

A physics-informed neural network for inversion of vadose zone water saturation profiles from seismic wave velocities

Introduction

Climate change, increasing pressure on water resources, and the need for sustainable agricultural practices have highlighted the importance of monitoring water storage and transfers within the shallow subsurface. In agricultural environments, the vadose zone controls the redistribution of infiltrated water, regulates plant water availability, and contributes to groundwater recharge processes. Accurate characterisation of water saturation profiles in this partially saturated zone is therefore essential for hydrological studies, agricultural management, and the monitoring of critical-zone processes [1].

Among near-surface geophysical methods [2], seismic techniques provide useful information on the hydromechanical state of soils. Compressional and shear wave velocities are sensitive to porosity, effective stress, capillary effects, and fluid saturation, making them relevant observables for the characterisation of partially saturated and/or potentially unconsolidated materials [3]. Previous studies have indeed demonstrated the potential of seismic imaging and seismic inversion for investigating the architecture and hydrological properties of the critical zone [4, 5]. However, the relationship between seismic velocities and water saturation remains strongly nonlinear and depends on several coupled hydraulic and mechanical parameters. A major difficulty arises from the inverse relationship between seismic wave velocities and water saturation. In partially saturated and potentially unconsolidated soils, identical or very similar compressional and shear wave velocity profiles may result from different combinations of hydraulic and mechanical parameters. The inversion problem is therefore intrinsically nonlinear and potentially non-unique. Furthermore, field measurements are affected by acquisition noise and inversion uncertainties, which further reduce the sensitivity of seismic observables to water saturation variations. Consequently, estimating water saturation profiles from seismic velocities cannot generally rely on a direct analytical inversion and requires the development of robust surrogate inverse models capable of learning complex nonlinear relationships, notably from large synthetic datasets.

Neural networks offer a natural data-driven solution when the forward problem is well described but the inverse relationship is difficult to solve explicitly [6, 7]. In a previous study [7], a feed-forward network was trained to map velocity profiles directly onto saturation profiles. While effective on noise-free data, this purely data-driven strategy proved sensitive to noise: left free to interpolate any profile shape, the network can fit non-physical, oscillatory solutions once the input velocities are corrupted, particularly within the capillary fringe where seismic sensitivity to saturation is highest.

This study addresses this limitation with a physics-informed neural network (PINN). Rather than predicting the saturation profile directly, the network estimates the small set of hydraulic parameters governing it, which are mapped back onto a physically consistent profile through the forward hydromechanical model itself. The training loss further embeds this forward model explicitly, constraining predictions to remain compatible with the observed velocities. This reparameterisation removes non-physical solutions by construction and improves robustness to noise.

Methodology

Unlike the direct profile-to-profile mapping of [7], the network introduced here predicts the van Genuchten shape parameters α and n , and, in the variable water table configuration, the water table depth w_t , rather than saturation values at each depth. The full profile is then reconstructed from these parameters using a forward hydrostatic model, which constrains predictions to physically valid, monotonic profiles regardless of noise level.

The forward model combines several physical descriptions commonly used for partially saturated porous media. Compressional and shear wave velocities are computed from the elastic properties of the porous medium using the Biot–Gassmann framework [8]. This approach relates the effective bulk and shear moduli of a porous material to the properties of its solid skeleton and pore fluids. Since the targeted en-

Table 1 Inversion performance of the PINN for the fixed and variable water table configurations, at two noise levels each.

Metric	Fixed w_t		Variable w_t	
	3% noise	8% noise	2% noise	5% noise
RMSE	8.02×10^{-3}	1.58×10^{-2}	1.14×10^{-2}	1.55×10^{-2}
MAE	2.02×10^{-3}	4.58×10^{-3}	2.31×10^{-3}	3.06×10^{-3}
R^2	0.9991	0.9967	0.9983	0.9967
MRE	3.54×10^{-3}	7.99×10^{-3}	4.08×10^{-3}	5.38×10^{-3}
MBE	-2.1×10^{-5}	-1.7×10^{-4}	1.0×10^{-4}	9.9×10^{-5}
CC	0.9996	0.9983	0.9991	0.9984

vironments correspond to partially unconsolidated agricultural soils, the elastic behaviour of the granular skeleton is described using a Hertz–Mindlin-type rock-physics model [9]. This description thus accounts for the dependence of elastic moduli on contact stiffness and effective stress. Saturation profiles follow the hydrostatic van Genuchten formulation [10]; water table depth and retention parameters are varied to generate a broad range of physically realistic profiles, from which the corresponding velocity profiles are computed.

The network is trained on velocity profiles corrupted with fresh noise drawn independently at every batch, with noise amplitude increased progressively (curriculum strategy) up to the target level. The loss combines (i) a data term on the predicted hydraulic parameters, (ii) a physics-consistency term comparing velocities recomputed from the predicted parameters to the noise-free input, and (iii) a direct term on the reconstructed saturation profile. This last term is essential since α and n are partially non-identifiable: distinct pairs can yield near-identical velocity profiles, in which case only a direct constraint on $S(z)$ recovers the profile of interest.

Results

The PINN was tested on independent synthetic sets representative of a shallow subsurface for the fixed ($w_t = 5$ m) and variable ($w_t \in [1, 9]$ m) water table configurations, at two noise levels each, with α and n drawn over their full physically admissible range. Table 1 reports the root-mean-square error (RMSE), mean absolute error (MAE), coefficient of determination (R^2), mean relative error (MRE), mean bias error (MBE), and correlation coefficient (CC) obtained over the full set of test curves. Figures 1 and 2 show, for each configuration, the RMSE distribution over the test curves (top) and reconstructed profiles against ground truth for representative cases (bottom).

Across all configurations, the PINN achieves $R^2 > 0.996$ and correlation coefficients above 0.998, with mean bias error remaining below 2×10^{-4} in absolute value – indicating that the predicted profiles are, on average, unbiased and tightly correlated with the true profiles even at the highest noise level tested. RMSE and MAE increase only moderately with noise amplitude, by approximately a factor of two between the lowest and highest noise levels in both configurations, while the variable water table configuration performs on par with the fixed configuration despite the additional degree of freedom introduced by w_t . These results represent a substantial improvement over the direct profile-regression strategy of [7], for which comparable noise levels produced markedly degraded, locally non-monotonic saturation profiles, particularly within the capillary fringe. Visual inspection of the reconstructed profiles in Figs. 1–2 confirms that the parameter-based reformulation systematically yields smooth, physically admissible saturation curves, in contrast with the oscillatory artefacts observed when the profile is predicted point-by-point under noisy conditions. The remaining error is concentrated in a small subset of curves associated with van Genuchten parameter combinations for which α and n are weakly identifiable from the velocity profiles alone, an intrinsic limitation of the inverse problem rather than of the network itself.

PINN characterization — fixed water table ($w_t=5.0$ m)

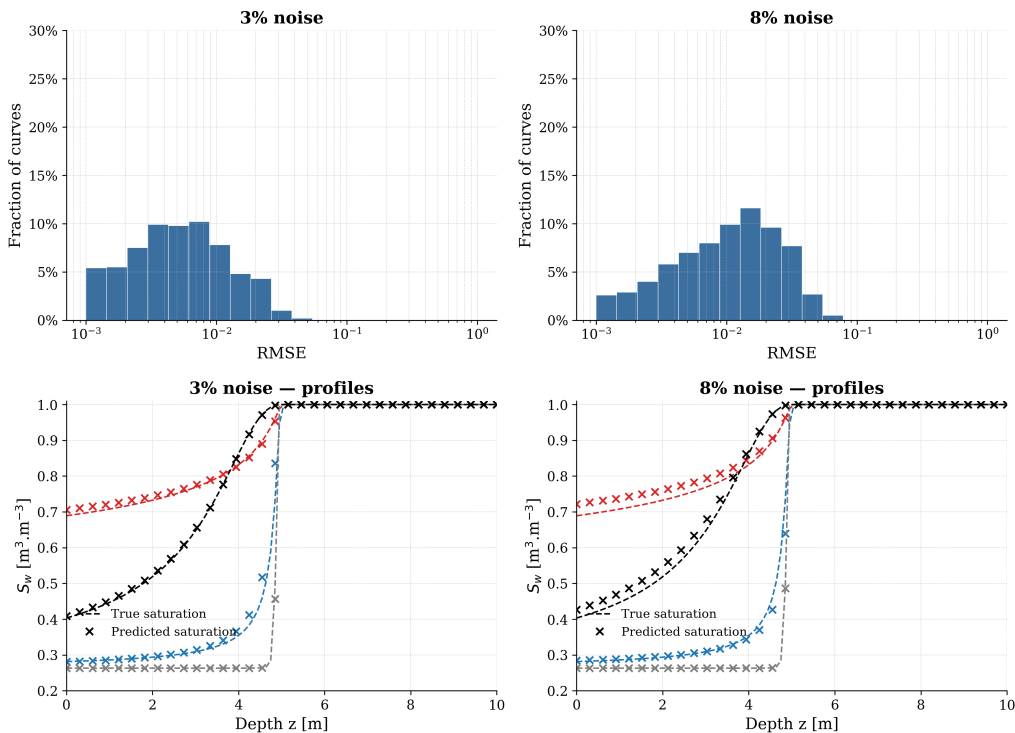


Figure 1 Fixed water table: RMSE histograms (top) and reconstructed vs. true saturation profiles (bottom) at 3% and 8% noise.

PINN characterization — variable water table

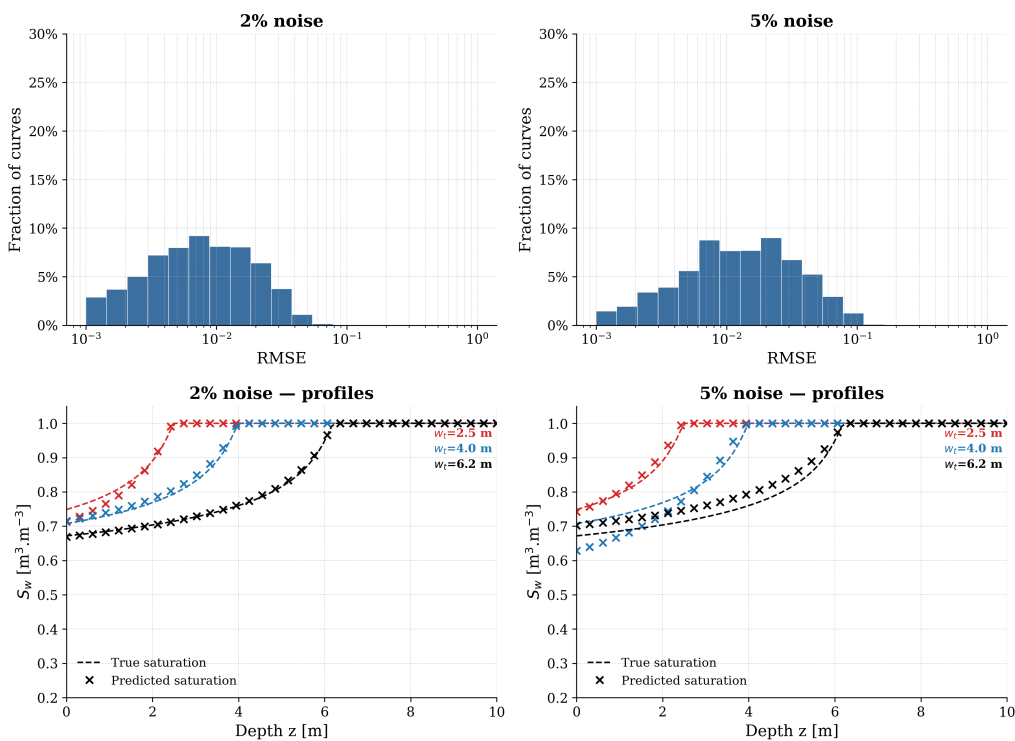


Figure 2 Variable water table: RMSE histograms (top) and reconstructed vs. true saturation profiles (bottom) at 2% and 3% noise.

Conclusions

This study proposes a PINN for inverting vadose zone water saturation profiles from seismic wave velocities. Building on [7], the network predicts hydraulic parameters rather than the profile itself, combined with a physics-consistency loss and a noise curriculum. The resulting model achieves $R^2 > 0.996$ on noisy synthetic data for both fixed and variable water table configurations, while systematically producing physically admissible profiles – unlike direct profile regression under noise. These results support further validation against field seismic data on agricultural plots.

Acknowledgements

The experimental component of this study was supported by the project AquaGeo ϕ (2023–2026), funded by the Provence-Alpes-Côte d’Azur Region (France), Avignon University, and the ANR Carnot Eau & Environnement program.

References

- [1] Stewart R. D. et al., 2025. Emerging issues and research opportunities in vadose zone processes, *Vadose Zone Journal*, 24-4, <https://doi.org/10.1002/vzj2.70030>
- [2] Binley A. et al., 2015. The emergence of hydrogeophysics for improved understanding of subsurface processes, *Water Resources Research*, 51-6, <https://doi.org/10.1002/2015WR017016>
- [3] Solazzi, S. et al., D., 2021. Surface-wave dispersion in partially saturated soils: The role of capillary forces. *Journal of Geophysical Research: Solid Earth*, 126(12), e2021JB022074. <http://dx.doi.org/10.1029/2021JB022074>.
- [4] Romero-Ruiz A. et al., 2018. A review of geophysical methods for soil structure characterization, *Reviews of Geophysics*, 56, <https://doi.org/10.1029/2018RG000611>
- [5] Pellorce L. et al., 2025. Time-lapse resistivity and seismic profiles (V_p , V_s) for Subsurface Characterization: A Case Study in a Well-Documented Agricultural Field, *EGU General Assembly*, Vienna, Austria, 2025, <https://doi.org/10.5194/egusphere-egu25-11068>.
- [6] Chen, Y., Saygin, E., 2021. Seismic inversion by hybrid machine learning. *Journal of Geophysical Research: Solid Earth*, 126(9), e2020JB021589. <http://dx.doi.org/10.1029/2020JB021589>.
- [7] Q. Didier et al., 2025. Neural network inversion of seismic wave velocities for vadose zone water content profile, *Applied Computing and Geosciences*, 27, <https://doi.org/10.1016/j.acags.2025.100285>
- [8] Biot, M., 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid: I. Low-Frequency Range, and II. Higher-Frequency Range. *Journal of the Acoustical Society of America*, 28(2), 168–191. <http://dx.doi.org/10.1121/1.1908239>.
- [9] Mavko, G., Mukerji, T., Dvorkin, J., 2009. *The Rock Physics Handbook: Tools for Seismic Analysis of Porous Media*. Cambridge University Press.
- [10] van Genuchten, M., 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, 44, 892–898. <http://dx.doi.org/10.2136/sssaj1980.03615995004400050002x>.