

Mapping density ratio over intercept-gradient space: a precomputed exact-Zoeppritz operator

Introduction

Estimating density from seismic amplitude variation with offset (AVO) remains one of the hardest tasks in quantitative interpretation. Density is only weakly expressed at the small-to-moderate incidence angles available in conventional gathers, and the amplitude calibration required to bring seismic data into the reflectivity domain is itself uncertain. Lehocki and Avseth (2020) addressed both difficulties by inverting uncalibrated near and far partial stacks jointly for the acoustic-impedance ratio (AI_r) and the density ratio (ρ_r) through the exact P-to-P Zoeppritz equation, exploiting the fact that the two ratios are inherently linked.

In the present work we extend that single-interface procedure into a precomputed map of all feasible interfaces. Applied to uncalibrated amplitude ratios, in which the unknown scaler cancels, the inversion does not return a single density ratio but an $AI_r - \rho_r$ relation: a nonlinear locus of (AI_r, ρ_r) pairs. This locus is set by the shape of the reflectivity – that is, by the intercept-gradient ($I - G$) pair of the cell. The key observation is that the absolute intercept then collapses the locus to a single point: it fixes AI_r and selects ρ_r on the locus. A calibrated $I - G$ pair therefore yields the density ratio directly. Solving this relation once for every cell turns the map into a data-free lookup, with no per-trace inversion.

Here we focus on the construction and physical parameterisation of this precomputed operator – a fixed, nonlinear rule from the $I - G$ plane to density ratio – building directly on the joint $AI_r - \rho_r$ exact-Zoeppritz inversion of Lehocki and Avseth (2020) and benchmarked against the direct density-ratio schemes of Lehocki et al. (2025a, 2025b). We describe how the intercept extent, the V_P/V_S and $\rho_r / AI_r (= DAR)$ ranges, and the admissible density-ratio window are fixed by physical consistency rather than convenience, and we identify the regimes over which the operator is well-posed.

Method and theory

Following Lehocki and Avseth (2020), the exact P-to-P reflection coefficients are reparameterised as $RC_{PP} = RC_{PP}(AI_r, \rho_r / AI_r, \gamma_1, \gamma_2, \theta)$, where $AI_r = AI_2 / AI_1$, $\rho_r = \rho_2 / \rho_1$, $\gamma_1 = V_{P1} / V_{S1}$, $\gamma_2 = V_{P2} / V_{S2}$, and the ratio $DAR = \rho_r / AI_r = V_{P1} / V_{P2}$. When $V_{P1} \leq V_{P2}$, this ratio equals the sine of the critical angle, a consequence of Snell's law. Assuming the unknown amplitude scaler is common to the two neighbouring angles, it cancels in the ratio, so the measured amplitude ratio equals the ratio of the underlying reflection coefficients; the $AI_r - \rho_r$ relation is then recovered without calibration, and a single intercept-calibration step later fixes the absolute scale (Lehocki and Avseth, 2020). An algebraic manipulation of the P-to-P Zoeppritz equation, where the ratio of the reflection coefficients at the two neighbouring angles is introduced, leads to a fourth-degree (quartic) polynomial in AI_r .

For a two-term response $RC(\theta) = I + G \sin^2(\theta)$ sampled at small angles, the consecutive-angle amplitude ratios are formed, and the quartic is solved over an exhaustive 3D grid of $(\gamma_1, \gamma_2, DAR)$. For a fixed (γ_1, γ_2) pair, each angle pair traces a curve in $AI_r - \rho_r$ space as DAR is varied; the curves from the different angle pairs intersect at a single consistent point. The locus of these intersections over the (γ_1, γ_2) grid defines the $AI_r - \rho_r$ relation of the cell, which we summarise with a low-order (linear and quadratic) polynomial fit (Figure 1).

Repeating this for every cell of the I-G grid yields maps of the fit coefficients and a precomputed density-ratio. The parameter ranges are fixed by physical consistency rather than convenience. Because $AI_r = \rho_r / DAR$, the chosen intercept envelope sets the admissible AI_r and therefore the required DAR range. An intercept span of $[-0.4, 0.4]$ (equivalent to AI_r span $[0.43, 2.33]$) is anchored by strong but realistic contrasts – for example the water / plexiglas interface, for which AI_r is approximately 2.2 (intercept approximately 0.38) – and demands DAR of about $[0.5, 2.0]$ together with a symmetric $\gamma_1 = \gamma_2$ range, since the same soft unit may be the upper or the lower layer. The density ratio itself is restricted to a rock-physically plausible window.

Results and discussion

Figure 1 shows the precomputed density-ratio maps (upper left and lower left subplots) and its fit-coefficient attributes for both a linear and a quadratic $AI_r - \rho_r$ relation. The maps are data-free: they depend only on the intercept-gradient coordinates, not on any survey.

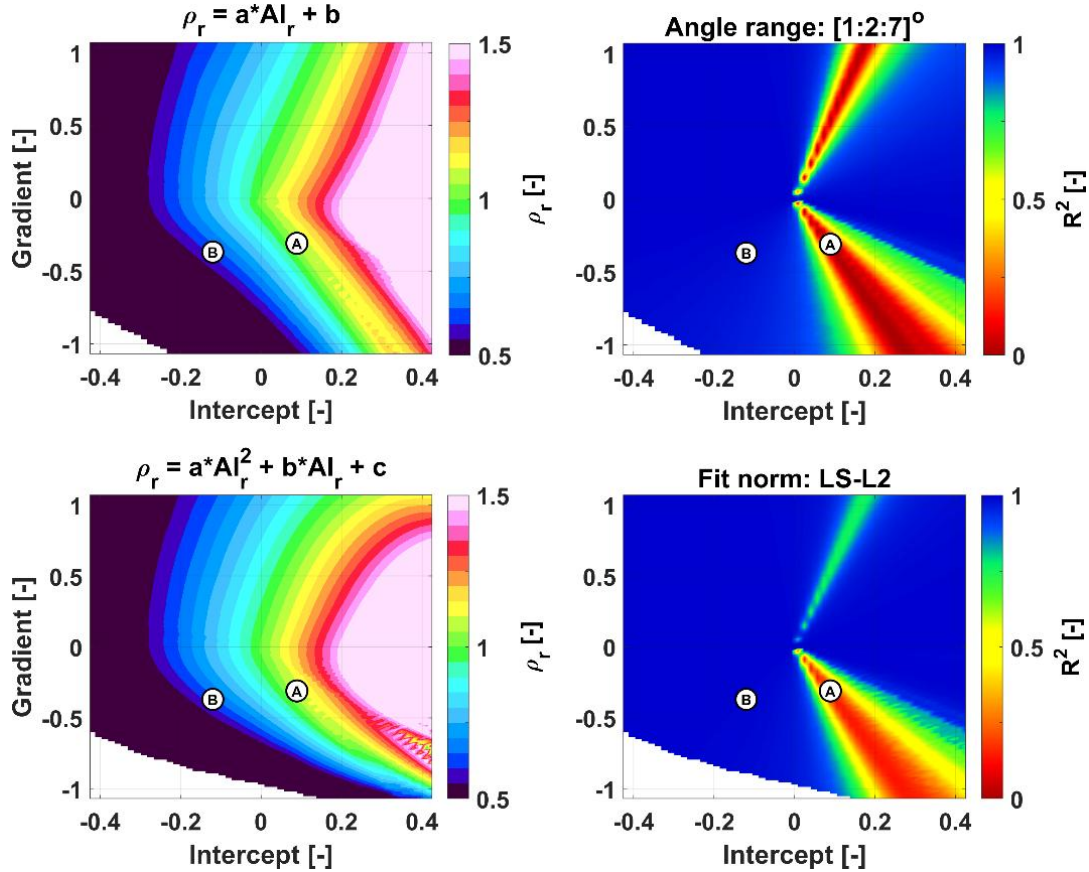


Figure 1 Precomputed density-ratio map over the intercept-gradient plane for a linear (top-left) and a quadratic (bottom-left) $AI_r - \rho_r$ fit, with the corresponding R^2 (surviving-model scatter) maps on the right. The two test interfaces of Figures 2 and 3 are marked A and B.

To probe where the operator is quantitatively reliable we forward-modelled two contrasting interfaces and inverted them at the same small angles used for the map (1-7 degrees). For both, the surviving models reproduce the true intercept-gradient pair almost exactly (Figures 2 and 3): the I – G pair is robustly constrained, independent of interface class. The density ratio, however, behaves differently on the two sides of the $\rho_r = AI_r$ line. For interface A, a class-I interface with $\rho_r < AI_r$ (brine side), the inverted density ratio closely matches the true value; for interface B, a class-III interface with $\rho_r > AI_r$ (the gas side, $DAR > 1$), it is biased toward the $\rho_r = AI_r$ line.

Importantly, this accuracy contrast is not predicted by the fit quality. Interface A lies in a low- R^2 region of the map yet its density ratio is accurately recovered, whereas interface B lies in a high- R^2 region yet is biased. The R^2 maps therefore measure the scatter of the surviving models - a useful quality-control layer that flags where the $AI_r - \rho_r$ relation is poorly resolved - but they are not a measure of density-ratio accuracy.

Consistent with Lehoccki et al. (2025a, 2025b), the two-term (curvature-free) reflectivity is sufficient for density-ratio recovery at these angles, where the exact-Zoeppritz inversion clearly outperforms approximate Shuey-type inversions. In practice the operator therefore recovers density ratio quantitatively on the brine-saturated background and acts as a robust hydrocarbon-anomaly indicator on the $\rho_r > AI_r$ side - the regime in which the prospective anomalies reside.

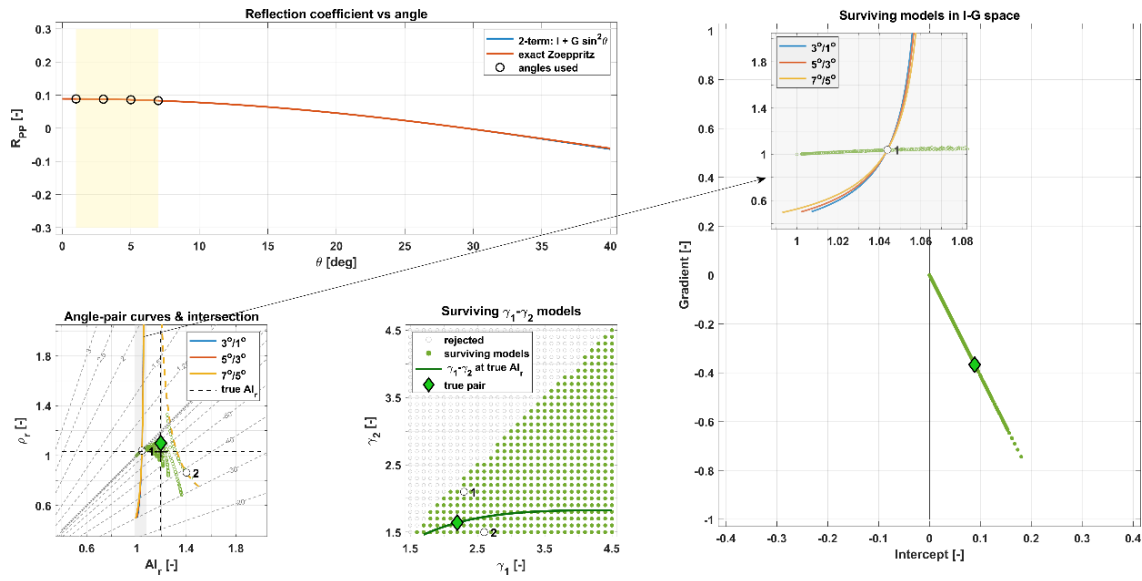


Figure 2 Inversion diagnostic for interface A (class I, $\rho_r < AI_r$), at the 1-7 degree angle range. Upper-left: reflection coefficient versus angle (two-term and exact Zoeppritz; angles used shaded). Upper-right: surviving models in I-G space, capturing the true (green diamond) intercept-gradient pair. Lower-left: the $AI_r - \rho_r$ angle-pair intersection (white circles) and the inverted density ratio (cross-hair); the inset (shown within the upper right subplot) magnifies the shaded box around point 1, showing that the three angle-pair curves genuinely intersect there despite nearly coinciding at these small angles. Lower-middle: the surviving $\gamma_1 - \gamma_2$ models with the true- AI_r relation (green curve) passing through the true $\gamma_1 - \gamma_2$ point. The inverted density ratio (horizontal dashed line in lower left subplot) closely matches the true value (diamond).

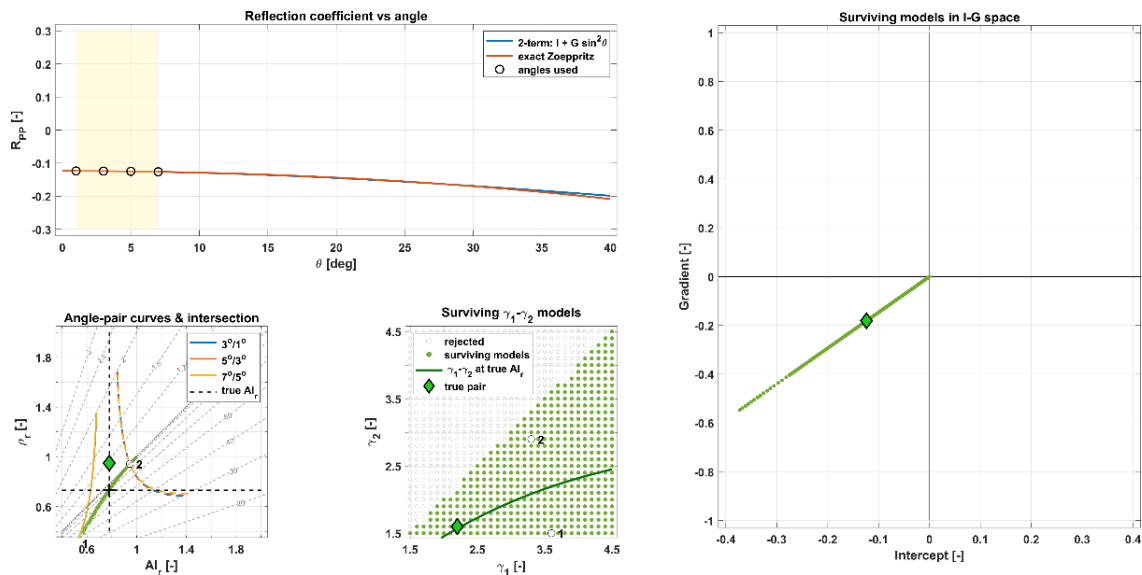


Figure 3 As Figure 2 but for interface B (class III, $\rho_r > AI_r$, i.e. $DAR > 1$). The intercept-gradient pair is again recovered, but the inverted density ratio (cross-hair) is biased toward the $\rho_r = AI_r$ line relative to the true value (diamond), illustrating the gas-side behaviour described in the text.

Figure 4 demonstrates the operator in application: the calibrated intercept and gradient at the Fuglen shale - Top Stø sand interface are passed through the quadratic operator of Figure 1, yielding the density-ratio map directly, without any per-trace Zoeppritz inversion. The mapped gas caps (black polygons) emerge as clear density-ratio anomalies: their intercept-gradient signature falls in the $\rho_r > AI_r$

region the fast map does not resolve quantitatively (whereas the direct inversions of Lehocki et al., 2025b, do).

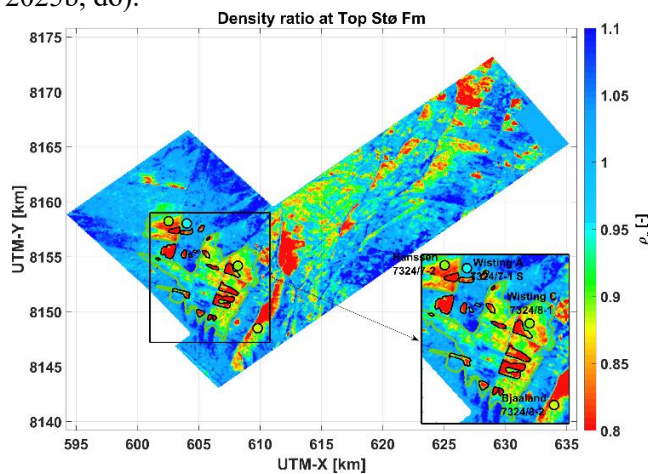


Figure 4 Density-ratio map at the Fuglen shale - Top Stø sand interface, Barents Sea (Wisting area). Black polygons outline the mapped gas caps; the operator highlights them as density-ratio anomalies, consistent with the well control of Lehocki et al. (2025b).

Conclusions

We have turned the joint $AI_r - \rho_r$ inversion of Lehocki and Avseth (2020) into a precomputed, data-free density-ratio operator over the intercept-gradient space, with parameter limits fixed by the physical consistency relation $DAR = \rho_r / AI_r$ rather than by convenience, and shown to be consistent with the direct inversions (Lehocki et al., 2025a, 2025b). The map (Figure 1) is built purely from ratios of reflection coefficients and is therefore data-free, whereas the field example (Figure 4) recovers the $AI_r - \rho_r$ relation from uncalibrated reflection amplitudes at Top Stø and applies a single calibration scaler to reach the $AI_r - \rho_r$ point.

Two contrasting interfaces show that the intercept-gradient pair is robustly recovered regardless of class, that density ratio is accurate on the brine-saturated trend ($\rho_r \leq AI_r$) and biased toward the $\rho_r = AI_r$ line on the hydrocarbon side ($\rho_r > AI_r$), and that the R^2 maps measure model scatter rather than accuracy. The operator is therefore best used as a fast, data-free quantitative density-ratio estimate on the background trend and as a robust anomaly indicator over prospective zones. Quantitative recovery in the $\rho_r > AI_r$ regime, and the contrast with the direct (Loris) inversion there, is the subject of ongoing work.

Acknowledgements

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