Capillary imbibition of shear-thinning fluids: from Lucas-Washburn to oscillatory regimes.

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The study of capillary imbibition has ramifications in many fields, such as energy, biology, process industry, and subsurface flows. Although the capillary rise of Newtonian liquids has been the subject of several studies since the seminal works of Lucas (1918)[1] and Washburn (1921)[2], its generalization to the case of non-Newtonian fluids is still an open question. To fill this gap, starting from first principles, we derive a transient one-dimensional model describing the rising dynamics of shear-thinning fluid, whose viscosity is described by the Ellis viscosity model. Our model identifies the scaling for the different imbibition regimes accounting for the interplay of inertial, gravity, and viscous non-Newtonian (i.e., the zero-shear-rate and the shear-thinning behaviour) effects. Specifically, the rising dynamics is described by the interplay of three dimensionless parameters: the Richardson number, the Ellis number, and the shear-thinning index. At early times the system follows a universal inertial regime, followed by two possible limiting regimes, i.e., the classical Lucas-Washburn and the oscillatory regimes. The competition between gravity, non-Newtonian, and inertial effects dictates the transition between the two.

We show that when the viscous effect dominates over inertia at late times, identifying a (time-dependent) scaling law for the effective viscosity that accounts for the interplay of the zero-shear-rate and shear-thinning effects leads to a generalization of the Lucash-Washburn theory. The rescaled trajectories toward equilibrium collapse over the same master curve and follow the classical 1/2 scaling law. On the contrary, when inertia dominates the later stage of the imbibition, the filling length oscillates around the equilibrium configuration. By means of linear control theory, we discuss in detail the physical mechanisms that lead to such oscillating behaviour. We also show that the use of the power-law viscosity model cannot be legitimized since, close to equilibrium, the model would become singular due to the unbounded growth of the viscosity leading to a misprediction of the later stages of the imbibition.

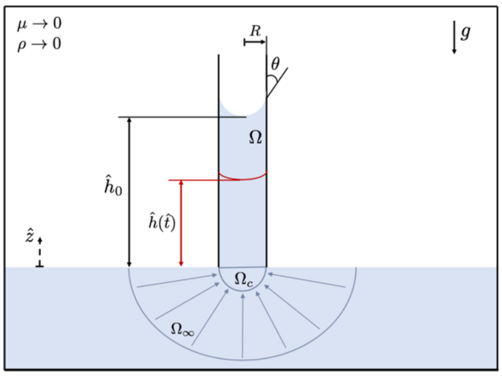

Figure 1: Sketch of the capillary in a capillary tube.

[1] Lucas, *Kolloid-Zeitschrift.* **23**, 15 (1918).

[2] Washburn, *Physical Review.* **27**, 273 (1921).