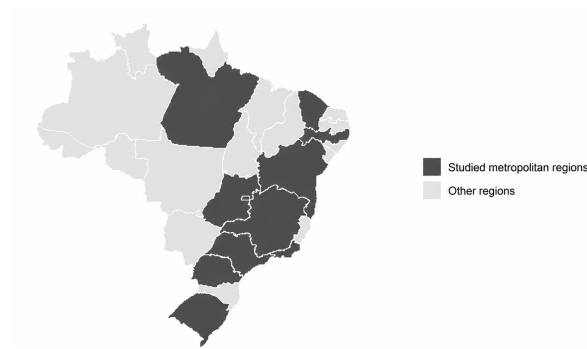
We use two main datasets in our research to construct the welfare-enhancing consumption bundle at the household level and to estimate locational price differentials. To construct the aggregate consumption bundle, we use consumption data from the Household Budget Survey (POF) for the years 2002/2003, 2008/2009, and 2017/2018. POF is a large-scale household survey that collects detailed information on household budgets, living conditions, and consumption patterns through household questionnaires, individual questionnaires, and expenditure records. Its level of granularity enables the research to discriminate expenditure at the level of goods and services consumed by each household in a given period of time, allowing for the reliable construction of the welfare-enhancing consumption bundle.

Expenditures are first temporally deflated to 2018 using the cumulative inflation calculated via the National Consumer Price Index (INPC) to make spending comparable over time. We then apply a spatial deflator, estimated from regional price data, to account for locational price differences.

The decision to use the INPC follows the standard practice in welfare and poverty analysis in Brazil since this index focuses on price variation for bundles consumed by households with income of up to 1 to 5 minimum wages, which permits us to make our analysis more representative of the median Brazilian household.

To correct for spatial deflation related to locational price differentials, we use data from a price vector for the reference year 2018 provided by the Institute for Applied Economic Research (IPEA), which supplies data on the average price within some selected metropolitan regions in Brazil at market prices. The metropolitan regions included in this analysis are shown below:

**Figure 1 – Analyzed regions**



The use of the Household Budget Surveys covering almost two decades of Brazil's living conditions allows for a comparative static analysis at the national level and also for regional disparities, offering a significant overview of the dynamics of well-being during that period. Some exploratory statistics may motivate our study, as Table 2, for instance, which provides an examination for the survey waves considered for household per capita expenditures by region:

**Table 2 – Average of metropolitan median per capita expenditures by region.**

| Region      | 2002/2003 | 2008/2009 | 2017/2018 |
|-------------|-----------|-----------|-----------|
| North       | 411.51    | 542.92    | 689.28    |
| Northeast   | 379.41    | 499.73    | 826.35    |
| Southeast   | 731.73    | 899.02    | 1280.24   |
| South       | 742.13    | 949.87    | 1442.97   |
| Center-West | 780.54    | 953.21    | 1652.76   |

**Source:** POF 2002/2003, 2008/2009 and 2017/2018.

*Note: Expenses with vehicle acquisitions were not considered.*

An initial descriptive analysis reveals that household spending is quite dispersed and unequal across regions, especially when considering the division between the South-Southeast axis and the rest of the country. For example, the highest per capita spending is more than twice as high as the lowest per capita spending in a region in 2018. Per capita household spending showed consistent growth across all regions, albeit to varying degrees. To further motivate the discussion, Table 3 presents the metropolitan price differential calculated via *weighted CPD*<sup>10</sup>. The estimation of regional price differences reveal how dispersed prices are in Brazil, with the disparity in values reaching approximately .15 p.p. between the most expensive and cheapest metropolitan regions. It is fair to point out that, as Gaddis (2016) emphasizes, nationally representative *market price survey data* are ideal sources for constructing an aggregate bundle, but they potentially suffer from substantial missing data. Therefore, by combining this data source with a method such as the *Country-Product Dummy*, which was originally conceived to address the problem of *missing data*, the results presented are more robust.

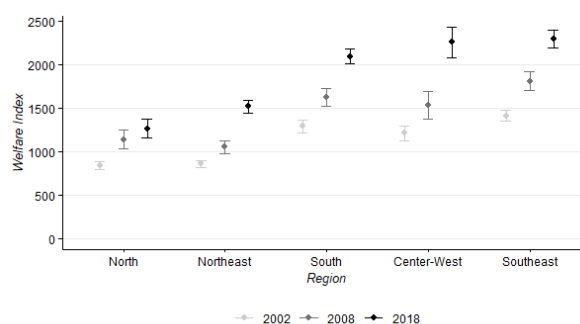
<sup>10</sup> Brasília is taken as numéraire in order to avoid perfect multicollinearity.

**Table 3 – Weighted Country-Product Dummy**

| Metropolitan region       | Coefficient | Standard error | $e^\beta$ |
|---------------------------|-------------|----------------|-----------|
| Belém                     | −0.158***   | 0.0234         | 0.854     |
| Vitória                   | −0.144***   | 0.0228         | 0.866     |
| Fortaleza                 | −0.143***   | 0.0224         | 0.866     |
| Recife                    | −0.14***    | 0.0225         | 0.869     |
| Salvador                  | −0.131***   | 0.0221         | 0.877     |
| Belo Horizonte            | −0.115***   | 0.0201         | 0.891     |
| Curitiba                  | −0.088***   | 0.0224         | 0.915     |
| Porto Alegre              | −0.071***   | 0.0234         | 0.931     |
| Goiânia                   | −0.069**    | 0.0219         | 0.932     |
| São Paulo                 | −0.064**    | 0.0236         | 0.937     |
| Rio de Janeiro            | −0.048**    | 0.0233         | 0.953     |
| Brasília                  | 0           | —              | 1         |
| <i>Observations</i>       | 2308        |                |           |
| <i>R<sup>2</sup> adj.</i> | 0.996       |                |           |
| <i>F statistic</i>        | 13296.41*** |                |           |

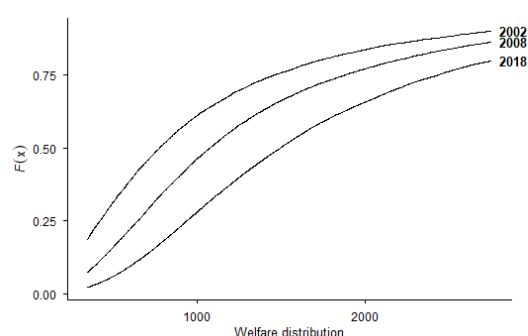
## V Results

The Household Welfare Index, given by equation II, was used to estimate household welfare at the consumption unit level. The result is shown in figure 2. The result shows how well-being in Brazil is mostly concentrated in the South-Southeast axis, even though the cost of living is higher in these regions. The North-Northeast regions consistently occupy the last positions across the years. Welfare improvements are heterogeneous across regions, but all regions display positive gains between 2002 and 2018. The timing of these gains differs considerably as some regions improved mainly during the 2000s, while others experienced larger gains during the 2010s. This is a result that will be discussed in subsection V.A.

**Figure 2 – Average welfare by region**

Beyond the estimated average well-being by region, it is essential to understand the behavior of the distribution of welfare at the national level during the period, as shown in figure 3:

**Figure 3 – Cumulative distribution of welfare**



The distribution reveals a first-order stochastic dominance (FOSD) characterized by  $F_{2018}(x) \leq F_{2008}(x) \leq F_{2002}(x)$ , which indicates that the 2018 welfare distribution dominates the 2008 distribution, which in turn dominates the 2002 distribution. This result indicates the presence of welfare gains for the entire population distribution within a comparable scale in real terms after considering the cumulative inflation of the period and the spatial variation of prices, indicating an improvement in welfare at the national level during this period. This signals a Pareto-like improvement at the distributional level, in the sense that the later distribution is preferred by any social welfare function that is increasing in household welfare; but first-order dominance does not reveal how the distribution of these gains occurs, that is, where the gains are more or less concentrated.

**Figure 4 – Intertemporal welfare dynamics in Brazil**

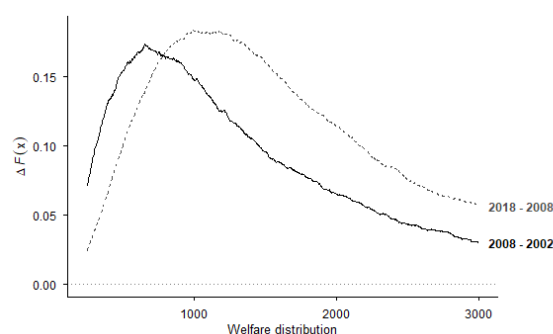
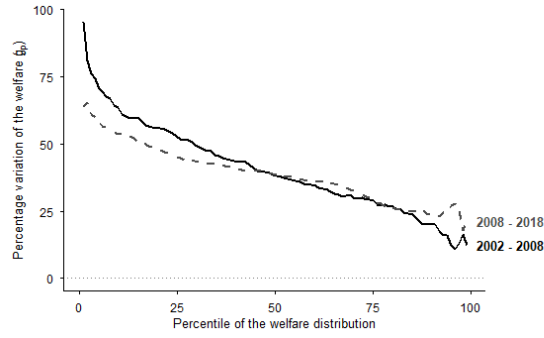


Figure 4 shows how, although well-being gains are positive across the population, their magnitude shows great heterogeneity for different segments of the population. In fact, what is observed is a considerable accumulation of welfare gains in the lower tail of the well-being distribution in Brazil, corresponding to the relatively poorer population. This phenomenon is observable for both the 2002-2008 and 2008-2018 time intervals, although it is more intense for the former. A visualization in percentage terms is possible using the growth incidence curve (GIC) to expand the understanding of the figure 4 for the reference period. The following result is shown in figure 5:

**Figure 5 – Growth Incidence Curve (GIC)**



The decreasing slope of the GIC reinforces the idea that the welfare gains in the reference period are concentrated in the lower tail of the welfare distribution, especially for the period 2002-2008, given the presence of a steeper downward slope. These relationships expose Brazil's dynamics during this period as one of well-being growth that, although encompassing the entire population, presents a significant disproportion of gains between social classes. This is evident when considering how the welfare gain among the poorest households is approximately 80 percentage points higher than among the richest households in the 2002–2008 period. This phenomenon is smoothed out in the period between 2008 and 2018, where the curve still indicates pro-poor growth, but with a smaller disparity in gains. This decrease in the welfare-gain disparity stems largely from a smaller increase in the well-being of the relatively poorer population in the second decade of the 2000s, as evidenced by the great similarity between the curves after crossing the threshold of the poorest 50%.

To understand the scale of gains beyond their dynamics, the concept of second-order dominance is applied to find the gains in monetary terms for each period, which provides a measurement of their magnitude. Letting  $Q_t$  represent the quantile distribution function of welfare for the reference year  $t$ , we can calculate the welfare gains in monetary terms as:

$$\frac{1}{6} \int_0^1 [Q_{2008}(p) - Q_{2002}(p)] dp = \frac{257.32}{6} = 42.89 \quad (14)$$

$$\frac{1}{10} \int_0^1 [Q_{2018}(p) - Q_{2008}(p)] dp = \frac{327.03}{10} = 32.7 \quad (15)$$

Therefore, when considering the annualized gains in well-being, it is observed that the period from 2002 to 2008 shows a gain 31.2% higher than the period from 2008 to 2018, that is, it not only represents a period of growth that is more pro-poor but also represents, in absolute terms, a higher accumulated growth in well-being.

## V.A Regional comparisons

We can perform a similar of examining differences between CDFs in a regional lenses, as shown in figure 6 by calculating  $F_{2002}(x) - F_{2008}(x)$  and  $F_{2008}(x) - F_{2018}(x)$  so that positive values indicate a rightward shift of the welfare distribution:

**Figure 6** – Difference between the CDF of equivalent household expenditures for Brazilian regions.

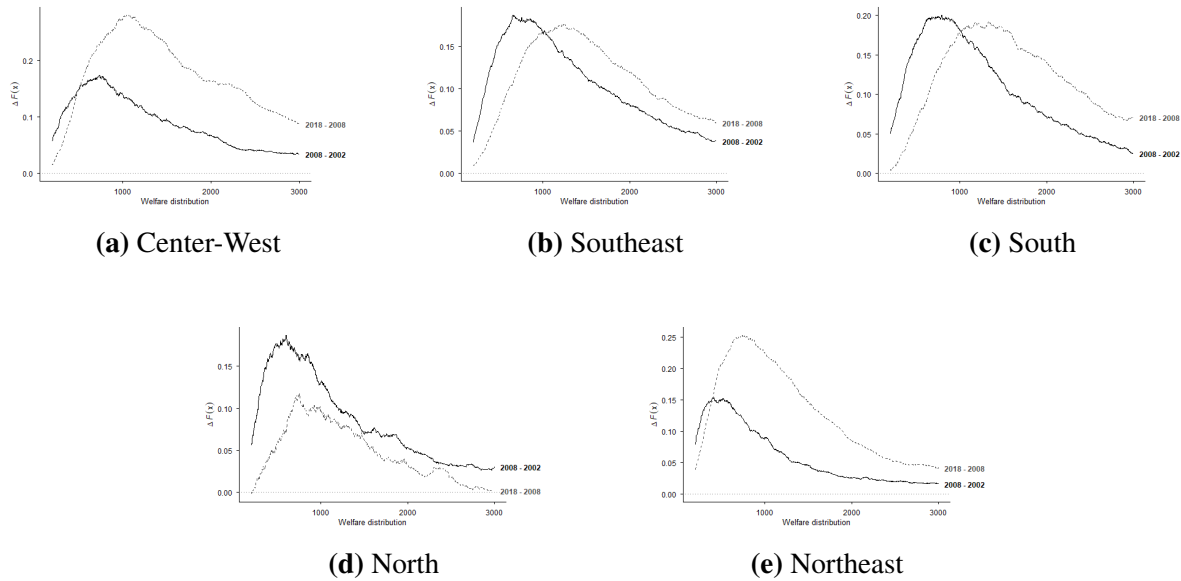
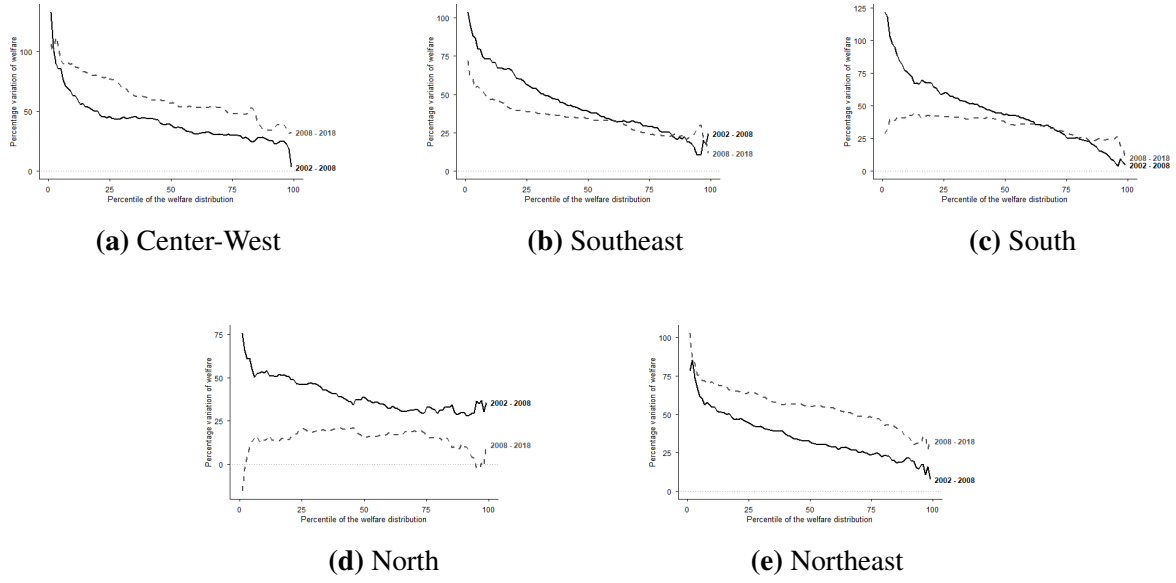


Figure 6 shows how observing only national dynamics overlooks a central aspect of Brazilian reality: regional disparities. Analyzing national dynamics by breaking them down into regions allows us to understand how growth can have greater or lesser magnitude depending on the region, and how different time periods can affect Brazilian macro-regions in different ways.

It is possible to observe consistent and distinct welfare gains in the North and Northeast regions during the reference period. For the other regions, the maximum gain of the lower tail of welfare distribution is significantly smaller. For this population, the gains appear to be higher during 2002-2008. This pattern is consistent with the expansion of redistributive and social policies during the 2000s that mark this period for the poorest segment of the population.

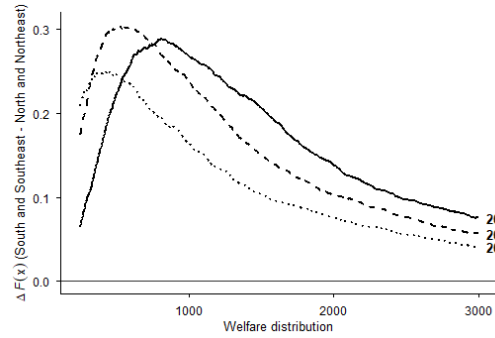
In the Southeast region, there is a concentration of welfare gains for the relatively poorer population, similar to the case in the Northeast, and again the poorest segment of the population showed superior gains for the period 2002-2008. Similarly, the South and Center-West also showed pro-poor growth for 2002-2008 and a less pronounced for 2008-2018, which allows us to see this as a persistent regularity among the regions. The GIC corroborates the results found:

**Figure 7 – Growth Incidence Curves for each region.**



The analytical exercise of exploring the dynamics of well-being within each region enriches the understanding of welfare evolution within regions, but it is still essential to understand the dynamics across the regions. One way to compare well-being between the North-Northeast and the South-Southeast is by studying the cumulative probability distribution of welfare  $F_i(S/SE) - F_i(N/NE)$ , where  $i$  is an index for the years 2002, 2008, and 2018. The result is shown in figure 8:

**Figure 8 – CDF difference between the southern and northern axes.**



The result shows how the welfare gap is particularly high for the poorest population, even when adjusting for price differentials, reflecting a previously discussed spatial concentration of poverty in Brazil, something that inevitably impacts well-being. The gap is also increasing for 2002-2008, while the period from 2008-2018 seems to have more distributive effects, with an increase in the gap specifically for the relatively wealthier population, and a decrease for the rest of society. To provide a cardinal measure of the regional welfare gap, we compute the average difference between the regional quantile functions. Let  $N$  represent the North-Northeast and  $S$

represent South-Southeast axis. We find the following results:

$$\int_0^1 [Q_{2002}(S) - Q_{2002}(N)] dp = 370.5274 \quad (16)$$

$$\int_0^1 [Q_{2008}(S) - Q_{2008}(N)] dp = 485.4934 \quad (17)$$

$$\int_0^1 [Q_{2018}(S) - Q_{2018}(N)] dp = 479.0663 \quad (18)$$

Therefore, although overall welfare inequality declined nationally, regional welfare disparities did not follow the same pattern even though the relatively poorest population in the country is concentrated in the North-Northeast regions. This suggests that pro-poor national growth did not fully translate into regional convergence between the North–Northeast and the South–Southeast axes.

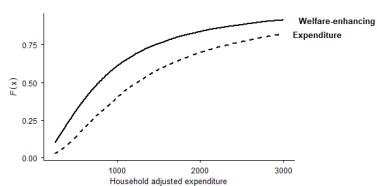
## VI Sensitivity analysis

Within the analytical scope of a welfare study, several decisions are made to understand well-being patterns, and often some decisions are partly normative or methodological, being inherently arbitrary. It is evident that the final results are therefore dependent on these decisions made throughout the process, making it essential to understand how relevant the assumptions are to the main results of well-being research. This investigation is particularly relevant when considering studies that use a predetermined equivalence scale, the construction of a consumption bundle, spatial deflation, and other criteria that directly affect the final results.

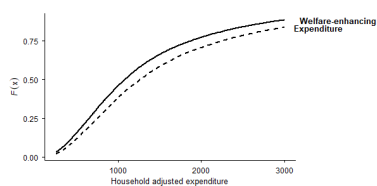
The first step in a sensitivity analysis is to observe the consistency of the results. This exercise is performed in Table 4:

**Table 4 – Analysis of first-order dominance for each criterion**

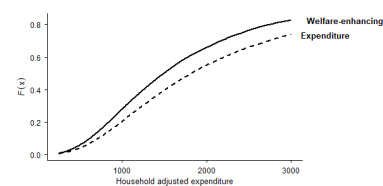
***Welfare enhancing bundle***



2002

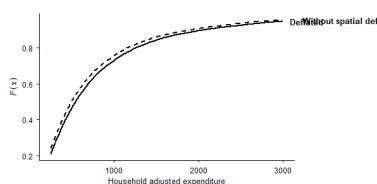


2008

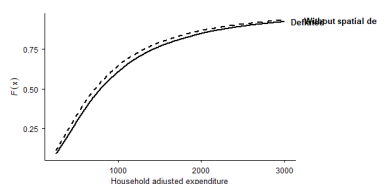


2018

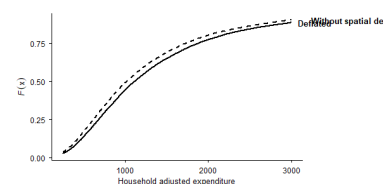
***Spatial deflation***



2002

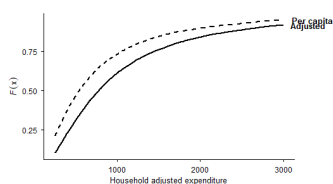


2008

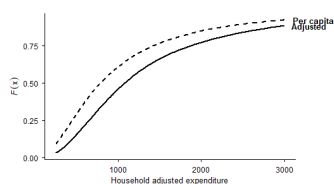


2018

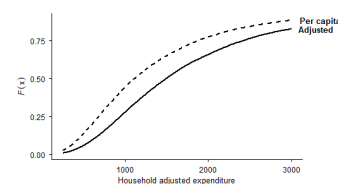
***Equivalence scale***



2002



2008



2018

The results can be considered consistent insofar as there is no crossing between the CDF curves when considering the construction of a welfare-enhancing bundle, spatial deflation, and the use of an equivalence scale. This implies that there is a first-order stochastic dominance between the curves, meaning that each of these choices results in a new distribution that does not disproportionately affect different segments of the population, which could bias the estimation results based on household consumption levels. Given that the results are consistent, it is important to understand whether they are robust to these methodological choices.

This analysis can be done by constructing a robustness matrix, as recommended by [Deaton and Zaidi \(2002\)](#), which presents the main results of the analysis and whether they hold up when these arbitrary choices are made relaxed. In this work, the main results are obtained by spatially deflating the expenditure of a specific consumption bundle with a specific equivalence scale, which therefore makes it necessary to relax each of these assumptions. When carrying out this process, the following is found:

**Table 5** – Robustness matrix for the stochastic dominance results

| Criterion                                     | Does first-order stochastic dominance across periods hold? | Is growth pro-poor? | Is regional inequality rising? | Is the welfare ranking unchanged? |
|-----------------------------------------------|------------------------------------------------------------|---------------------|--------------------------------|-----------------------------------|
| Without spatial deflation                     | ✓                                                          | ✓                   | ✓                              | <i>x</i>                          |
| Without welfare-enhancing bundle restrictions | <i>x</i>                                                   | ✓                   | <i>x</i>                       | <i>x</i>                          |
| Per capita                                    | ✓                                                          | ✓                   | ✓                              | ✓                                 |
| OECD I                                        | ✓                                                          | ✓                   | ✓                              | ✓                                 |
| OECD II                                       | ✓                                                          | ✓                   | ✓                              | ✓                                 |
| LIS                                           | ✓                                                          | ✓                   | ✓                              | ✓                                 |

Without spatial deflation, nominal expenditures overstate welfare in high-price regions, particularly in regions such as Southeast, which present cities such São Paulo and Rio de Janeiro. As a result, the regional welfare ranking changes, even though the main temporal and distributional conclusions remain unchanged.

In contrast, the use of a welfare-enhancing consumption bundle is central to some of the study’s conclusions. When the raw POF consumption aggregate is used, growth remains pro-poor. However, first-order stochastic dominance between 2002 and 2008 no longer holds, since the CDFs cross. This is also reflected in the Growth Incidence Curve, which becomes negative from the seventh welfare decile onward, indicating welfare losses among the upper part of the distribution. Similarly, interregional inequality no longer follows the baseline pattern: it remains roughly stable between 2002 and 2008 and falls marginally by 2018, demonstrating the centrality of this adaptation to the results found.

## VII Conclusion

This study concludes that welfare dynamics in Brazil throughout the 21st century were characterized by a generalized Pareto-like growth for all income brackets. While growth was strictly positive for the entire distribution at a nationwide scope, it was substantially greater for the poorest, particularly during the 2000s (2002–2008), with more moderate pro-poor trends persisting into the subsequent decade.

These results at a national scale can be complemented by a significant spatial heterogeneity for regional outcomes. Our findings highlight that despite the generalized improvement, the structural welfare gap between the historically wealthier South-Southeast axis and the North-Northeast remains constant and even marginally increased. This suggests that the convergence often observed in nominal income earnings has not translated into a convergence of household well-being when spatial price differentials and consumption patterns are properly accounted for. It is also perceivable that growth patterns exhibit significant variation across different regions and time frames, confirming the relevance of the spatial component for the welfare determinants.

From a methodological standpoint, the main conclusions of this work remained unchanged with the usage of different equivalence scales and spatial deflation, but were sensitive to the construction of a welfare-enhancing consumption bundle. This sensitivity analysis, therefore, reinforces the insufficiency of usual well-being proxies and emphasizes the importance of filtering non-utility-generating expenditures to accurately evaluate population well-being.

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