

Multiple-nullspace shuttling in time-lapse full waveform inversion

Introduction

Constructing time-lapse images is crucial to properly monitoring reservoir changes, helping to control oil production and CO₂ storage. However, this characterisation relies on noisy, limited data collected at different calendar times (Lumley, 2001). Time-lapse models can be constructed using full-waveform inversion (FWI) (Virieux and Operto, 2009), which uses the entire seismic wavefield to extract subsurface information. However, the small, spatially localised nature of time-lapse variations and the non-repeatability of effects make this estimation challenging. Non-repeatability issues can be associated with differences in acquisition geometry and/or climatic factors (Zhou & Lumley, 2021b).

Several 4D FWI schemes have been proposed to address these challenges in the context of reservoir monitoring. In this regard, the parallel strategy (Lumley, 2003) estimates the baseline and monitor models separately using the same initial model. In contrast, the sequential strategy (Asnaashari et al., 2012) uses the result of the baseline inversion as the initial model in the monitor estimation. The central-difference approach (Zhou & Lumley, 2021a) employs four FWI workflows by combining two sequential inversions and averaging them to obtain a final time-lapse estimate. Recently, the weighted-average strategy (Mardan et al., 2023) was proposed to improve the accuracy of the central-difference method while reducing the number of FWI workflows from four to three. This strategy introduces a weighting factor to compensate for the opposite-sign artefacts mentioned above. In addition, joint 4D FWI formulations solve a combined objective function for baseline and monitor data, augmented with target-oriented regularisation terms (Ayeni & Biondi, 2010).

Null-Space Shuttle Overview

Null-space shuttling (NSS) extends the FWI workflow by exploring model updates that preserve the data misfit while modifying the baseline–monitor model difference according to a secondary objective function. NSS operates within the set of plausible models that satisfy the inversion constraints equally well by exploring the effective null space. This method has recently been used in viscoelastic FWI (Keating and Innanen, 2021) and adapted for 4D acoustic FWI (Keating and Innanen, 2024).

The main aspects of NSS can be summarised as follows. Let $\mathbf{m}_B \in \mathbb{M}$ and $\mathbf{m}_M \in \mathbb{M}$ denote the inverted baseline and monitor models, with corresponding misfit levels ϵ_B and ϵ_M , respectively. The effective null-space $\mathbb{N}$ is a subspace of the model space $\mathbb{M}$ spanned by models which satisfy the conditions: $\mathbb{N} = \{\mathbf{m}_B : \phi(\mathbf{m}_B \leq \epsilon_B) \cap \mathbf{m}_M : \phi(\mathbf{m}_M \leq \epsilon_M)\}$, where $\phi(\mathbf{m})$ is the FWI objective function evaluated for the model $\mathbf{m}$. To obtain these models, we solve a constrained optimisation problem to find model updates $\delta\mathbf{m}_B$ and $\delta\mathbf{m}_M$ that modify the 4D difference while keeping $\mathbf{m}_B + \delta\mathbf{m}_B$ and $\mathbf{m}_M + \delta\mathbf{m}_M$ within $\mathbb{N}$. With that in mind, we propose a Multiple-Shuttle Null-Space (MSNS) formulation where a secondary objective has two complementary terms: (i) a minimising term to suppress differences outside the reservoir, and (ii) a maximising term to enhance differences inside the reservoir (Hu et al., 2024).

Let $\Omega \in \mathbb{R}^2$ denote the model physical domain and $\Omega_R \in \Omega$ the reservoir region. The secondary objective function is defined as:

$$\Psi(\mathbf{m}) = \int_{\Omega} w(\mathbf{x}) \mathcal{L}(\Delta\mathbf{m}) d\mathbf{x} - \int_{\Omega_R} (1 - w(\mathbf{x})) \mathcal{L}(\Delta\mathbf{m}) d\mathbf{x}$$

where $\Delta\mathbf{m} = (\mathbf{m}_M + \delta\mathbf{m}_M) - (\mathbf{m}_B + \delta\mathbf{m}_B)$ is the baseline-monitor difference and $w(\mathbf{x}) \in [0,1]$ are weights used to emphasise the region outside of the reservoir. $\mathcal{L}$ is the Huber norm used to attenuate non-subsurface 4D noise and to increase reservoir contrast.

The proposed MSNS updates each term of the secondary objective function as follows. First, the baseline model is updated in two sequential stages: **(1)** minimising 4D differences (first term) and **(2)** maximising the 4D response (second term). Second, the monitor model is treated analogously, with the **(3)** minimisation of the 4D differences followed by **(4)** maximisation of the 4D response. All four steps can be executed in the most general scenario. In addition, two alternative strategies are also considered: a three-step formulation comprising **(1)**, **(3)**, and **(4)**, and a reduced formulation restricted to the minimisation steps **(1)** and **(3)**, which is very similar to the conventional NSS method. Since all these alternatives explore the null space differently while maintaining comparable FWI misfits for the baseline and monitor datasets, the spread in their 4D differences provides a qualitative uncertainty envelope around the time-lapse estimate.

Numerical Experiments

We evaluate the MSNS strategy using a synthetic dataset of marine towed streamers representative of Brazilian pre-salt conditions. The acquisition includes 316 sources spaced ~50 m apart and 321 receivers spaced 25 m apart, producing offsets ranging from 50 m to 8000 m. To simulate realistic nonrepeatability effects, small random perturbations were applied to the source (up to 50 m in X and 5 m in Z) and receiver positions (up to 10 m in X and 5 m in Z). These variations introduce acquisition-related 4D noise that is challenging to suppress. Synthetic data were generated using an acoustic, constant-density wave equation with a 20 Hz Ricker wavelet for both modelling and inversion, ensuring numerical consistency within the model and allowing us to isolate the effects of the shuttling strategies. Figures 1a and 1b present the baseline model and the imposed production-related anomaly used to construct the monitor model. The initial model of Fig. 1c is used in all FWI runs. These provide inputs from which baseline–monitor differences are computed and subsequently modified by the MSNS exploration method.

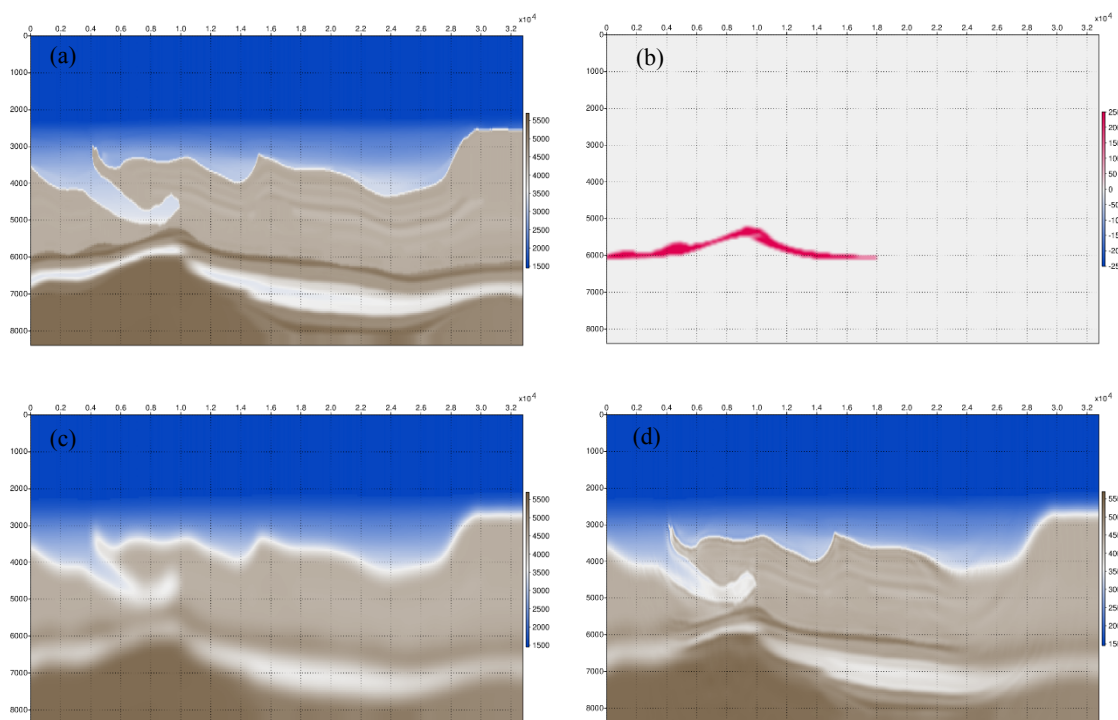


Figure 1 (a) Baseline velocity model. (b) Production-induced velocity anomaly superimposed on the baseline model to generate the monitor model. (c) Initial velocity model used as the starting model for all full-waveform inversion (FWI) experiments. (d) FWI result for the baseline survey, providing the reference subsurface image used for subsequent time-lapse difference analysis.

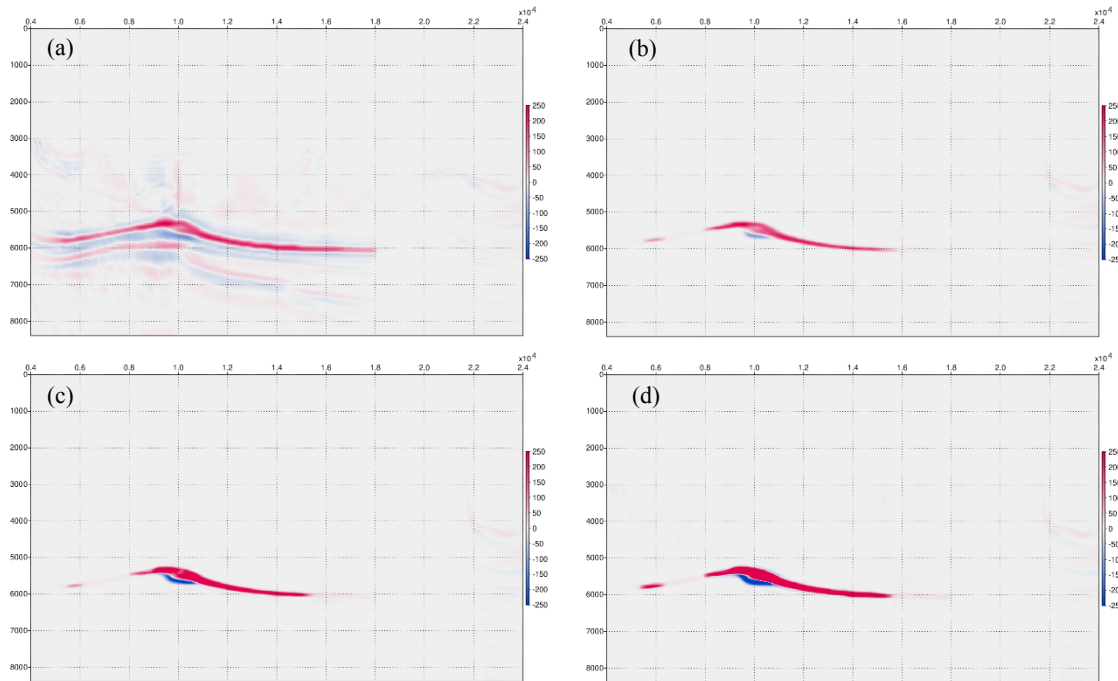


Figure 2 (a) 4D signal estimated by parallel baseline–monitor FWI and used as input to the MSNS workflow. Reconstructed anomalies after applying (b) the 2-step (classical NSS), (c) the 3-step, and (d) the 4-step MSNS strategies.

Figure 1d shows the inversion result for the baseline survey, providing the reference subsurface model from which time-lapse differences are subsequently computed. Figure 2a displays a zoomed-in view of the production-related anomaly recovered using the parallel time-lapse inversion. This localised 4D signal serves as a starting point for the MSNS procedures, which then suppress or enhance selected components of the baseline–monitor difference to explore the range of admissible time-lapse responses under an equal-misfit constraint.

Panels (b), (c) and (d) of Fig. 2 summarise the behaviour of the MSNS variants. Fig. 2b shows the 2-step minimisation-only shuttle (classical NSS), which suppresses non-reservoir differences in both baseline and monitor models and therefore produces the smallest 4D anomaly. It defines the lower bound of our uncertainty envelope. Fig. 2c corresponds to the 3-step shuttle, which still minimises differences outside the reservoir but applies the maximisation term only to the monitor, selectively enhancing reservoir-related contrasts and yielding a stronger upper-bound scenario dominated by monitor variability. Finally, Fig. 2d presents the full MSNS configuration, in which the alternating 4-step scheme is applied, producing the widest admissible range of 4D responses while preserving the overall FWI misfit.

All three variants reduce the baseline and monitor FWI objective functions, confirming that shuttling can preserve or even improve data misfit while reshaping 4D differences. We observe that the 2-step variant yields the strongest suppression outside the reservoir, with a ~99% reduction in the second term of the secondary objective function. The 3-step variant continues suppressing (~97% reduction) artefacts outside of the reservoir while enhancing the monitor model by increasing the second term of the secondary objective function by a factor of 2.16. When the 4-step configuration is used, the suppression outside the reservoir is slightly less severe (~90% reduction). However, the combined effect of all steps maximises the spread of plausible 4D responses within the equal-misfit constraint and yields the highest value of the total secondary objective function.

Conclusions

We demonstrate that the proposed MSNS framework provides a flexible and robust mechanism for exploring admissible 4D models within the model space that satisfy baseline and monitor FWI misfit constraints. Our experiments show that even under modest non-repeatability conditions, the method effectively separates non-reservoir artefacts from true reservoir-related changes. Our results indicate that the MSNS approach is an effective framework for uncertainty estimation in 4D FWI. It can suppress spurious time-lapse differences arising from acquisition non-repeatability while enhancing reservoir-focused signals in a controlled and interpretable manner.

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