Moment of Inertia Measurement of a CubeSat Through Object Motion Tracking of Trifilar Pendulum Oscillations

Bas Stijnen (1), David McKeown (1), Joseph Thompson (1), Eghan Somers (1)

(1) University College Dublin School of Mechanical and Materials Engineering, University College Dublin, Belfield, Dublin 4, Ireland. Eircode: D04 V1W8, +353 1 716 1584,
Bas.Stijnen@ucdconnect.ie, David.McKeown@ucd.ie, Joseph.Thompson@ucd.ie

ABSTRACT

The experimental determination of small satellites' Moments of Inertia (MoI) can be performed using a low-cost, easy-to-manufacture and easy-to-operate test setup. In this paper, a method is presented that makes use of a trifilar pendulum and motion detection capabilities of the open-source 3D computer graphics software tool Blender®. Different ways to analyse the motion data and eventually calculate the MoI tensor are presented.

The proposed testbed uses a camera to capture the pendulum's motion. The video can then be analysed using the step-by-step procedure outlined in this paper. The motion tracking data extracted from the video can be analysed, to find temporal data using one of the proposed methods, and from this, the MoI of the test object can be calculated. To determine the full inertia tensor of a test object multiple test orientations are required, this procedure and its corresponding equations are outlined. The testbed is used to measure the inertia tensor of the Educational Irish Research Satellite (EIRSAT-1) [1], which is a 2-unit CubeSat [2].

1 Introduction

Most satellite project follows similar steps, from requirements definition and preliminary design to the final design, manufacturing, assembly, test and verification, launch, and operation. At each step, a milestone review is required with the status of the design verified against requirements. However, this is commonly a non-trivial exercise when considering the spacecraft's Attitude Determination and Control Systems (ADCS). This difficulty is compounded by the small budgets commonly available for CubeSat projects. The replication of the space environment and hardware testing of spacecraft dynamics becomes prohibitively expensive and time-consuming. As a result, hardware testing is replaced exclusively with computer simulations and reliance on Computer-Aided Design (CAD) models. It is however possible to measure properties such as Moment of Inertia (MoI) using low-cost test setups as will be outlined in this paper.

Relying on CAD models for MoI calculation is often deficient as complex parts, such as harnessing, are frequently modelled incorrectly, over simplified, or omitted. This limits the usefulness of the model estimations which are required by launch authorities, or to identify the spacecraft dynamics for control purposes. There are numerous ways to determine the MoI of an object experimentally [3]. In this paper, the focus will be on the MoI measurement using a trifilar pendulum testbed. As an example object under test, the 2U CubeSat, EIRSAT-1 [1], will be used. The satellite was designed and built as part of the ESA Education Fly Your Satellite! (FYS) programme.

2 Trifilar Pendulum design

A trifilar pendulum was designed for CubeSat Mass Moment of Inertia measurement based on the requirements and rationale presented in Table 1. Each requirement has an identifier, a description and a verification strategy, the different verification methods are; by Design (D), by Test (T), by Inspection (I) and by Analysis (A) [4].

Table 1: High-level design requirements, their rationale and verification method

Req #	Requirement	Ver
REQ-MOI-01	Test setup shall support mountings for CubeSat sizes from 1U to 12U.	D,I
Rationale	To create a test setup that can support multiple missions without redesign.	
REQ-MOI-02	The test setup shall be portable.	D
Rationale	Due to limited cleanroom space, the test setup will be moved in and out of cleanroom area to make space for other test setups when required	
REQ-MOI-03	The test setup shall be compatible with use in a cleanroom.	D
Rationale	Due to cleanliness requirements of satellites the setup should be easy to clean	
REQ-MOI-04	The test setup design shall restrict the ratio between the radius of the pendulum platform (R) and the length of the cables (L) such that $R/L < 0.214$.	D, I
Rationale	To keep the error in the measurement as low as possible. In [5] Previati suggests keeping the ratio R/L as small as possible.	
REQ-MOI-05	The test setup shall have an accurate means (<5% error) to measure the oscillations of the platform.	D,T,A
Rationale	Mass moment of Inertia (MoI) of an object under test is found based on the oscillation data.	
REQ-MOI-6	The test setup shall be of low cost (< 1k €).	D
Rationale	The test setup has been designed with CubeSat projects in mind where frequently the budgets for Mechanical Ground Support Equipment (MGSE) are small.	

The requirements outlined in Table 1 formed the basis of the design of the MoI test setup which can be seen in Figure 1. The frame is constructed from 20mm x 20mm aluminium extruded beams and aluminium top and bottom plates of 8 mm thickness. Aluminium was chosen to facilitate easy cleaning for use inside a cleanroom and to keep the overall weight as low as possible and allow easy relocation of the testbed, in compliance with requirements REQ-MOI-02 and REQ-MOI-03. The top plate has mounting holes for the cables on which the pendulum table is suspended. The bottom plate allows for the mounting of a GoPro camera with the lens aligned with the centre of the platform for recording the motion of the platform to comply with requirement REQ-MOI-05, see section 3. The platform is made from a 10 mm aluminium with an Isogrid [6] cut out on the bottom to reduce weight while maintaining sufficient stiffness, see Figure 4. Mounting holes were added to the platform to easily secure different holding stands of standard-sized CubeSats (1U to 12U) and to facilitate holding satellites in the necessary orientations, in order to comply with REQ-MOI-01. The most significant design parameters are listed in Table 2. From REQ-MOI-1 the size of the table was derived (R = 225mm) in order to comply with REQ-MOI-04 the length of the wires (L) where rounded off to 1250 mm.

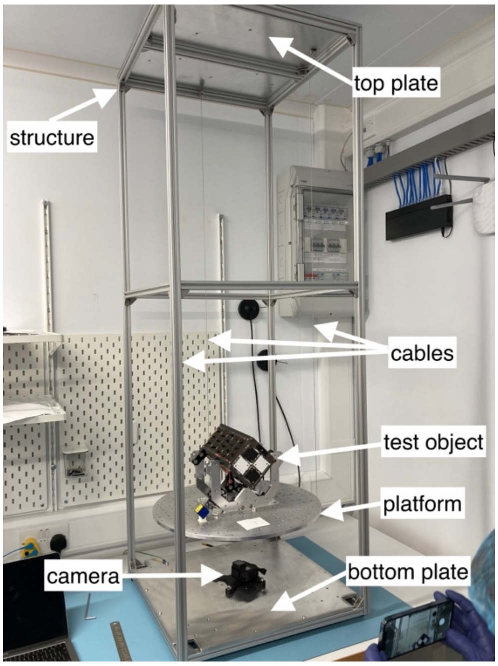

Figure 1: Picture of the Trifilar Pendulum, a GoPro Camera on the bottom below the platform and EIRSAT-1 on the platform in a 45 degree orientation

Table 2: Final design parameters

Design parameter	Symbol	Measurement and unit
Radius of the plate	R	225 mm
Length of the cable	L	1250 mm
Mass of the platform	$M_{platform}$	2693 g
Overall height of structure	H	1500 mm

The total cost of the test setup is estimated to be €450, excluding the cost of the camera. This could be reduced significantly if the pendulum was suspended from an existing structure, for example, a low ceiling. The platform, cables, and cable attachments cost approximately €100, with the platform isogrid milled in-house. Alternatively, a platform could be made from solid aluminium or polylactic acid (PLA) through additive manufacturing. A schematic of the test setup can be seen in Figure 2 in which dimensions L, H and R are illustrated.

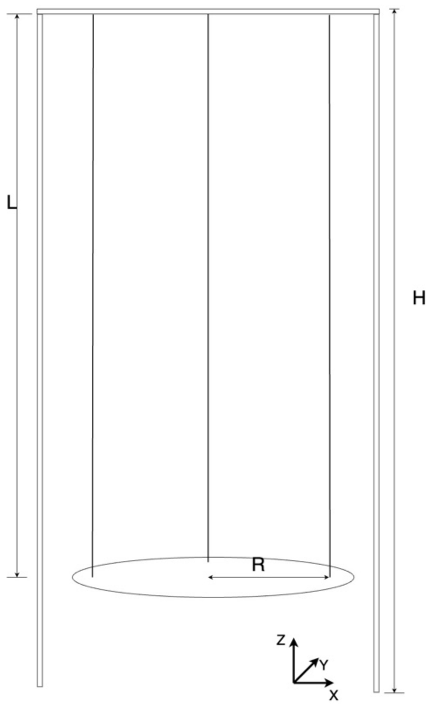

Figure 2: Diagram of the pendulum setup

3 Video Analysis

The video analysis of the platform motion was performed in the open-source software Blender®. Blender® is a free 3D creation suite which supports modelling, animation, simulation, rendering, rigging, composing and motion tracking. The software is cross-platform compatible with Linux, Windows, and Macintosh [7]. Blender® has a built-in Python interpreter, which allows automation data extraction from motion tracking conversion to a comma-separated value (CSV) file for further analysis.

Motion tracking in Blender® is extremely easy to use without the need for training models. A captured video can be directly loaded into Blender® and separated into individual frames for motion tracking of features in the video. The motion of different markers placed on the platform allows the period of oscillation to be determined.

The tracking data can be extracted from Blender® in x and y coordinates per frame, the position is measured from the lower left corner of the scene which has coordinate (0,0) and the coordinates are a ratio between the real image size and the Blender® scene size in pixels [7].

The platform is free to oscillate and is not constrained by the x and y directions (z is orthogonal to the plate and pointing upwards, see Figure 2). An unwanted side effect of this platform sway (translational motion in x and y) the pendulum motion is not started with pure rotation about the z-axis. To account for this sway the centre of the pendulum should be tracked, to allow the removal of this swaying motion whilst performing the MoI analysis.

3.1 A detailed description of performing motion tracking in Blender®

The first step is to load the video file into Blender® and create an image sequence from the video, to prevent synchronisation problems. Each image in the sequence is a frame of the video. If the framerate on the camera is stable this will provide the most accurate time measure. To do this the following steps should be performed:

step 1. Open Blender and navigate to the movie sequencer by pressing ‘shift’ + ‘F8’.
step 2. Now click add and in the drop-down menu chose ‘movie’.
step 3. A pop-up window will open showing your computes file structure, chose the movie that belongs to your test run.
step 4. Delete the audio track, select the light green audio track and press ‘x’.
step 5. Refer to Figure 3 and click on ‘A’ output properties, in this menu check if the following has been correctly determined: ‘B’ if Blender has identified the frame rate correctly. ‘C’ the number of frames you want to use for your experiment. At ‘D’ the output folder is specified and at ‘E’ the file format (recommended to use PNG), colour dept ‘8’ and compression can be set to 0.
step 6. If all the output parameters are satisfactory, separating the video into frames can be started by pressing ‘ctrl’ + ’F12’.

Blender® will now automatically run the video and save every individual frame as a .png image in the specified output folder.

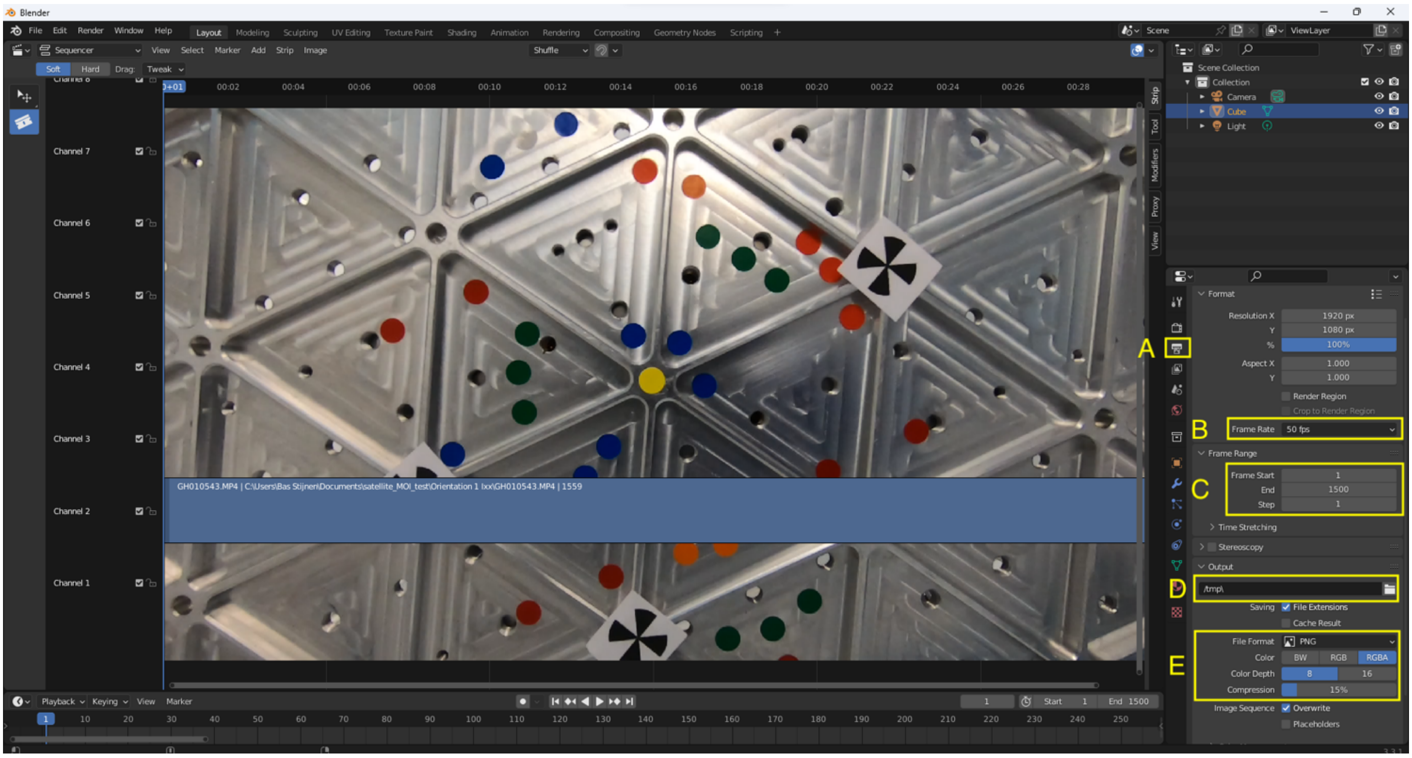

Figure 3: Screenshot from Blender's movie sequencer

As can be seen in Figure 4 the bottom of the platform has markers in the shape of coloured dots and motion tracking markers. These markers can be used to record the motion of the pendulum.

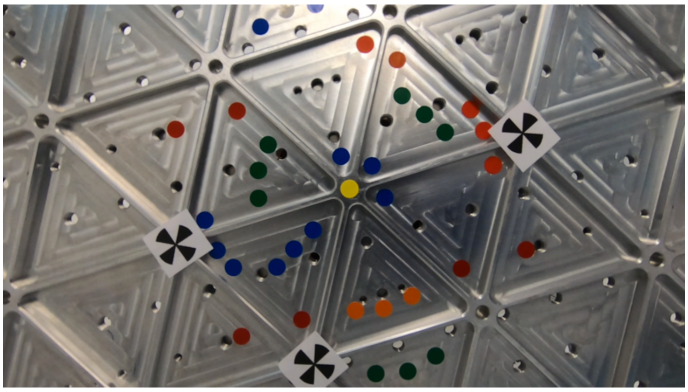

Figure 4: Camera snapshot used for motion tracking.

step 7. Now Navigate to the movie clip editor by pressing shift+F2

step 8. Click the open button and navigate to the image sequence, select the first image, and click open.

step 9. In Figure 5 start by clicking set scene frames in box A, make sure the number in B represents the total number of frames in the image sequence generated in step 6.

step 10. In Box A now click prefetch and see the time bar in section C become light purple. This will significantly speed up the tracking.

step 11. In box D select the type of track, for simple coloured dots selecting ‘location’ is sufficient, this is Blender's fastest and most simple motion-tracking algorithm for tracking markers select ‘location and rotation’. ‘Key frame’ is automatically selected.

step 12. Hold ‘ctrl’ and click the left mouse button on the feature that you would like to track. A white square as can be seen in box E appears, this is called the ‘pattern area’, you can move it by holding ‘g’ and moving your mouse, you can scale it with ‘s’ and rotate with ‘r’. The pattern area defines what object is going to be tracked.

step 13. By pressing ‘alt’ + ‘s’ a search box is created (which is the outer white square in box E) this is the area where Blender® is going to search in the next frame for this object.

step 14. By using the track menu on the right (box F) one can align the centre of the pattern area with the centre of the object to be tracked.

step 15. The tracking is now setup and can be started by either holding ‘ctrl’ and ‘arrow to the right’ or by holding ‘ctrl’ and pressing ‘t’.

step 16. Blender® shows the user the track as overlayed small red dots connected with a line (by default it will show 15 dots, but this number can be increased or decreased).

step 17. After the track is successfully completed, navigate to the text editor by pressing ‘shift’ + ‘F11’

step 18. Use the Python script BlenderCoordinateCSVScript.py [8] to extract the x and y positions of the centre of the pattern area per frame from Blender® and save them to a .CSV file.

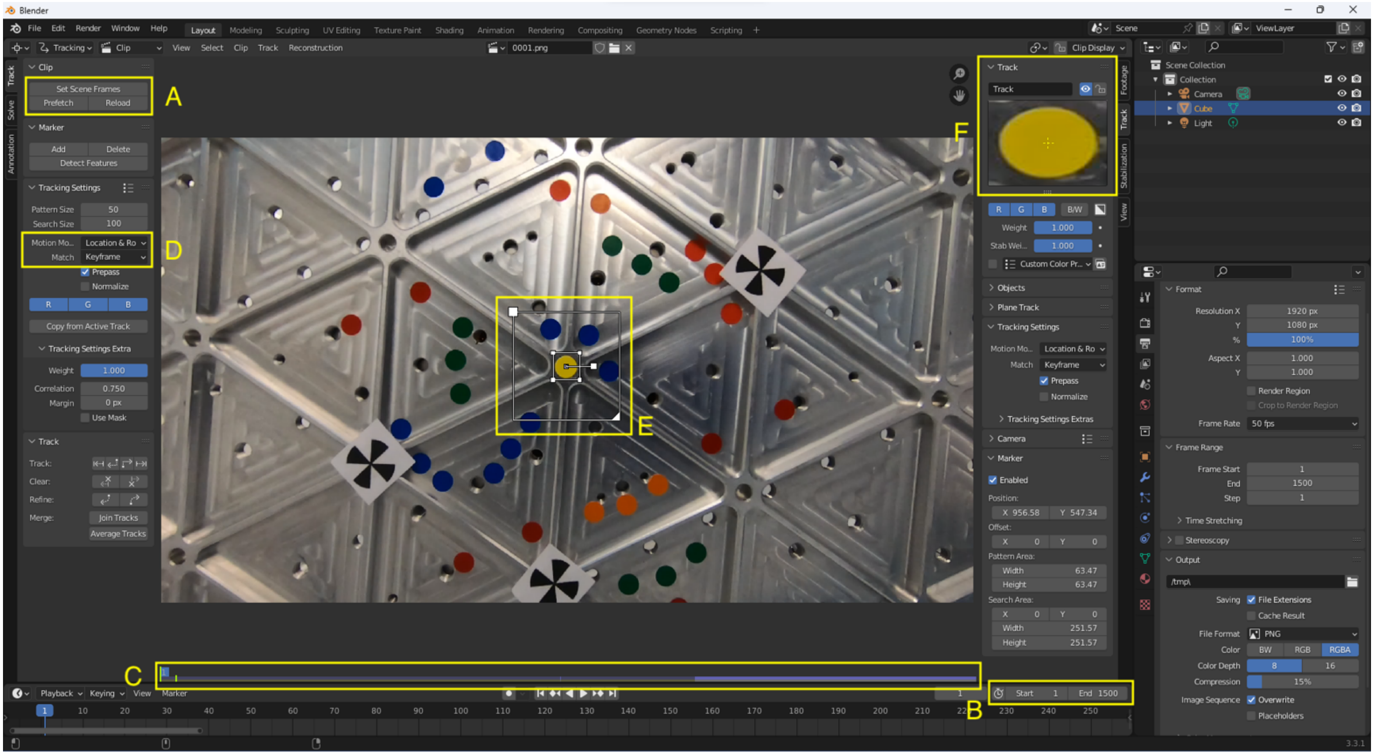

Figure 5: Screenshot of Blenders Movie Clip Editor

Care should be taken in identifying which csv file belongs to the centre dot so re-naming the file after it has been generated is good standard practise. Steps 7 to 18 should now be repeated until the motion of at least one other outlying marker dot (i.e a marker dots not at the centre) is captured. These will be used to calculate the period of oscillation, where the track of the centre dot is used to account for any unwanted sway due to the translational motion in the x and y direction. To have reliable results it is advised to track at least three outlying dots/markers.

3.2 Blender outcome analysis

If all the steps and recommendations in section 3.1 are followed four .CSV files will be generated. One .CVS file corresponding to the motion of the centre dot and three .CSV files corresponding to the motion of outlying dots. Each .CSV contains three columns, the frame number in column 1 the x coordinate and a y coordinate. The coordinates will be relative to the dimensions of the frame measured from the bottom left corner.

4 Moment of Inertia calculation

Once the motion of the pendulum is captured and the data exported to a CSV file, the sway correction can be performed. The x and y coordinate of the centre of the plate should be subtracted, from the tracked motion of each outlying marker dot. If the platform's motion was pure rotation the centre marker dot should be stationary, so by subtracting the motion of the platform centre for each individual data point, the effect of the sway can be eliminated using equations 1 and 2, where x_{dot} and y_{dot} are the x and y position of one of the outlying marker dots, x_{plate} and y_{plate} are the plate's centre marker dot x and y positions, and x_{nom} and y_{nom} are the resulting normalised for translational motion coordinates. The result is illustrated in Figure 6, where the raw motion of an outlying dot can be seen in the left plot and the normalised data can be seen on the right.

$$x_{nom} = x_{dot} - x_{plate} \tag{1}$$

$$y_{nom} = y_{dot} - y_{plate} \tag{2}$$

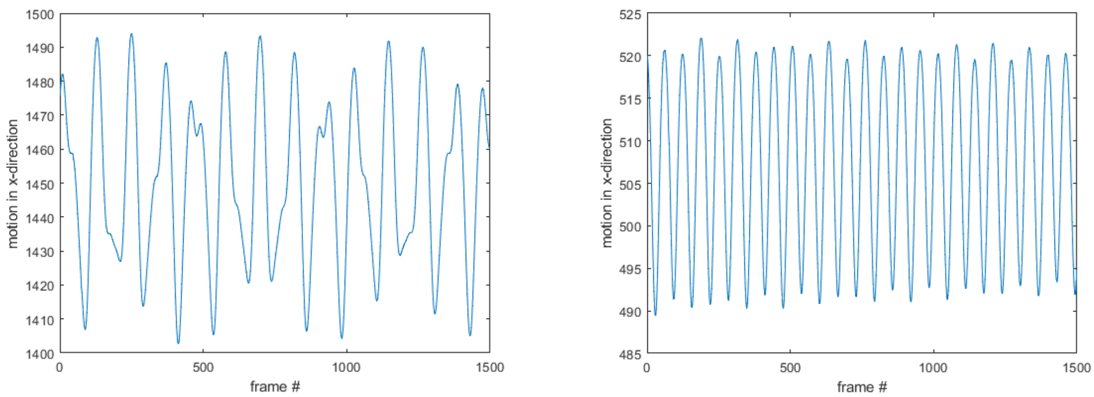

Figure 6: Table motion with sway (left) and corrected for sway (right)

The next step is to convert the motion of the outlying marker dot from cartesian x and y to polar coordinates, This can be achieved by using equation 3, where θ is the angle in polar coordinates. The resulting motion is plotted in Figure 7. Furthermore, the frame rate of the recording is converted to time in seconds using equation 4, where frame # is the current frame number and FPS is the frames per second at which the video was originally made. For example, if the video was made with a frame rate of 30 frames/second, the data point corresponding to frame 30 occurred 1 second after the start of the experiment.

$$\theta = \tan^{-1}\frac{y_{nom}}{x_{nom}} \tag{3}$$

$$t = \frac{frame\ \#}{FPS} \tag{4}$$

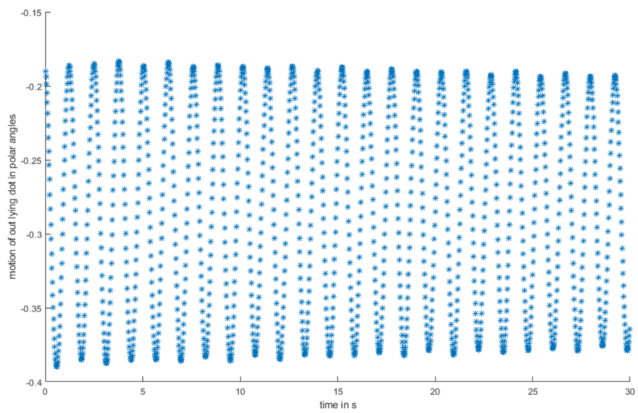

Figure 7: motion of the pendulum in polar angles vs time

From Figure 7 the period of oscillation can be determined. This can either be done with a peak search, zero crossing search, a curve fit, or a Fourier Transform. Examples of curve fitting and Fourier transform can be seen in Figure 8. After obtaining the period (τ) and measuring the mass (m) of both the platform $m_{platform}$ and the test object m_{object}, the radius of the platform (R) and the length of the wires (L) by using equation 5, derived in [9], the MoI can be calculated. First the MoI of the platform without the test object should be calculated using only equation 5, where τ is the period of oscillation of only the platform, g is the acceleration due to gravity. If the MoI of the platform is already known (from previous test or design) equation 6 can be used directly, where τ is now the period of oscillation of the combined platform and object configuration, the mass is the sum of the two masses. The result of the equation before the minus sign is the combined MoI of the platform and the object, by subtracting the MoI of the platform the MoI of the object is found.

$$I_{platform} = \frac{g \cdot m_{platform} \cdot R^2 \cdot \tau^2}{4 \cdot \pi^2 \cdot L} \tag{5}$$

$$I_{object} = \frac{g \cdot R^2 \cdot \tau^2}{4 \cdot \pi^2 \cdot L} \cdot \left(m_{platform} + m_{object}\right) - I_{platform} \tag{6}$$

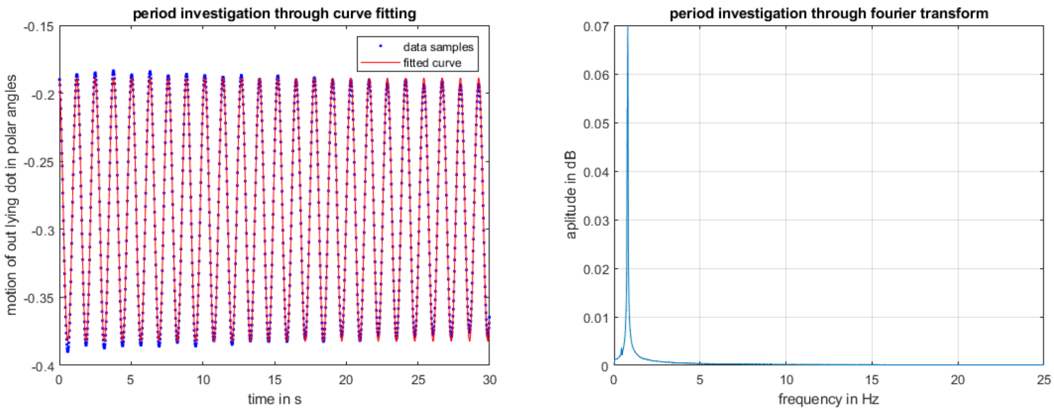

Figure 8: Example of period investigation through curve fitting (left) and Fourier Transformation (right)

An example of finding the period the oscillations using curve fitting is shown in the left plot of Figure 8, a least squares cost function is employed using the MATLAB® fminsearch function [10]. The algorithm requires two inputs: a function the user tries to fit and initial parameters. The function that is fitted for this particular example can be seen in equation 7, where A is an initial guess for the sine function Amplitude, B is the period, and C is the offset from the x-axis.

$$fit = A \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{B}\right) + C \tag{7}$$

In order to perform the period investigation a Fast Fourier Transform (FFT) can be done as shown on the right in Figure 8, this transform was performed using MATLAB®'s built-in FFT function which uses equations 8 and 9 to perform the transform [11].

$$X(k) = \sum_{j=1}^{n} X(j) W_n^{(j-1)(k-1)} \tag{8}$$

$$W_n = e^{\frac{-2 \cdot \pi \cdot i}{n}} \tag{9}$$

By finding the peak in the FFT to determine the frequency (f) and using equation 10 the period (τ) can be found.

$$\tau = \frac{1}{f} \tag{10}$$

The MoI value measured corresponds to the axis of the platform oscillation. To measure a different axis, the object simply needs to be rotated on the platform so that a different axis aligns with the z-axis of the platform. To measure the MoIs of 3 axes and the corresponding Products of Inertias (PoIs), measurements in at least six different orientations need to be taken. These are about 3 orthogonal axes for the MoIs and most simply in three orientations with the satellite rotated 45 degrees. A diagram of 9 possible test orientations is shown in Figure 9. To determine the PoI as outlined in [12] equations 9, 10 and 11 can be used. Where I_{xx}, I_{yy}, and I_{zz} are the MoIs around the 3 orthogonal axes, $I_{xy(45)}$, $I_{xz(45)}$ and $I_{yz(45)}$ are the MoIs with the satellite rotated 45 degrees (orientations 4, 7 and 8 in Figure 9). The MoI is measured around an axis 'A' which lies in the X-Y plane but is offset 45 degrees from the X-axis and 45 degrees from the Y-axis (and similar for X-Z and Y-Z). P_{xy}, P_{xz} and P_{yz} represent the PoIs corresponding to these planes. The Inertia tensor can be completed as $P_{xy} = P_{yx}$, $P_{xz} = P_{zx}$ and $P_{yz} = P_{zy}$.

$$P_{xy} = \frac{I_{xx} + I_{yy}}{2} - I_{xy(45)} \tag{11}$$

$$P_{xz} = \frac{I_{xx} + I_{zz}}{2} - I_{xz(45)} \tag{12}$$

$$P_{yz} = \frac{I_{yy} + I_{zz}}{2} - I_{yz(45)} \tag{13}$$

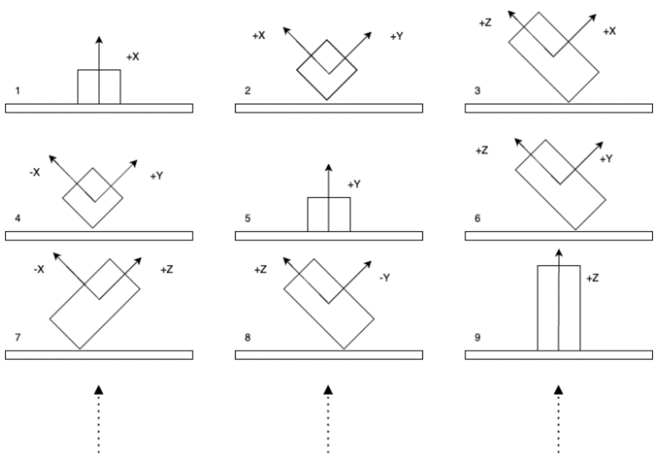

Figure 9: Nine test orientations required for complete MoI/PoI calculation

5 Conclusion

To verify that the ADCS system of a satellite is correctly sized and to predict its behaviour in orbit and at deployment it is important to know the Moments of Inertia and the Products of Inertia of the integrated flight model. This paper presented a step-by-step guide to performing a cost-effective and uncomplicated way to measure the Moment of Inertia of small satellites. The measurement setup was based on a trifilar pendulum, video capture and motion tracking data analysis using Blender®. Methods for extracting the period of oscillations from the data were presented and related to the MoI using the equations of motion of the trifilar pendulum. Furthermore, a way to calculate the PoIs from the experimentally determined MoIs in six orientations was outlined. Hence the complete process from object to fully defined inertia tensor is completed.

6 Bibliography

[1] D. M. E. J. e. a. Murphy D, "EIRSAT-1: The Educational Irish Research Satellite," in *Proceedings of the international Astronautical Congress, IAC*, Bremen, 2018.

[2] The CubeSat Program, "CubeSat Design Specification Rev. 14," California Polytechnic State University (Cal Poly), San Luis Obispo, CA, 2020.

[3] G. G. a. C. Delprete, "Some Considerations on the Experimental Determination of Moments of Inertia," *Meccanica ,* vol. 29, pp. 125-141, 1994.

[4] J. S. G. S. Peter Fortescue, Spacecraft Systems Engineering, Chichester: Wiley, 2011.

[5] G. Previati, "Large oscillation of the trifilar pendulum: Analytical and experimental study," *Elsevier Mechanism and Machine Theory,* vol. 156, no. 104157, 2021.

[6] Mc Donnell Douglas Astronautics Company, "Isogrid Design Handbook," Huntington Beach, Ca , 1973.

[7] Blender, "Blender," Blender, [Online]. Available: blender.org. [Accessed 24 04 2023].

[8] Susu, "Blender StackExchange," forum, [Online]. Available: https://blender.stackexchange.com/questions/65518/export-track-markers-to-csv. [Accessed 24 05 2023].

[9] J. L. N. A. S. Du bois, "Error analysis in Trifilar Inertia Measurements," *Exp Mech,* pp. 533-540, 2009.

[10] Matlab, "Mathworks Help centre fminsearch," Matlab, 2023. [Online]. Available: https://uk.mathworks.com/help/matlab/ref/fminsearch.html. [Accessed 25 05 2023].

[11] Matlab, "Matworks Help Centre fft," Matlab, 2022. [Online]. Available: https://uk.mathworks.com/help/matlab/ref/fft.html. [Accessed 12 05 2023].

[12] R. B. Kurt Wiener, "Using the "Moment of Inertia Method" to Determine Product of Inertia," in *51st Annual Conference of the Society of Allied Weight Engineers*, Hartford, Connecticut, 1992.