

Water column sound speed estimation through Travel Time Tomography with *a priori* information

Introduction

When considering oceanic seismic inversion, most modern techniques use acoustic waves in the water column (Li et al. 2024), with this signal crossing and being transformed by both the water and ground environments. While the speed of sound in water is generally assumed constant around 1.5km/s, this approximation doesn't hold in large environments (Medwin et al. 1999), with it being influenced by environmental properties such as pressure, temperature, and salinity (Medwin et al. 1999). While preliminary works related these environmental quantities to the speed of sound (Del Grosso 1974; McDougall and Barker 2011), modern approaches treat this estimation as an inversion problem; furthermore, the maritime setting allows the addition of *a priori* information regarding the environment to this estimation process.

In this work, we propose combining these two approaches (*a priori* information, and the travel time tomography) for an enhanced sound speed field estimation in oceanic environments, aiming to refine the prior data using *in situ* travel time measurements. In a real-life-like simulation, we compare three proposed methods with the traditional Travel Time Tomography (TTT), and with the benchmark of directly using the prior field without refining or inversion. While all three outperform the non-informed TTT, none of the proposed approaches successfully refine the prior field, achieving similar or worse sound speed RMS errors.

Fundamentals of sound speed estimation

We are interested in estimating the two-dimensional sound speed field in the oceanic water column, using travel time as our exclusive measured quantity, and incorporating *a priori* available information regarding the environment into the model. Due to its mathematical simplicity and versatility, we will employ the Travel Time Tomography (Ali et al. 2019) technique.

We consider M measurements between devices, and N cells in a 2D environment, for which their sound slowness (reciprocal of speed) will be modeled. By letting $\mathbf{t}_{\text{obs}}$ be a vector of M travel time measurements, $\mathbf{R}$ a forward operator of distances (an $M \times N$ matrix), and $\mathbf{s}$ a vector of the modeled slowness for each of the N cells, the traditional TTT is done iteratively. The $(i + 1)$ -th solution $\mathbf{s}_{i+1}$ is obtained through $\mathbf{s}_{i+1} = \arg\min_{\mathbf{s}} \|\mathbf{R}_i \mathbf{s} - \mathbf{t}_{\text{obs}}\|^2$, where $\mathbf{R}_i$ (for the i -th iteration) is achieved via an Eikonal solver (Ganster 2025).

It is also common to add regularization to the previous formulation, with different objectives. Usually, this is achieved through a quadratic regularization structure $\alpha \|\mathbf{D}(\mathbf{s} - \mathbf{r})\|^2$, where $\mathbf{D}$ is a regularization matrix, $\mathbf{r}$ is a reference vector, and α is a regularization parameter. In this work, we are interested in two types of regularization: smoothness (Ali et al. 2019), which penalizes abrupt changes in the slowness vector $\mathbf{s}$; and reference penalization (Aki and Lee 1976), penalizing the difference of the model against a prior slowness vector $\mathbf{s}_{\text{obs}}$.

Using *a priori* information

While TTT is usually employed with a smoothness regularization, the setting of ocean water sound speed estimation enables employing *a priori* environmental information; we will consider the use of World Ocean Atlas (Garcia et al. 2024) data (of salinity and temperature along depth) allied with the TEOS-10 model (McDougall and Barker 2011). The possibility of using *a priori* information is particular — but

not unique — to this oceanic environment, as its patterns can be predictable (Medwin et al. 1999) but not constant (Del Grosso 1974; McDougall and Barker 2011)(due to physico-chemical oceanic interactions), allowing these prior measurements to be refined.

Extending the TTT with a priori information

We will consider three approaches that extend the traditional TTT, enabling *a priori* information usage:

Trade-off: Inspired by (Aki and Lee 1976), this is a natural extension of the traditional TTT + smoothness regularization framework, adding a secondary regularization term penalizing the distancing to a reference.

EOF: Based on the framework proposed by (Radhakrishnan and K 2024), where Empirical Orthogonal Functions (EOFs) were constructed based on the *a priori* information, and employed in the estimation of the sound speed field. We also extended it to the 2D scenario and adjoined it with a Laplacian (smoothness) regularization, ensuring consistency between columns.

Munk: One last method is based on the Munk profile (Munk 1974), modifying the modeling in (Wu et al. 2023) to the 2D case, and also adding a reference regularization to the iterative process.

Simulation and Results

The presented techniques will be compared through simulated real-life-like scenarios, being employed in the inverse map of estimating the sound speed field. Our implementation used yearly WOA temperature and salinity data (Garcia et al. 2024) with (0.25°, 0.25°, 50m) spatial (latitude, longitude, depth) resolution, and the TEOS-10 model through the GSW (McDougall and Barker 2011) Python package. We consider 9 profiles near 36.375°S, 50.375°W up to a depth of 1.5km, with 31 vertical slices. The true sound speed field, is shown in figure 1a, along with the considered device configuration: sources (yellow circles) along the ground; and receivers along the surface and in a column central column, emulating two data acquisition trips. Figure 1b also shows the respective 1D vertical slices of figure 1a, for a better visualization of the differences between the profiles.

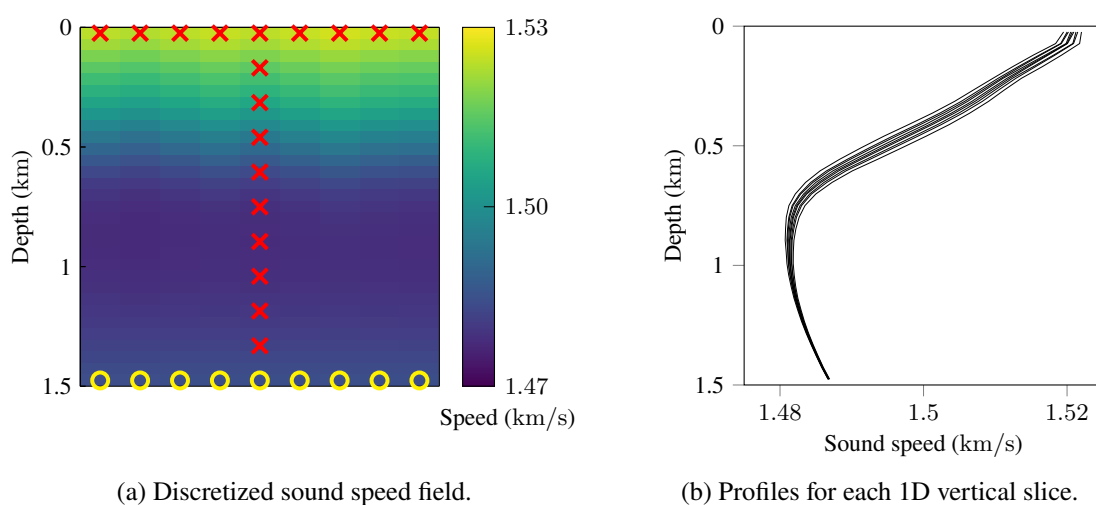


Figure 1: Considered environment, and devices (sources and receivers) configuration.

The three previously exposed methods (trade-off, EOF, and Munk) will be compared among them,

and with two benchmarks: the traditional TTT with smoothness regularization, for comparisons of performance gains when adding *a priori* knowledge; and the direct use *a priori* of this *a priori* information without further refining, allowing comparisons of whether the techniques successfully improved it.

We are primarily interested in the RMS sound speed error between the estimated and true sound speed vectors; note that the metrics utilize the speed, not the slowness. We will also use the normal (non-RMS) sound speed error. In the estimation of the forward map $\mathbf{R}_i$, the EikonalFM Python package (Ganster 2025) was used.

Results

For the proposed simulations, we show the RMSE metric in figure 2a, and error boxplots (for each technique and along depth) in figure 2b. From the results in figure 2a, we see that the Munk profile technique didn't achieve decent results, having errors nearly $10\times$ greater than the other methods. Meanwhile, the two other proposed (along with the mean benchmark) outperformed the traditional TTT. This indicates that employing *a priori* data improves the performance; however, the gains are small, of less than 0.5m/s .

Comparing against the mean (non-refining) benchmark, we see that all methods had worse performance than this benchmark. Therefore, these techniques — which employ this *a priori* data — don't necessarily refine the prior information, achieving the same performance at best (with the trade-off), or deteriorating the sound speed field in the worst case.

We can also see that the error for all (except Munk's parameterization) is concentrated above 750m depth, where the original profiles (in figure 1b) vary most rapidly. While the error between 100m and 750m was slightly reduced, the iterative process caused prominent errors very near the surface (above 100m), indicating that this minimization (and in particular the smoothness regularization, since this also appears for the traditional TTT) may not be the most suited.

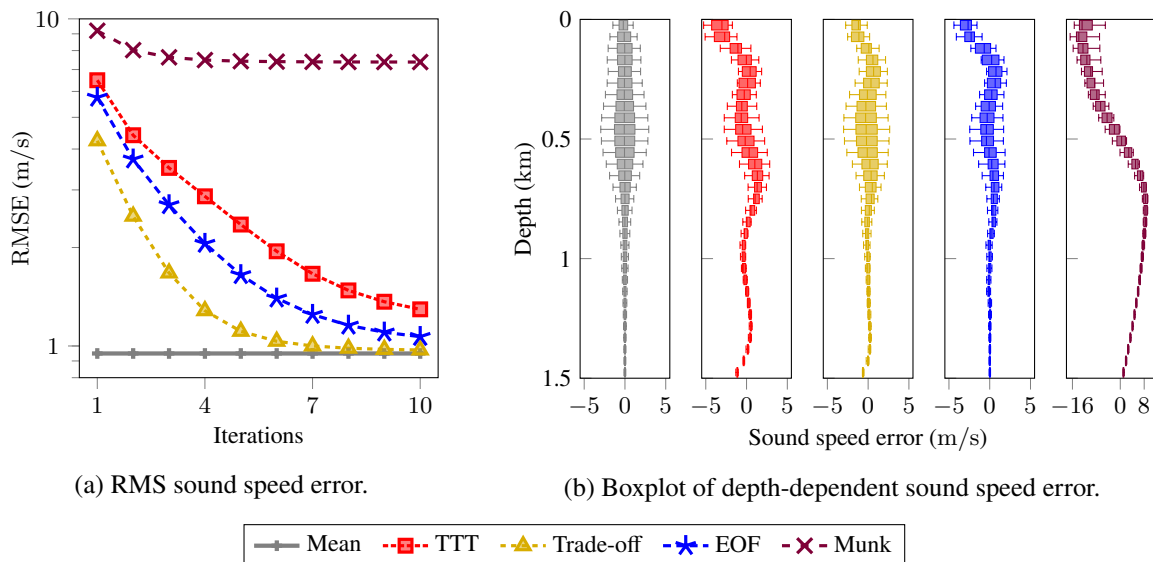


Figure 2: Sound speed error metrics of the iterative estimation process.

Conclusion

In this work, we brought three different minimization schemes that adapt the Travel Time Tomography (TTT) structure to include *a priori* information, modifying them to the 2D case when necessary, focusing

on estimating the sound speed field in water. Through simulations with real data, we showed that two of the three shown techniques outperformed the TTT without prior information. However, neither of the explored frameworks was successful in refining the *a priori* field, leading to a worse (or at most equal) performance when compared to directly using the prior data without any further iterative processes.

Acknowledgment

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