Application of Post-Widder Inversion Formula to the Calculation of Relaxation Spectrum from Relaxation Modulus

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In linear viscoelasticity, the relaxation spectrum is one of the most important viscoelastic functions, because most other viscoelastic functions can be calculated from it, and it helps to understand the relaxation mechanism of materials. Relaxation spectrum cannot be measured directly and must be calculated from other measurable viscoelastic functions. However, the calculation of relaxation spectrum is a typical ill-posedness problem that measurement errors in the experiment are apt to be magnified in relaxation spectrum. Therefore, it is necessary to develop an algorithm that avoids the error magnification. The purpose of this study is to obtain the relaxation time spectrum $H(\lambda)$ from the relaxation modulus *G(t)* by use of the Post-Widder inversion theorem.

Since the relation between G(t) and $H(\lambda)$ can be transformed as a pair of Laplace transform, the determination of $H(\lambda)$ is equivalent to the inversion of Laplace transform. Tschoegl[1] has studied the inversion of the Laplace transform by use of the Post-Widder theorem, which helps to express relaxation spectrum as an infinite series of $d^nG/d\log^n t$. However, since his study was conducted in pre-computer days, high-order derivatives of experimental data were not available, the truncation up to the 4^{th} derivatives were calculated. Recently, Lee and Cho[2] developed a numerical algorithm for n^{th} order derivatives of positive functions such as storage and loss modulus by use of B-spline regression and Leibniz theorem for differentiation.

In this study, the inversion of Laplace transform was calculated using the Post-Widder inversion formula without the limit of differentiation order. This is the generalization of the work of Tschoegl[1]. This generalization is possible because we derived a recursive equation which transforms the Post-Wider inversion formula to the series of $d^nG/d\log^n t$. The recursive equation is essential in coding the numerical method.

[1] Tschoegl, N. W. *Rheol. Acta*. **10**, 582 (1971).
[2] Lee, J. H.; Cho, K. S. *Korea-Aust. Rheol. J.* **34**, 187 (2022).