

Mitigating Keystone-Induced Aliasing in Push-broom Imaging Spectrometers via PSF Engineering

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ABSTRACT

Space instruments are typically designed to maximize signal-to-noise ratio by minimizing the f-number. In diffraction-limited systems, this results in a point spread function (PSF) that is smaller than the detector pixel, leading to undersampling that violates the Nyquist sampling criterion. This is particularly problematic for push-broom imaging spectrometers, where keystone distortion causes spectra to deviate slightly from detector rows. As a result, spectral aliasing artifacts arise, coupled to across-track spatial variation. These systematic errors are especially challenging because both the amplitude and the shape of the keystone distortion influence the outcome; reducing the keystone amplitude alone does not necessarily improve performance. In this work we introduce an optical distortion model to assess systematically the impact of keystone distortion. Our results show that the most effective way to reduce systematic error is to broaden the PSF. While detector defocusing can achieve this, aliasing artifacts persist and the system becomes highly sensitive to axial misalignment. We demonstrate that a more robust approach is to homogeneously suppress high spatial frequencies in the optical transfer function, producing an “Airy beam”-like PSF (not to be confused with an Airy disk). This tailored PSF reduces keystone-induced artifacts by up to $7\times$ in both artificial and realistic scenes.

1. INTRODUCTION

Earth-observation push-broom imaging spectrometers have become indispensable tools for monitoring atmospheric composition, vegetation, water quality, and surface properties. In particular, the increasing demand for quantitative trace-gas retrievals places stringent requirements on instrument performance, requiring high spatial resolution to resolve localized emission sources while simultaneously maintaining high radiometric sensitivity.¹ Achieving these objectives drives instrument designs toward large apertures, fast optical systems, and high optical throughput.

These optical requirements, however, are accompanied by practical constraints imposed by detector technology. Detector pixel dimensions cannot be reduced indefinitely because they determine the optical etendue, influence detector noise characteristics, and affect achievable signal-to-noise ratio. As a result, modern space-borne imaging spectrometers frequently combine optics with detector pixels that are considerably larger than the diffraction-limited spot.

When the diffraction-limited point spread function (PSF) becomes smaller than the detector pixel, the optical image is undersampled and the Nyquist sampling criterion is violated.² Although the optical system faithfully transfers high spatial frequencies, the detector cannot sample them correctly, causing spatial frequencies beyond the Nyquist limit to fold back into the sampled image. This phenomenon, known as aliasing, introduces systematic measurement errors that cannot be removed during data processing because the lost information is irreversibly mixed with the sampled signal.

One optical aberration that becomes particularly problematic for push-broom imaging spectrometers under undersampled conditions is keystone distortion. In a push-broom spectrometer, the telescope images an across-track (ACT) slit onto the detector in the column direction, while a dispersive element separates the incoming light spectrally in the row direction. The along-track (ALT) spatial dimension is built up sequentially as the spacecraft moves along its flight path. Keystone is a geometric distortion in the across-track direction where spectra get a wavelength-dependent shift relative to the detector rows.³ While these shifts are generally small, they vary continuously across the spectral dimension and are inherently wavelength dependent.

In a well-sampled imaging system, such (sub-pixel) displacements primarily result in small geometric registration errors. In an undersampled system, however, they interact with the aliased spatial frequencies present in the scene. Since each wavelength effectively samples a slightly different aliased realization of the spatial scene, spatial contrast couples into the measured spectra (Level-1B data product). The resulting spectral differences do not originate from the observed surface or atmosphere, but from the interaction between keystone distortion and undersampling.

Retrieval algorithms interpret the aliased spectral variations as physical absorption features, leading to errors in the retrieved quantities (L2-product) such as trace-gas concentrations. Because the errors originate in the optical sampling process rather than detector noise, they cannot be mitigated by increasing integration time or averaging multiple observations. This raises the central question addressed in this work: How can optical design reduce keystone-induced aliasing in undersampled imaging spectrometers?

To investigate this question, we consider a representative instrument based on the specifications of TANGO-Carbon.⁴ The hypothetical instrument for this study employs a 30 km swath, a ground instantaneous field of view (giFoV) of 60 m $\times$ 150 m, and operates over the 1590–1670 nm short-wave infrared spectral range for CO₂ and CH₄ retrieval, with 15 μ m detector pixels. Assuming an f/2 optical system, the diffraction-limited PSF has a full width at half maximum (FWHM) of approximately 3.3 μ m, corresponding to an undersampling factor of roughly nine relative to the detector pixel size. This configuration provides a realistic and challenging test case in which keystone-induced aliasing is expected to become a significant source of systematic retrieval error.

This paper addresses the problem from both a physical understanding and an optical engineering perspective. The first part develops a distortion model describing how keystone distortion interacts with undersampling to generate spectral aliasing. Using this model, we quantify the resulting spectral and retrieval errors as functions of keystone amplitude, keystone shape, and PSF width. We further demonstrate that conventional peak-to-valley keystone specifications alone are insufficient to predict retrieval performance, as instruments with identical keystone amplitudes can produce substantially different aliasing behaviour depending on the detailed distortion profile and optical transfer characteristics.

The second part investigates whether the optical system itself can be engineered to suppress aliasing while maintaining optical throughput. Rather than accepting the diffraction-limited PSF as fixed, we explore the use of phase aberrations to deliberately broaden and reshape the PSF through PSF engineering.⁵ Several engineered PSFs are evaluated using their point spread functions, optical transfer functions, encircled energy distributions, and through-focus behaviour. Among the investigated designs, a cubic-phase PSF is shown to provide the most favourable balance between alias suppression and optical performance. Finally, we demonstrate that this engineered PSF significantly reduces keystone-induced spectral errors in both synthetic scenes and realistic Earth-observation imagery, illustrating how optical design can improve retrieval accuracy in undersampled imaging spectrometers.

2. MODELING KEYSTONE DISTORTION

This section investigates the impact of keystone distortion on Level-2 retrieval accuracy. We first introduce a geometric distortion model that describes the wavelength-dependent image distortion, followed by a synthetic benchmark scene that isolates aliasing effects. Finally, the influence of keystone amplitude and point spread function (PSF) width is quantified to identify the dominant contributors to retrieval error.

2.1 Geometric distortion model

An imaging spectrometer maps object coordinates onto detector coordinates through an optical projection that is generally wavelength dependent. The detector coordinates (x', y') are therefore expressed as a curvi-linear transformation T of the ideal image coordinates (x, y) : $T : (x, y) \rightarrow (x', y')$. See Figure 1A. The geometric distortion model used in this work consists of two components: keystone distortion in the spatial direction and smile distortion in the spectral direction. Keystone is modelled as a polynomial distortion in the spectral direction,

$$y' = y (1 + K_1 x + K_2 x^2 + \dots), \quad (1)$$

where the coefficients K_i determine the curvature.

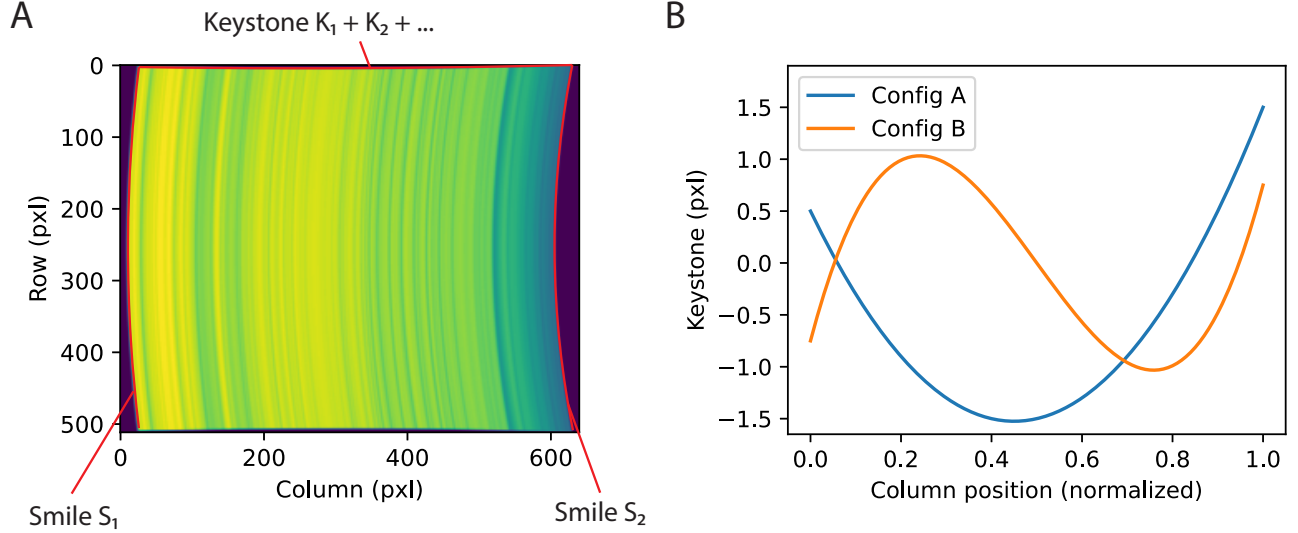


Figure 1. A) Geometric distortion illustration, indicating the smile and keystone distortion. B) Different keystone distortion shapes for Configuration A and B.

Smile is represented by a quadratic displacement function,

$$x' = x + y^2 \left[S_1 \left(\frac{1}{2} - x \right) + S_2 \left(\frac{1}{2} + x \right) \right], \quad (2)$$

where S_1 and S_2 describe the distortion at the smallest and largest wavelength, respectively. This formulation allows two smile amplitudes, which scale linearly and continuously in between the wavelengths.

The transformation remains physically valid provided the Jacobian remains non-zero,

$$J = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{vmatrix} \neq 0, \quad (3)$$

ensuring that the mapping remains one-to-one throughout the detector. Additionally, energy conservation follows directly from the Jacobian,

$$\iint I(x, y) dx dy = \iint I(x', y') J dx' dy'. \quad (4)$$

In the remainder of this paper, the analysis focuses on keystone distortion but the smile distortion is included. For both the forward model, as well as the L1 processor, the interpolation is done by cubic-spline interpolation. Throughout this work, we investigate 2 different keystone configurations: a second order (quadratic) curvature (Config A) and a third order curvature (Config B) as shown in Figure 1B. These profiles have a Peak-Valley (PV) distance of 3 and 2 pixels respectively. For the smile distortion, we choose a smile of 15 pixels (left) and 25 pixels (right) for both keystone distortions.

2.2 Artificial benchmark scene

To isolate aliasing effects from natural scene variability, a synthetic benchmark scene was constructed consisting of alternating Lambertian surface reflectances of 0.2 and 0.3, see Figure 2A, resulting in an ideal detector image shown in Figure 2B. The stripes have a spatial period of 500 m, or about 8 detector pixels. This represents a benchmark scene for aliasing while remaining representative of medium-contrast transitions encountered in agricultural and urban environments.

The synthetic scene is propagated through the complete instrument model, including optical distortion and detector sampling, to generate a Level-1A image (digital counts on detector grid). The synthetic data is further

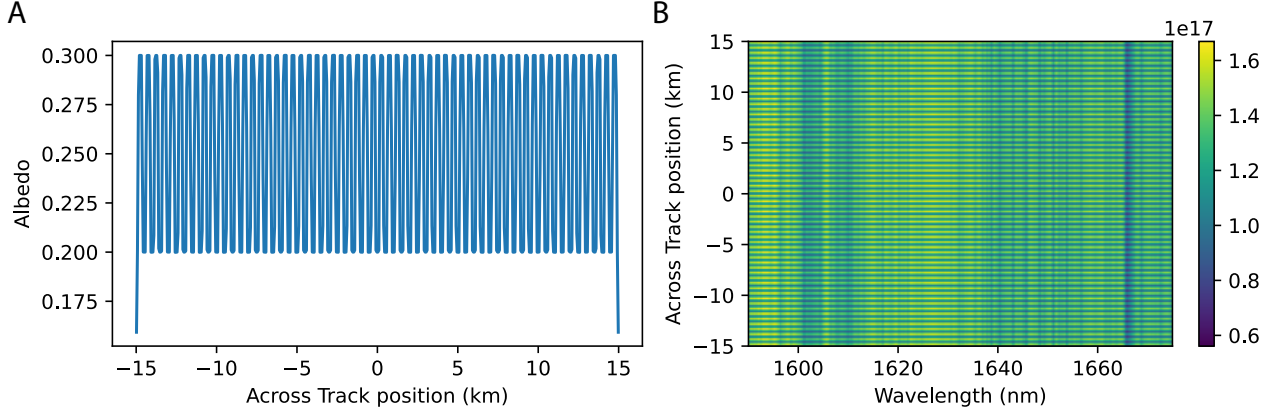


Figure 2. A) Keystone evaluation benchmark scene. The albedo oscillates between 0.2 and 0.3 every 500 m (8 detector pixels) with a rectangular function. B) Ground truth L1B data product for (A). Unit are in $\text{ph}/(\text{s sr m}^2 \text{ nm})$

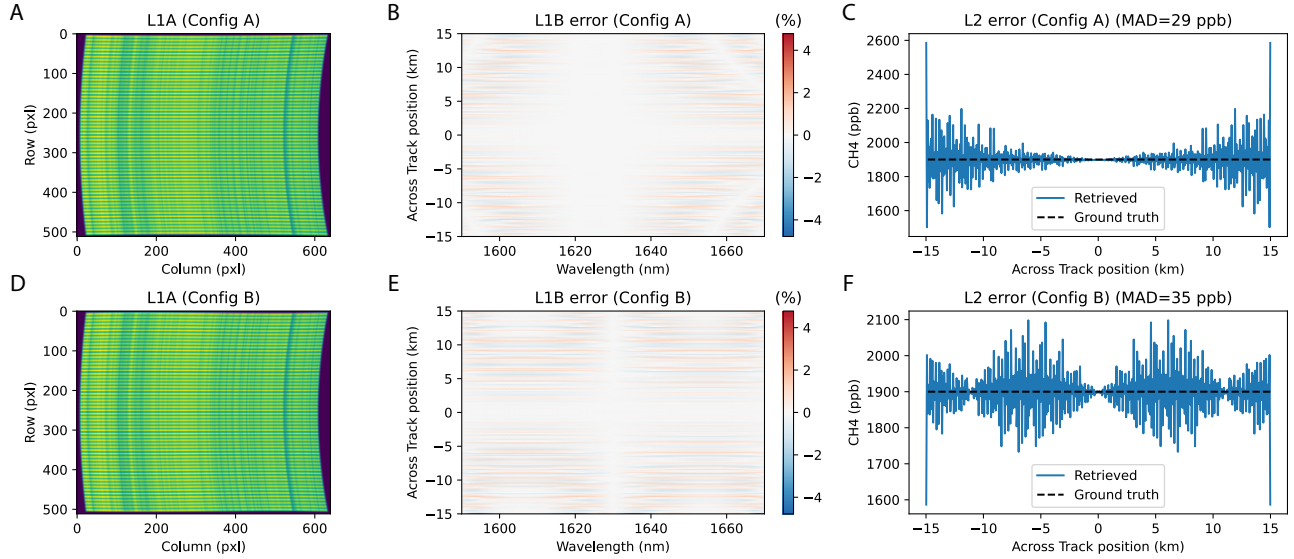


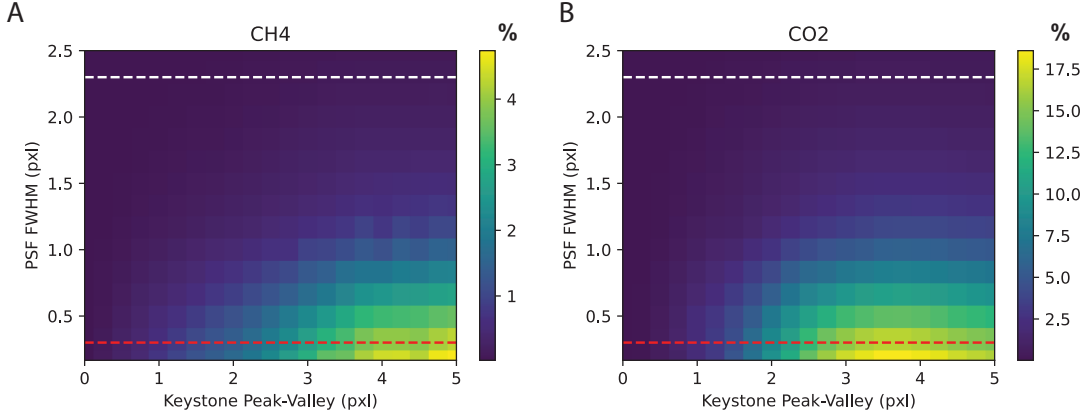
Figure 3. A) L1A product for the reference scene for keystone configuration A. B-C) L1B and L2 error of (A). D-F): Same as (A-C) but for keystone configuration (B).

processed in Level-1B (radiance on wavelength and ACT position grid) and Level-2 trace-gas amount fractions. We applied a perfect L1-processor with no additional error sources and ignore stochastic noise.

Figure 3 shows the simulated Level-1A detector image (in counts), the resulting Level-1B error (in radiance), and the retrieved methane concentrations for the two keystone configurations. The Level-1B error exhibits a characteristic Moiré pattern, demonstrating that the observed artifacts originate from spatial aliasing rather than interpolation errors or detector noise. These aliasing artifacts propagate directly into the Level-2 retrieval, producing localized retrieval spikes at the transitions between adjacent albedo regions. The retrieval error over the complete scene was quantified using the median absolute deviation (MAD), providing a robust measure that is insensitive to isolated outliers.

Configuration A produces a methane retrieval error of 29 ppb (1.5%), while configuration B results in 35 ppb (1.8%). Although configuration A exhibits a larger peak-to-valley keystone than configuration B, its retrieval error is lower. This observation demonstrates that conventional peak-to-valley specifications alone are insufficient to predict retrieval performance. The benchmark scene therefore illustrates the central hypothesis of this work: retrieval errors are primarily determined by the interaction between keystone distortion and undersampling rather than by the geometric distortion itself.

Config A



Config B

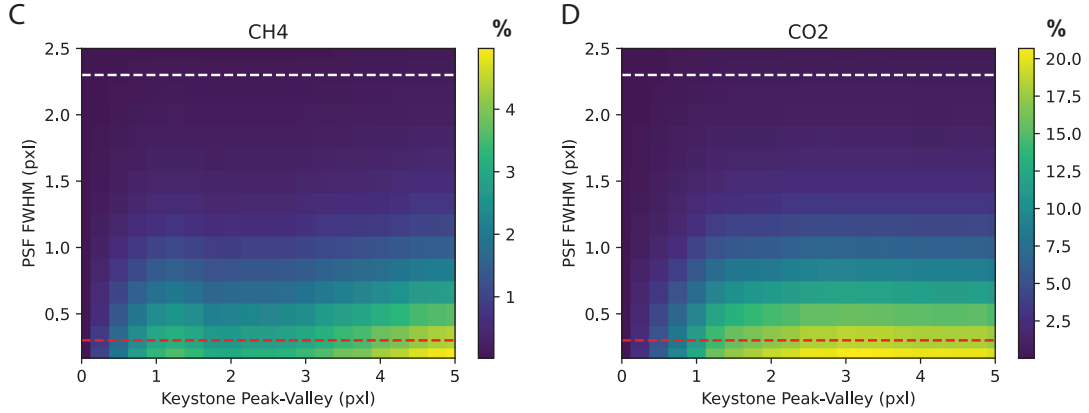


Figure 4. Error in % in CO_2 and CH_4 retrieval for the two keystone configurations as function of PSF FWHM and Peak-Valley amplitude. The dashed red line indicates the diffraction limited PSF size and the dashed white line indicates the Nyquist sampling criterion.

2.3 Influence of keystone distortion

Having established the origin of the retrieval artifacts, we next assess the sensitivity of the retrieval error as a function of both keystone distortion and PSF width (Figure 4) for both keystone profiles (Config A and B).

From Figure 4 it can be seen that reducing the peak-to-valley keystone does not consistently reduce the retrieval error. Significant improvements are only observed once the PV distortion becomes smaller than approximately one detector pixel. Furthermore, the two distortion profiles with nearly identical peak-to-valley amplitudes can produce markedly different retrieval errors. These results indicate that the commonly used peak-to-valley specification does not uniquely determine the sampling behaviour of the instrument and is therefore a poor predictor of Level-2 retrieval error.

In contrast, increasing the PSF width reduces the retrieval error consistently for both keystone configurations. Broadening the PSF suppresses spatial frequencies above the detector Nyquist frequency before sampling, thereby reducing the aliasing that couples spatial structure into the measured spectra. Unlike tightening geometric distortion tolerances, this behaviour is largely independent of the detailed keystone profile.

These results suggest that keystone-induced retrieval errors should primarily be regarded as a sampling problem rather than a geometric distortion problem. Consequently, engineering the optical transfer function to provide controlled optical low-pass filtering offers a more robust mitigation strategy than further reducing geometric distortion alone. This observation forms the motivation for the remainder of this paper, where the diffraction-limited PSF is engineered to suppress spatial aliasing while preserving optical throughput.

3. ENGINEERING THE POINT SPREAD FUNCTION

Here we investigate whether the PSF, and its Fourier counterpart, the Optical Transfer Function (OTF) can be deliberately modified to suppress spatial frequencies above the detector Nyquist frequency. This section first introduces the Fourier optics model used to evaluate different pupil phase functions. Subsequently, several candidate phase profiles are compared and assessed.

3.1 Fourier optics model

The optical system is modelled using scalar Fourier optics,⁶ assuming coherent propagation through a circular pupil. The complex pupil function is defined as

$$P(\rho_x, \rho_y) = A(\rho_x, \rho_y) \exp[-i\phi(\rho_x, \rho_y)], \quad (5)$$

where (ρ_x, ρ_y) denote normalized pupil coordinates, A is the pupil transmission, and $\phi(\rho_x, \rho_y)$ is the applied phase function. Unless stated otherwise, the pupil transmission is unity inside the circular aperture and zero elsewhere.

The image-plane electric field is obtained by the Fourier transform of the pupil function,

$$E(\xi, \eta) = \mathcal{F}\{P(\rho_x, \rho_y)\}, \quad (6)$$

where (ξ, η) are image coordinates expressed in units of λ/NA . The normalized point spread function (PSF) is subsequently given by

$$\text{PSF}(\xi, \eta) = \frac{|E(\xi, \eta)|^2}{\iint |E(\xi, \eta)|^2 d\xi d\eta}. \quad (7)$$

The optical transfer function is computed as the normalized autocorrelation of the pupil function,

$$\text{OTF}(f_x, f_y) = \frac{\iint P(\rho_x, \rho_y) P^*(\rho_x - f_x, \rho_y - f_y) d\rho_x d\rho_y}{\iint |P(\rho_x, \rho_y)|^2 d\rho_x d\rho_y}, \quad (8)$$

where (f_x, f_y) denote spatial frequencies in units of NA/λ .

Throughout this work the OTF is used to quantify the spatial bandwidth of the optical system, while the PSF is used to evaluate the spatial redistribution of optical energy. In addition, the encircled energy and through-focus behaviour are used to compare the robustness of the investigated phase functions.

3.2 Phase functions

The objective of the phase mask is to redistribute optical energy such that spatial frequencies above the detector Nyquist frequency are suppressed without reducing the optical throughput. Several candidate phase profiles were investigated, selected based on their optical properties and practical relevance, see Figures 5 and 6.

Defocus Defocus represents the simplest method of increasing the PSF size and serves as a natural reference. The phase function is

$$\phi_{\text{defocus}} = a(\rho_x^2 + \rho_y^2), \quad (9)$$

where a is the phase amplitude. Although defocus broadens the diffraction-limited PSF, it rapidly degrades image quality and exhibits a strong dependence on focus position. Consequently, only modest reductions in aliasing are expected.

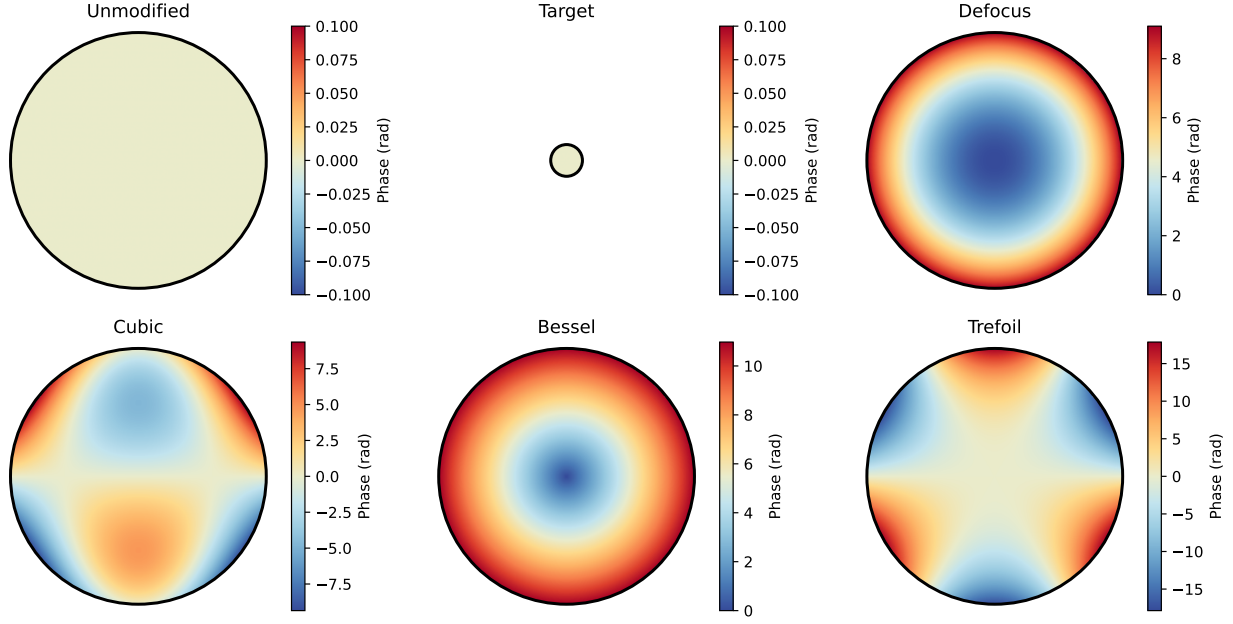


Figure 5. Six examples of potential phase functions

Cubic phase The cubic phase profile is defined as

$$\phi_{\text{cubic}} = a (\rho_y^3 + 3\rho_x^2\rho_y - \rho_y), \quad (10)$$

where the linear term removes the average wavefront tilt. Cubic phase masks are widely used in wavefront coding and Airy-beam generation because they produce an extended depth of focus while preserving optical throughput.⁷ The broadened PSF acts as an optical low-pass filter, making this phase profile a promising candidate for suppressing aliasing.

Trefoil The trefoil phase function is

$$\phi_{\text{trefoil}} = a (\rho_y^3 - 3\rho_x^2\rho_y). \quad (11)$$

Trefoil was selected following a preliminary optimization over Zernike modes, which indicated that this aberration provides an effective redistribution of optical energy while maintaining a relatively compact PSF. Since cubic phase can be expressed as a linear combination of coma and trefoil, it provides an interesting comparison between a classical Zernike aberration and the (engineered) cubic phase profile.

Bessel phase The conical phase function

$$\phi_{\text{Bessel}} = a\sqrt{\rho_x^2 + \rho_y^2} \quad (12)$$

produces a Bessel-like beam.⁸ Similar phase masks are commonly employed in light-sheet microscopy because they generate a narrow central lobe that remains nearly invariant over an extended propagation distance. In the present application, the corresponding increase in depth of focus is accompanied by a controlled broadening of the PSF, thereby reducing the transfer of high spatial frequencies.

Reduced aperture As an ideal reference, the pupil diameter was reduced to directly limit the optical bandwidth,

$$A(\rho_x, \rho_y) = \begin{cases} 1, & \rho < \rho_c, \\ 0, & \rho \geq \rho_c, \end{cases} \quad (13)$$

where ρ_c is selected to obtain the desired diffraction-limited cut-off frequency. This approach provides the ideal anti-aliasing behaviour but simultaneously reduces the optical throughput and therefore represents the theoretical lower bound rather than a practical solution.

Higher-order spherical aberrations were not considered because they primarily redistribute energy toward the outer regions of the PSF, producing extended wings with limited suppression of high spatial frequencies. Likewise, astigmatism and higher-order astigmatic aberrations were excluded because they introduce strong directional and field-dependent behaviour, resulting in anti-aliasing performance that depends on image orientation.

To enable a fair comparison between the investigated phase functions, the amplitude of each phase profile was optimized such that each profile produced the same effective optical bandwidth of 0.1 at $0.2 \lambda/\text{NA}$. This ensures that the radial averaged OTF of each engineered PSF goes through this point. See Figure 9A. This matches a diffraction limited PSF with a $8\times$ reduced aperture. Therefore, if this target OTF response can be met, the sampling would (almost) satisfy the Nyquist sampling criteria.

3.3 Best anti-aliased PSF

The resulting PSFs, OTFs and through-focus plots are shown in Figure 6, 7 and 8. As expected, the diffraction-limited PSF has a broad frequency support and is therefore very small. The target PSF, with the reduced aperture, exhibits a broad PSF which contains 84% of the energy inside the airy-disk radius. However, the reduced aperture size would result in a $64\times$ additional reduction in intensity and is therefore not a viable solution. As the depth-of-focus of a beam depends on the NA^2 of the system, the target PSF has a $64\times$ larger depth of focus as can be seen in Figure 8.

The defocus PSF and the Bessel PSF show very similar behaviour. Both PSFs contain a centre focus point, a large surrounding annulus and both have an oscillation (contrast inversion) in the OTF (see Figure 9). In the through-focus image (Figure 8) a clear positional dependency is observed, making both phase sensitive to axial alignment. For these reasons, both of these options are not attractive solutions.

The cubic and trefoil PSFs exhibit stronger fringes compared to the defocus and Bessel PSF, but have a better suppression of the high spatial frequencies. The cubic phase contains a comparable total amount of encircled energy as the target PSF and also has a similar radial increase (see Figure 9B). The trefoil PSF in contrast only contains 58%, which would result in a substantial reduction of contrast in the image. The cubic phase also outperforms the trefoil PSFs in the through-focus behaviour as it exhibits an excellent focus through-out a larger depth of focus.

For these reasons, it appears that the cubic phase shows the best compromise between OTF suppression, encircled energy and through-focus behaviour. We note however, that a different choice of trefoil amplitude might increase the encircled energy (at the expense of OTF suppression) and therefore remains an interesting candidate.

3.4 Engineered Instrument Spectral Response Function

We next investigate whether the engineered cubic phase PSF also produces a suitable instrument spectral response function (ISRF). The ISRF is obtained by convolving the image-plane PSF with the entrance slit function in the spectral direction. Throughout this work the slit is modelled as an ideal rectangular aperture with a corresponding ALT giFoV of 150 m (see the instrument design described in Section 1), which is half of the smear distance due to the ground speed of 300 m. Figure 10 compares the ISRF obtained using the engineered cubic phase mask with the reference ISRF corresponding to the reduced-aperture target pupil introduced in Section 3. The engineered ISRF is referred to throughout the remainder of this paper as the *anti-aliased ISRF* (AA-ISRF).

It appears that, although the cubic phase PSF exhibits pronounced interference fringes, these fringes are largely removed by convolution with the slit function. The resulting ISRF is considerably smoother than the original PSF and exhibits no significant oscillatory structure in the spectral direction. Second, the spectral profile of the AA-ISRF closely follows that of the target ISRF. In the ACT direction, ISRF remains relatively sharp, but is overall similarly broad to the target ISRF.

These observations indicate that the apparent complexity of the engineered PSF does not translate into an equally complex instrument response. Instead, the finite slit width naturally smooths the oscillatory features,

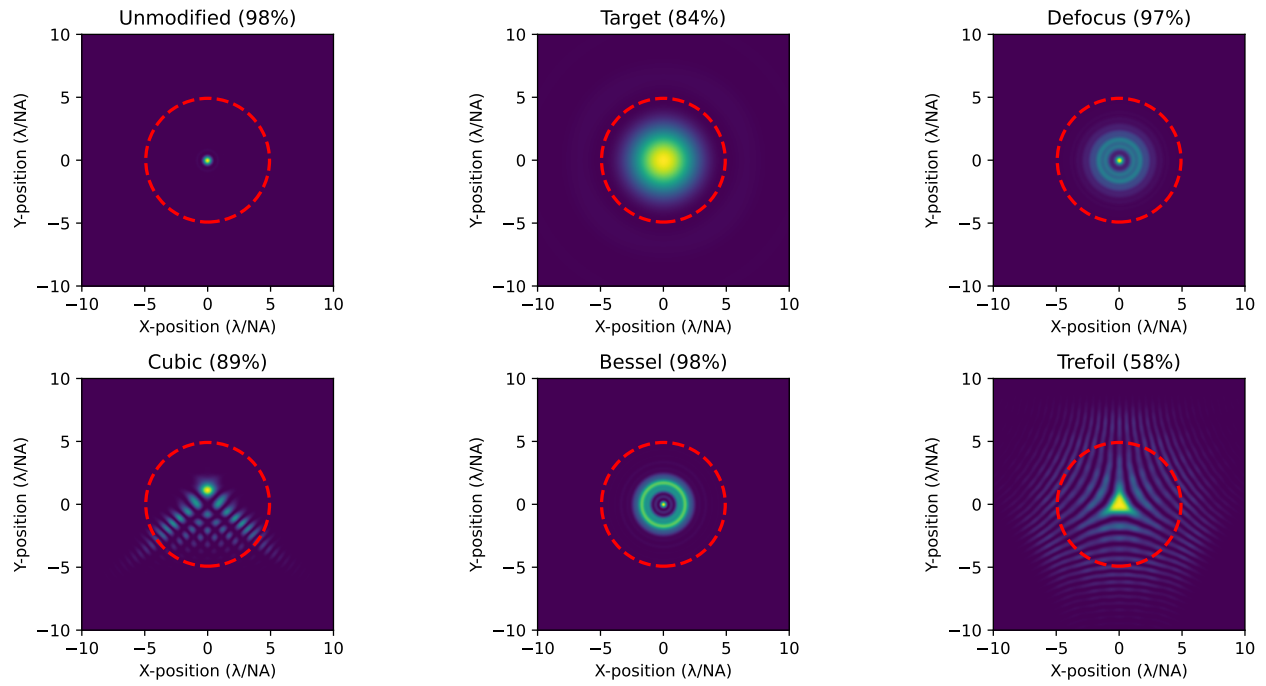


Figure 6. Point-Spread Functions belonging to the phase functions of Figure 5. The red dashed line indicates the airy-disk radius for the target PSF ($8\times$ reduced aperture). The %-value indicates the encircled energy inside this radius normalized to the pupil. Note that for the Target-PSF the reduced aperture results in an additional $64\times$ reduction in intensity.

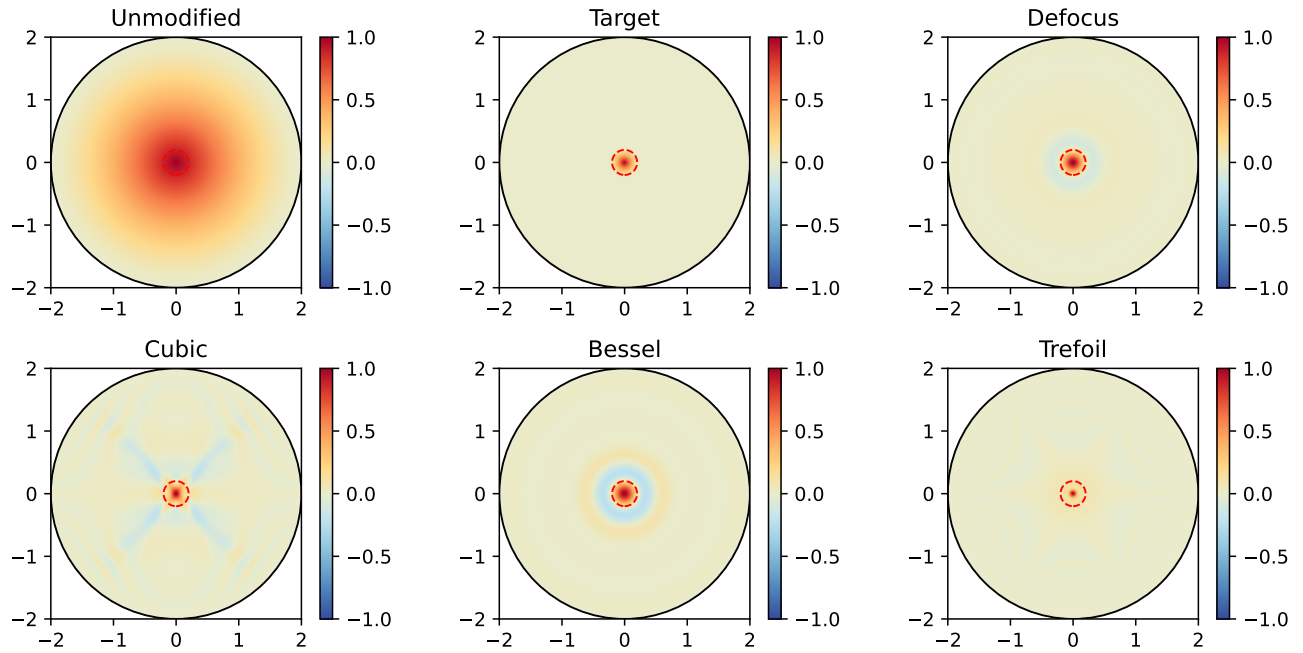


Figure 7. Optical Transfer Functions belonging to the phase functions of Figure 5. The axes are in units of NA/λ . The red dashed line indicates the cut-off spatial frequency of $0.1 \text{ NA}/\lambda$.

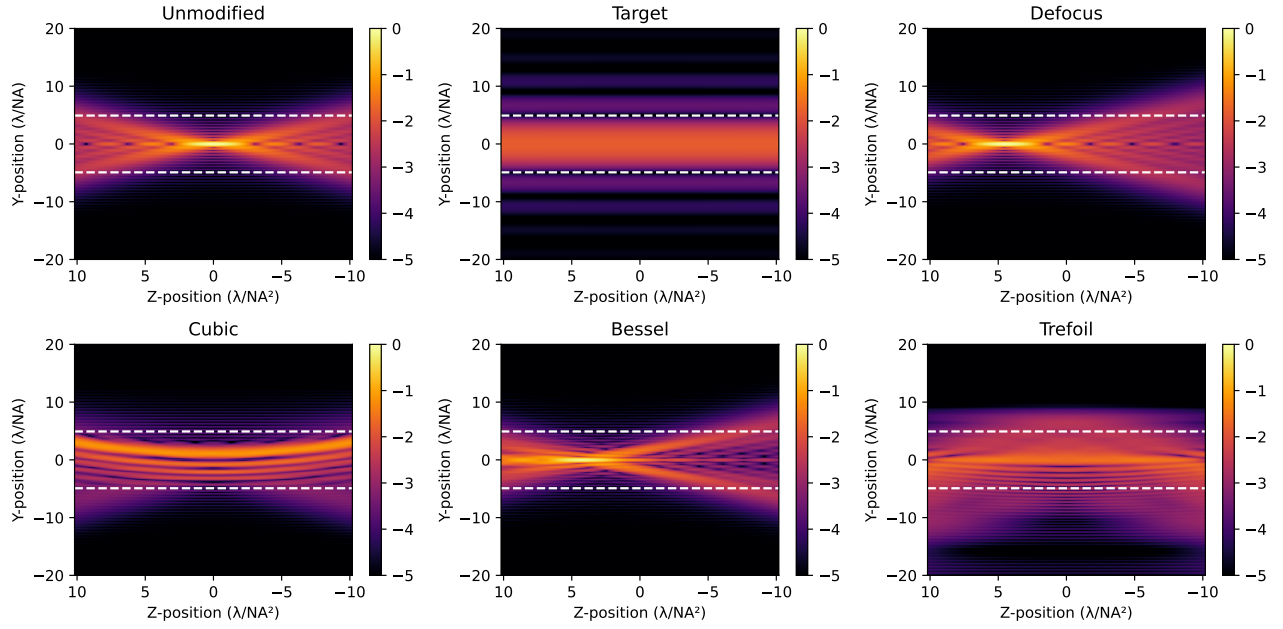


Figure 8. Through-focus point spread functions belonging to the phase functions of Figure 5. The white dashed lines indicate the airy-disk radius of the target PSF.

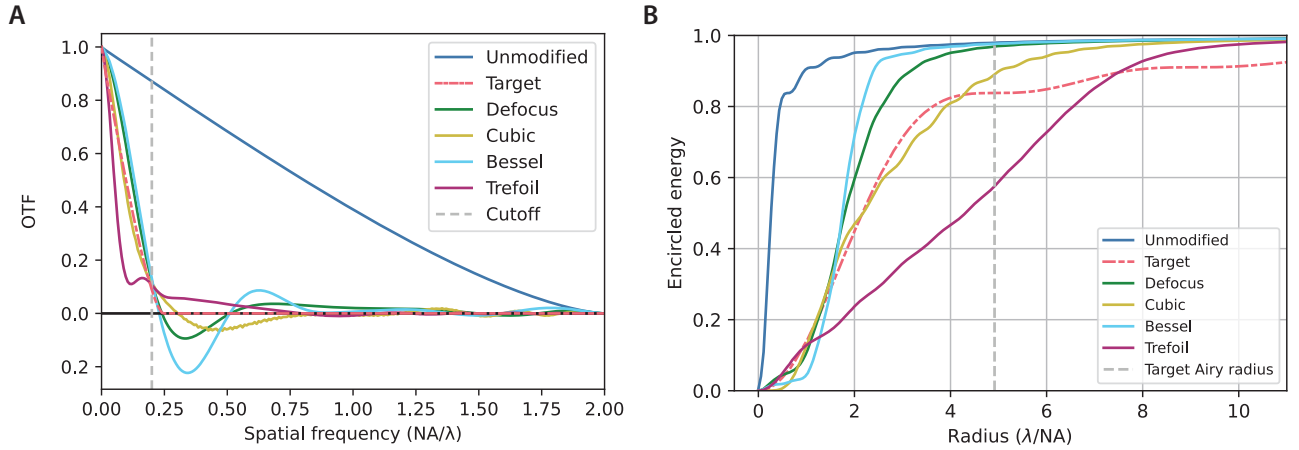


Figure 9. A) Radially averaged OTF for the 6 phase profiles. The red dashed lines indicates the cut-off frequency. B) The encircled energy as function of radius. The black dashed line indicates the target airy radius.

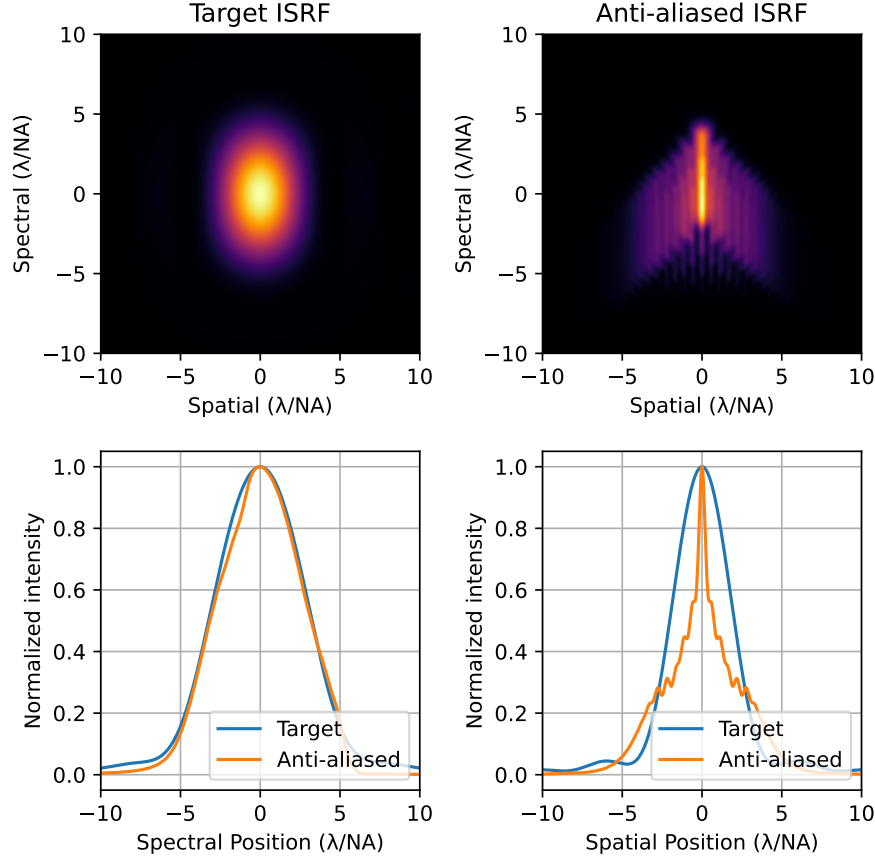


Figure 10. Comparison between the target ISRF, consisting of a PSF with $8\times$ smaller aperture and the ISRF engineered to be anti-aliased. Cross-section are shown in the spectral and spatial direction.

producing an ISRF that approximates the ideal anti-aliased response while maintaining the throughput of the full optical aperture.

4. IMPACT OF THE ENGINEERED PSF ON KEYSTONE-INDUCED RETRIEVAL ERRORS

We next evaluate the resulting reduction in keystone-induced retrieval errors on L2 level, where the AA-ISRF is compared with the diffraction-limited optical system introduced in Section 3. For this analysis we used the most favourable keystone distortion, configuration A, with a peak-valley amplitude of 3 detector pixels.

4.1 Artificial benchmark scene

The challenging benchmark scene introduced in Section 2 provides a systematic approach for evaluate the influence of aliasing. Figure 11 compares the retrieved methane concentration obtained using the diffraction-limited ISRF and the AA-ISRF. For the diffraction-limited instrument, the undersampled image produces characteristic oscillatory retrieval artifacts due to the albedo transitions. These artifacts originate from the interaction between the wavelength-dependent keystone distortion and spatial frequencies above the detector Nyquist frequency. As demonstrated in Section 2, these frequencies become aliased into the measured spectra and subsequently propagate into the Level-2 retrieval.

Replacing the diffraction-limited PSF by the engineered AA-ISRF strongly suppresses these artifacts. The retrieved methane field becomes considerably smoother for the artificial benchmark scene. The residual retrieval

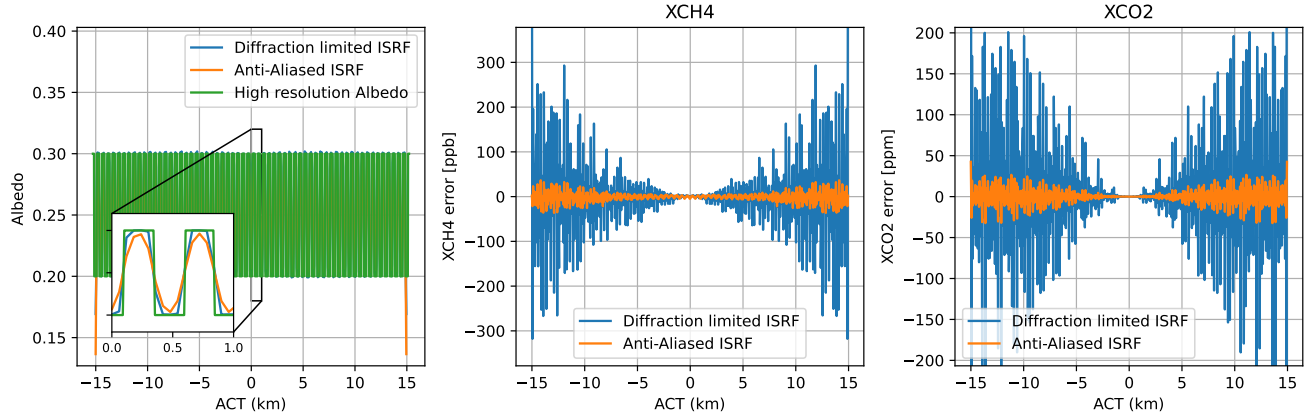


Figure 11. Retrieved methane and CO₂ for the diffraction limited ISRF and anti-aliased ISRF applied to the benchmark scene.

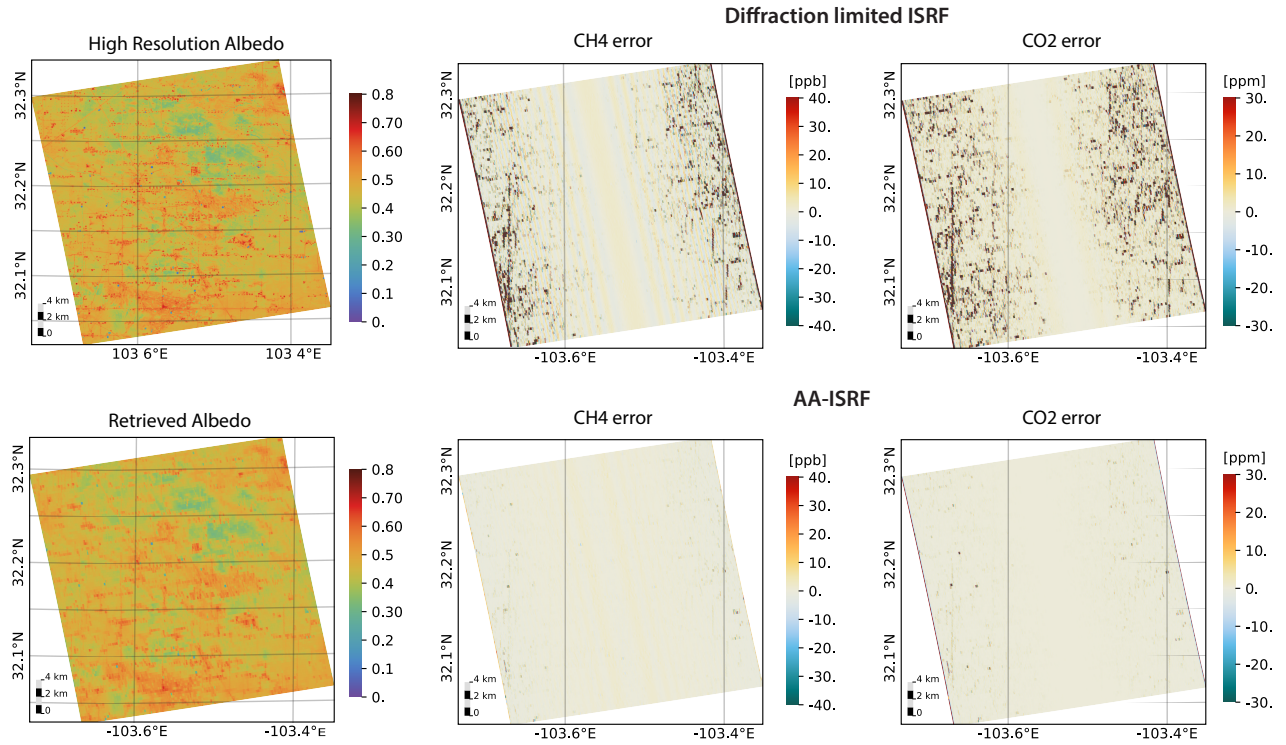


Figure 12. Retrieved methane and CO₂ for the diffraction limited ISRF and anti-aliased ISRF for the realistic scene.

error for methane, as quantified by the MAD was reduced by the AA-ISRF from 40.9 ppb (2.15%) to 6.8 ppb (0.36%), corresponding to an improvement of 6X. For CO₂ the improvement was even larger, where a 7.2-fold improvement is observed (47.8 ppm vs 6.6 ppm).

4.2 Realistic Earth-observation scenes

The performance of the engineered PSF is also evaluated using realistic Earth-observation imagery representative for a targeted trace-gas mission. For this we took a scene in the Permian Basin, New Mexico in the United States. Figure 12 compares the Level-2 errors. Similar to the artificial benchmark scene, the diffraction-limited instrument exhibits structured retrieval artifacts that correlate strongly with both the keystone error and the high-contrast surface features such as field boundaries, roads and urban structures as seen in the high resolution

albedo map. Not all of these features can be observed in the retrieved albedo map as the instrument is not capable of measuring sub-Nyquist spatial features.

The AA-ISRF substantially suppresses these spatially correlated artifacts. In particular, retrieval residuals around sharp albedo transitions are significantly reduced, while homogeneous regions remain essentially unchanged. In addition to the keystone induced error, an oscillation artifact can be observed in the ACT direction (west-east) near the track in the middle, especially for CH_4 . This error is caused by the spectral smile of the distortion, which is not further discussed in this work.

In the New Mexico scene, the corresponding retrieval precision improves from 4.0 ppb (0.21%) to 1.0 ppb (0.05%) in the case for CH_4 and improves for CO_2 more than 7-fold from 3.7 ppm (0.89%) to 0.89 ppm (0.12%).

5. DISCUSSION AND CONCLUSION

The presented results demonstrate that keystone-induced retrieval errors are fundamentally determined by the keystone error in combination with the spatial bandwidth of the optical system. Once the diffraction-limited PSF becomes substantially smaller than the detector pixel, the optical transfer function determines how strongly spatial scene structures are coupled into the measured spectra.

This observation has two important implications. First, conventional peak-to-valley keystone specifications provide only limited predictive power for retrieval performance (unless the peak-to-valley keystone is < 0.5 detector pixel). Instruments exhibiting different geometric distortions with similar peak-to-valley amplitudes can produce substantially different retrieval errors.

Second, engineering the point spread function provides an effective means of reducing aliasing while preserving the full collecting aperture. Rather than reducing the pupil diameter and sacrificing throughput, phase engineering redistributes optical energy such that spatial frequencies above the detector Nyquist frequency are naturally attenuated before sampling.

The proposed approach is generally applicable to diffraction-limited imaging spectrometers operating in undersampled conditions. The method is independent of the retrieval algorithm and can therefore be incorporated at the instrument design stage to improve the robustness of future Earth-observation missions.

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