Feasible Region of Lamination Parameters for Double-Double Laminate

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1 General Introduction

Double-double laminates (DD) proposed by Tsai [1] are a new family of composite laminates which consist of a repeat of a four ply sub-laminate. The usage of DD laminates avoids several issues in the design of composite laminates. Homogenization in the thickness direction is one of the key advantages that allows DD laminates to be much more lightweight compared to QUAD layups [1]. This is because when a laminate is homogenized, which is achieved by stacking the sub-laminate, its in-plane-out-of-plane coupling can be intrinsically ignored. As a result, the optimization design of DD laminates can be performed without imposing the mid-plane symmetry rule reducing a larger number of plies. While traditional QUAD laminates are heterogeneous, and it requires at least 120 plies or 15 mm to be homogenized.

Tsai[1] concluded that DD-laminates could achieve a similar performance to most QUAD designs by taking advantage of their larger design spaces. However, the design space of DD-laminate has never been found. York[2] discovered that the lower bound of the design space of lamination parameters ξ_1^A, $\xi_2^A$and ξ_1^D, $\xi_2^D$for general two-angle laminates can be represented by parabolas. However, the upper bound and design spaces for other lamination parameters, especially for ξ_{1-4}^Bwhich relate to the in-plane-out-of-plane coupling and key parameter to determine the homogenization, have not been discussed. Therefore, the extension of the homogenization of a DD-laminate is hard to define mathematically, resulting in sacrificing design space for the near homogenized laminates.

Taking advantage of repetition, formulations for obtaining the lamination parameters of DD laminates are derived, based on which lamination parameters can be expressed in simple forms. With the aid of these formulations, feasible regions of any combination of two lamination parameters can be obtained analytically. Based on the obtained feasible regions, clear insight into the homogenization of a DD laminate can be visualized. Furthermore, the effect of the number of plies on the homogenization is investigated based on the homogenization criterion defined in [1]. The work proposed offers practical solutions in making design, manufacturing, and testing simpler and more competitive

2. Lamination parameters of General DD-laminate

Lamination parameters were first introduced by Tsai et al in 1968 [3] enabling the stiffness matrix to be represented by a linear function and significantly reducing the number of design variables. As the sub-laminate of DD-laminates has only four angles, namely two angles with opposite signs or four different angles [1] (i.e. unbalanced DD-laminates), lamination parameters can be derived with respect to the four angles $\theta_1, \theta_2, \theta_3, \theta_4$.

$$\xi_j^A = \left\{\frac{1}{4} f_j(\theta_1) + \frac{1}{4} f_j(\theta_2) + \frac{1}{4} f_j(\theta_3) + \frac{1}{4} f_j(\theta_4)\right\} \quad 2$$

$$\xi_j^B = \left\{-\frac{3}{2n} f_j(\theta_1) - \frac{1}{2n} f_j(\theta_2) + \frac{1}{2n} f_j(\theta_3) + \frac{3}{2n} f_j(\theta_4)\right\} \quad 3$$

$$\xi_j^D = \left\{\left(\frac{1}{4} + \frac{3}{n^2}\right) f_j(\theta_1) + \left(\frac{1}{4} - \frac{3}{n^2}\right) f_j(\theta_2) + \left(\frac{1}{4} - \frac{3}{n^2}\right) f_j(\theta_3) + \left(\frac{1}{4} + \frac{3}{n^2}\right) f_j(\theta_4)\right\} \quad 4$$

where trigonometric functions $f_1(\theta) = \cos(2\theta), f_2(\theta) = \cos(4\theta), f_3(\theta) = \sin(2\theta), f_4(\theta) = \sin(4\theta)$ and n is the number of plies.

3. Feasible regions of DD-laminate

Compared to QUAD layups, the design complexity of DD-laminates is significantly reduced due to the simple stacking concept. The feasible regions of any DD laminate can be found by using Lagrange multipliers and angle constraints. Taking Staggered 1 which has sub-laminates with the stacking sequence $[+\phi/-\psi/-\phi/+\psi]$ [1]. The feasible regions for $\xi_{1-4}^{B}$are found to be

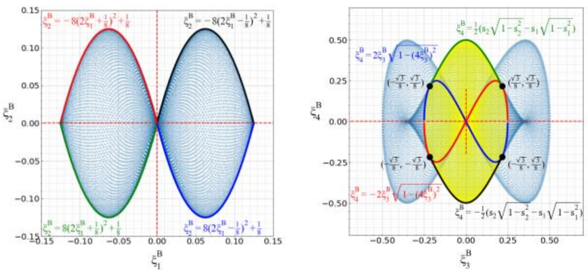

Fig 1. Feasible regions for 8 plies laminate for staggered 1 $[+\phi/-\psi/-\phi/+\psi]$

4. Homogenization criterion

For practical design, the coupling stiffness B is an important concern as it causes warping within a laminate. For QUAD laminates, the coupling stiffness is forced to be zero by stacking laminas symmetrically, resulting in high thickness laminates. Due to the nature of DD laminates, their coupling stiffness B cannot be eliminated. However, the coupling effect can be significantly reduced by increasing the number of plies. Such a phenomenon is known as homogenization.

To find the limit of homogenization, a criterion of 2% of master ply [1] can be drawn into feasible regions as shown below for ξ_{1-4}^{B}.

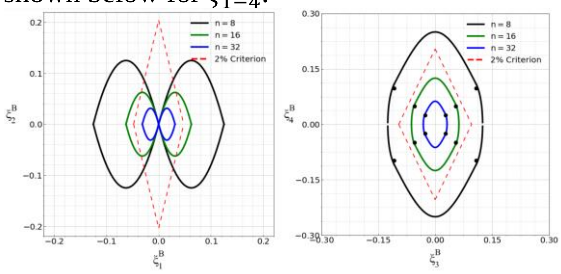

Fig. 2. Feasible region of $\boldsymbol{\xi_j^B}$ with 2% homogenization criterion.

For the feasible regions out of this homogenization criterion envelop, other types of DD laminates staggered 3 can be employed instead to cover this area, and 16 plies/4 repeats are enough for DD families to reach homogenization. Thus, DD laminates can still retain the maximum design space in bending and extension stiffness while having the minimum or no effect on coupling stiffness. By such means, staggered 1 can reach homogenization at 16 plies/4 repeats using staggered 3 as replacements on the unreachable area as shown in Fig.3.

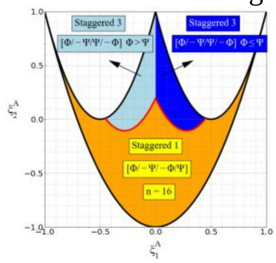

Fig. 3. A tailored feasible region for a combination of staggered 1 and 3.

5. Conclusions

In this study, novel formulations for obtaining the lamination parameters of DD laminates are presented, based on which the feasible regions of the combinations of any two lamination parameters are analytically obtained using Lagrange multipliers. The proposed method is further validated against one of the popular DD laminates with the stacking sequence of the sub-laminate as [+ϕ/-ψ/-ϕ/+ψ]. Besides, based on the simple expressions of lamination parameters, feasible regions for any two lamination parameters can be obtained analytically using the method of Lagrange multipliers. Finally, the homogenization criterion of 2% has been drawn in the feasible region of DD laminates, and combination sequences of staggered 1 and 3 have been proved to be able to reach all the design space covered by the 2% homogenization criterion with only 16 plies/4 repeats.

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