

Geostatistical Elastic Inversion for Complex Geology

Introduction

Seismic inversion at reservoir scale is a complex process, with many techniques using a Bayesian framework to assess uncertainties, both in the parameterisation of the inversion through the definition of probability density functions (PDFs) and in the results by drawing samples from the posterior distribution. However, Bayesian formulations only provide a set of equations, not algorithms, and practical implementations often struggle to achieve good performance on large datasets.

We propose here to formulate a geostatistical elastic inversion as a variant of kriging. Geostatistics provide a strong mathematical basis for integrating various types of information into a single model. However, most techniques have been developed for point-based information such as well logs rather than for data with complex spatial dependencies such as seismic data. Classical implementations also often require approximations (e.g., neighbourhood search schemes) to handle large datasets.

Our approach builds on expressing kriging as the solution of a stochastic partial differential equation (SPDE). This provides a mathematically robust approach to the solution, allowing a flexible elastic parametrisation while handling in a rigorous manner various types of geological constraints, such as non-stationary relationships, that are particularly important in modelling complex geological settings. Additionally, this approach can be used to generate stochastic simulations for uncertainty assessment.

Bayesian seismic elastic inversion

Following Buland and Omre (2003), the seismic inversion is formulated as a Bayesian inverse problem. When assuming a log-normal distribution of the elastic parameters to invert, usually P- and S-impedances or P- and S-velocities and density (defined by their mean μ_m and covariance Σ_m) and a normal distribution of the seismic likelihood (mean 0 and covariance Σ_s), the posterior distribution is a normal distribution. Its mean μ_p and covariance matrix Σ_p have analytical formulations:

$$\begin{aligned}\mu_p &= \Sigma_p(\Sigma_m^{-1}\mu_m + G'\Sigma_s^{-1}s) = \mu_m + \Sigma_m G'(G\Sigma_m G' + \Sigma_s)^{-1}(s - G\mu_m) \\ \Sigma_p^{-1} &= \Sigma_m^{-1} + G'\Sigma_s^{-1}G\end{aligned}$$

The matrix G represents the forward modelling of seismic amplitudes from elastic data. It is formed by combining a linearisation of the reflectivity equations (either Fatti or Aki-Richards) and a 1D trace-by-trace convolution by the seismic wavelet for each partial angle stack.

A practical implementation of this framework requires computing the inverse of a matrix of the size of the domain to invert (3D grid). Various approximations have been proposed. For example, Escobar et al. (2006) used a trace-by-trace decomposition similar in principle to the sequential Gaussian simulation (SGS) technique. However, to achieve practical computing times, an approximation of the random sampling was used, at the cost of a slight bias. Our approach avoids these issues by using a rigorous algorithm for kriging of large datasets.

Kriging of large data sets with SPDE

Kriging is a well-known linear interpolator usually expressed as a linear problem. In simple kriging, the vector of interpolation weights at each location is obtained by inverting the matrix of covariances between the locations of the known data points. The spatial correlation between points is assumed to be entirely modelled by a two-points covariance function (usually through a variogram). While this can be an issue when using only sparse data to model complex geology (e.g., channelised environments constrained only by wells), for dense data, such as seismic, this assumption is considered acceptable. The size of the matrix to invert is the number of data points, which is not practically feasible for very large data sets.

Alternatively, kriging can also be expressed as a Bayesian update problem, as the posterior mean (or expectation) of a Gaussian Random Field (GRF) Z of prior covariance Σ updated by known values Y :

$$E(Z|Y) = \Sigma M' (M \Sigma M')^{-1} Y$$

where M is a transfer operator that selects the locations of the domain where data points are available (i.e., where Y is defined). Solving this equation still requires inverting a matrix the size of the full domain. However, Lindgren et al. (2011) have shown that such a GRF can be approximated as the solution of a stochastic partial differential equation (SPDE) defined on a mesh, with an approach akin to finite elements. Pereira et al. (2022) have further shown that the inverse Q of the covariance matrix Σ can be approximated by a sparse matrix of known terms dependent on the mesh shape. With τ being the modelling error introduced by the mesh, the posterior mean can then be expressed as:

$$E(Z|Y) = \Sigma M' (M \Sigma M' + \tau^2 I)^{-1} Y = (\tau^2 Q + M' M)^{-1} M' Y$$

This is a large linear problem tractable using any numerical solver, such as the conjugate gradient. Additionally, this can be implemented using only matrix-vector products with the matrices being implicitly defined, which makes it suitable for seismic scale datasets.

Geostatistical seismic inversion framework

The different formulations of the posterior means of the seismic inversion problem (μ_p) and of the SPDE kriging system ($E(Z|Y)$) are similar. By replacing the transfer operator M of the kriging with the forward model G of the seismic inversion, it becomes possible to use the SPDE approximation for seismic inversion. In addition to being applicable to large data sets, this approach maintains the flexibility of the geostatistical framework. It is applicable to any geological grid (e.g., stratigraphic grid) and allows for non-stationary parameters such as the covariance (variogram ranges or anisotropy orientation), the prior model uncertainty, or the seismic noise level.

Additionally, the forward model G can be reformulated for any consistent set of elastic parameters, making it possible to invert for combinations of attributes such as P-impedance, Vp/Vs ratio, and density. Such alternative elastic parameterisations are better suited to complex geologies, both because there is less inter-dependency between attributes and because the geological interpretation (e.g., facies classification) is often performed in these spaces.

Finally, random samples can be efficiently drawn from the posterior distribution by solving the SPDE kriging equation for randomly perturbed data (Pereira et al. 2022). This makes it possible to easily generate any number of realisations from the posterior distribution, allowing uncertainty quantification at fine spatial scale.

Challenges of a complex sandstone reservoir

The method was applied to a synthetic data set derived from the well-known Marmousi data set. The selected part of the model contains a sequence of alternating sands and shales, either as continuous layers or isolated lenses. A rock physics model was used to generate elastic attributes corresponding to different rock and fluid properties. The resulting model shows strong variations of elastic attributes, including layers where density decreases while P-velocity increases. This anti-correlation does not match the usual trend (e.g., Gardner's law) and is often challenging to invert accurately, as demonstrated by several North Sea studies of remobilised complex sand bodies (Aziez et al. 2024).

The posterior mean of the elastic inversion was computed on synthetic seismic data up to 40 degrees, starting from a low-frequency prior model. Results are shown in Figure 1. The inversion was first performed for P-velocity, S-velocity and density. A good match is obtained for all attributes, and the seismic is correctly reproduced. However, small deviations from the truth are visible, particularly for density. Comparing inversion results with the prior model shows that the mean absolute error (MAE) to the truth is reduced by 74% for P-impedance, 45% for Vp/Vs ratio but only 15% for density.

In a second step, the inversion was performed for P-impedance, Vp/Vs ratio and density. This elastic parameterisation allows better separation of the elastic attributes, and in particular, the density is visibly improved. The P-impedance is only slightly improved (MAE reduced by 77%), showing that the parameterization has relatively little impact on the acoustic part of the inversion. However, the improvement is clear for the Vp/Vs ratio (MAE reduced by 63%) and density (MAE reduced by 58%).

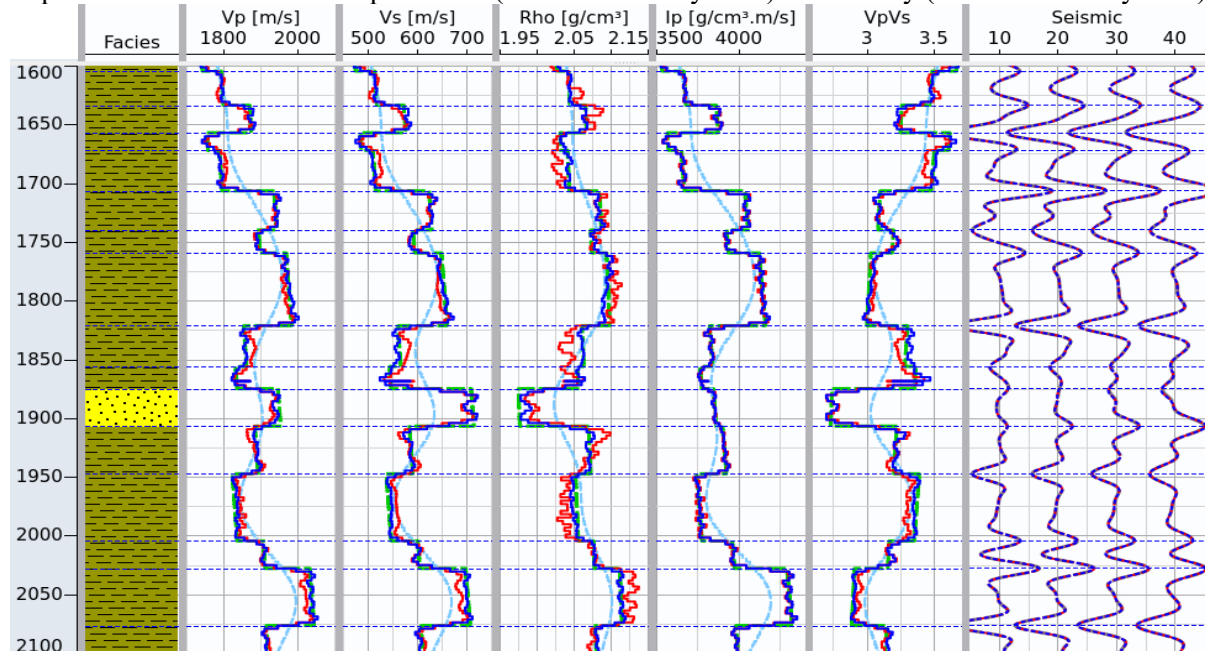


Figure 1 Posterior mean of the geostatistical inversion on a part of the Marmousi data set. P-velocity and density anti-correlation can be observed in the sand layer (around 1900 ms). Truth in green, prior model in light blue. Red curves show the inversion in P-velocity, S-velocity and density (P-impedance and Vp/Vs ratio derived from these). Dark blue curves show the inversion in P-impedance, Vp/Vs ratio and density (P- and S-velocities derived from these). Both results match the seismic (rightmost track).

Next, stochastic realisations were drawn from the posterior distribution to explore the uncertainty of the inversion around the posterior mean (Figure 2). As expected, the spatial distribution of the realisations around the posterior mean reproduces the specified variogram parameters, while preserving the match with the seismic data.

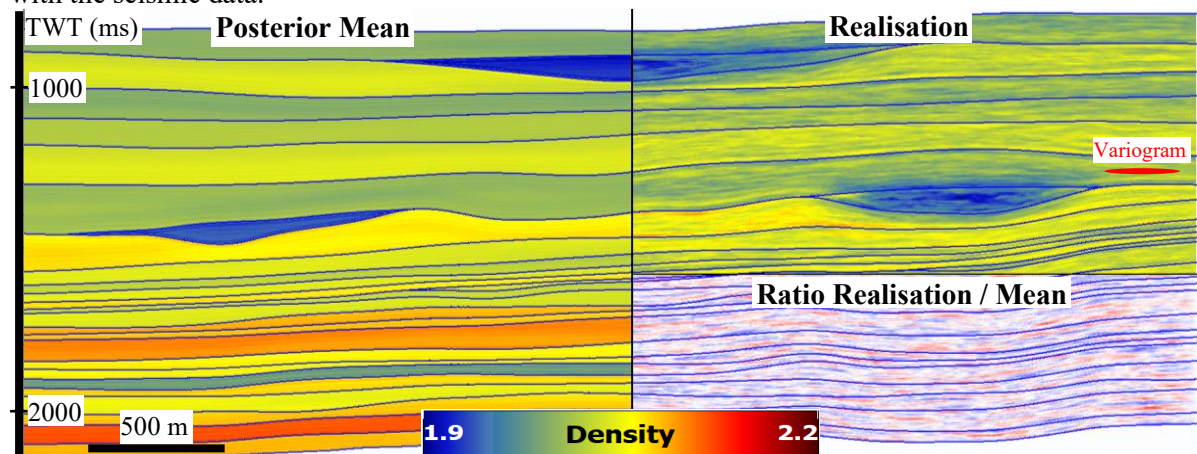


Figure 2 Composite section showing the posterior mean of the geostatistical inversion (left), one sample from the posterior distribution (top right) and the ratio to the posterior mean (bottom right). Vertical and lateral variogram ranges are illustrated in the red ellipse.

Modelling remobilised sand complexes in the Norwegian North Sea

The method was then applied to a part of the North Viking Graben (NVG) dataset, a dual-azimuth seismic in the Norwegian North Sea (Buriola et al. 2023). The main challenge is the characterisation of hydrocarbon and water-filled injectite sand complexes, located in the Hordaland and Rogaland groups. Preliminary results of the posterior mean of the geostatistical elastic inversion, performed directly in P-impedance and Poisson's ratio, are shown in Figure 3.

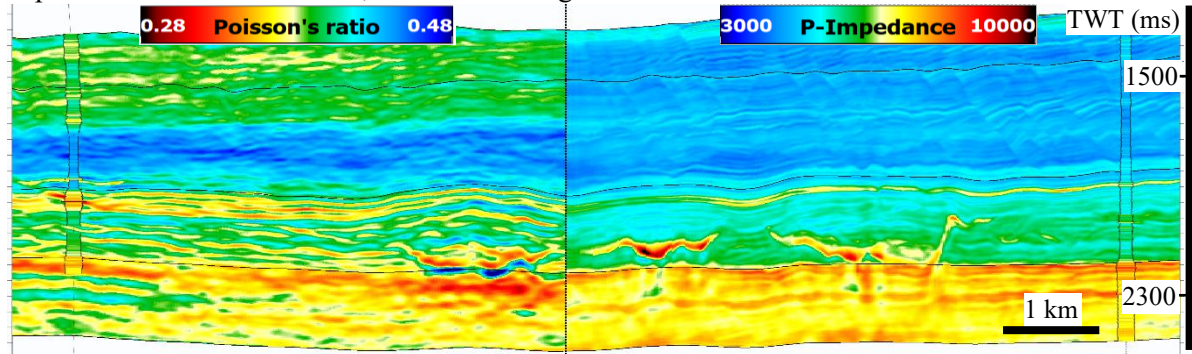


Figure 3 Composite section of Poisson's ratio (left) and P-impedance (right) between two wells. Complex shapes that can be interpreted as remobilised sand complexes are visible in the lower interval.

Conclusions

A geostatistical elastic inversion framework is presented, grounded in a Bayesian Gaussian formulation and solved via an SPDE approximation to the kriging system. The approach is scalable to large datasets, supports both deterministic posterior mean estimation and stochastic sampling of the posterior distribution, and accommodates flexible elastic parametrisations alongside non-stationary geostatistical constraints. On synthetic data the proposed approach is shown to accurately retrieve density and fluid-sensitive attributes in a geologically complex setting. Application to a North Sea injectite dataset yields geologically coherent results consistent with the expected morphology of remobilised sand complexes.

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