

4D Bayesian linearized inversion of post-stack seismic data and time-shifts for V_P estimation

Introduction

Over the past decades, time-lapse (4D) seismic acquisition has matured into a common practice for proactive reservoir monitoring and production management. By offering spatial insights into dynamic reservoir behavior, its integration into production workflows optimizes field development strategies and contributes to higher recovery factors (Lumley, 2001; Johnston, 2013).

The quantitative use of 4D seismic data, for example in seismic inversion workflows, presents several challenges such as acquisition reproducibility, geomechanical effects, seismic noise or artifacts, and potential ambiguities. Despite these challenges, several studies have been published in the literature, providing valuable insights for reservoir monitoring (MacBeth et al., 2006; Grana and Mukerji, 2015; Rosa et al., 2020; Côte et al., 2023; Exterkoetter et al., 2024).

Due to petro-elastic effects, pressure and fluid variations during production affect the elastic properties of the reservoir. These changes in elastic properties are translated into seismic amplitude variations. Specifically, changes in seismic wave velocities also translate to different traveltimes of the reflection events, known as 4D seismic time-shifts. The use of these time-shifts as additional data in the inversion workflows is presented as a promising approach, resolving ambiguities when seismic amplitudes alone are considered (Trani et al., 2011; Côte and MacBeth, 2026).

In this work, we first propose a novel joint linearized forward model of post-stack seismic amplitudes and 4D time-shifts. Then, based on this forward model, we solve the joint inverse problem using both data types for time-lapse P-wave velocity (V_P) estimation based on Bayesian linear inversion theory (Buland and Omre, 2003). We validate the methodology on a synthetic dataset built based on a real reservoir model, where the time-lapse effects are simulated under realistic production conditions.

Methodology

The objective of our proposed methodology is to estimate time-lapse P-wave velocities from 4D post-stack seismic amplitudes and time-shifts simultaneously. To achieve this goal, we first derive a linear forward model that defines the relationship between these properties. In this work, the monitor survey is assumed to be time-aligned with the baseline survey, utilizing pre-estimated time-shifts derived through a registration workflow outside the scope of this abstract.

Post-stack convolutional seismic modeling is defined by the convolution of a wavelet with the zero-incidence reflectivity series, computed for each layer as:

$$R = \frac{\Delta I_P}{2I_P} = \frac{1}{2} \left(\frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right), \quad (1)$$

where I_P is the P-impedance, V_P is the P-wave velocity and ρ is the density.

Applying differentiation rules to the Gardner's relation $\rho \approx kV_P^{1/4}$, we have that $\Delta \rho / \rho \approx 1/4 \Delta V_P / V_P$. Replacing this approximation in Equation 1, we obtain

$$R \approx \frac{5}{8} \frac{\Delta V_P}{V_P}. \quad (2)$$

As proposed by Buland and Omre (2003), the convolutional seismic model can be represented as a linear matrix operation. de Figueiredo et al. (2014) adapted this linear forward model specifically for post-stack inversion. Following the same approach with the reflectivity from Equation 2, we obtain the following seismic forward model:

$$s \approx \frac{5}{8} \mathbf{W} \mathbf{D} \ln V_P + \mathbf{e}_s, \quad (3)$$

where s is a vector with a post-stack seismic trace, V_P is the P-wave velocity of the corresponding trace, $\mathbf{W}$ is the convolutional matrix with a given wavelet, $\mathbf{D}$ is a first-order differentiation operator, and $\mathbf{e}_s$ is

the seismic noise.

For simplicity, we present the methodology assuming two time-lapse seismic surveys with corresponding P-wave velocities V_{P_1} and V_{P_2} associated with the baseline and monitor surveys, respectively. The variation between these velocity values, $\delta V_P = V_{P_2} - V_{P_1}$, is related to production operations and also related to time-shifts observed in the seismic events between the surveys.

Because our seismic modeling of Equation 3 is written in terms of $\ln V_P$, we apply the natural logarithm to the equation $\delta V_P = V_{P_2} - V_{P_1}$ for a given point at the trace and rearrange the terms to obtain the following relation

$$\frac{V_{P_1} + \delta V_P}{V_{P_1}} = \frac{V_{P_2}}{V_{P_1}} \longrightarrow \ln \left(1 + \frac{\delta V_P}{V_{P_1}} \right) = \ln(V_{P_2}) - \ln(V_{P_1}). \quad (4)$$

Assuming that the monitor is time-aligned with the baseline survey, for a specific point on the trace, by definition, we have that $V_{P_1} = dz_1/dt$ and $V_{P_2} = dz_2/(dt + \Delta t)$, where dt is the seismic sampling interval, dz is the depth interval, and Δt is the vertical delta time-shift at that specific point. Assuming $dz_1 \approx dz_2$ and replacing these definitions in Equation 4, we obtain:

$$\ln \left(1 + \frac{\delta V_P}{V_{P_1}} \right) = \ln \left(\frac{dz_2/(dt + \Delta t)}{dz_1/dt} \right) \approx \ln \left(\frac{dt}{dt + \Delta t} \right). \quad (5)$$

From Equations 4 and 5, we can obtain a direct relation between the variation of the logarithm of velocities and the delta time-shift, which can be extended for the entire seismic trace using vector notation as

$$\boldsymbol{\tau} = \ln(V_{P_2}) - \ln(V_{P_1}) + \mathbf{e}_\tau, \quad (6)$$

where the vector $\boldsymbol{\tau}$ is the term $\ln(dt/(dt + \Delta t))$ computed for all the points of the trace, and $\mathbf{e}_\tau$ is an error term vector associated with warping methods for estimating the time shift.

Assuming two seismic time-lapses s_1 and s_2 , we can write the following joint linearized forward model that encapsulates both Equations 3 and 6, accounting for seismic amplitudes and time-shift simultaneously:

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{e}, \quad (7)$$

with

$$\mathbf{d} = \begin{pmatrix} s_1 \\ s_2 \\ \boldsymbol{\tau} \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \mathbf{W}\mathbf{D} & \mathbf{0} \\ \mathbf{W}\mathbf{D} & \mathbf{W}\mathbf{D} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} \ln(V_{P_1}) \\ \ln(V_{P_2}) - \ln(V_{P_1}) \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} \mathbf{e}_{s_1} \\ \mathbf{e}_{s_2} \\ \mathbf{e}_\tau \end{pmatrix} \quad (8)$$

Assuming that all the observed errors $\mathbf{e}_{s_1}$, $\mathbf{e}_{s_2}$ and $\mathbf{e}_\tau$ of the vector $\mathbf{e}$ are Gaussian with zero mean and covariance matrix $\boldsymbol{\Sigma}_e$, we can solve the inverse problem based on Bayesian linear inversion theory (Buland and Omre, 2003). The solution is the multivariate Gaussian posterior distribution of the model parameters $\mathbf{m}$ given the observed parameters $\mathbf{d}$:

$$p(\mathbf{m}|\mathbf{d}) = N(\mathbf{m}; \boldsymbol{\mu}_{m|\mathbf{d}}, \boldsymbol{\Sigma}_{m|\mathbf{d}}), \quad (9)$$

with mean and covariance matrix given by

$$\boldsymbol{\mu}_{m|\mathbf{d}} = \boldsymbol{\mu}_m + \boldsymbol{\Sigma}_m \mathbf{G}^T (\mathbf{G} \boldsymbol{\Sigma}_m \mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1} (\mathbf{d} - \mathbf{G} \boldsymbol{\mu}_m), \quad (10)$$

and

$$\boldsymbol{\Sigma}_{m|\mathbf{d}} = \boldsymbol{\Sigma}_m - \boldsymbol{\Sigma}_m \mathbf{G}^T (\mathbf{G} \boldsymbol{\Sigma}_m \mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1} \mathbf{G} \boldsymbol{\Sigma}_m \quad (11)$$

where $\boldsymbol{\mu}_m$ is the prior mean defined by the low frequency models of the model parameters, and $\boldsymbol{\Sigma}_m$ is the prior covariance matrix accounting for prior variances and spatial correlation. From the posterior distribution, we can compute the maximum a posteriori (MAP) solution based on the Equation 10, which is the solution that maximizes the posterior distribution.

Application

We validate the methodology on a synthetic case based on the UNISIM-I benchmark model (Avansi and Schiozer, 2015). It is a realistic dataset since the benchmark is based on the 3D reservoir model of the

Namorado Field, Brazil, and the monitoring effects are simulated under realistic production conditions. The reservoir properties are defined in a 3D geocellular model in the depth domain considering two surveys, the baseline in 2013 and the monitor in 2024.

Using the soft sand rock physics model with Gassmann's equation and the petrophysical properties, the reference P-wave velocity and P-impedance of both time-lapses are calculated and converted to a regular seismic grid in two-way travel time (TWT) domain in *ms*. The seismic data is then computed using the convolutional model based on the P-impedance to compute the reflectivity series (Equation 1). Figure 1a) and b) show the baseline and monitor seismic data, respectively. Since each time-lapse is converted to TWT using its own velocity model, we can observe the time-shift effect between the two seismic data.

The time-shift field is estimated using a cross-correlation-based algorithm and is used to align the monitor data with the baseline survey. Figure 1d) shows the estimated time-shift compared to the reference one shown in Figure 1c). In the time-shift field, we can observe negative values related to a positive velocity variation during production, caused by water replacing oil. With the baseline seismic data, the time-shift aligned monitor data, and the estimated time-shift data, the methodology is applied to the dataset by computing the MAP solution of Equation 10. This equation gives the solution of the base-line P-wave velocity and the velocity variation between the time-lapses, which are shown in Figure 1f) and h), respectively, in comparison to the reference properties in Figure 1e) and g).

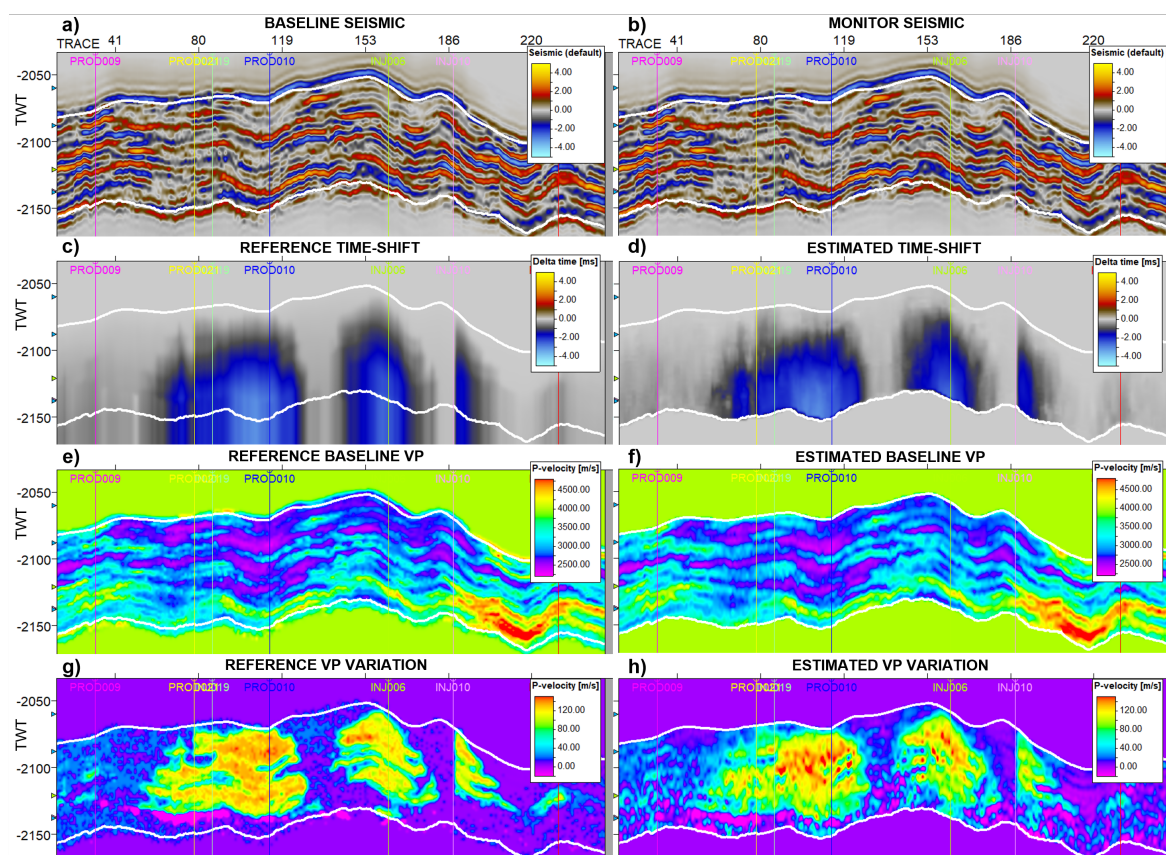


Figure 1: Input baseline and monitor seismic data in a) and b); reference and estimated time-shift data in c) and d); reference and estimated baseline P-wave velocities in e) and f); and reference and estimated velocity variation between baseline and monitor in g) and h).

To analyze the 3D distribution of the results, we convert the P-wave velocity variation to the depth domain, extract its maximum value between two horizons, and compare the result with the variation of water saturation along a layer of geocellular grid. This comparison is shown in Figure 2, where we see that the high variations in velocity highlight the regions where we have high variations in saturation. These results are in agreement with the production conditions, where the oil is replaced by water.

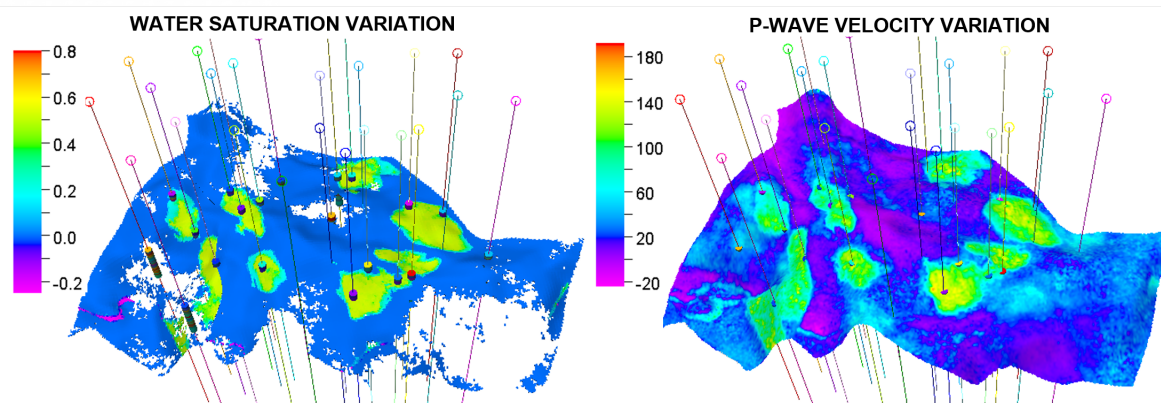


Figure 2: Variation of water saturation in the reference geological model, in comparison to the estimated of P-wave velocity variation.

Conclusion

In this work, we presented a joint linearized forward model and a Bayesian inversion framework that simultaneously uses post-stack seismic amplitudes and time-shifts to estimate time-lapse P-wave velocities (V_P). The application to the synthetic UNISIM-I benchmark demonstrates that the estimated results are in good accordance with the reference properties, validating the methodology for future applications to real data. Although we assumed the same wavelet for both seismic data, the methodology can be easily adapted to work with different wavelets. Additionally, while we presented the workflow for two time-lapse surveys for simplicity, extending the methodology to any number of time-lapses is straightforward. For future work, we intend to generalize the methodology to invert for V_P , V_S , and ρ from angle-stacked seismic data.

Acknowledgements

We acknowledge Petrobras, LTrace and the SER of the University of Wyoming for the support.

1 References

- Avansi, G. and Schiozer, D. [2015] UNISIM-I: synthetic model for reservoir development and management applications. *Int. J. Model. Simul. Pet. Ind.*, **9**, 21–30.
- Buland, A. and Omre, H. [2003] Bayesian linearized AVO inversion. *Geophysics*, **68**(1), 185–198.
- Côte, G., Amini, H. and MacBeth, C. [2023] Bayesian inversion of 4D seismic data to pressure and saturation changes: Application to a west of Shetlands field. *Geophysical Prospecting*, **71**(2), 292–321.
- Côte, G. and MacBeth, C. [2026] Time-Lapse Time-Shift Assimilation Resolves Vertical Ambiguities in 4D Seismic Inversion to Water and Gas Saturation Changes. **2026**(1), 1–5.
- Exterkoetter, R., Dutra, G.R., de Figueiredo, L.P., Bordignon, F., Neto, G.M.S. and Emerick, A.A. [2024] Feature Extraction in Time-Lapse Seismic Using Deep Learning for Data Assimilation. *SPE Journal*, **29**(04), 2014–2025.
- de Figueiredo, L.P., Santos, M., Roisenberg, M., Neto, G.S. and Figueiredo, W. [2014] Bayesian framework to wavelet estimation and linearized acoustic inversion. *IEEE Geoscience and Remote Sensing Letters*, **11**(12), 2130–2134.
- Grana, D. and Mukerji, T. [2015] Bayesian inversion of time-lapse seismic data for the estimation of static reservoir properties and dynamic property changes. *Geophysical Prospecting*, **63**(3), 637–655.
- Johnston, D.H. [2013] *Practical Applications of Time-lapse Seismic Data*. Society of Exploration Geophysicists.
- Lumley, D.E. [2001] Time-lapse seismic reservoir monitoring. *Geophysics*, **66**(1), 50–53.
- MacBeth, C., Floricich, M. and Soldo, J. [2006] Going quantitative with 4D seismic analysis. *Geophysical Prospecting*, **54**(3), 303–317.
- Rosa, D.R., Santos, J.M.C., Souza, R.M., Grana, D., Schiozer, D.J., Davolio, A. and Wang, Y. [2020] Comparing different approaches of time-lapse seismic inversion. *Journal of Geophysics and Engineering*, **17**(6), 929–939.
- Trani, M., Arts, R., Leeuwenburgh, O. and Brouwer, J. [2011] Estimation of changes in saturation and pressure from 4D seismic AVO and time-shift analysis. *Geophysics*, **76**(2), C1–C17.