

Anisotropy in Wave-Equation-Based AVO Modelling and Inversion

Introduction and Theory

Vertical Transverse Isotropy (VTI) and Horizontal Transverse Isotropy (HTI) are forms of anisotropy that can be accounted for in seismic Amplitude-Vs-Offset (AVO) modelling and inversion. The VTI model is based on a horizontally layered medium where the properties depend on the inclination (angle of propagation vector with the vertical) of plane waves propagating through the stack while obeying Snell's law, i.e. with constant horizontal slowness, or ray-parameter. Apart from the angle dependency of the properties, this model is in agreement with the horizontally layered earth model assumed in AVO modelling and inversion. For HTI there is a contradiction between the horizontal property variation caused by vertical layering, and the standard AVO model of horizontal layering. However, it is very simple to split-up the AVO image-gathers in different azimuth bins, provided different azimuths are available, and invert them independently, leading to azimuth dependent properties, which should serve the purpose of finding azimuths that are perpendicular to vertical cracks. As this process will use standard AVO inversion technology it is already available and will not be further discussed in this paper.

For VTI, on the other hand, we need an extension of the current AVO data models, which will allow for inclination dependent properties. We do this based on the δ and ε parameters of Thomsen's weak anisotropy approximation (Thomsen, 1986):

$$\begin{aligned} v_p(\vartheta) &\approx v_p(0) \left(1 + \delta \sin^2 \vartheta \cos^2 \vartheta + \varepsilon \sin^4 \vartheta \right) \\ v_s(\psi) &\approx v_s(0) \left[1 + \frac{v_p^2(0)}{v_s^2(0)} (\varepsilon - \delta) \sin^2 \psi \cos^2 \psi \right] \end{aligned} \quad (1)$$

where we deliberately used different symbols for the inclination angle in the P -wave velocity v_p and the S -wave velocity v_s . In linear, primary reflectivity-based AVO models, all reflected waves are either P -waves, or S -waves, whereas all incident waves are P -waves, which makes it easy to translate the inclination angles ϑ and ψ to a ray-parameter p :

$$\sin \vartheta = p v_p \quad , \quad \sin \psi = p v_s \quad (2)$$

However, for full elastic wave-equation-based (WEB) AVO modelling and inversion (Gisolf et al. 2017), the total wavefield is always a combination of P -waves and S -waves. Since v_p and v_s directly relate to P -wave and S -wave propagation, this leads automatically, by inserting Eq. (2) into Eq. (1), to:

$$\begin{aligned} v_p(p) &\approx v_{p_0} \left(1 + \delta p^2 v_{p_0}^2 (1 - p^2 v_{p_0}^2) + \varepsilon p^4 v_{p_0}^4 \right) \\ v_s(p) &\approx v_{s_0} \left[1 + (\varepsilon - \delta) p^2 v_{p_0}^2 (1 - p^2 v_{s_0}^2) \right] \end{aligned} \quad (3)$$

Because we are dealing with weak anisotropy, and to keep the equations linear in δ and ε , we only use the zero-angle velocities on the right-hand side of Eq. (3). Eq. (3) can be used directly to compute the Zoeppritz coefficients used by the Kennett algorithm (Kennett, 1983) for full elastic AVO modelling of PP data and PS data. Full elastic WEB-AVO modelling and inversion is using the elastic compliances κ (compressibility = $1 / K$) and M (shear compliance = $1 / \mu$) that can directly be obtained from Eq. (3)

$$\begin{aligned} \kappa(p) &= \frac{1}{\rho} \left\{ v_{p_0}^2 \left[1 + \delta p^2 v_{p_0}^2 + (\varepsilon - \delta) p^4 v_{p_0}^4 \right]^2 - \frac{4}{3} v_{s_0}^2 \left[1 + (\varepsilon - \delta) p^2 v_{p_0}^2 (1 - p^2 v_{s_0}^2) \right]^2 \right\}^{-1} \\ M(p) &= \frac{1}{\rho} \left\{ v_{s_0}^2 \left[1 + (\varepsilon - \delta) p^2 v_{p_0}^2 (1 - p^2 v_{s_0}^2) \right]^2 \right\}^{-1} \end{aligned} \quad (4)$$

We could linearise Eq. (4) in δ and ε , but we didn't do that to keep δ and ε exactly as defined by Thomsen.

Full Elastic Modelling and Inversion of VTI AVO data

The two elastic AVO modelling methods we are demonstrating in this paper are the Kennett method and the integral formulation of the full elastic wave-equation (WEB-AVO). Both can account for VTI anisotropy with the help of Eqs. (3) and (4) respectively. For modelling we could also use the linearised primary reflectivity based methods, provided one takes the anisotropy into account both in the reflection coefficients (Rueger, 1998), as well as in the propagation of incident and reflected waves (Eq. (3)). Whether the last method will give a good modelling results depends entirely on the validity of neglecting all multiple scattering and multiple mode-conversion, but the purpose of the present paper is to demonstrate how anisotropy can be accounted for in full elastic applications.

In Figure 1 we show a set of logs from the Viking Graben, donated to industry and academia by Mobil many years ago. The Thomsen parameters shown are not part of that data-set, but arbitrarily chosen by the author, based on anisotropy curves that are sometimes provided with other sets of logs. We use these VTI parameters purely to demonstrate that VTI anisotropy can be modelled, and therefore inverted for, in full elastic wave-equation based methods.

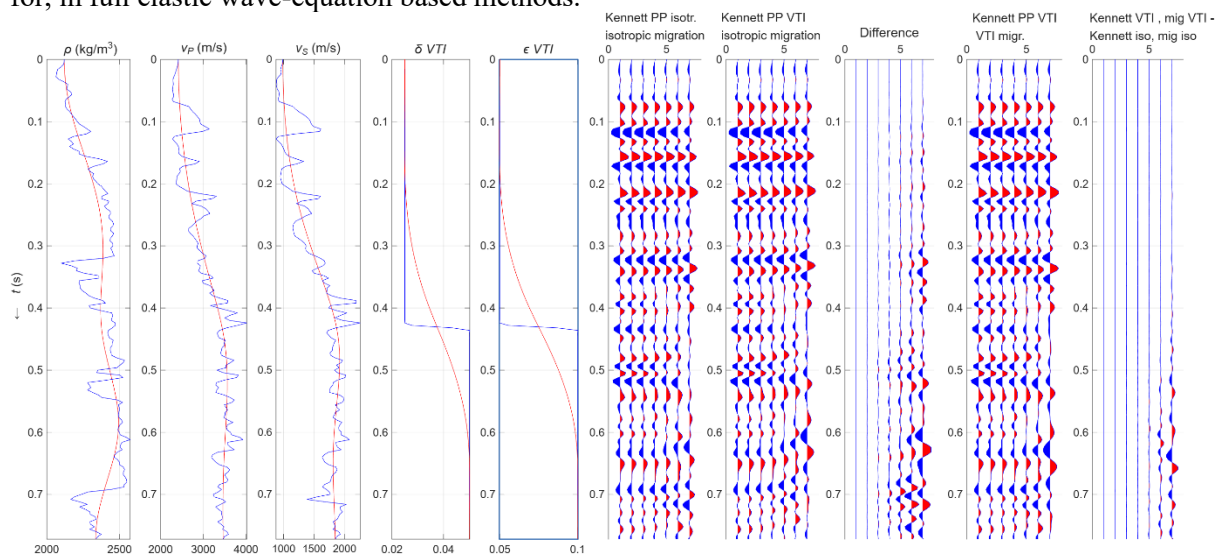


Figure 1 Kennett modelling of PP data, with and without VTI anisotropy. The data panels 6 and 7 show isotropically modelled and VTI modelled PP data in the τ/p domain, both migrated with the same isotropic migration velocities. In panel 8 we see the difference. The data panel 9 show the VTI modelled data, this time migrated with VTI migration velocities. The difference with the isotropically modelled and isotropically migrated data is shown in panel 10. The red curves in panels 1-5 are background curves, not used by Kennett modelling, but used as starting model for WEB VTI inversion. The blue curves in panels 1-3 are the zero-angle properties, anti-alias filtered for 3 m resampling, but still with .5 ft sampling. The VTI migration velocities are base on all red background curves.

In panels 6 and 7 in Figure 1 we see the Kennett modelling result without, and with, VTI anisotropy, based on the δ and ϵ curves shown in panels 4 and 5. Both data-sets were plane-wave migrated with the same isotropic migration velocities. From the difference shown in panel 8 we conclude that the modelled anisotropy has significantly impacted the data. In panel 9 we show the VTI modelled data, but this time plane-wave migrated with anisotropic migration velocities, based on the background δ and ϵ shown in red. The difference between panel 9 and panel 6 is shown in panel 10. This clearly shows that in this example the anisotropy has mainly affected the kinematics and only shows minor reflection amplitude effects. Given the δ and ϵ used for modelling, this is not surprising, but the point is that full elastic modelling, like Kennett, or WEB-AVO, make no distinction between kinematics and dynamics, whereas time domain reflectivity based inversion only accounts for the dynamic effects. In Figure 2 we show the same VTI Kennet result as shown in Figure 1, panel 7, and compare it with the WEB-AVO modelling based on the same parameters. The very minor differences are mainly caused by the fact that Kennett was run on 5 m anti-alias filtered, but not re-sampled properties, whereas for WEB-AVO these properties were actually resampled to a 3 m grid.

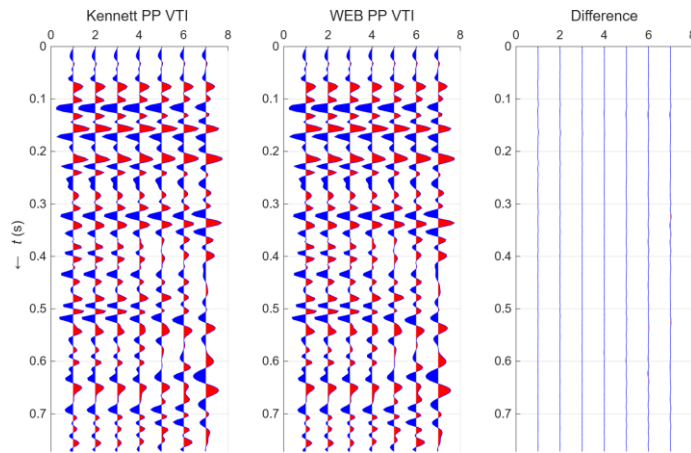


Figure 2 Comparison of full elastic modelling in the presence of VTI anisotropy using the Kennett method, with the full elastic wave-equation based (WEB-AVO) method. As pointed out in the caption of Figure 1, Kennett was applied to the anti-alias filtered properties without resampling. For the WEB modelling the anti-alias filtered properties were re-sample to 3 m sampling.

We will now look at AVO inversion of this data-set. We will only use WEB-AVO inversion, as Kennett on a fine grid is very time-consuming when used as modelling engine for inversion. WEB-AVO inversion is based on the integral representation of the 1.5 D full elastic wave-equation. It is an iterative scheme where in every iteration a higher order of multiple scattering is accounted for. The inversion is based on a series expansion of the wavefield in powers of the property contrasts against their backgrounds. It is a non-linear inversion scheme where all internal multiple scattering and internal multiple mode-conversions are accounted for, leading to properties that come out with a wider spatial bandwidth than obtained from linearised primary reflectivity based AVO inversions (Aki et al. 2002).

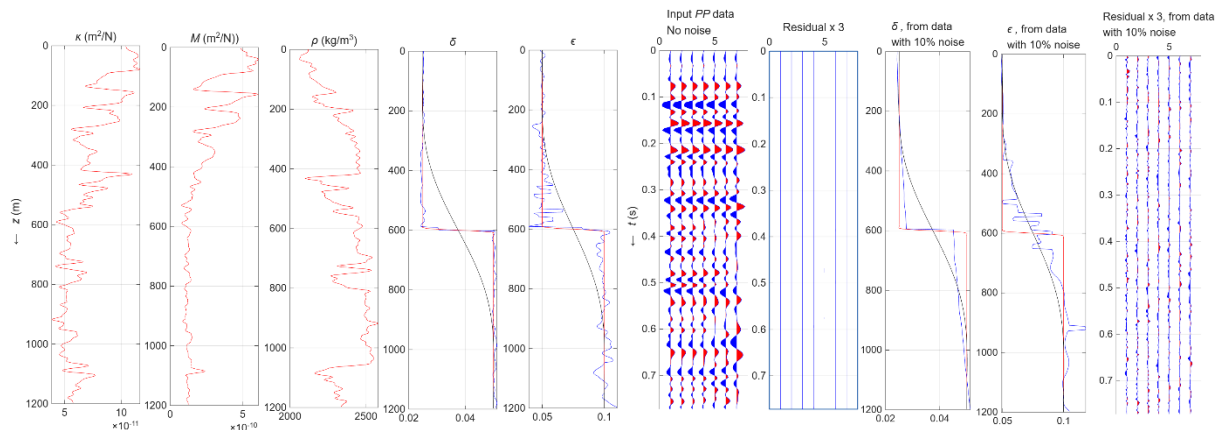


Figure 3 Inversion of noise free image gather (panel 6), for δ and ϵ only (panels 4 and 5), where κ , M and ρ are known. In panels 4 and 5 blue curves are inversion results, red curves are background/starting models, black curves are the true values. In panels 8 and 9 the same result, but now obtained from data with 10 % random noise added. Panels 7 and 10 show the residuals $\times 3$ of the noise-free and the noise-added inversions respectively.

In Figure 3, panels 4 and 5, we see the results for δ and ϵ of WEB-AVO inversion of the VTI data without additive noise, when ρ and the zero-angle κ and M values are known. In panel 7 the residual is shown. When 10 % Gaussian noise is added to the data, the result is shown in panels 8 and 9, with the residual shown in panel 10. From this experiment it is clear that from PP data δ is much better determined than ϵ . When we look at Eq. (1) it is clear that ϵ is only relevant for S-waves and the higher angles that are usually not present in AVO inversion. On the other hand, δ seems rather robust against random noise. It should be noted that for the inversion for δ and ϵ we used the smooth background model as starting model. The big question is where this information is going to come from. In normally logged wells there is no direct measurement of the Thomsen parameters. However, with the help of Backus averaging (Liner et al. 2006) we can have a rough estimate of the anisotropy introduced by fine layering only, which could serve as a prior estimate for inversion for δ and ϵ . For the logs shown in

this paper the natural layer-based VTI anisotropy is rather weak and wouldn't serve the purpose of demonstrating what WEB-AVO inversion for anisotropy parameters can do.

Finally we show the result of inverting the data for zero-angle κ and M , and ρ , when the anisotropy parameters are known. Also shown in Figure 4 is the result of the same inversion when anisotropy is ignored altogether, which significantly impacts (cf. Figure 1) the quality of κ , M and ρ results.

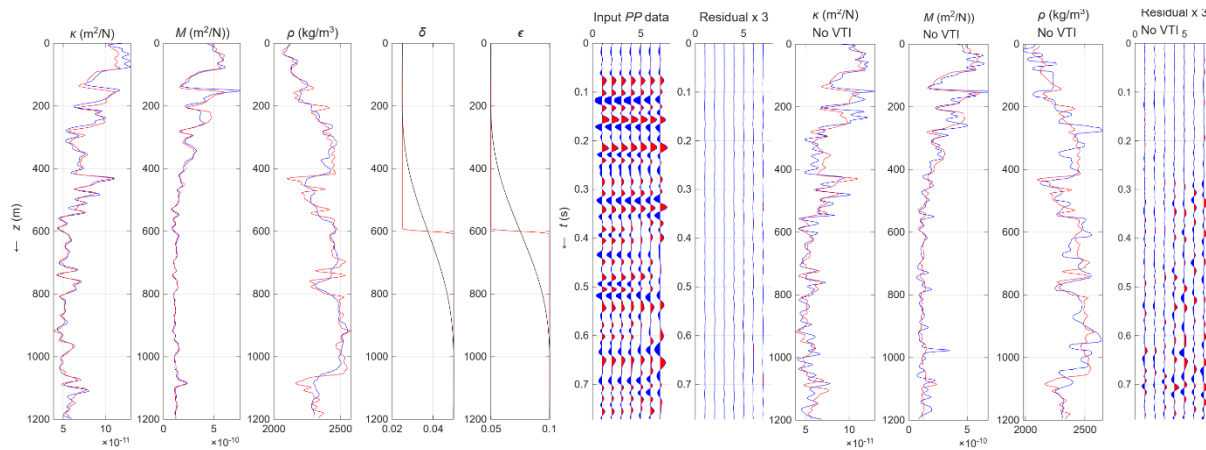


Figure 4 In panels 1-3 the results inversion of VTI anisotropic data, for κ , M and ρ , when the Thomsen parameters describing the anisotropy are known. In panels 4-6 we show the inversion result of the same data, when the anisotropy is ignored altogether.

The relevance of all this for real data inversions is that we first invert the seismic at the well location, for the Thomsen parameters, with known properties from the well, followed by inversion away from the well for the properties away from the well, assuming known anisotropy parameters determined at the well location. Simultaneous inversion for the properties and the anisotropy parameters is an under-determined problem that will not yield good results. The way we treat anisotropy is similar to how we deal with the wavelet, determined at the well location and kept constant away from the well.

Conclusions

Full elastic modelling of image gathers in a 1.5D VTI anisotropic earth can be carried out with the help of the Kennett algorithm, or the full elastic wave-equation based (WEB-AVO) method.

For AVO inversion we can use the WEB-AVO method, inverting for the VTI Thomsen parameters at the well location, and inverting for the zero-angle properties, in the presence of known VTI anisotropy, away from the well.

Although the 1.5D AVO data model is not directly suitable to describe HTI anisotropy according to its strict definition, it is simple to invert data from different azimuth bins separately and fit a simple curve as a function of azimuth to the results. This is relevant for finding crack orientations.

References

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