

Bayesian Elastic Seismic Inversion Using Ensemble with Multiple Data Assimilation Applied to the Santos Basin Pre-Salt

Introduction

Elastic seismic inversion converts prestack amplitudes into quantitative estimates of P-wave impedance (I_p), S-wave impedance (I_s), and density, supporting lithological interpretation, facies discrimination, and uncertainty assessment. In Santos Basin pre-salt carbonates, these attributes are especially important because of strong facies heterogeneity and depositional complexity. Acoustic impedance alone may not distinguish reservoir carbonates from non-reservoir facies such as magnesium-rich clays; integrating I_s , V_p/V_s , and I_p-I_s is therefore essential for reliable reservoir characterization and facies discrimination (Mello and Lupinacci, 2022).

The problem remains challenging because the elastic-property–amplitude relationship is nonlinear, while seismic data have limited bandwidth, noise, wavelet uncertainty, and low-frequency-model inaccuracies. These factors make solutions non-unique and motivate probabilistic methods. Conventional Bayesian inversion often linearizes the forward model and assumes Gaussian distributions to obtain analytical solutions (Buland and Omre, 2003). Although efficient, this may oversimplify the problem and inadequately represent nonlinear elastic–seismic relationships.

Ensemble data assimilation offers an alternative. ES-MDA combines Bayesian updating with ensemble-derived statistics to iteratively condition multiple realizations to observed data. Its posterior ensemble samples the posterior distribution, enabling joint estimation of elastic properties and uncertainty. Introduced for reservoir history matching by Emerick and Reynolds (2013), ES-MDA was later adapted to seismic inversion with nonlinear forward models, including the full Zoeppritz equations (Liu and Grana, 2018).

This study applies a nonlinear Bayesian ES-MDA workflow to broadband OBN data from the Búzios Field, Santos Basin pre-salt. It estimates elastic properties and uncertainties while honoring seismic observations and prior geological information, aiming to improve reservoir characterization and support risk-informed decisions.

Ensemble Smoother with Multiple Data Assimilation (ES-MDA)

Within a Bayesian framework, seismic inversion can be formulated as the estimation of the posterior probability distribution of the subsurface model conditioned to the observed seismic data. The posterior distribution is obtained through Bayes' theorem:

$$p(\mathbf{m} \mid \mathbf{d}_{obs}) = \frac{p(\mathbf{d}_{obs} \mid \mathbf{m}) p(\mathbf{m})}{p(\mathbf{d}_{obs})}$$

where $p(\mathbf{m} \mid \mathbf{d}_{obs})$ is the posterior probability distribution of the model parameters $\mathbf{m}$ given the observed seismic data $\mathbf{d}_{obs}$, $p(\mathbf{m})$ is the prior probability, $p(\mathbf{d}_{obs} \mid \mathbf{m})$ is the likelihood function and $p(\mathbf{d}_{obs})$ is a normalization constant.

Rather than seeking a single optimal solution, stochastic Bayesian inversion aims to characterize a set of plausible subsurface models that are consistent with both prior geological information and seismic observations. In the ES-MDA, covariance is represented through an ensemble of realizations. Each realization corresponds to a possible geological model containing P-wave impedance (I_p), S-wave impedance (I_s) and density (ρ). Collectively, these realizations define the prior distribution and quantify the uncertainty associated with the subsurface before the assimilation of seismic information.

For each ensemble member, the forward operator transforms the elastic model into a predicted seismic response. At each assimilation step, the mismatch between observed and predicted seismic data is used to update all realizations simultaneously through a Kalman-type correction based on the model–data cross-covariance, the predicted-data covariance, and the prescribed data-error covariance. The ES-MDA update equation is

$$\mathbf{m}_j^{k+1} = \mathbf{m}_j^k + \mathbf{C}_{md}^k (\mathbf{C}_{dd}^k + \alpha_k \mathbf{C}_D)^{-1} [\mathbf{d}_{\text{obs},j}^k - g(\mathbf{m}_j^k)].$$

where $\mathbf{m}_j^k$ is the model vector of realization j at assimilation step k , and $\mathbf{m}_j^{k+1}$ is the updated model. The term $g(\mathbf{m}_j^k)$ represents the predicted seismic response, while $\mathbf{d}_{\text{obs},j}^k$ is the perturbed observation vector. The matrices $\mathbf{C}_{md}^k$ and $\mathbf{C}_{dd}^k$ are, respectively, the model–data cross-covariance and predicted-data covariance estimated from the ensemble; $\mathbf{C}_D$ is the data-error covariance matrix; and α_k is the covariance-inflation factor for assimilation step k .

A key characteristic of ES-MDA is the distribution of the assimilation process into multiple consecutive updates. Instead of conditioning the models to the observations in a single step, the algorithm performs a sequence of assimilations using inflated data-error covariance matrices. The inflation coefficients must satisfy the condition:

$$\sum_{k=1}^{N_a} \frac{1}{\alpha_k} = 1$$

where N_a is the number of assimilation steps and α_k is the inflation coefficient associated with assimilation step k . A common choice is to use constant inflation factors throughout the assimilation process: $\alpha_k = N_a$, which automatically satisfies the previous condition. This inflation strategy improves numerical stability, reduces the risk of ensemble collapse, and allows the inversion to progressively incorporate seismic information while preserving geological variability within the ensemble.

After the final assimilation step, the posterior ensemble provides estimates of the elastic properties and their associated uncertainties. Statistical volumes, including posterior averages, standard deviations, percentiles, and non-parametric probability density functions, can be extracted directly from the updated realizations. Therefore, ES-MDA provides both elastic-property estimates and a quantitative characterization of their uncertainty.

Application in the Búzios Field

The ES-MDA inversion was applied to broadband ocean-bottom-node (OBN) seismic data from the Búzios Field, one of the largest producing fields in the Santos Basin pre-salt province. The dataset is characterized by a broad frequency bandwidth, high signal-to-noise ratio, full-azimuth coverage, and ultra-far-offset information, making it well suited for elastic seismic inversion. Four partial stacks corresponding to incidence-angle ranges of 3–12°, 12–21°, 21–27° and greater than 27° were used as input.

The prior ensemble was generated using a Relative Geological Time (RGT) model constructed from five interpreted stratigraphic horizons, allowing the elastic properties to be distributed consistently with the depositional framework. Well-log data from 36 wells were integrated with an interval-velocity model derived from full-waveform inversion (FWI), which was used as a secondary variable to constrain the lateral distribution of the low-frequency background model. This model was subsequently perturbed to generate 500 geostatistical realizations. Vertical correlations among the elastic parameters were imposed through a Cholesky decomposition of the prior covariance matrix, whereas lateral continuity was introduced using three-dimensional correlated noise controlled by field-scale

variograms. This procedure produced an ensemble that preserved the main geological trends while representing plausible spatial variability within the reservoir.

Zero-phase statistical wavelets were estimated independently for each partial stack. Their scaling factors were optimized by minimizing the amplitude-energy mismatch between well-based synthetic traces and the observed seismic data. Uncertainty in the seismic observations was represented by the data-error covariance matrix C_D , assuming independent errors among the partial stacks while accounting for the uncertainty associated with noise, acquisition limitations, and forward modeling. Four assimilation steps were performed using covariance-inflation factors of 1.875, 3.75, 7.5 and 15. To reduce computational cost, a truncated singular value decomposition (SVD) was applied. The data and covariance matrices were then projected onto the reduced subspace defined by the dominant singular vectors, substantially decreasing the dimensionality of the inverse problem and improving the numerical stability of the Kalman-gain calculation.

The posterior distribution was summarized through the average and standard deviation of the 500 updated realizations, as shown in Figure 1. The posterior average volumes reflect the combined contribution of the seismic information and the spatial variability inherited from the geostatistical simulations used to construct the prior ensemble, whereas the posterior standard deviations represent the spatial uncertainty of the estimates. Higher uncertainty is observed in the northern portion of the field. This pattern may be partly influenced by the greater geological heterogeneity of the northern sector, which can increase the variability of the elastic response and, consequently, the uncertainty of the inversion results.

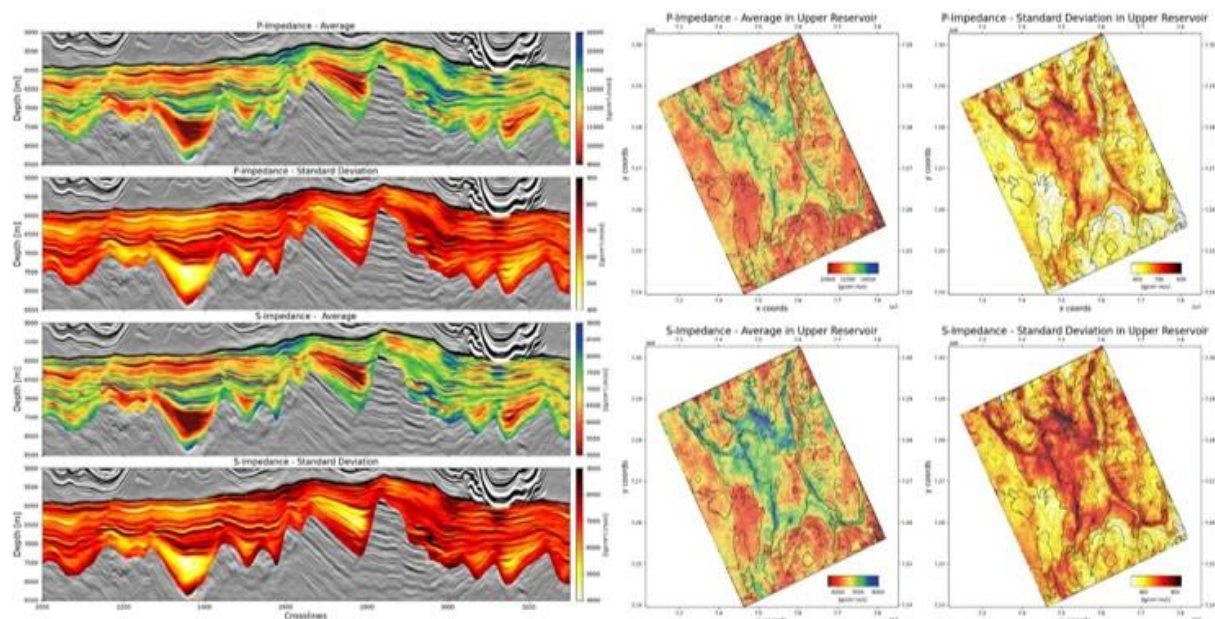


Figure 1 Posterior ensemble statistics from the ES-MDA elastic seismic inversion. The left panels show, from top to bottom, the posterior average and standard deviation of acoustic impedance, followed by the posterior average and standard deviation of the S-impedance along a representative inline section. The right panels display interval-averaged maps of the posterior average and standard deviation for acoustic impedance (top) and S-impedance (bottom) within the upper reservoir.

At wells, the posterior average is generally closer to the measured logs than the prior average, while most observations remain within the posterior uncertainty interval. This behavior is illustrated in Figure 2, which compares measured logs, prior and posterior estimates, and their associated uncertainty ranges. Across all 36 wells, correlation coefficients reached an average of 0.86 for P-impedance and 0.85 for S-impedance.

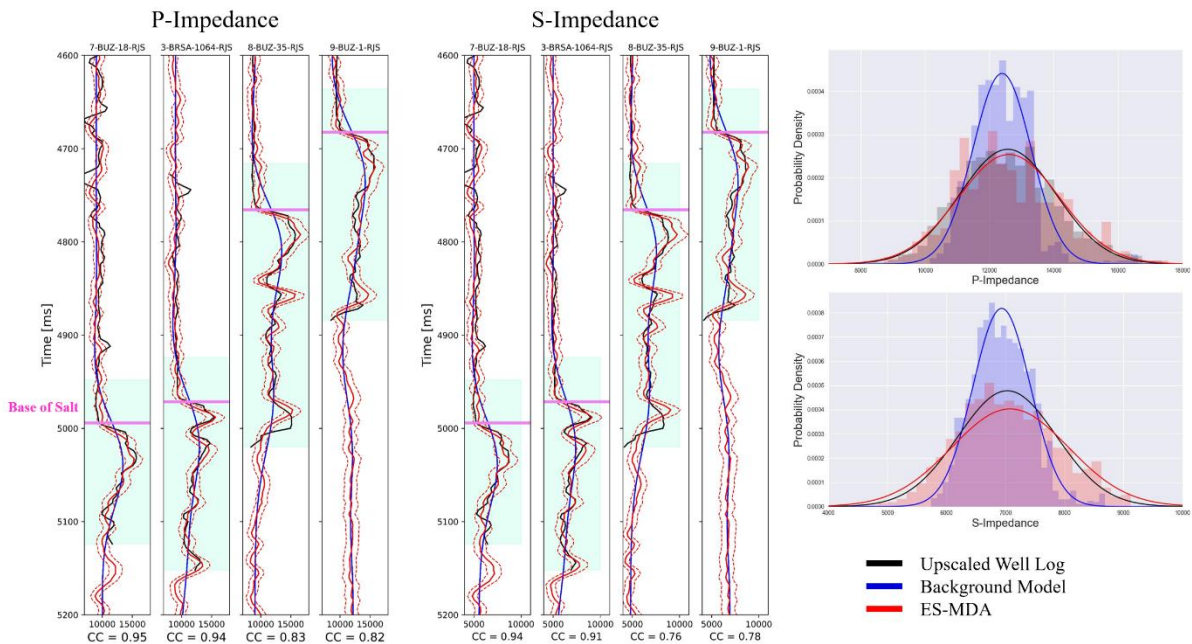


Figure 2 Well-based assessment of the ES-MDA posterior ensemble. The left panels compare the measured well logs with the prior average, posterior average and uncertainty interval at representative wells for acoustic impedance and S-impedance.

Conclusions

The application of ES-MDA demonstrated its capability to estimate elastic properties while explicitly quantifying uncertainty within a nonlinear Bayesian framework. Overall, the results indicate that ES-MDA is a robust approach for probabilistic elastic seismic inversion in pre-salt carbonate reservoirs. These characteristics make ES-MDA a promising framework for reservoir characterization and decision-making in complex and heterogeneous geological settings.

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