

Enhanced fast track inversion combining amplitudes and velocity estimate from FWI

Introduction

In the early phase of exploration, the main focus is to gain a regional understanding, to generate hypothesis of the subsurface and stratigraphy, to generate leads, and warning of shallow hazards. The wells in the region are few or non-existent, there are large regions to investigate, and no specific target. In this phase the goal of seismic inversion is to provide the best possible image of the subsurface. Images created from seismic amplitudes lack the low frequent part, traditionally there has been a spectral gap between the low frequent velocity model and the seismic amplitudes, further in absence of well data a statistical wavelet with an unknown amplitude level is used as a proxy for the link between reflectivity and seismic amplitudes. With the advent of full waveform inversion (FWI) the spectral gap is closed, and there is a spectral overlap. We propose a method for automatic estimation of the scaling factor of the statistical wavelet utilizing the spectral overlap between amplitudes and FWI, and a method for merging the two types of data if both are present. The approach provides high resolution velocity models utilizing velocities from FWI, seismic amplitudes, and the assumption of a blocky earth from total variation inversion. The purpose of this approach is to find suitable proxy models for the velocity to integrate the two types of data. We do not claim that the model accounts for the full physics of the problem, but that the model is sufficiently accurate in many cases, and that it is easy to assess and understand the shortcomings of the approach. In the same way a relative inversion using a statistical wavelet gives a quick first pass on inversion, the proposed approach provides an updated velocity suitable in the early phase of exploration. In an example study we create a high frequent model consistent with FWI and amplitudes where the aggregated kinematic effect on velocity is in the order of 1ms when comparing inversion result to the original FWI model. Besides the interpretive value of the high-resolution result, the approach enables an automation of the workflow generating highly realistic synthetic data relevant to local geology, as described in Evensen et al. (2023, 2025).

Method and Theory

Let $\mathbf{m}$ denote the velocity in the logarithmic domain. For the proxy models we propose

$$\mathbf{d}_V = \Psi_V \mathbf{m} + \epsilon_V \quad (1)$$

$$\mathbf{d}_A = \Psi_A \mathbf{D}_t \mathbf{m} + \epsilon_A \quad (2)$$

Where expression (1) gives the relation to the FWI data where, $\mathbf{d}_V$ is the log transform of the FWI-velocity, Ψ_V is the point spread function of the FWI velocities, and ϵ_V is an error term, and in expression (2) $\mathbf{d}_A$ is the seismic amplitudes Ψ_A being the wavelet, $\mathbf{D}_t$ being differentiation in time, and ϵ_A is an error term. Both Ψ_V and Ψ_A are assumed to have a symmetric Toeplitz form.

The modeling error where the velocity has replaced the acoustic impedance in expression (2), essentially corresponds to moving the contribution of the density into the error term. In sedimentary rocks the variability in velocity often dominates the density by a factor 3-4 when comparing coefficient of variation. Consider now the intercept stack, if we assume that the relation of where ρ and V_p follows a Gardener type of relation, we further have that

$$\log \rho = \log \alpha + \beta \log V_p + \epsilon_\rho \quad \Rightarrow \quad \log AI = (1 + \beta) \mathbf{m} + \log \alpha + \epsilon_\rho$$

where ρ, V_p, AI is the density, p-velocity and acoustic impedance, α, β are the coefficients in Gardners relation. The addition to the error in expression (2) is thus smaller than that of the variation of velocity to that of density, as for the factor $(1 + \beta)$, this is absorbed in the scaling of the wavelet Ψ_A .

In the full stack a similar argument can be made for the elastic impedance using a mudrock relation in addition. The part which do not correlate to the velocity contributes to the error term.

To derive point spread functions, we approximate these as 1D pulses, thus the goal then becomes to

estimate frequency components of Ψ_V and Ψ_A . Letting $\widehat{\cdot}$ denote the Fourier transform we get the following energy relations:

$$\begin{aligned} |\widehat{D_t d_V}|^2 &= |\widehat{\Psi_V}|^2 \cdot |\widehat{D_t m}|^2 + |\widehat{D_t \epsilon_V}|^2, \\ |\widehat{d_A}|^2 &= |\widehat{\Psi_A}|^2 \cdot |\widehat{D_t m}|^2 + |\widehat{\epsilon_A}|^2. \end{aligned}$$

The calibration of $\widehat{\Psi_V}(\omega)$, and $\widehat{\Psi_A}(\omega)$ is automated by splitting the frequency domain in two, at a threshold f_I . Below this threshold we assume that the point spread function of the FWI data capture the velocity, i.e. $|\widehat{d_V}(\omega)|^2 \approx |\widehat{m}(\omega)|^2$, above this threshold we assume that the spectrum of the velocity is white, i.e. $E\{|\widehat{D_t m}(\omega)|^2\} \approx \text{const}$. In addition, we assume that it is a smooth transition in both point-spread-functions across the threshold. Below the threshold ($0 \leq |\omega| < f_I$), we have the governing equations:

$$|\widehat{\Psi_V}(\omega)|^2 |\widehat{D_t m}(\omega)|^2 \approx |\widehat{D_t m}(\omega)|^2 \Rightarrow \widehat{\Psi_V}(\omega) \approx 1 \quad (3)$$

$$|d_A(\omega)|^2 \approx |\widehat{\Psi_A}(\omega)|^2 |\widehat{D_t m}(\omega)|^2 \Rightarrow |\widehat{\Psi_A}(\omega)|^2 \approx \frac{|d_A(\omega)|^2}{|D_t d_V(\omega)|^2}. \quad (4)$$

Above the threshold ($|\omega| > f_I$), we get the governing equations:

$$|D_t d_V(\omega)|^2 \approx \text{const} \times |\widehat{\Psi_A}(\omega)|^2 \Rightarrow |\widehat{\Psi_V}(\omega)|^2 \approx |D_t d_V(\omega)|^2 / \text{const} \quad (5)$$

$$|d_A(\omega)|^2 \approx \text{const} \times |\widehat{\Psi_A}(\omega)|^2 \Rightarrow |\widehat{\Psi_A}(\omega)|^2 \approx |d_A(\omega)|^2 / \text{const} \quad (6)$$

Where "const" in expressions (5) and (6) is the level of the specter of the velocity. We estimate "const" by the continuity assumption on the boundary for the velocity relation, thus $|D_t d_V(f_I)|^2 \approx \text{const} \times 1$. This automatically gives the normalization of the statistical wavelet, $\widehat{\Psi_A}(\omega)$. The expression for the energies used above, do not consider the error term. The error is reduced by smoothing in the frequency domain (i.e. tapering in time domain) and averaging over multiple traces, as is common in estimation of a statistical wavelet. The systems in expression (1) and (2), are thus identified, and we are free to choose the form of inversion. In our setting where we aim at screening, we use total variation regularization for inversion. This approach gives a velocity model presentation with low frequencies from the FWI, intermediate frequencies from the amplitudes, and high frequencies from blocky nature of the TV-regularization. Total variation inversion optimizes the expression,

$$\frac{\mu_V}{2} \|d_V - \Psi_V m\|_2^2 + \frac{\mu_A}{2} \|d_A - \Psi_A D m\|_2^2 + \|m\|_{TV}$$

where μ_V and μ_A are weight factors for velocity and amplitude data respectively, the total variation norm has the form:

$$\|m\|_{TV} = \sum_{i=1}^N \sqrt{|\nabla_x m_i|^2 + |\nabla_y m_i|^2 + |\nabla_z m_i|^2},$$

and can be solved using the ADMM (Boyd et al 2011).

In practice, the inversion is run on a set of large 3D cubes by generating overlapping 3D windows that then can be inverted separately. In that case, statistical wavelets can be extracted for each window, thus capturing time and space variance of the frequency content in the data.

Examples

The data analyzed is from the NCS and consists of 25Hz FWI velocities and full stack amplitudes, sampled at 4ms. The threshold f_t , was set to 15Hz (essentially the peak energy from FWI data). Both types of point-spread-functions were estimated in four intervals of 1000ms. The resulting estimates are shown in Figure 1. It is clearly seen that the estimate of the PSF for the velocity is stable at all depths, whereas the frequency content in the amplitudes, is reduced with depth. We observe that the shallow part (0-1000ms) has less energy which is natural given the data is ocean bottom seismic.

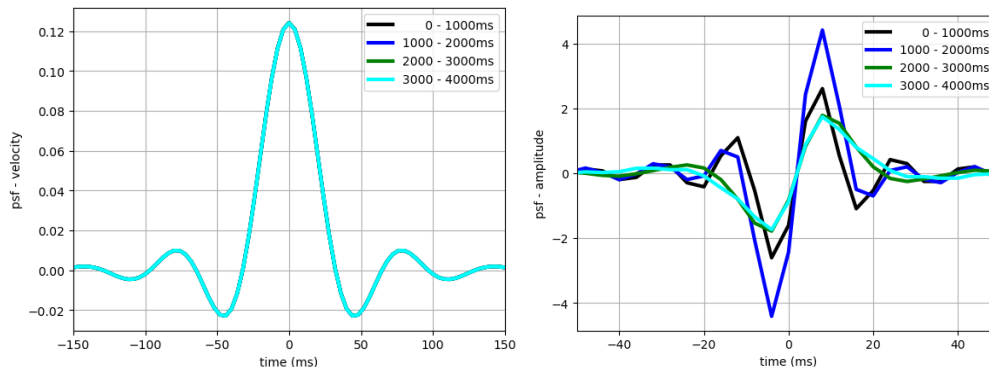


Figure 1 Estimated point spread function for the velocity data to the left, and the amplitude data to the right. Point spread function is estimated for four depth bands indicated in the figure. For the velocity all are on top of each other.

In Figure 2 we see details of a typical inversion result, using the methodology. It is easy to see the increased resolution in the velocity model. The zoomed in figure a thin layer appearing in the velocity even though invisible on the FWI velocities. The kinematic effect in the two velocity models a difference in the vertical TWT in of the order of 1ms, thus the approach preserves the kinematic properties well. It is reasonable that the amplitudes do not change the kinematics at scale since they only carry information about intermediate frequencies. The parameter μ_V determines how well the kinematic properties are preserved. Increasing this parameter will improve the preservation of kinematic properties at the cost of matching amplitude data and the blockiness of the inversion.

Conclusions and future extensions

In an acoustic FWI setting, we have derived a simple method for merging seismic amplitude data and FWI velocities. The validity of the approach depends on the assumptions governing expression (1) and (2), and our approach to identifying the systems. The current approach does not claim to capture full physical reality but has a set of pragmatic assumptions that are suited for the early phase of exploration, or to create realistic high frequency velocity models as input to feasibility studies in mature areas. We show that under these assumptions we do not need a well to calibrate the seismic wavelet. FWI velocities provide the information required to scale the statistical wavelet.

In this article we have focused on estimation of the velocity. The approach for integrating the FWI velocities in expression (1), can be directly integrated into any standard AVO inversion methodology using the linearized likelihood of Buland and Omre (2003). The approach lends itself to the framework of sampling-free Bayesian inference for local refinement described in Semin-Sanchis, and Kolbjørnsen (2023). This provides a framework for integration of FWI velocities in the standard workflows for AVO data, i.e. prospect risking, property estimation in reservoirs, and 4D responses of pressure and saturation changes.

Acknowledgements

We would like to thank Aker BP and production license PL167 for permission to publish data.

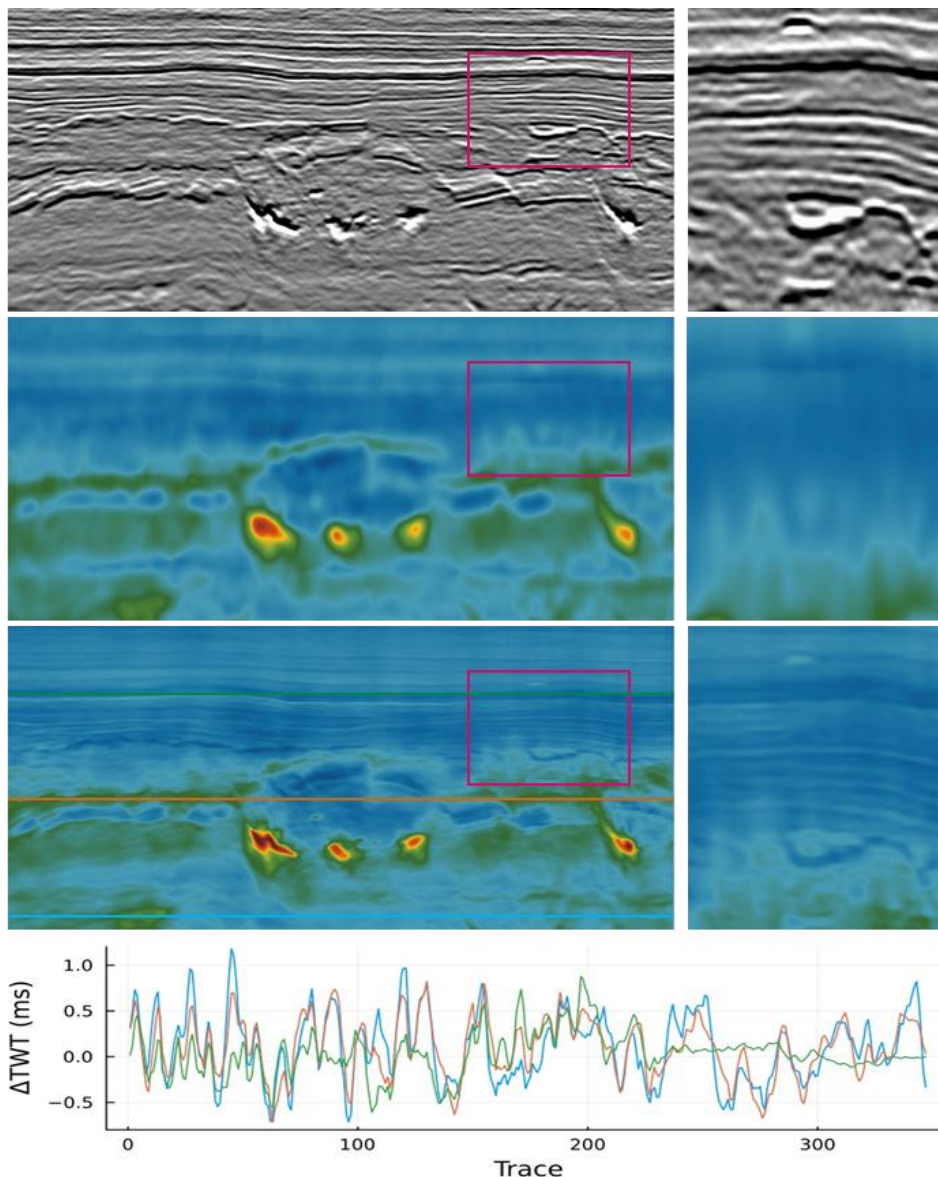


Figure 2 Inversion result using OBN data from the North Sea. Top image: Seismic data used in the inversion. Middle image: Existing velocity model. Bottom image: Inverted velocity using low frequent velocity model and the stacked seismic volume. The magenta window shows the zoomed section to the right. The bottom plot denotes differences in TWT between input and inversion results at the indicated levels (green, orange, and blue line).

References

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