Viscoelastic continuum models based on evolving relaxed states

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Viscoelastic materials are ubiquitous in industrial processes involving polymers and mixtures of macromolecules and various suspending fluids. An interest in these materials dates back to the very origin of rheological studies. The characterization of viscoelastic fluids makes use of several rheometric techniques combined with efforts toward a physical understanding of the microscopic origin of the material response¹²³.

We present a model for viscoelastic materials based on the observation that the microscopic arrangement of molecules determines the state at which the system would converge in the absence of applied forces. Differently from what happens for solids, this state can evolve in time as a result of deformations and stresses. Moreover, it is not always possible to represent that state by means of a global configuration of the system. We rather seek a way to express a local state of deformation for which elastic stresses should vanish. This is accomplished by introducing a relaxed deformation tensor.

The main difference between our model and classical ones is in the use of two coupled tensorial evolutions equations: one for the current state of deformation and another one for the relaxed deformation tensor. Instead of postulating equations directly for the stress or for a conformation tensor, we generate the elastic contribution to the material response by a suitable combination of current and relaxed deformation tensors. In particular, we make use of a relative logarithmic strain as a measure of elastic deformations.

The modeling effort focuses on the evolution of the relaxed state. We take as a basic request that, if we keep the material in a static configuration, then the relaxed deformation tensor should converge to the current deformation tensor. We propose a model that, with common assumptions, is able to present significant differences between the evolution under simple shear or extensional flows. After studying the influence of material parameters on fundamental rheometric experiments, we also consider the modeling of finite-extensibility effects and their consequences on the predicted behavior.

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