EXPERIMENTAL PARAMETER IDENTIFICATION FOR 3D NONLINEAR VISCOELASTIC MATERIAL MODEL

Liva Pupure[1], Janis Varna[1,2], Zainab Al-Maqdasi[2] and Leonids Pakrastins[1]

[1] Department of structural engineering, Riga Technical University, Kipsalas 6A, LV-1048, Riga, Latvia, Liva.Pupure@rtu.lv, Leonids.Pakrastins@rtu.lv, www.rtu.lv
[2] Department of Engineering Sciences and Mathematics, Lulea University of Technology, Lulea, University Campus, SE-97751, Sweden, zainab.al-maqdasi@ltu.se, www.ltu.se

Keywords: *Time-dependent behavior, viscoelasticity, creep, 3D*

ABSTRACT

There is a growing demand from the industry for nonlinear 3-dimensional material models and simulation tools. Nonlinearity in material response can be attributed to different phenomena – viscoelasticity, viscoplasticity and/or damage. In previous studies, authors have looked at viscoplastic behavior in 3D and developed material models with different complexity for the 3D cases. The first model assumes that all the nonlinearity functions are direction independent, and the second model assumes direction dependency. In this study, the viscoelasticity is analyzed in the axial and lateral directions in order to investigate the applicability of those developed material models. The results show high direction independence for viscoelastic nonlinearity functions for studied high-density polyethylene with and without different amounts of graphene nanoplatelets. The obtained information will be used in ongoing work to further adjust the nonlinear material model and implement the obtained expressions in a finite element code.

1 INTRODUCTION

In recent years the need for more precise material models is increasing. This has been driven by the development of bio-based composites and other novel materials with high nonlinearity or the need to use composites in more demanding environments, e.g., elevated temperature and humidity and in changing material state (the degree of cure, physical and chemical aging, etc.). With the development of more advanced nonlinear material models, materials could be used more efficiently, allow composites to be used in new applications, improve the manufacturing process and have better shape distortion predictions, thus reducing the overall manufacturing costs.

The most widely used nonlinear viscoelastic (VE) material model has been developed by Scapery [1-4]. In [5], it was modified to include the Zapas model for viscoplasticity (VP) [6] and damage. The said model has been widely used to simulate different materials' one-dimensional (1D) time-dependent behavior [7-9]. Due to the complexity of these material models, it is not enough to just develop them. Creating a simulation tool, such as finite element modeling, is crucial. This would be a convenient way for anyone interested in modeling time-dependent behavior to have the ability to analyze this behavior without understanding the complex mathematics behind these models.

It is, moreover, not enough to use 1D material models since most real-life applications have 3-dimensional (3D) stress states [10]. Most of the material models, and subsequently their simulation tools, are developed for 1D material cases. Even if some of the models are developed for 3D cases, methodologies for experimental parameter identification for these models are lacking. In the best-case scenario, there is a theoretical description of experimental parameter identification [11].

In order to simulate material behavior in 3D, the nonlinearity of an isotropic material has to be studied not only in the loading direction but also in the transverse direction. In the case of anisotropic material, the number of experiments significantly increases because each direction has to be studied separately. Another challenge for the nonlinear modeling of complex geometrical structures is the need to find the nonlinear functions and parameters for the model in both loading modes - tension and compression.

In [10], two material models for nonlinear viscoelasticity have been proposed. The main uncertainty is that the empirical nonlinearity functions in the Schapery model are direction-dependent or independent (scalars). If these functions are direction independent, then the experimental parameter identification is relatively straightforward, and it is similar to the 1D case.

If these functions are direction dependent, each function is a vector with six components. The experimental characterization is significantly more complex and requires a multi-axial viscoelastic relaxation test. Multi-axial tests are complex by themselves. Thus, performing a multi-axial relaxation test (or creep test) would be even more challenging. However, this approach could be justified for anisotropic materials since the response of such materials in different directions significantly varies.

The initial study on viscoplasticity [12] of various material systems showed that one scalar number could characterize the direction dependencies. This current aims to analyze the viscoelasticity of the 3D material model. Similar to the viscoplastic strain, each nonlinearity function will be analyzed with respect to the direction. If possible, the functions will be simplified to obtain fewer direction independent variables. The possibility of introducing a proportionality coefficient for each or some of the functions that could account for their direction variability will be considered.

2 MATERIAL MODEL

Most widely used nonlinear material models for composites are based on ideas developed by Schapery [1-4]. His models are thermodynamically consistent and have high adaptational versatility; thus, they can be used for various materials and conditions. Schapery's nonlinear viscoelastic material model has two forms: stress formulation and strain formulation and for 1D cases are presented below:

$$\varepsilon_{VE} = \varepsilon_{el} + g_1 \int_0^t \Delta S(\psi - \psi') \frac{d(g_2\sigma)}{d\tau} d\tau \tag{1}$$

$$\text{Where } \psi = \int_0^t \frac{dt\prime}{a_\sigma} \quad \text{and} \quad \psi' = \int_0^\tau \frac{dt\prime}{a_\sigma} \tag{2}$$

$$\sigma = E_r \varepsilon + h_1(\varepsilon) \int_0^t \Delta E(\psi_\varepsilon - \psi_\varepsilon{}') \frac{d(h_2\varepsilon)}{d\psi_\varepsilon{}'} d\psi_\varepsilon{}' \tag{3}$$

$$\text{Where } \psi_\varepsilon = \int_0^t \frac{dt'}{a_\varepsilon} \quad \text{and consequently} \quad \psi_\varepsilon{}' = \int_0^\tau \frac{dt'}{a_\varepsilon} \tag{4}$$

In (1), the initial elastic response ε_{el}, generally speaking, may be a nonlinear function, g_1 and g_2 are nonlinearity functions, and a_σ is the shift factor. Some materials behave or have a region of linear VE, where $g_1 = g_2 = a_\sigma = 1$. Schapery showed that the linear VE creep compliance could be written in the form of the Prony series [1-2]:

$$\Delta S(\Psi) = \sum_i C_i \left(1 - exp\left(-\frac{\Psi}{\tau_i}\right)\right) \tag{5}$$

In eq. (3) ε is a function of strain invariants, E_r is an equilibrium modulus, C_i and τ_m are coefficients in the Prony series, and a, h and g are empiric functions that depend on strain, material state and environmental conditions.

The transient part of the VE strain formulation response is characterized by $\Delta E(\psi_\varepsilon)$ which does not depend on stress and has a form of the Prony series [3-4],

$$\Delta E(\psi_\varepsilon) = \sum_i E_i exp\left(-\frac{\psi_\varepsilon}{\tau_i}\right). \tag{6}$$

The eq. (3) has been rewritten and adapted for 3D cases in [10]. Within this study, two different material models are analyzed with respect to the nonlinearity of strain components. In one case, all strain components show the same nonlinearity dependence is the same, while in the other, more complex case, where each strain component may have a different response to a material state, environment and/or is a loading dependent function. Although the second model is more flexible, it has a significantly higher number of experimentally determined functions and parameters. Moreover, the parameter identification requires one bi-axial viscoelastic relaxation test.

Within the current study, the viscoelasticity in loading (subsequently noted as "axial") direction and contraction (notation in the text is "lateral") direction are analyzed separately. The analysis of the creep and subsequent recovery curves is done by using the following expressions:

$$\varepsilon^{j}_{creep} = \varepsilon^{j}_{el} + g^{j}_{1} g^{j}_{2} \sum_{i} C^{j}_{i} \left(1 - exp\left(-\frac{t}{a^{j}_{\sigma}\tau_i}\right)\right) \tag{7}$$

$$\varepsilon^{j}_{rec} = g^{j}_{2}\sigma \sum_{i} C^{j}_{i} \left(1 - exp\left(-\frac{t_1}{a^{j}_{\sigma}\tau_i}\right)\right) exp\left(-\frac{t - t_1}{\tau_i}\right) \tag{8}$$

The experimental procedure in detail is presented in [9,13]. The obtained results of the fitting of equations (7) and (8) are presented in section results and discussions. The fitting was done on High-density polyethylene (HDPE) and HDPE with 2%, 6% and 15% nanoplatelets (notation HDPE+X%, where X represents the amount by weight of graphene nanoplatelets). Detailed information on these materials can be found in [14,15]. This study concluded that all four material systems are nonlinear VE even at relatively low stress levels in both directions, as can be seen from creep compliance curves presented in Fig 1. for HDPE+6% for both directions.

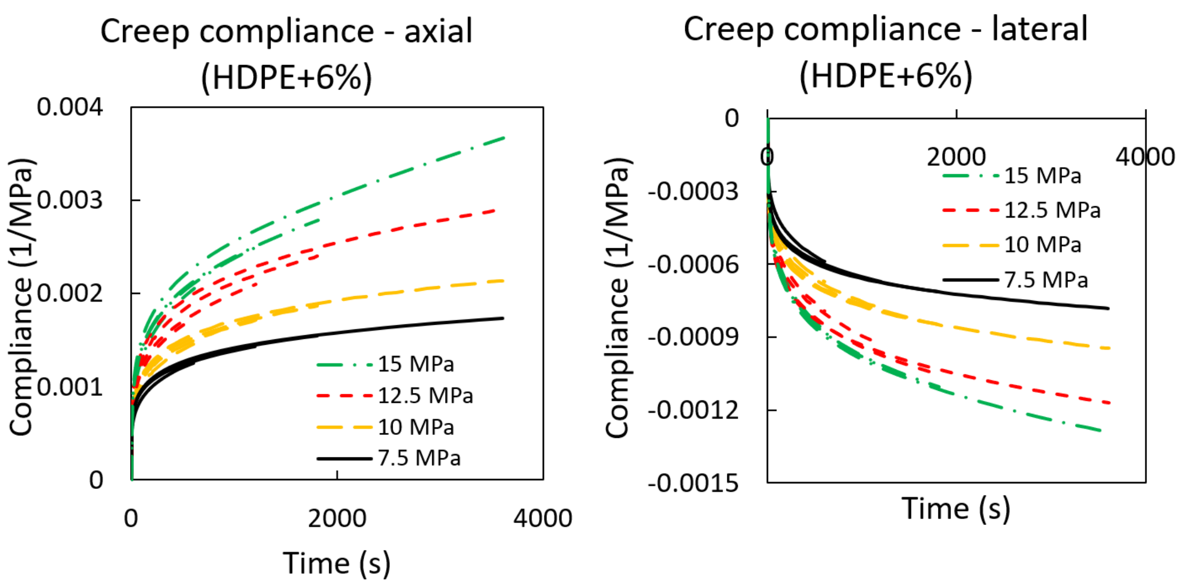

Figure 1: Viscoleastic creep comploances ΔS_{11} (left) (axial direction) and ΔS_{12} (right) (lateral direction) for HDPE+6%

3 RESULTS AND DISCUSSION

Firstly, for the analysis of the VE, the coefficients C_i in the Prony series have to be identified. These values were obtained from the lowest stress level of 7.5 MPa using equations (7)-(8) and the procedure described in [9-10,12,13]. The retardation times τ_i in the Prony series are chosen arbitrarily, but the largest value should be by about a decade larger than the duration of the creep test conducted. The C_i values have to be the same for all stress levels. Initially, the axial and lateral direction was analyzed separately, and the corresponding coefficients are presented in Table 1. The ratio in the coefficients of lateral and axial direction was observed to be relatively stable. Thus a coefficient of proportionality was introduced that could be used to find C_i in the lateral direction from the values of the axial direction. The coefficients are presented in Table 2. The Coefficients in the Prony series obtained using the proportionality coefficients are presented in Table 3.

τ_i, (s)	C_i, (MPa/%)							
	HDPE		HDPE+2%		HDPE+6%		HDPE+15%	
	Axial	Lateral	Axial	Lateral	Axial	Lateral	Axial	Lateral
5	0.03371	0.01347	0.02376	0.01417	0.02341	0.01158	0.00522	0.00252
200	0.06729	0.02402	0.06570	0.02829	0.06736	0.03316	0.02564	0.01173
1500	0.03310	0.01034	0.03705	0.01303	0.02420	0.00996	0.01025	0.00515
9000	0.09017	0.02471	0.08616	0.02848	0.06883	0.02521	0.02307	0.00662

Table 1: Coefficients in Prony series, obtained by best fit for each direction separately

HDPE	HDPE+2%	HDPE+6%	HDPE+15%
0.34	0.43	0.44	0.43

Table 2: Coefficients of Proportionality between the axial and lateral direction of the Prony series

τ_i, (s)	C_i, for Lateral direction(MPa/%)							
	HDPE		HDPE+2%		HDPE+6%		HDPE+15%	
	Initial	Coef.	Initial	Coef.	Initial	Coef.	Initial	Coef.
5	0.01347	0.01132	0.01417	0.01015	0.01158	0.01033	0.00252	0.00226
200	0.02402	0.02259	0.02829	0.02807	0.03316	0.02972	0.01173	0.01108
1500	0.01034	0.01111	0.01303	0.01583	0.00996	0.01068	0.00515	0.00443
9000	0.02471	0.03028	0.02848	0.03682	0.02521	0.03037	0.00662	0.00997

Table 3: Final coefficients in Prony series for lateral directions using values presented in table 2 (notation: Coef.) and the initial values presented in table 1 (notation: Initial)

The effect of using proportionality coefficients in Table 2 and the subsequent values of the Prony series on the overall fit of the curves can be seen in Figures 2-3, where the fitting curves for lateral compliance for both cases: using initial values and the values using the proportionality coefficient are presented. It can be seen that the changes in the overall fit are negligible. This indicates that the use of a single coefficient to obtain the values of the Prony series in the lateral direction can be used in simulations without sacrificing the integrity of the final result. It can also be seen that for some stress levels, the fit improved slightly when using proportionality coefficients, and it was somewhat worsened for other cases. Since the difference in C_i values was not uniform for all the τ_i values, it can be expected that fit might improve for some cases but have a slightly less agreeable fit for other stress levels. Larger differences between values of the Prony coefficient were observed for higher stress levels. If the differences were analyzed for each τ_i separately, the overall trend was that the proportionality coefficient was slightly decreasing with increases τ_i values (with few exceptions). Additionally, the values in Table 2 are close to the Poisson's ratio of HDPE that is between 0.4-0.45, which would be rational since the coefficients in the Prony series represent the linear viscoelasticity, thus could be linked somehow to Poisson's effect that often is assumed time-independent. It has to be noted that similar trends were observed for all materials and all stress levels.

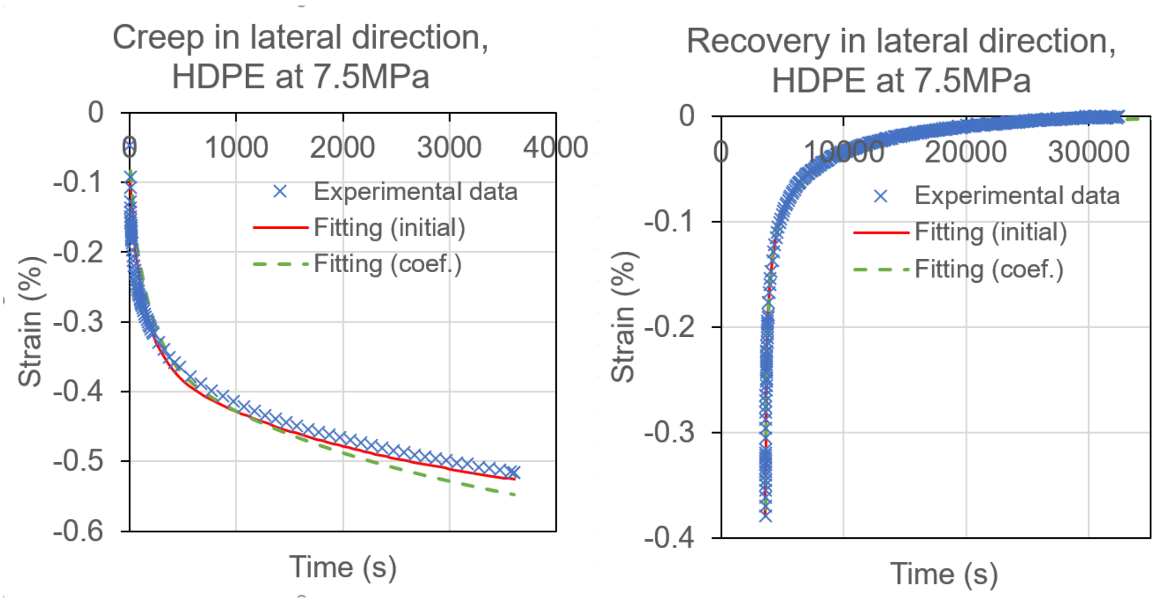

Figure 2: Fitting curves for HDPE at 7.5 MPa for creep (left) and recovery (right) parts using best fit for each direction (notation “initial”) and using proportionality coefficients (notation “coef.”)

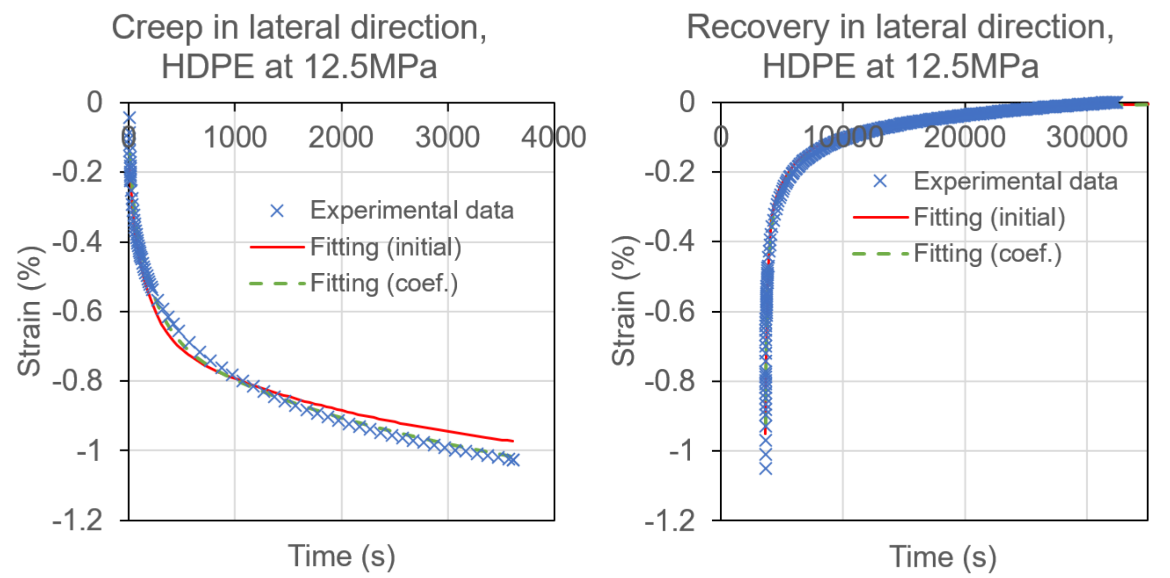

Figure 3: Fitting curves for HDPE at 12.5 MPa for creep (left) and recovery (right) parts using best fit for each direction (notation “initial”) and using proportionality coefficients (notation “coef.”)

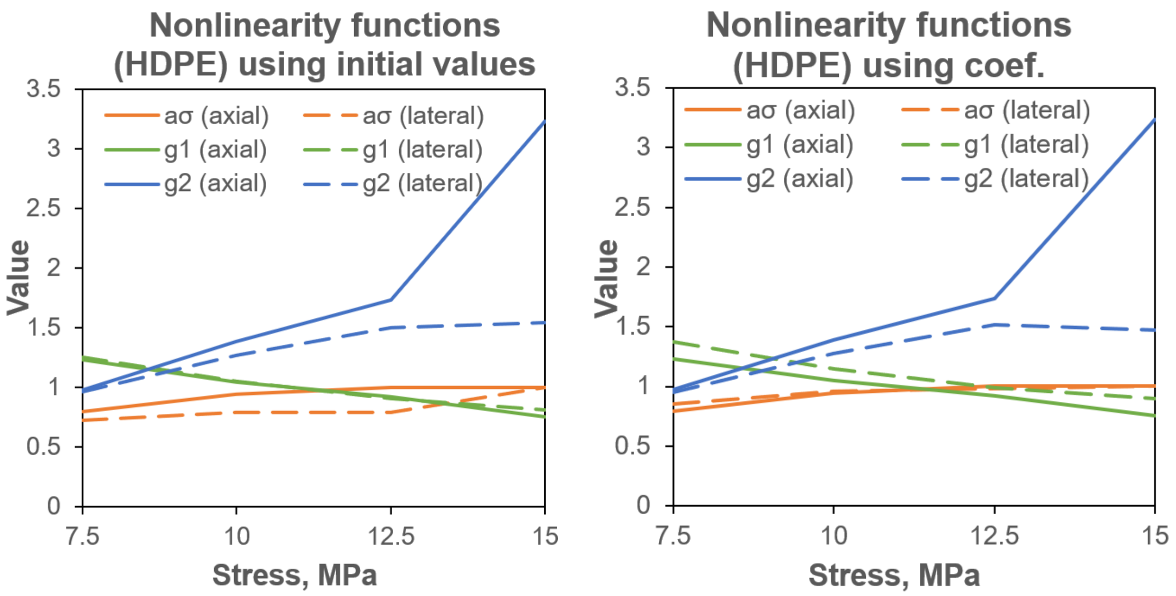

Figure 4: The nonlinearity functions a_σ, g_1 and g_2 for HDPE using Initial (left) and coefficient (right) values of the Prony series. The solid lines represent the axial direction and the dashed lateral direction.

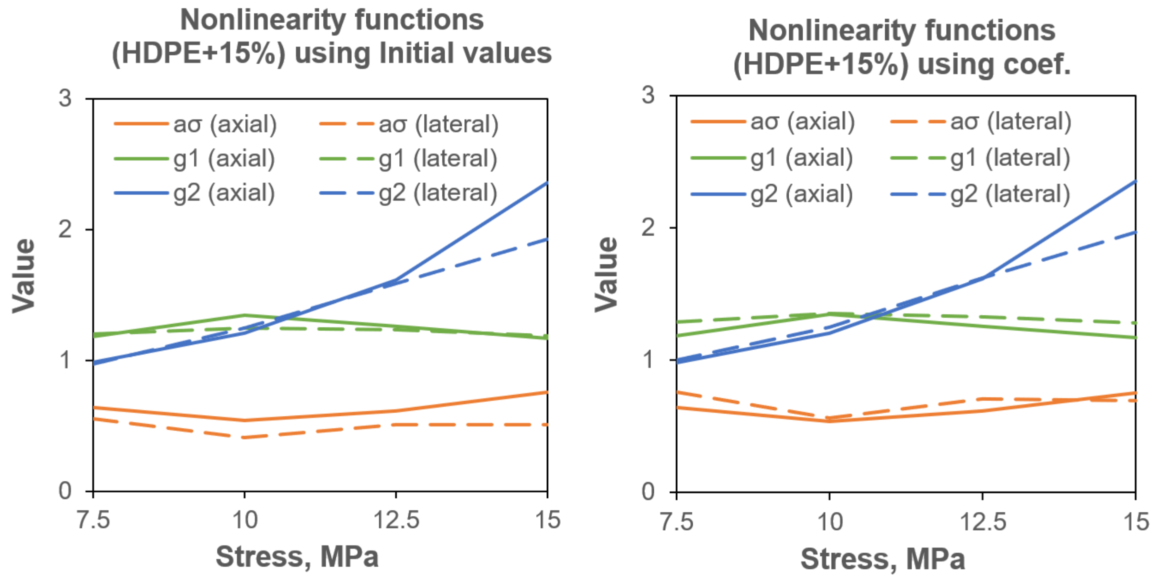

Figure 5: The nonlinearity functions a_σ, g_1 and g_2 for HDPE+15% using Initial (left) and coefficient (right) values of the Prony series. The solid lines represent the axial direction and the dashed lateral direction.

After the Prony coefficients are obtained, it is possible to obtain nonlinearity functions using equations (7)-(8) and the least square method. Each direction was analyzed separately, and the obtained nonlinearity values for all stress levels for HPDE and HDPE+15% are presented in Figure 4-5. Both the initial values and values obtained with the proportionality coefficient of the Prony series have been analyzed. It can be seen that there is an overall trend between the lateral and axial directions. The graphs show small deviations for the nonlinearity functions a_σ and g_1 between axial and lateral directions. This is a strong indication that these functions are direction independent. It can also be seen that these functions have a slightly better agreement when the proportionality coefficient is used to obtain values for the Prony series in the lateral direction. The function a_σ is a time shift function, thus indicating the same time dependency of VE in both directions. The function g_1 affects the result only in the creep part of the experiment, as seen from eq. (7)-(8), where in the recovery part, this function is not present.

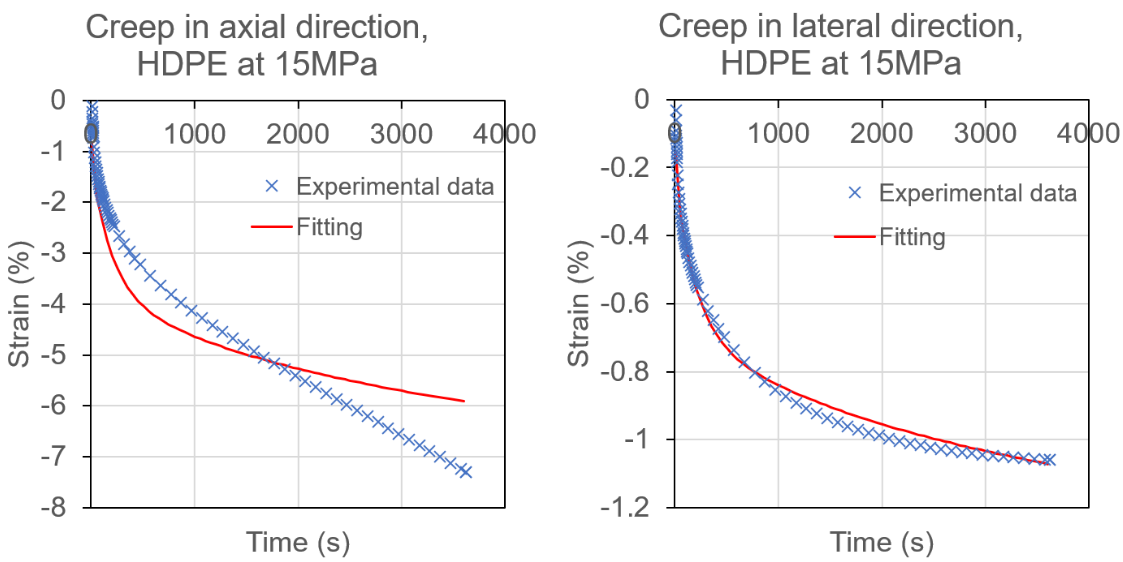

Figure 6: The experimental data and simulation at 15 MPa for axial (left) and lateral (right) direction for HDPE.

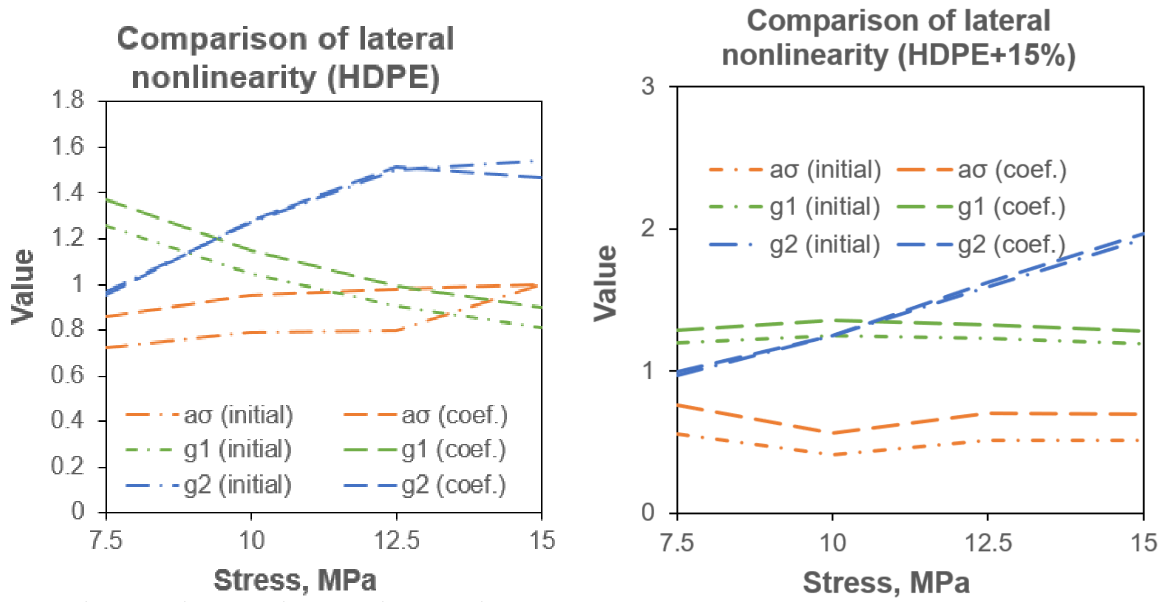

Figure 7: The comparison of nonlinearity functions a_σ, g_1 and g_2 obtained using initial coefficients in the Prony series (initial) and values obtained by proportanilty coefficient (coef.) for HDPE (left) and HDPE+15% (right).

Nonlinearity function g_2 shows a small deviation up until 15 MPa, where the curves deviate significantly from each other. This nonlinearity function is present in both the creep and recovery parts, as seen from eq. (7)-(8). These deviations at higher stress levels can be linked to the overall fit of the curves and can be due to the fact that the stress level of 15 MPa was close to creep failure. Thus there are different mechanisms involved that VE material models can not characterize. Near to material failure, the VE strain curves are significantly steeper than curves at other stress levels. Thus, the obtained fit of the curve is not as good as for the other stress levels. It could be observed in the fitting curves for 15MPa, that the overall fit was not as good as for the lowest stress levels (see Fig. 6).

The effect on the used Prony coefficients are presented in Figure 7, when comparison of both methods – coefficients obtained by the best overall fit for lateral and acial direction (notation “initial”) and by using the proportionality coefficient (notation “coef.”). It can be seen that the used method to obtain the Prony coefficients has negligible effect on time-shift factor a_σ. The change in Prony coefficients have a shifting affect on g_1 and similar trend can also bee seen for nonlinearity function g_2. This is expected since all nonlinearity functions are somewhat connected, thus if we change slightly the Prony series coefficients, best mathematical fit can only be achieved if the nonlinearity functions are also slightly adjusted to account for this change.

In [13], it was concluded that adding the 6% and 15% of graphene nanoplatelets significantly decreased the viscoplastic and viscoelastic strains. It can be seen in Fig. 8 that the overall viscoelastic strain in the creep part of the curve for HDPE+15% is three times lower than that of pure HDPE. The creep curves for HDPE+15% also did not show a significant acceleration of viscoelastic strain at 15MPa as seen for the pure HDPE (see Fig. 6).

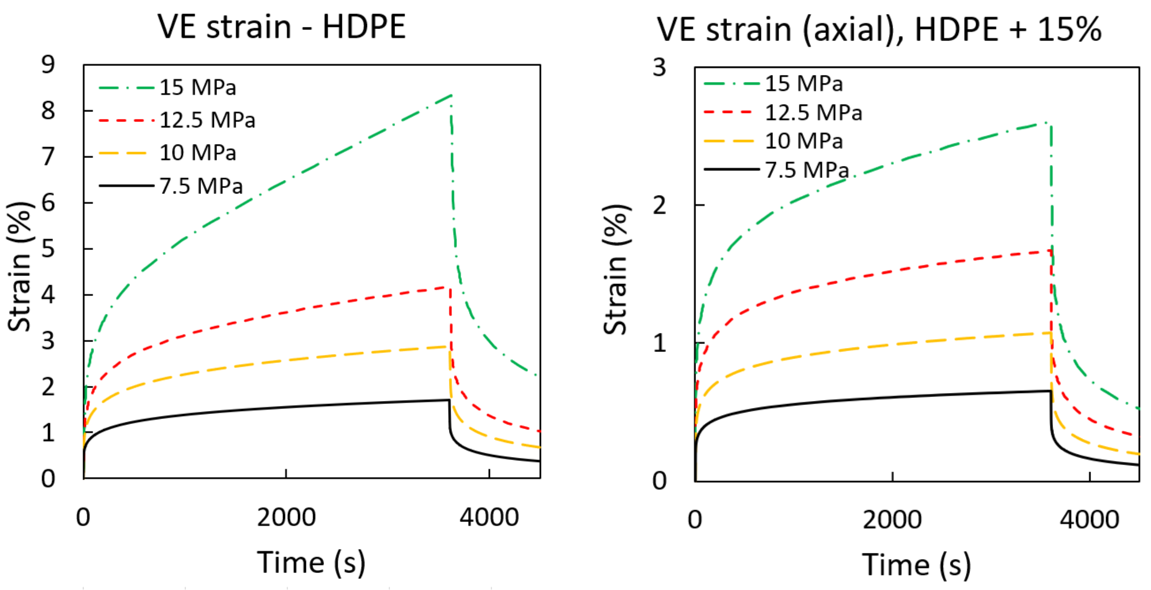

Figure 8: The VE strain in the axial direction for HDPE (left) and HDPE+15% (right).

It can be concluded that the nonlinearity in the lateral direction can be obtained from the functions of axial direction and a single coefficient. These findings will significantly simplify the experimental characterization needed for the nonlinear material model for 3D. It will not require a full experimental characterization of each direction, but only one test, to determine the proportionality coefficient. There is an indication that this coefficient might be linked to Poisson's coefficient, but a more comprehensive analysis needs to be performed for conclusive proof. However, these materials are isotropic. Anisotropic materials are expected to be more complex, and it is more probable that each direction has distinctively different nonlinear VE behavior. In these cases, the 3D implemented model within the finite element code would be of great help. For example, for composite materials consisting of two isotropic materials, the nonlinear VE could be obtained by experimentally characterizing each constituent separately and then using representative volume elements and the obtained properties of constituents, the nonlinear viscoelastic properties of anisotropic composite could be obtained. Furthermore, this model could be used for multiple scales – not only on the micro level bet also on the laminate level. Such an approach would be significantly more effective than experimentally characterizing each composite or laminate separately. Of course, to have conclusive proof that this is not a specific material behavior but an overall trend, the assumption has to be also confirmed on different isotropic materials.

4 CONCLUSIONS

The current study investigated the nonlinear VE behavior of four materials – pure HDPE and HDPE with 2%, 6% and 15% of graphene nanoplatelets as an example of isotropic materials. The materials exhibited nonlinear VE behavior already at low-stress levels. Thus, more advanced nonlinear VE models have to be applied to simulate their behavior.

The initial analysis showed the best overall fit of creep compliance in both directions (axial and lateral) using different coefficients in the Prony series. Since the values of Prony series of axial and lateral direction showed a proportionality, a coefficient was calculated as an average value of all the proportionality for all τ_i. The values of lateral direction Prony coefficients was calculated using this coefficient and used in simulations of creep and recovery curves. The use of this method did not significantly reduce the overall quality of the fit of creep strains in axial and lateral directions. Since using proportional coefficients for both directions would significantly reduce the complexity of the 3D nonlinear VE material model, it is justified to use these values instead of the initial, mathematically obtained coefficients.

By analyzing the nonlinear viscoelastic functions, the time-shift factor a_σ and g_1 showed small and negligible deviation between axial and lateral directions, thus indicating the direction independency for these functions. The highest differences were observed in function g_2 at stress levels close to rupture, where different mechanisms are involved and the initial fit was also significantly worse. This could indicate that the values for the function g2 is direction independent, at least at lower stress levels. This indicated, that the materials lateral VE characteristic values and funtions can be obtained using only a single coefficient.

ACKNOWLEDGEMENTS

Liva Pupure's work within this study has been supported by the European Regional Development Fund within the activity 1.1.1.2 "Post-doctoral Research Aid" of the Specific Aid Objective 1.1.1 "To increase the research and innovative capacity of scientific institutions of Latvia and the ability to attract external financing, investing in human resources and infrastructure" of the Operational Programme "Growth and Employment" (No.1.1.1.2/VIAA/4/20/646).

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