

Acoustic Impedance Inversion with Hybrid Tikhonov-TV Regularisation for Imaging the Japan Trench Outer-Rise Sediments

Introduction

The outer-rise region of the Japan Trench is shaped by the bending of the Pacific Plate as it descends into the subduction zone. This flexure generates bending-related faults and triggers petit-spot volcanism, whose associated magmatic intrusions penetrate the overlying sedimentary sequence and disturb the smectite-rich pelagic clay layer (Fujie et al., 2020). Quantitative characterisation of this structure requires rock-property information that acoustic impedance (AI) inversion can recover directly from seismic reflection data.

AI inversion is inherently ill-posed, and the choice of regularisation should reflect the geological character of the target (Gholami, 2015). Tikhonov regularisation favors smooth impedance models, matching gradual compaction-driven trends but oversmoothing sharp contrasts. Total Variation (TV) regularisation preserves sharp boundaries but introduces staircase artifacts in smoothly varying intervals. We therefore adopt the hybrid Tikhonov-TV scheme (Gholami and Gazzola, 2022), which decomposes the impedance into a smooth and a piecewise-constant component and recovers them jointly, making it naturally suited to this piecewise-smooth structure.

We apply AI inversion to a two-dimensional multi-channel streamer (MCS) reflection profile acquired by JAMSTEC across the Japan Trench outer-rise. Processing followed a standard flow including predictive deconvolution, velocity analysis, NMO correction, stacking, band-pass filtering, and Kirchhoff post-stack time migration. As no well-log control is available, the wavelet was estimated directly from the data using the ℓ_1 -norm method of Gelpi et al. (2019). Seismic reflection data are band-limited and lack the very low frequencies needed to recover the absolute level and long-wavelength trend of the impedance model. This missing low-wavenumber component, $\mathbf{X}_{low}$, must therefore be supplied from an independent source. Here, it was derived from the FWI velocity model of Górszczyk and Amirzadeh (2026), obtained from OBS data collected in the same area, and converted to impedance using the Gardner relation. Three regularisation strategies — Tikhonov, TV, and the hybrid Tikhonov-TV — are assessed on synthetic data and then applied to two contrasting zones: (i) a thick-sediment zone with undisturbed stratigraphy, and (ii) a thin-sediment zone where magmatic intrusion has disrupted the sedimentary column.

Method

Wavelet Estimation

Since no well-log data are available, the wavelet phase is estimated directly from the seismic data using the ℓ_1 -norm minimisation method of Gelpi et al. (2019). The method assumes that the wavelet phase can be approximated by a constant rotation ϕ , and that the reflectivity series is sparse. The zero-phase wavelet $\mathbf{w}^0$ is first constructed by averaging the amplitude spectra over all available traces within a given time-distance window and performing the inverse Fourier transform. For each candidate phase rotation ϕ , the rotated wavelet is obtained via the Hilbert transform, and sparse-spike deconvolution is then performed trace by trace by minimising:

$$J(\mathbf{r}) = \frac{\lambda}{2} \|\mathbf{W}\mathbf{r} - \mathbf{s}\|_2^2 + \|\mathbf{r}\|_1, \quad (1)$$

where $\mathbf{s}$ and $\mathbf{r}$ represent seismic trace and reflectivity, respectively. $\mathbf{W}$ is the Toeplitz convolution matrix formed from the wavelet, and $\lambda > 0$ is the regularisation parameter. The optimal phase, $\hat{\phi}$, is the rotation that minimises the mean ℓ_1 -norm of the deconvolved traces across all available traces.

Acoustic Impedance Inversion

Following the two-step strategy of Gholami (2015), once the reflectivity $\mathbf{r}$ has been estimated, the acoustic impedance z is recovered through the continuous-earth model assumption (Foster, 1975):

$$r[j] = \frac{1}{2} \log \frac{z[j+1]}{z[j]}. \quad (2)$$

Introducing the log-impedance $x[j] := \frac{1}{2} \log z[j]$ renders a linear relation: $\mathbf{r} = \mathbf{D}\mathbf{x}$, where $\mathbf{D} \in \mathbb{R}^{(n-1) \times n}$ is the first-order vertical difference operator and n is the number of samples along a trace. Recovering $\mathbf{x}$ from the band-limited and noisy estimate $\mathbf{r}$ is ill-posed. Including the error in the estimated reflectivity as $\mathbf{r} = \mathbf{D}\mathbf{x} + \mathbf{e}$ with $\mathbf{e} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$, the impedance estimate reads

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{\alpha}{2} \|\mathbf{D}\mathbf{x} - \mathbf{r}\|_2^2 + \mathcal{R}(\mathbf{x}), \quad (3)$$

where the first term enforces consistency with the estimated reflectivity, weighted by $\alpha > 0$, and $\mathcal{R}(\mathbf{x})$ is the regulariser. The impedance model then follows from $\mathbf{z} = \exp(2\mathbf{x})$. The choice of $\mathcal{R}$ depends on the prior assumption made about the model, specifically the expected statistical distribution. The three regularisers below correspond to three such assumptions.

regularisation

Tikhonov regularisation rests on the assumption that the model derivative is small and normally (Gaussian) distributed, which favors smooth solutions through $\mathcal{R}(\mathbf{x}) := \frac{1}{2} \|\mathbf{L}_2 \mathbf{x}\|_2^2$, where $\mathbf{L}_2$ denotes the second-order derivative operator.

TV regularisation assigns a Laplace (sparse) prior to the model gradient ($\mathbf{L}_1 \mathbf{x}$) through $\mathcal{R}(\mathbf{x}) := \|\mathbf{L}_1 \mathbf{x}\|_1$, where $\mathbf{L}_1$ is the first-order derivative operator. TV preserves sharp boundaries at the cost of staircase artifacts in smoothly varying regions (Gholami, 2015).

Hybrid Tikhonov-TV regularisation (Gholami and Gazzola, 2022) decomposes the model as $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$, where $\mathbf{x}_1$ is a piecewise-constant component regularised by TV and $\mathbf{x}_2$ is a smooth component regularised by Tikhonov:

$$\min_{\mathbf{x}_1, \mathbf{x}_2} \frac{\alpha}{2} \|\mathbf{D}[\mathbf{x}_1 + \mathbf{x}_2] - \mathbf{r}\|_2^2 + \underbrace{\|\mathbf{L}_1 \mathbf{x}_1\|_1}_{\text{blocky: sparse}} + \underbrace{\frac{\beta}{2} \|\mathbf{L}_2 \mathbf{x}_2\|_2^2}_{\text{smooth: Gaussian}}. \quad (4)$$

Here $\beta > 0$ is the balancing parameter weighting the two regularisation terms relative to each other. The two priors carry a direct geological reading: $\mathbf{x}_1$ captures the sharp impedance contrasts at intrusive and lithological contacts (isolated strong reflectors), whereas $\mathbf{x}_2$ captures the gradual, compaction-driven background trend of the pelagic sediments. The limiting behavior is instructive: as $\beta \rightarrow 0$ the minimisation on TV term dominates and the smooth component ($\mathbf{x}_2$) absorbs the signal structure. On the other hand, as $\beta \rightarrow \infty$ the formulation asymptotically reduces to TV regularisation. Rather than tuning β by hand, we set it adaptively by treating the sparse (non-Gaussian) entries of the total gradient as statistical anomalies, whose magnitude is characterised robustly via the median absolute deviation (Gholami and Gazzola, 2022). The problem is solved within an alternating direction method of multipliers (ADMM) framework. This formulation is better suited to piecewise-smooth models such as the outer-rise sedimentary sequence, where gradual impedance transitions are interrupted locally by a sharp contrast with oceanic crust below the sediments. All three methods are evaluated on synthetic data before application to the academic field dataset.

Result

The three regularisation schemes were assessed on synthetic data generated in GO-3D-OBS model (Górszczyk and Operto (2021); Figure 1), confirming that the hybrid Tikhonov-TV regularisation most closely recovers the original AI model. For the field data, the inversion was applied to two contrasting zones: a thick-sediment zone unaffected by petit-spot volcanism (Figure 2), where the sedimentary layering is clearly resolved, and a thin-sediment zone where magmatic intrusions have disrupted and alternated the sedimentary sequence (Figure 3). The recovered impedance models reveal a clear contrast between the two zones. In the thick-sediment zone, high impedance values are observed at depth, consistent with the presence of intact basaltic oceanic crust beneath an undisturbed sedimentary sequence. In the thin-sediment zone, these high impedance values are largely absent, and the impedance distribution is generally lower and more heterogeneous throughout the column. This is consistent with the interpretation of Fujie et al. (2020) that the acoustic basement in this zone does not represent the top of the intact oceanic crust, but rather magmatic intrusions within the sediments associated with petit-spot volcanism, which have disturbed and thermally metamorphosed the original sedimentary sequence.

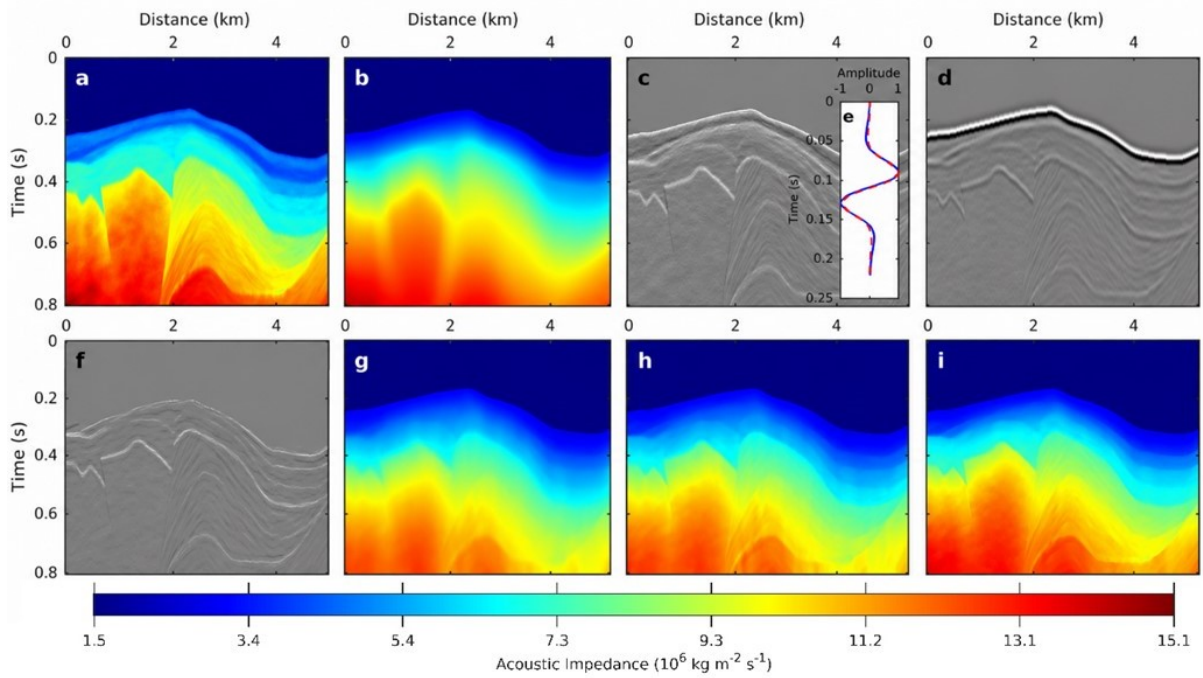


Figure 1 AI inversion of GO_3D_OBS synthetic model. (a) Original model. (b) Low-wavenumber trend of the AI model obtained by smoothing the model in (a). (c) Original reflectivity model. (d) Seismic section. (e) Wavelet: Blue and red lines are the original and estimated wavelets respectively. (f) Recovered reflectivity model. (g)-(i) AI models recovered with Tikhonov, TV and Tikhonov-TV regularisation respectively.

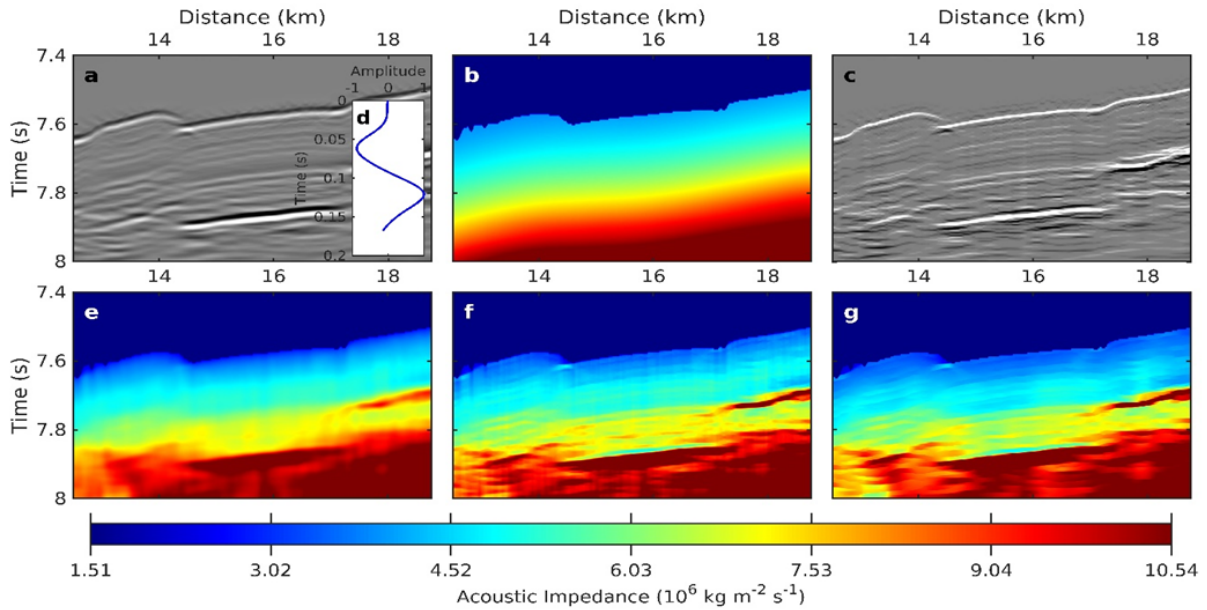


Figure 2 AI inversion from thick-sediment area. (a) Seismic section. (b) Low-wavenumber trend of the AI model. (c) Reflectivity model. (d) Estimated wavelet. (e)-(g) AI models recovered with Tikhonov, TV and Tikhonov-TV regularisation, respectively.

The key to resolving these features is the joint use of the two priors: by letting a TV-constrained component carry the sharp intrusive contacts while a Tikhonov-constrained component carries the smooth compaction trend, the hybrid inversion captures both the blocky and the gradual structure that a single regulariser would blur together. This makes regularised AI inversion a quantitative tool for distinguishing intact from volcanically disturbed sedimentary sequences in the outer-rise region.

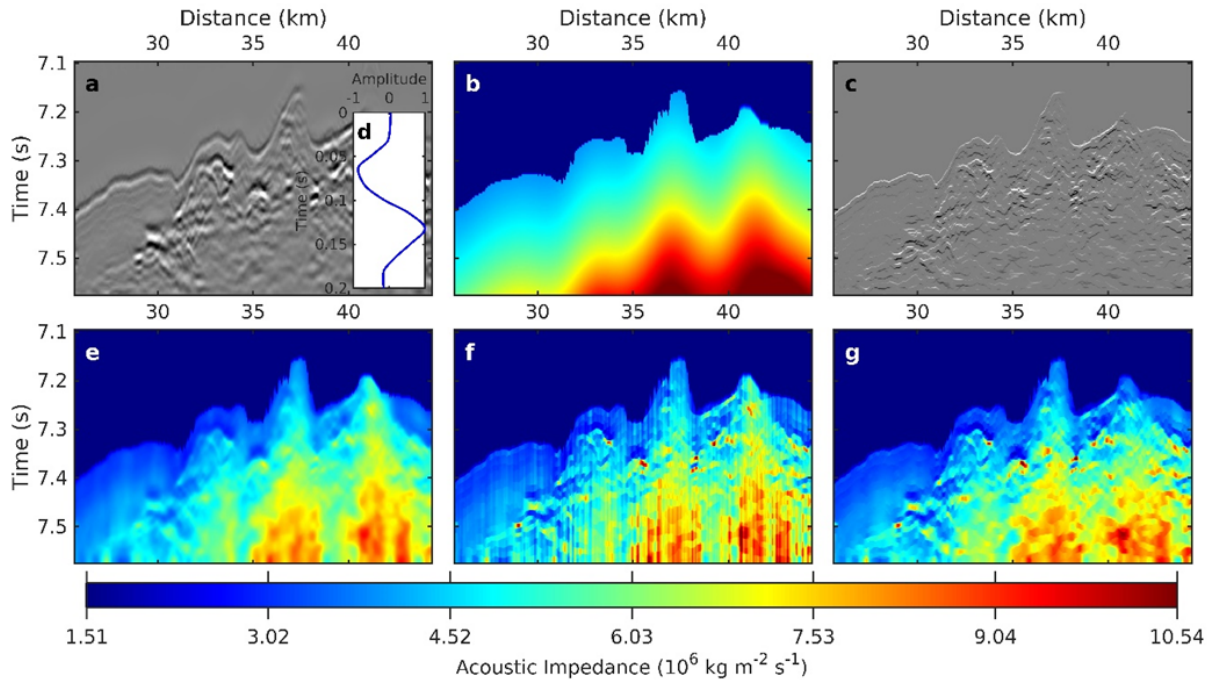


Figure 3 AI inversion from alternated-sediment area. (a) Seismic section. (b) Low-wavenumber trend of the AI model. (c) Reflectivity model. (d) Estimated wavelet. (e)-(g) AI models recovered with Tikhonov, TV and Tikhonov-TV regularisation, respectively.

Conclusions

We applied post-stack time migration and regularised acoustic impedance inversion to MCS profile to characterise the shallow sedimentary structure of the Japan Trench outer-rise. By decomposing the model into smooth and piecewise-constant components with an adaptively balanced parameter, the hybrid Tikhonov-TV scheme recovered more accurate impedance results than Tikhonov or TV alone in both synthetic and field tests. Crucially, letting the data set the balance between each regularisation component removes manual-tuning subjectivity and adapts the regularisation to the local geology, so smooth compaction trends and sharp intrusive contacts are each constrained by the prior that suits them.

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