

iStein: Probabilistic Implicit Full-Waveform Inversion of the Viking Graben Field Dataset

Introduction

Full-waveform inversion (FWI) is the reference technique for high-resolution subsurface velocity estimation, exploiting the complete information content of recorded wavefields to image structures relevant to hydrocarbon exploration, CO₂ storage and geothermal assessment. Conventional deterministic FWI, however, returns a single velocity model and provides no measure of the reliability or non-uniqueness of the solution. Because the inverse problem is ill-posed, strongly non-linear and corrupted by noise, decision-critical interpretation tasks — well placement, reservoir characterization and risk assessment — require not only a best-fitting model but a quantitative assessment of its uncertainty.

Bayesian formulations address this need by targeting the posterior distribution over models. Markov chain Monte Carlo is asymptotically exact but prohibitively expensive, since every proposal demands a full wave-equation solve, whereas parametric variational inference is cheaper but restricts the posterior to a chosen family (Zhang and Curtis, 2020). Stein Variational Gradient Descent (SVGD; Liu and Wang, 2016) occupies an attractive middle ground: it is a deterministic, particle-based scheme that requires neither burn-in nor parametric assumptions. Its performance nonetheless deteriorates rapidly in high dimensions and, when applied directly in the velocity-grid space, it requires both very-low-frequency data and an initial model close to the solution to avoid particle collapse and cycle skipping (Corrales et al., 2025).

In parallel, implicit neural representations — and Sinusoidal Representation Networks (SIRENs; Sitzmann et al., 2020) in particular — have proven powerful reparameterizations for FWI: their periodic activations represent sharp contrasts and smooth gradients simultaneously, and their differentiability integrates seamlessly with automatic-differentiation wave solvers. Existing implicit FWI schemes, however, lack principled uncertainty quantification; Monte Carlo dropout (Gal and Ghahramani, 2016) yields largely epistemic uncertainties that do not faithfully reflect the true posterior variability of the inverse problem (Izzatullah et al., 2024).

We present iStein (implicit Stein Variational Gradient Descent), a probabilistic FWI framework that performs SVGD in the compact latent space of a latent-conditioned SIREN. This combination preserves the regularizing power of implicit representations while keeping the particle dynamics well-conditioned and inexpensive, yielding uncertainty estimates that are calibrated against the seismic data misfit. After summarizing the methodology, we focus on its application to the publicly available Viking Graben marine dataset, validating the recovered model against well-log measurements and analyzing the resulting uncertainty map and data fit.

Method

iStein parameterises the P-wave velocity field as a continuous function of spatial coordinates through a SIREN, rather than on a discrete grid. Each member of an ensemble is encoded by a low-dimensional latent vector z that is concatenated with the input coordinates and decoded by a single SIREN backbone whose weights are shared across all ensemble members. Consequently, the diversity of the ensemble is carried entirely by the latent vectors, and inference can be performed in a space of a few tens of dimensions rather than in the million-dimensional space of grid nodes or network weights.

The workflow comprises two phases (Figure 1). In the first phase, the shared SIREN is pre-trained on Gaussian Random Field (GRF) realizations, which endows the latent space with a geologically meaningful geometry; a single latent vector and the shared weights are then jointly optimized to minimize the seismic data misfit, placing the solution in the basin of attraction of a data-consistent model. In the second phase, the shared weights are frozen and an ensemble of latent particles, initialized by GRF-derived perturbations of the warm-start solution, is evolved by SVGD.

The SVGD update transports the particles through a reproducing-kernel Hilbert space, balancing a data-driven attraction term — obtained by back-propagating the misfit gradient through the wave solver and the SIREN decoder to the latent vector — against a repulsive term that prevents particle collapse and

maintains ensemble diversity. Because the particles live in the low-dimensional latent space, the radial-basis-function kernel evaluations are cheap and the repulsive force remains effective for moderate ensemble sizes, while the frozen decoder guarantees that every particle maps to a geologically plausible velocity model. The final ensemble $\{v_i\}$ approximates the posterior, from which the pointwise mean and standard deviation are extracted as the model estimate and its uncertainty. The forward problem is solved with the PyTorch-based acoustic engine Deepwave (Richardson, 2018), whose automatic differentiation provides the end-to-end gradients and the employed loss function is the standard L_2 norm.

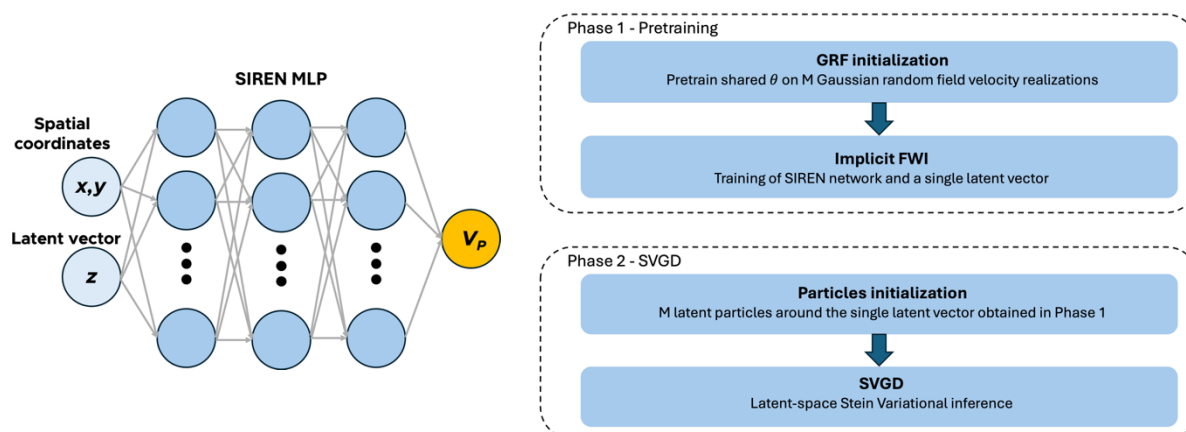


Figure 1 Left: latent-conditioned SIREN, mapping spatial coordinates and a latent vector z to velocity. Right: the two-phase iStein workflow — GRF pre-training and deterministic implicit-FWI warm-start (Phase 1), followed by latent-space SVGD posterior sampling (Phase 2).

Field data application: Viking Graben

The Viking Graben dataset is a publicly available 2-D marine line acquired in the North Sea (Keys and Foster, 1998) and a standard benchmark for real-data FWI, owing to its well-constrained geological context and the availability of well logs for independent validation. The line spans approximately 30 km and comprises 1001 shots, each with 120 channels, 6s of recording and a 4ms sampling interval; the common-midpoint spacing is 12.5m, the maximum offset is 3237m and the water depth is about 370m. The relatively short offset limits the penetration of diving waves and the illumination of deep structures, while strong water-bottom multiples contaminate the wavefield — a demanding test for any inversion scheme, especially in the absence of a high-quality initial model.

Prior to inversion, a band-pass filter with a 12 Hz high-cut was applied to observed and simulated gathers to restrict the inversion to the most informative frequency band, and a trace-by-trace amplitude normalisation was used in the misfit. The shared SIREN backbone has three hidden layers of 96 features; the latent dimension is set to $d = 32$, reflecting the predominantly layered character of the Viking Graben geology, which requires fewer degrees of freedom than a salt-bearing setting. The resulting network has roughly 56,000 trainable parameters, far fewer than the more than 700,000 model grid nodes, so the implicit representation acts as a compact regulariser rather than over-parameterising the problem. The ensemble comprises $N = 50$ particles, and both phases ran for 1000 iterations using the Adam optimiser with a warm-up followed by cosine annealing. Crucially, iStein converged directly from a simple 1-D gradient starting model without multiscale frequency scheduling; source batching and gradient checkpointing controlled the memory footprint of the large domain. All experiments were run on a single NVIDIA RTX 6000 GPU.

Figure 2 shows the posterior mean velocity model and the pointwise standard deviation. The mean model reveals a plausible North Sea sedimentary succession: a shallow water layer near 1500 m/s passes into a low-velocity near-surface zone and then into a broadly layered column whose velocity increases smoothly from about 1700 m/s in the shallow sediments to more than 3500 m/s below 3 km

depth. The principal velocity interfaces are laterally continuous across the full 30 km line, consistent with the sub-horizontal layering expected in this setting. The standard-deviation map is physically interpretable: uncertainty is low in the shallow section (above ~1.5 km), where direct arrivals and primary reflections at near-to-mid offsets provide strong constraints, and grows progressively below ~2 km as illumination from the short-offset geometry weakens; a mild lateral increase towards the line ends mirrors the reduced fold there.

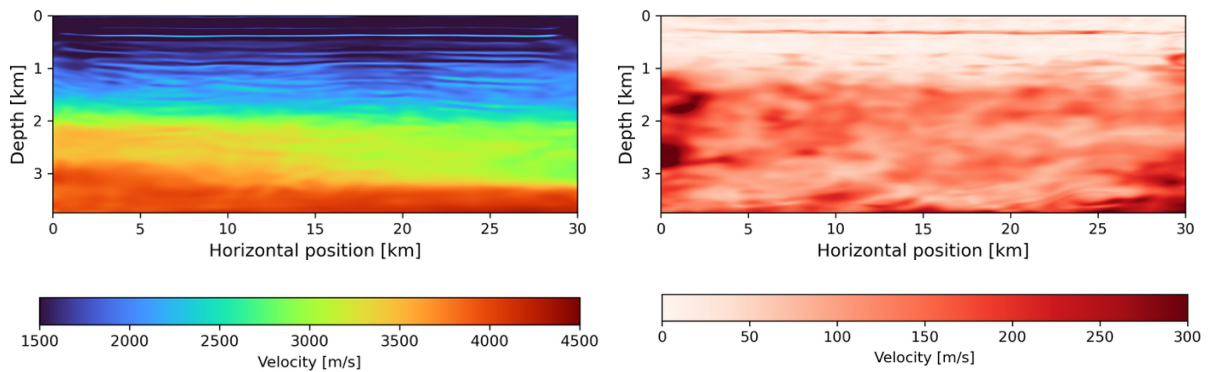


Figure 2 *iStein* result for the Viking Graben line. Left: posterior mean P-wave velocity. Right: pointwise posterior standard deviation across the particle ensemble.

The most stringent validation is provided by the two well logs available in the dataset. Figure 3 compares the 1-D velocity profiles extracted from the posterior mean at the two well locations against the logged velocities. Over the depth interval where the seismic data carry meaningful information (approximately 1–3.5 km), the inverted profiles track the major stratigraphic boundaries and reproduce the compaction-driven velocity trend characteristic of the North Sea. Crucially, the logged velocities lie predominantly within the 95% posterior confidence interval, whose width grows with depth as the seismic constraints weaken.

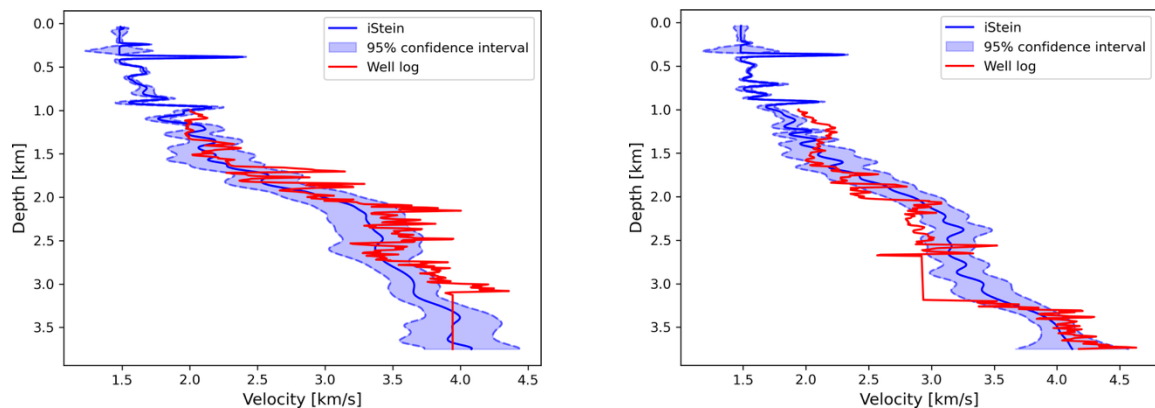


Figure 3 Independent well-log validation. *iStein* 1-D velocity profiles (blue) extracted from the posterior mean at the two well positions, with the shaded band showing the 95% posterior confidence interval, compared with the measured well logs (red).

Finally, Figure 4 compares observed shot gathers (black) with data modelled on the posterior mean (red) for two representative shots, at 3737 m and 18737 m along the line. The primary reflected arrivals are matched closely at near-to-mid offsets and early-to-intermediate times, confirming that the posterior mean is genuinely data-consistent and not merely a statistically plausible model. The residuals visible at far offsets and late times are consistent with the limited illumination of the deep structure by the short-offset acquisition — precisely the region that the standard-deviation map of Figure 2 flags as least certain, so that the data fit and the uncertainty estimate corroborate one another.

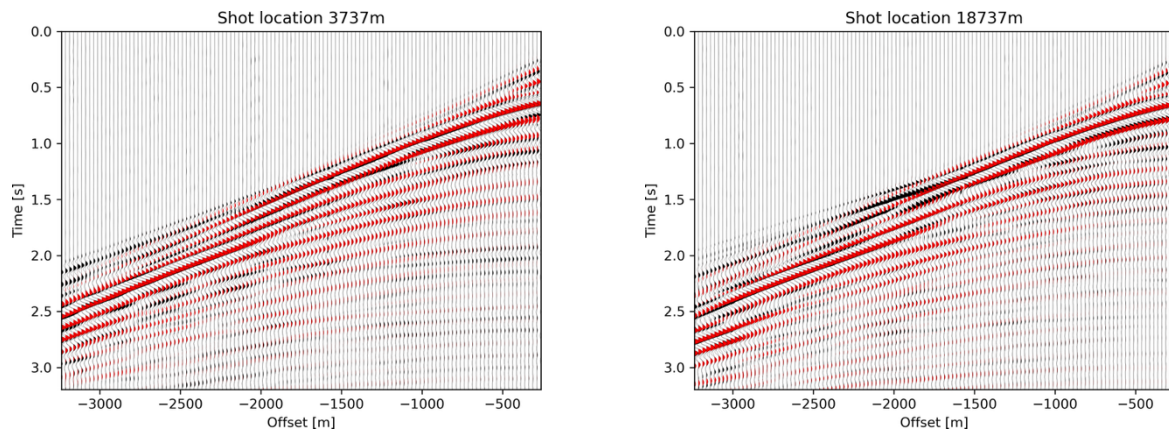


Figure 4 Waveform fit. Observed (black) and predicted (red) shot gathers modelled on the posterior mean for shots at 3737 m (left) and 18737 m (right).

Conclusions

We have applied iStein, a probabilistic implicit FWI framework, to the Viking Graben field dataset. By evolving an ensemble of latent particles with SVGD through a frozen, GRF-pre-trained SIREN decoder, iStein delivers both a velocity model and a calibrated, spatially coherent uncertainty estimate while converging from a simple 1-D starting model without multiscale frequency scheduling — conditions under which classical model-space SVGD typically fails. The posterior mean agrees with two independent well logs and reproduces the observed waveforms, and the standard-deviation map identifies the deep, poorly illuminated regions as least certain, in agreement with the far-offset data residuals. These results demonstrate that latent-space Stein inference is a practical route to uncertainty-aware FWI on real data, with direct relevance to risk-aware subsurface characterization. Future work will extend the framework to elastic forward modelling and to time-lapse monitoring applications.

References

- Corrales, M., Berti, S., Denel, B., Williamson, P., Aleardi, M. and Ravasi, M. [2025] Annealed Stein variational gradient descent for improved uncertainty estimation in full-waveform inversion. *Geophysical Journal International*, 241(2), 1088–1113.
- Gal, Y. and Ghahramani, Z. [2016] Dropout as a Bayesian approximation: representing model uncertainty in deep learning. In: *Proceedings of the 33rd International Conference on Machine Learning*, 1050–1059.
- Izzatullah, M., Alali, A., Ravasi, M. and Alkhalifah, T. [2024] Physics-reliable frugal local uncertainty analysis for full-waveform inversion. *Geophysical Prospecting*, 72(7), 2718–2738.
- Keys, R.G. and Foster, D.J. [1998] A data set for evaluating and comparing seismic inversion methods. In: *Comparison of Seismic Inversion Methods on a Single Real Data Set*, SEG, 1–12.
- Liu, Q. and Wang, D. [2016] Stein variational gradient descent: a general purpose Bayesian inference algorithm. In: *Advances in Neural Information Processing Systems*, 29, 2378–2386.
- Richardson, A. [2018] Deepwave: a PyTorch-based wave propagation and FWI package. Software, <https://github.com/ar4/deepwave>.
- Sitzmann, V., Martel, J.N.P., Bergman, A.W., Lindell, D.B. and Wetzstein, G. [2020] Implicit neural representations with periodic activation functions. In: *Advances in Neural Information Processing Systems*, 33, 7462–7473.
- Zhang, X. and Curtis, A. [2020] Seismic tomography using variational inference methods. *Journal of Geophysical Research: Solid Earth*, 125(4), e2019JB018589.