

Poststack Impedance Inversion in Attenuating Media Using Genetic Algorithm with Annealing and a $Z - Q$ Reflectivity Model

Introduction

Seismic energy propagating into subsurface geological layers experienced wave attenuation and velocity dispersion phenomenon in a wide range of frequency spectrum and spatial scales, i.e., from global-scale oscillations to high-frequency laboratory measurements. Numerous physical mechanisms, commonly grouped into elastic effects (such as scattering losses and geometric spreading that preserve total wave energy) and inelastic mechanisms (caused by the conversion of wave energy into heat), are responsible for the seismic wave attenuation (Gurevich and Carcione, 2022). From an exploration geophysics perspective, the inelastic attenuation of the P-wave caused by the pore fluid and elastic heterogeneities of the reservoir rock is of special interest and has been studied by many geophysicists (Ehsan et al., 2016; Khalid and Ahmed, 2016; Mavko et al., 2020). Although seismic attenuation influences seismic waveforms and amplitudes, it is often neglected in conventional parameter-estimation and acoustic impedance inversion methods, in addition to a limited number of previous studies (Chapman et al., 2006; Oliveira et al., 2009; Ahmed et al., 2023, 2024b).

Previous studies have shown that seismic attenuation can generate measurable normal-incidence reflection responses even in the absence of elastic impedance contrasts. Using poroelastic and viscoelastic theories, Quintal et al. (2009) demonstrated that wave-induced fluid flow in partially saturated thin layers can produce strong, frequency-dependent reflection coefficients, while Ren et al. (2009) analytically investigated the frequency dependence of normal-incidence reflectivity at interfaces involving dispersive, patchy-saturated media, identifying characteristic low-frequency bright spots, dim-outs, and phase reversals. Despite their physical rigor, these approaches present limitations for seismic-scale inversion. Both formulations rely on explicit frequency-dependent analysis and require assumptions regarding layer thickness, saturation geometry, or non-attenuating background media, parameters that are difficult to constrain from broadband seismic data. As a result, their direct application to post-stack seismic inversion is computationally expensive and sensitive to poorly known rock-physics inputs.

In this work, we adopt a constant Q viscoacoustic parameterization and derive a normal-incidence reflectivity model explicitly in terms of acoustic impedance (Z) and quality factor (Q). The resulting $Z - Q$ formulation avoids explicit frequency dependence and thin-layer assumptions and can be easily integrated into standard seismic forward modeling and inversion workflows for reservoir-scale property estimation. Poststack acoustic impedance inversion is implemented using a global optimization method, i.e., elitist genetic algorithm combined with a simulated annealing schedule. The genetic algorithm provides global exploration of the model space (Sen and Stoffa, 2013) and robustness against algorithms based on local minima (Ahmed et al., 2022, 2024a, 2026), while annealing progressively reduces the perturbation scale to improve convergence.

Theory and Methods

The theory and method part comprises numerous parts. In a first step, a normal incidence reflectivity equation is derived in the formulation $Z - Q$. The second part describes the rock physics theory used to estimate the P-wave seismic attenuation, and in the end, we discuss the seismic forward and inversion workflow adopted to predict the seismic P-wave impedance.

To address this limitation of the elastic impedance equation, we adopt a constant Q model and formulate the normal incidence reflectivity directly in terms of acoustic impedance Z and quality factor Q . Under the assumption of weak seismic attenuation and nearly constant Q over the seismic data bandwidth, the complex acoustic impedance of layer j is given as

$$Z_j^* = Z_j(1 + i\alpha_j), \quad \alpha_j = \sqrt{Q_j^2 + 1} - Q_j, \quad (1)$$

where $Z_j = \rho_j V_{Pj}$ denotes the real (lossless) acoustic impedance and Q_j controls intrinsic attenuation.

For a normally incident P-wave propagating from an upper medium (1) into a lower medium (2), the continuity of displacement and normal stress at the interface leads to the complex reflection coefficient.

$$R = \frac{Z_2^* - Z_1^*}{Z_2^* + Z_1^*}. \quad (2)$$

Substituting the above impedance parameterization yields a closed-form $Z - Q$ reflectivity equation that depends only on (Z_1, Z_2, Q_1, Q_2) . In the limit $Q \rightarrow \infty$, this formulation reduces exactly to the classical elastic reflectivity equation. Seismic attenuation is parameterized using two models: 1) Waters and Waters (1981) equation, and 2) a constant Q model that assumes a wave-induced fluid-flow mechanism in porous media Mavko et al. (2020). Total attenuation is the sum of the attenuation due to fluid and elastic rock heterogeneities, and the order of magnitude of seismic wave attenuation is controlled by the modulus difference between the high- and low-frequency limits of the elastic modulus (M_∞ and M_0 , respectively).

Forward modeling is performed using a convolutional poststack process in which a zero-phase Ricker wavelet is convolved with the reflectivity trace. The zero-phase Ricker wavelet is modified to incorporate intrinsic attenuation using a constant Q -based amplitude decay operator which is applied in the frequency domain.

$$\hat{w}_Q(\omega) = \hat{w}(\omega) \exp\left(-\frac{\omega T_{\text{eff}}}{2Q_{\text{eff}}}\right), \quad (3)$$

where $\hat{w}(\omega)$ and $\hat{w}_Q(\omega)$ denote the Fourier spectra of the original and attenuated wavelets, respectively, ω is the angular frequency, Q_{eff} represents an effective quality factor, and T_{eff} is the effective propagation time. The attenuated wavelet is obtained by applying the inverse Fourier transform and convolving it with the $Z - Q$ reflectivity series to generate synthetic seismic traces.

The Genetic-algorithm optimization workflow for the seismic poststack inversion is shown in Figure 1. The inversion process starts with the generation of an initial population ensemble around the low-frequency impedance model. For each candidate model, synthetic traces are modeled using a convolution-based seismic forward model, and the fitness of a candidate is evaluated.

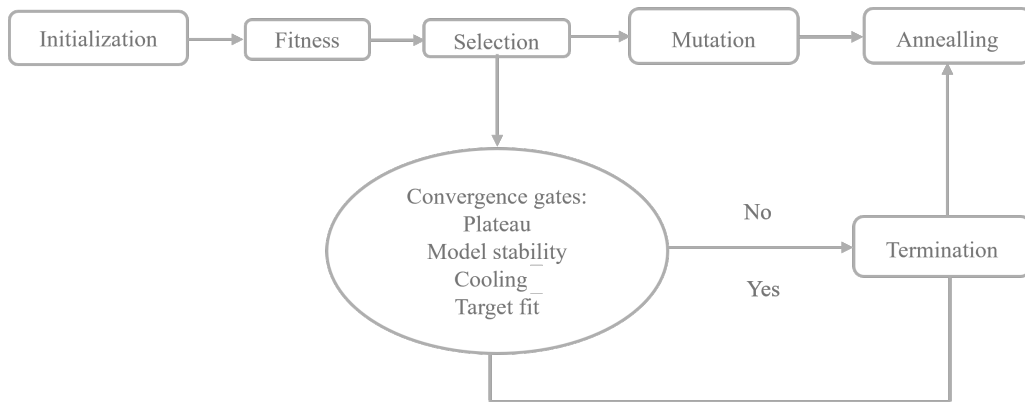


Figure 1 This workflow describes the optimization loop of the inversion algorithm, from initial population generation to convergence control.

In the next step, elitist selection criteria are adopted, whereby a fixed fraction of the highest-scoring candidate models is preserved at every generation. The median of the elite population is used to update the guiding trend, ensuring that favorable model characteristics are retained while maintaining diversity in the population. Then, new candidate models are generated by perturbing this updated trend. Simulated annealing is incorporated by progressively reducing the amplitude and spatial correlation length of the stochastic perturbations with increasing generation number. This annealing schedule enables broad exploration of the model space during early iterations and gradually focuses the search around promising regions, thereby enhancing convergence and stability. The algorithm is iteratively continued until one

or more convergence criteria are satisfied. These include an improvement in the objective function over successive generations, stabilization of the best impedance model, the achievement of a predefined similarity threshold, or the completion of the annealing schedule. Upon convergence, the best-performing impedance model is selected as the final inversion result.

Applications

This section describes 1D and 2D applications of the method, which utilizes the $Z - Q$ formulation of reflectivity and the inversion process, along with the convergence matrices. In the first numerical example, we adopt a very simple case to model Q , such as an empirical model (Waters and Waters, 1981) in which $Q = 10.76 \times V$, while P-wave velocity (V) is in km/s . The quality factor is shown in Figure 2a, and the reference model Z is obtained from the P-wave velocity and density logs, as shown in Figure 2b. Figure 2b compares the initial, true, and inverted models, and the inversion method successfully recovers the predicted impedance. Figures 2c - 2e show the performance matrices convergence of similarity, objective function convergence, and local and global seismic similarity measure, respectively. Overall, these curves illustrate that the inversion method progressively improves the match between observed and modeled seismic data, with both the objective function convergence and similarity measure curves increasing toward 1 as the generations advance. Their convergence close to one indicates that the population has stabilized around a consistent solution that closely reproduces the seismic response.

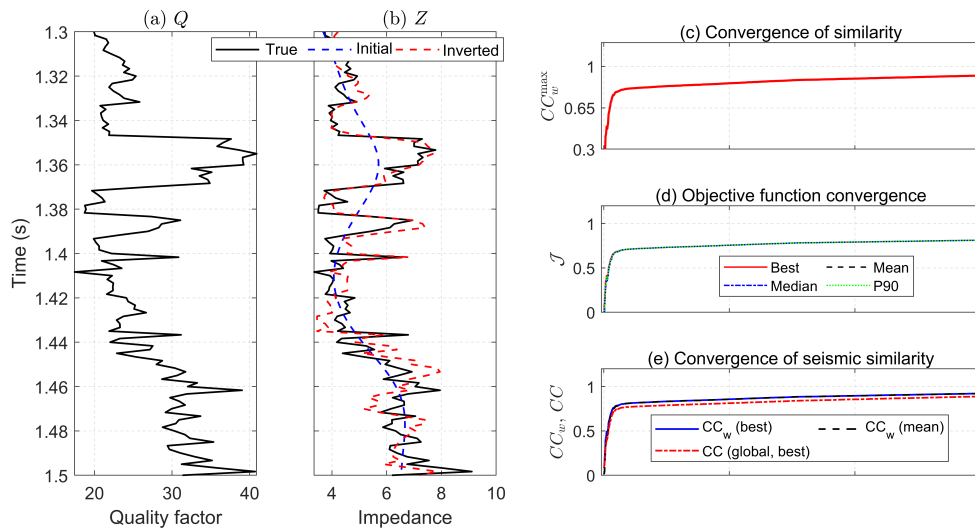


Figure 2 Comprises on (a) quality factor, (b) comparison of the initial, true, and inverted impedance models, convergence of (c) similarity, (d) objective function, and (e) seismic similarity.

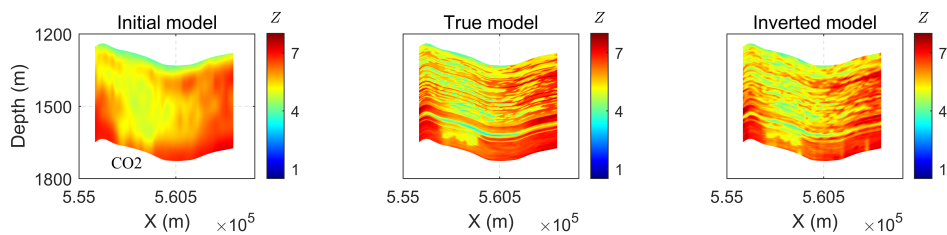


Figure 3 Comparison of the initial, true, and inverted models. The injection point is labeled with CO_2 in the initial model.

In the second example, we test the inversion on a 2D example modeled from the Smeaheia field reservoir, a potential CO_2 storage cite in the North Sea. In a first step, rock physics modeling is performed assuming a wave-induced fluid flow phenomenon to model a 2D Q - model, and then inversion is applied trace-by-trace. Figure 3 shows the initial, true, and inverted impedance models. Overall, the results show a closer match with the true models.

Conclusions

This work presents a derivation of a newly physically consistent $Z^* - Q$ reflectivity equation describing the normally incident seismic P-wave propagation in attenuating media. The effect of seismic wave attenuation on normal incident reflectivity is accounted for by using a constant model Q . The experiments show that the higher attenuation results in a phase change and magnitude loss in the single. It incorporates the contrasts in acoustic impedance and seismic wave attenuation across the geological layers in seismic reflection amplitudes and overcomes the limitations of elastic normal incidence reflectivity models. Acoustic impedance inversion is then implemented using a global optimization method, i.e., elitist genetic algorithm with simulated annealing. The results of synthetic and field data applications show that the inversion method using the new formulation provides geologically consistent Z models.

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