

A freeform optics 12U CubeSat for Earth observation

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ABSTRACT

This paper presents the optical design of a hyperspectral 12U CubeSat dedicated to Earth observation. The optical payload occupies 8U of the total volume, featuring a two-mirror off-axis Schwarzschild configuration. Operating in a push-broom acquisition mode, the instrument achieves hyperspectral imaging through a compact design utilizing a linear variable filter positioned directly on top of a silicon CMOS image sensor. This configuration covers a spectral range from 400 nm to 900 nm, spanning the visible to near infrared spectrum.

A major constraint of CubeSats is their restricted volume, which limits the aperture size and reduces the incoming light flux, potentially compromising signal quality. To ensure the detected signal meets the required operational thresholds despite these aperture limitations, a dedicated radiometric model was developed to evaluate the system performance.

Furthermore, the limited physical volume of the payload can also affect and degrade the optical quality. To overcome these limitations, freeform optics based on Zernike polynomials were integrated into the design to minimize aberrations and optimize imaging performance.

Finally, a tolerance analysis was performed to validate the manufacturability of the proposed optical system.

Keywords: CubeSat, Earth observation, freeform optics, Zernike polynomials

1. INTRODUCTION

CubeSats have transformed space exploration, offering a faster and cheaper alternative to traditional large scale satellites for both Earth observation and Solar System exploration. The main advantages of these platforms include lower manufacturing costs, shorter development timelines, and reduced mass. Additionally, they can be deployed in satellite constellations or launched as secondary payloads, significantly improving access to space [1, 2].

The main drawback of CubeSats is their restricted volume, which causes the entrance pupil diameter to be less than 100 mm, limiting the incoming light flux and compromising the signal quality, mainly for hyperspectral applications. To verify that the collected signal meets the operational requirements, a dedicated radiometric model was developed.

The compact design of CubeSats can also limit the optical performance, making it challenging to satisfy mission specifications using standard optical elements. To overcome these limitations, freeform optics (FFOs) were introduced into the design. By definition, FFOs are optical surfaces that lack any axis of rotational invariance within or beyond the optical part. Unlike traditional rotationally symmetric optics, freeform surfaces offer additional degrees of freedom in both design and fabrication, enabling the realization of more complex surface geometries [3, 4, 5]. Moreover, they can correct optical aberrations over a wider field of view (FOV) and reduce the total number of optical elements by combining multiple optical functions into a single surface. As a result, FFOs enable system miniaturization, reducing both payload volume and mass, while maintaining high optical performance [6].

The goal of this study is to present the optical design of a hyperspectral CubeSat for Earth observation in the visible to near infrared spectral range, using FFOs. The telescope features a two-mirror off-axis Schwarzschild configuration.

The paper is structured as follows: Section 2 outlines the overall system architecture, including the radiometric model and the optical design limitations using conventional optics. Section 3 introduces the FFOs approach and the optimized optical performance achieved with freeform surfaces. Section 4 reports the tolerance analysis of the final design.

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2. OPTICAL DESIGN

System overview

The objective of this research is the development of a hyperspectral camera for Earth observation in the visible to near infrared spectral range (400 – 900 nm), designed for operation in a polar Low Earth Orbit (LEO). The optical payload is composed of a two-mirror system and a Linear Variable Filter (LVF) [7], positioned directly on top of a 2D CMOS sensor. The operational concept is illustrated in Figure 1 and relies on observing the Earth's surface using the push-broom acquisition technique with a LVF in front of a bidimensional sensor: as the satellite travels along its orbital trajectory, sequential 2D image frames are acquired by the hyperspectral camera. Along the cross-track (spatial) axis, pixels within a given row record spatial information at the same wavelength. Conversely, along the along-track (spectral/flight) direction, the scene is sampled across different wavelengths. By stitching together successive frames captured during flight, a hyperspectral data cube of the observed Earth surface is reconstructed [8].

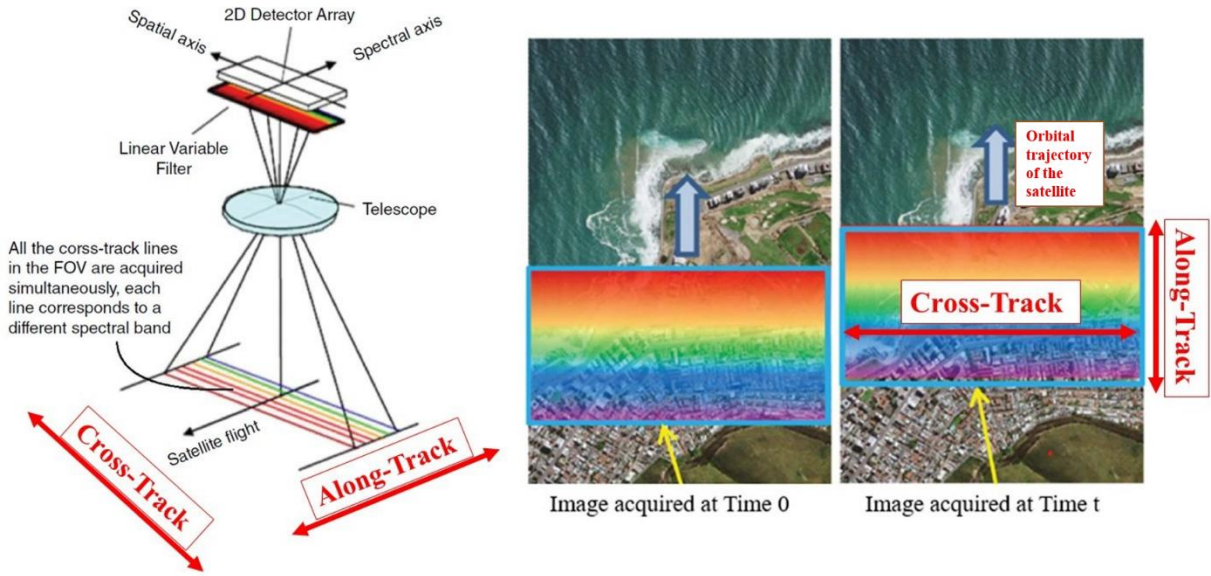


Figure 1: Illustration of the operational concept. As the satellite orbits the Earth, sequential 2D image frames are acquired by the hyperspectral camera. Along the cross-track axis, pixels within a given row record spatial information at the same wavelength. Conversely, along the along-track axis, the scene is sampled across different wavelengths. By stitching together successive frames captured during flight, a hyperspectral data cube of the observed Earth surface is reconstructed. Here, the cross-track axis refers to the spatial direction perpendicular to the satellite motion, while the along-track axis corresponds to the spectral direction along the flight path [8].

Radiometric model

In order to define the telescope parameters, a dedicated radiometric model was developed to evaluate the signal to noise ratio (SNR), defined as the ratio between the output signal recorded by each pixel and the total noise, which includes photon shot noise, readout noise, and dark current shot noise. The output signal is calculated by modelling the incoming radiation from the target object together with the optical properties of the system, such as mirror reflectivity, filter transmittance, and detector Quantum Efficiency (QE) [9].

A trade-off between the SNR and orbital parameters (satellite altitude, velocity, and Ground Sampling Distance (GSD)) was carefully evaluated to satisfy the initial requirements, reported in Table 1, and ensure an SNR greater than 20 across the entire spectral range.

Table 1: Instrument global requirements.

Spectral range	VIS – NIR
Dimension	12U (8U for the optical part)
GSD	< 50 m
Orbit	Polar LEO
SNR	> 20
FOV along the spectral direction	> 10°
MTF	>50% at Nyquist frequency

The results of the radiometric model are reported in Figure 2, which demonstrates that the SNR remains above 20 throughout the spectral range of interest. The resulting orbital and instrument parameters derived from the radiometric model are summarized in Table 2.

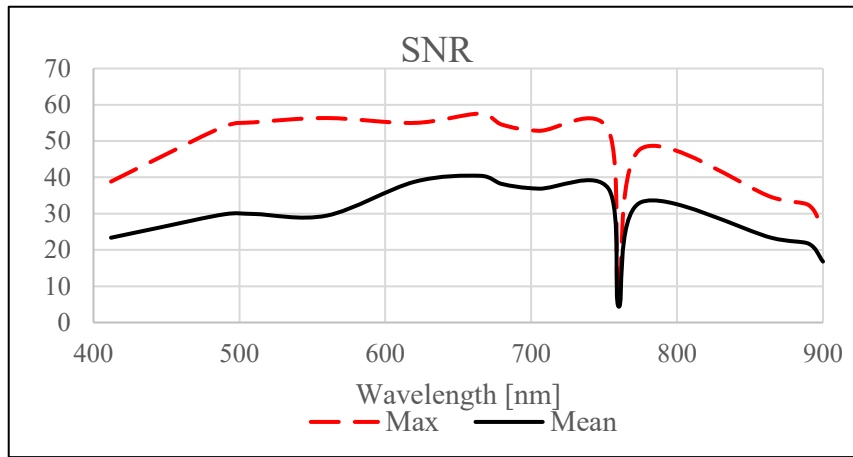


Figure 2: Results of the radiometric model. The SNR remains higher than 20 across the entire spectral range of interest. The dashed red line refers to the maximum Earth radiance, while the black straight line to the mean Earth radiance.

Table 2: Orbital and instrument parameters.

Parameter	Value
Resolution	2048 × 2048 px
Pixel pitch	5.5 μm
Orbital height	450 km
GSD	41.3 m
Dimension	12U (8U for the optical part)
Effective focal length	60 mm
Entrance pupil diameter	36 mm
F#	1.67
FOV along the spectral direction	12°
FOV along the spatial direction	4°
Spectral range	400 – 900 nm

Preliminary optical design

The selected optical layout is based on a two-mirror off-axis Schwarzschild configuration with a convex primary mirror (M1), defined as the aperture stop, and a concave secondary mirror (M2) [10]. In this first design, conventional aspheric mirrors have been considered.

While the nominal design parameters listed in Table 2 specify an entrance pupil diameter of 36 mm, calculated to guarantee an SNR > 20 at a 450 km orbital altitude, a scaled down version with a 24 mm aperture was designed to build a prototype and evaluate its performance in a laboratory environment. The main objective is to verify that experimental parameters, such as MTF and spot diagrams, correspond to the simulated values. The downscaled optical configuration, shown in Figure 3, remains structurally identical to the 36 mm baseline analysed by Senter et al. [7], with changes restricted only to the reduced physical dimensions of both mirrors. This downscaled design was adopted primarily for budgetary reasons and to shorten mirror fabrication times; in any case, the reduced photon flux resulting from the smaller aperture can be easily compensated for in a laboratory environment by increasing the exposure time, thereby preserving the target SNR. Hereafter, only the 24 mm entrance aperture layout will be discussed.

Figure 4 shows the optical layout illustrating the FOV along both the spatial and spectral directions, while Table 3 reports the specifications of the design.

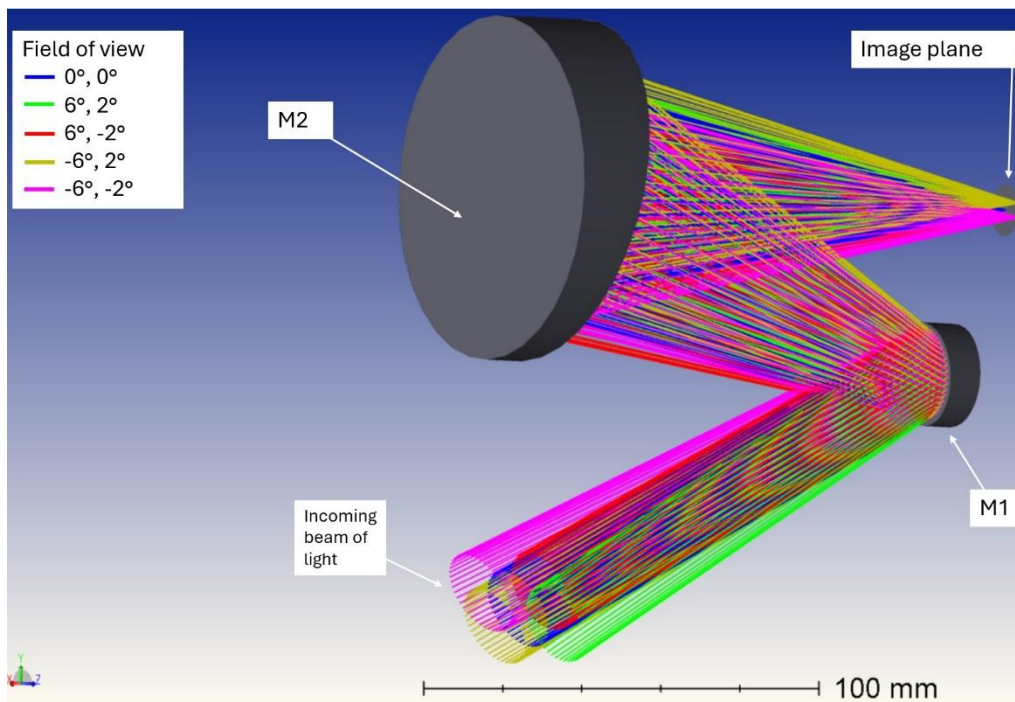


Figure 3: Optical layout of the off-axis Schwarzschild configuration. M1 and M2 refer to the primary and secondary mirrors, respectively. The FOV is $12^\circ \times 4^\circ$.

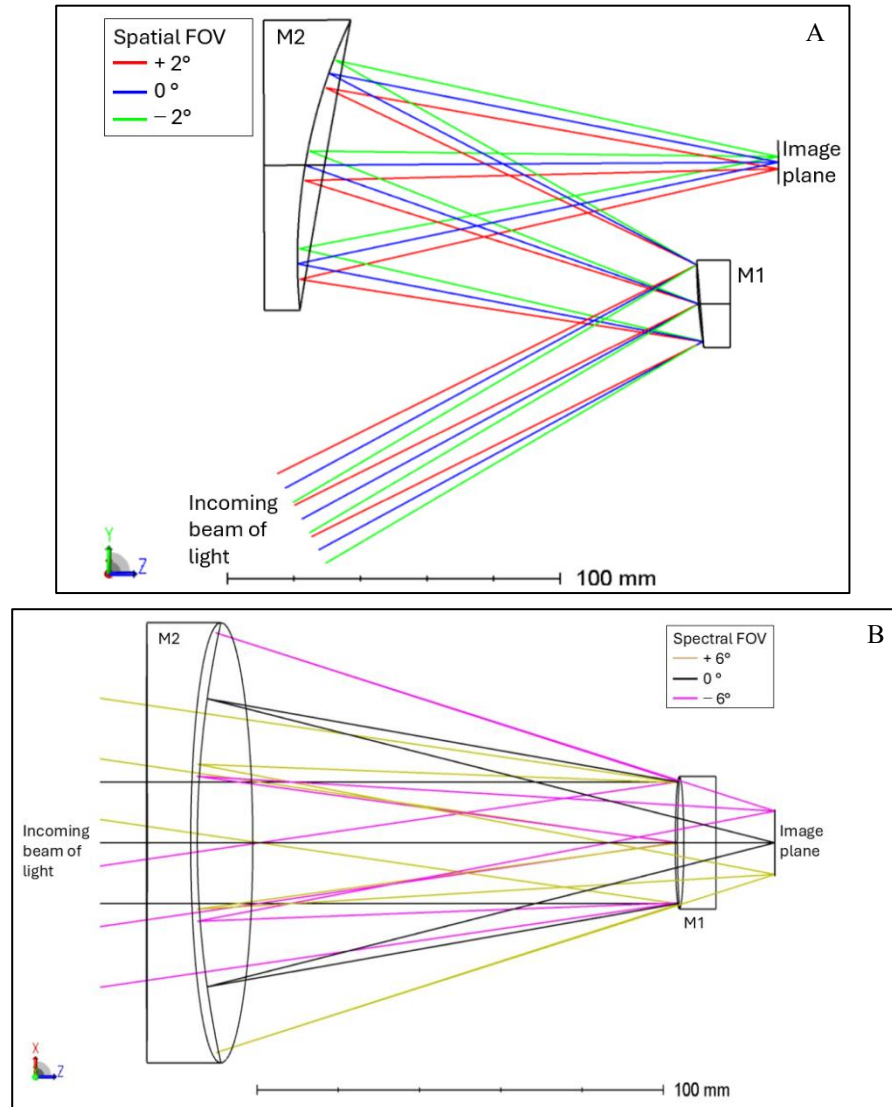


Figure 4: Optical layout of the off-axis two-mirror Schwarzschild configuration. (A) tangential plane, corresponding to the spatial direction on the sensor (B) sagittal plane, which corresponds to the spectral (along track) direction on the sensor.

Table 3: Design characteristics of the off-axis Schwarzschild configuration. The thickness refers to the distance between the vertexes of two consecutive surfaces: in this case to the distance between M1 and M2 (-120 mm), and M2 and the image plane ($+144.6$ mm).

	Radius of curvature [mm]	Thickness [mm]	Conic constant
M1 (Stop)	+169.7 (convex)	-120	+5.49
M2	+169.7 (concave)	+144.6	+0.18

The spot diagrams, optical MTF, and ensquared energy plots are presented in Figure 5. As can be observed, the optical performance is not satisfactory. The spot diagrams are larger than the detector pixel size ($5.5 \mu\text{m}$); the black square box in the spot diagram grid measures $20 \mu\text{m}$ in size. Additionally, the optical MTF falls below 50% at the Nyquist frequency (90 lp/mm) across a large portion of the FOV, while the ensquared energy within $5.5 \mu\text{m}$ remains below 80% over much of the field. This degraded image quality indicates that conventional optical surfaces are insufficient to compensate for

aberrations in this off-axis configuration. To overcome these limitations and correct the residual aberrations, FFOs were introduced into the design.

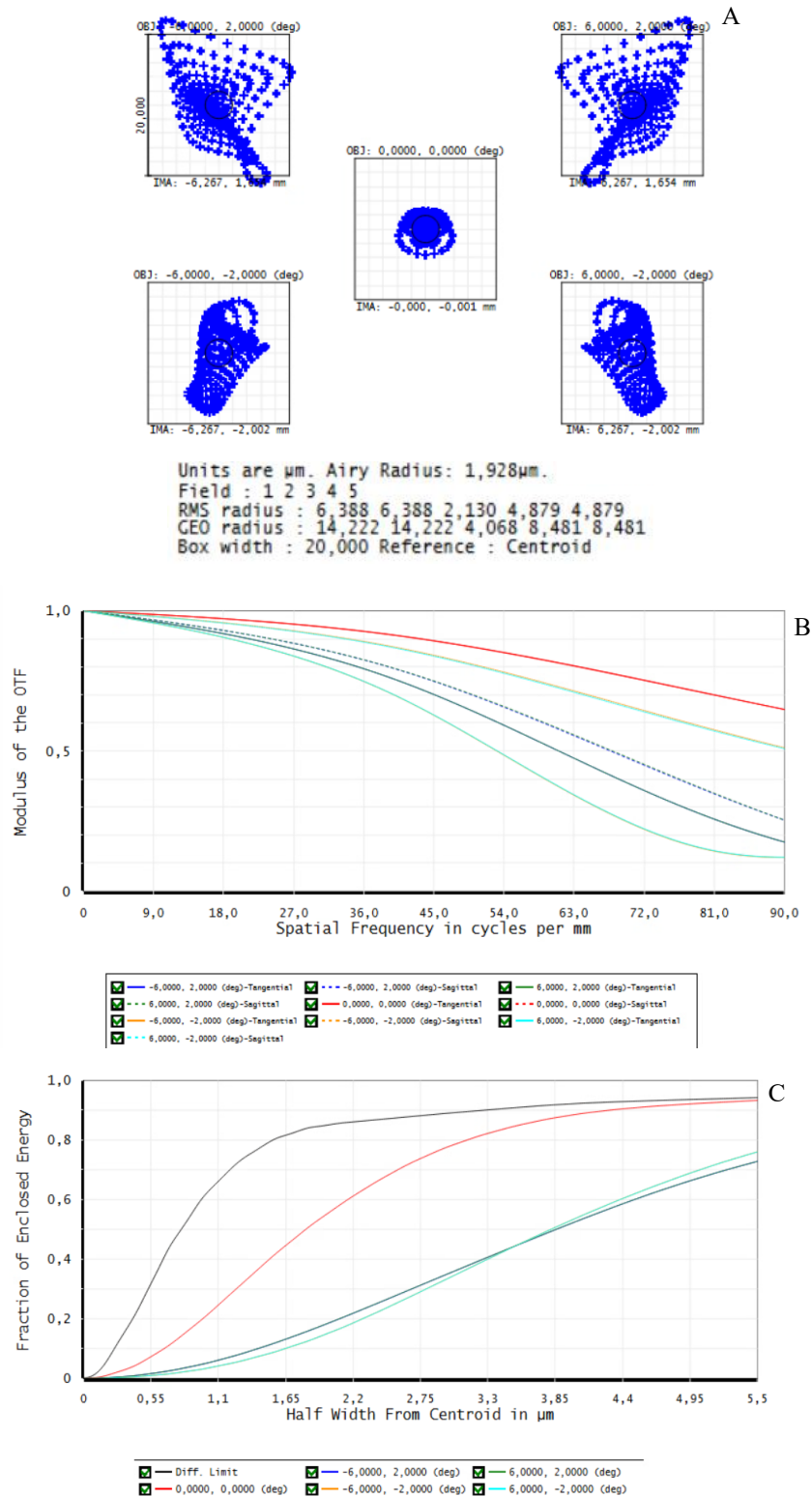


Figure 5: (A) Spot diagrams: the spots are larger than the pixel size of 5.5 μm . The black square box is 20 μm in size. The Airy radius refers to $\lambda = 633 \text{ nm}$. (B) optical MTF: lower than 50% at 90 lp/mm (Nyquist frequency) over large part of the FOV, calculated over the 400 – 900 nm. (C) Ensquared energy: less than 80% at 5.5 μm over much of the FOV, evaluated from 400 to 900 nm.

3. FREEFORM OPTICS

Theory of freeform optics

A freeform surface can be mathematically described by several different functions [11]; in this work, Zernike polynomials (hereafter referred to as *Zernikes*) using the FRINGE indexing convention were selected. The sag departure from the base surface varies as a function of both the radial coordinate ρ and the clocking angle θ , and the total surface sag $z(\rho, \theta)$ based on the Zernike expansion is expressed as:

$$z(\rho, \theta) = \frac{c\rho^2}{1 + \sqrt{1 - (1 + k)c^2\rho^2}} + \sum_{j=1}^J C_j Z_j(u, \theta)$$

where $\rho = \sqrt{x^2 + y^2}$ denotes the radial coordinate in a xyz Cartesian reference frame, k is the conic constant, c is the base surface curvature, $u = \frac{\rho}{\rho_{max}}$ is the normalized dimensionless radial coordinate, θ is the clocking angle measured counterclockwise from the x -axis, Z_j are the Zernike polynomials, C_j are the Zernike coefficients representing the weight of each polynomial, and j is the FRINGE index starting from 1 (where Z_1 corresponds to piston).

The first term of the equation represents the base conic surface, while the second term refers to the higher order Zernikes describing the freeform departure [12]. Figure 6 illustrates the first 15 FRINGE Zernike terms, with their corresponding aberration names specified in the caption.

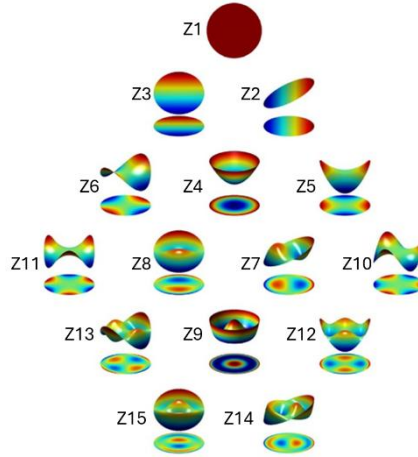


Figure 6: FRINGE Zernike polynomials up to 15th term. Z_1 piston, $Z_{2/3}$ tilt, Z_4 defocus, $Z_{5/6}$ astigmatism, $Z_{7/8}$ coma, Z_9 spherical, $Z_{10/11}$ trefoil, $Z_{12/13}$ secondary astigmatism, $Z_{14/15}$ secondary coma.

Zernikes were chosen because they form a complete set and are orthogonal in sag over the continuous unit circle. This orthogonality ensures that each surface is described by a unique set of coefficients without mutual cross talk or cancellation [13]. Consequently, the coefficient values provide direct physical insight into the surface shape and are directly related to the optical aberrations [14].

Nodal Aberration Theory (NAT) [15] provides an analytical framework to describe the aberration field behaviour in optical systems without rotational symmetry, such as the unobscured off-axis Schwarzschild design investigated in this work. In non rotationally symmetric systems, optical aberrations exhibit a multinodal field dependence rather than a single field centred node. To visualize and evaluate these complex field dependent behaviours across the entire FOV, Full Field Displays (FFDs) are used to illustrate both the magnitude and orientation of individual aberration contributions at discrete field points (an example is shown in Figure 7).

Combining NAT with FFD analysis makes it possible to understand the origin and field dependence of the aberrations when Zernikes are integrated into the optical layout [12]. Fuerschbach [16] demonstrated that applying a Zernike term to an optical surface located at the aperture stop introduces field constant aberrations across the entire FOV. Conversely, applying the same Zernike term to a surface positioned away from the stop generates field dependent aberration

contributions, causing the aberration magnitude and orientation to vary across different points of the FOV. Placing a higher order Zernike term away from the stop not only makes that term field dependent but also induces a combination of lower order aberration fields across the FOV.

Consequently, the joint application of NAT and FFDs enables a strategy to identify which specific Zernike polynomial terms should be integrated into the optical layout and where, in order to effectively target and minimize aberrations in the optical design. It is also important to consider the degeneracy constraint that must be fulfilled while integrating freeforms in the optical system [14].

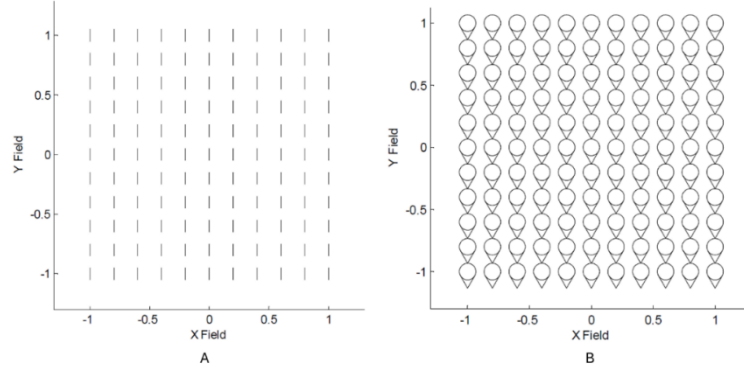


Figure 7: Full Field Display examples, illustrating the magnitude and orientation of individual aberration contributions across the FOV ($1^\circ \times 1^\circ$ in this example) at discrete field points (11×11 field matrix in this case). (A) Field constant astigmatism; (B) field constant coma.

Application of freeform optics into the off-axis Schwarzschild layout

Once the system was unobscured and fundamental parameters such as effective focal length, system volume constraints, and entrance pupil diameter were established, the integration of freeform optics began. The design process started from a first order layout composed entirely of spherical mirrors. Starting with spherical optical surfaces rather than conic mirrors was an intentional design choice: in the initial on-axis setup, rotationally symmetric aberrations were present; tilting the surfaces to achieve an unobscured design introduced off-axis aberrations that could naturally compensate for these preexisting rotational contributions due to spherical mirrors.

The FFDs corresponding to the initial layout with spherical mirrors are shown in Figure 8, displaying defocus, astigmatism, and coma. The field plots revealed that astigmatism and coma were the primary limiting aberration across the FOV. To address this, Z_5 (astigmatism) and Z_8 (coma) were added to both mirrors. Following this, conic constants were introduced to the base surfaces, yielding a substantial overall reduction of the aberrations. Finally, to refine system performance and satisfy the target threshold of MTF $> 50\%$ at Nyquist frequency, higher order terms (Z_{11} , Z_{12} , and Z_{15}) were integrated into both mirrors, while the back focal length and surface tilts were adjusted. The FFDs of the final optimized design are illustrated in Figure 9.

In summary, Zernike terms up to the 15th FRINGE index were considered, specifically selecting those associated with the dominant aberrations: Z_5 , Z_8 , Z_{11} , Z_{12} , and Z_{15} . Higher order terms were excluded due to their negligible contributions. Both mirrors were set as freeform to increase the available degrees of freedom and to distribute the aberration correction across both elements, thereby reducing the sag departure on each individual mirror. Table 4 lists the Zernike FRINGE sag coefficient values (in mm) extracted from the Lens Data Editor for both mirrors, along with their corresponding polynomial functions.

Table 4: Zernike FRINGE sag coefficients C_j of both mirrors, along with the Zernike Polynomials Z_j .

	Zernike Coefficient C_j of M1 [mm]	Zernike Coefficient C_j of M2 [mm]	Zernike Polynomial Z_j	Name
Z5	$-1.00382 \cdot 10^{-3}$	$6.60024 \cdot 10^{-4}$	$u^2 * \cos(2\theta)$	Astigmatism
Z8	$-7.22415 \cdot 10^{-4}$	$-1.67183 \cdot 10^{-4}$	$(3u^2 - 2) * u * \sin\theta$	Coma
Z11	$-9.59521 \cdot 10^{-5}$	$8.69637 \cdot 10^{-4}$	$u^3 * \sin(3\theta)$	Trefoil / Elliptical coma
Z12	$1.28043 \cdot 10^{-3}$	$-1.87384 \cdot 10^{-3}$	$(4u^2 - 3) * u^2 * \cos(2\theta)$	Secondary astigmatism
Z15	$-4.80706 \cdot 10^{-4}$	$-1.28110 \cdot 10^{-4}$	$(10u^4 - 12u^2 + 3) * u * \sin(\theta)$	Secondary coma

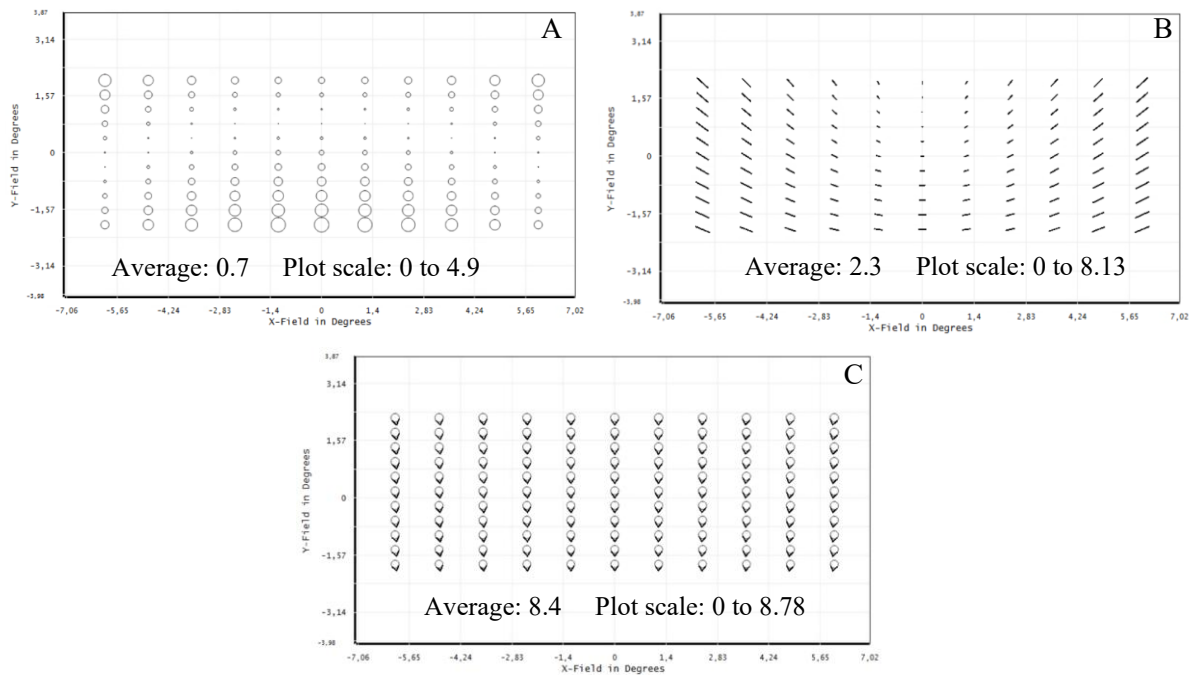


Figure 8: FFDs of the optical system prior to adding Zernike terms. (A) defocus, (B) astigmatism, (C) coma.

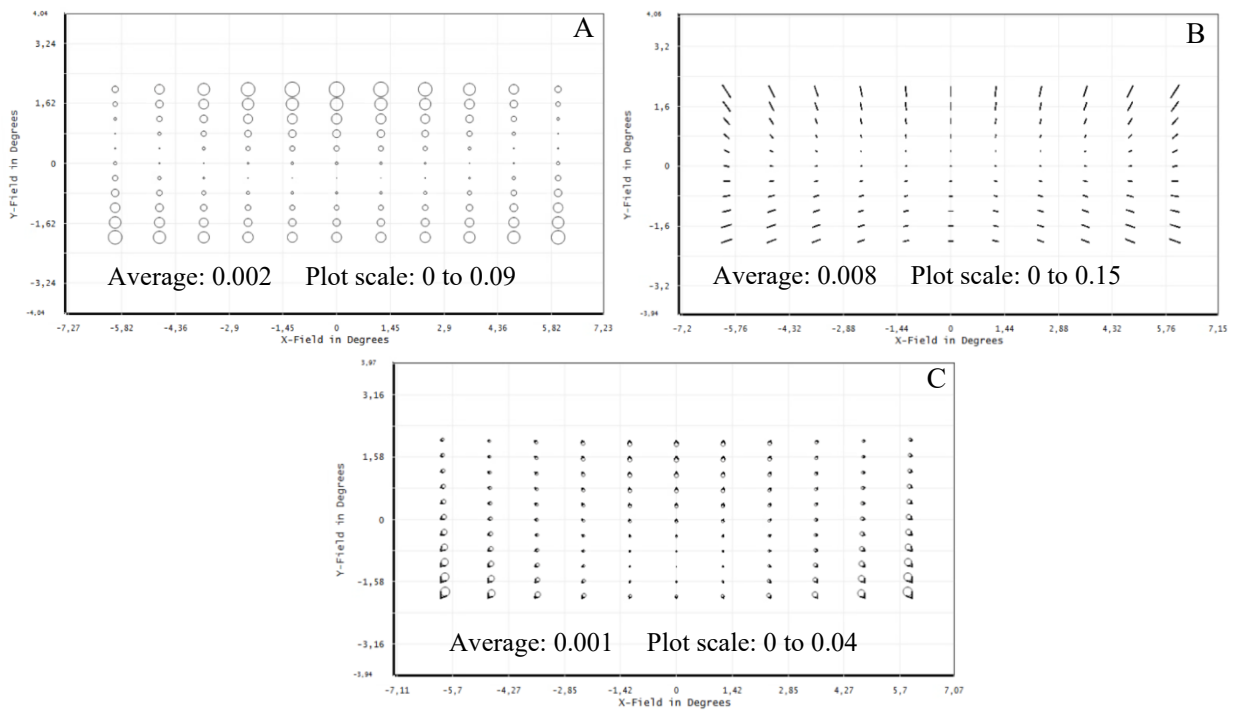


Figure 9: FFDs after optimization with freeform optics. (A) defocus, (B) astigmatism, (C) coma.

Results

The spot diagrams, optical MTF, and ensquared energy plots for the optimized freeform design are presented in Figure 10. It is important to note that while the optimization was performed across the entire FOV, only the extreme and most aberrated field points ($\pm 2^\circ$, $\pm 6^\circ$) are shown for clarity. In these spot diagrams, the black square box corresponds to the detector pixel size of $5.5\ \mu\text{m}$, clearly demonstrating that the spots are well contained within the individual pixel area. Additionally, the optical MTF exceeds 60% at 90 lp/mm, while the ensquared energy within a $5.5\ \mu\text{m}$ size remains above 90% across the field. Consequently, the introduction of freeform surfaces has successfully enhanced the optical performance, achieving pixel limited, high quality imaging across the entire FOV.

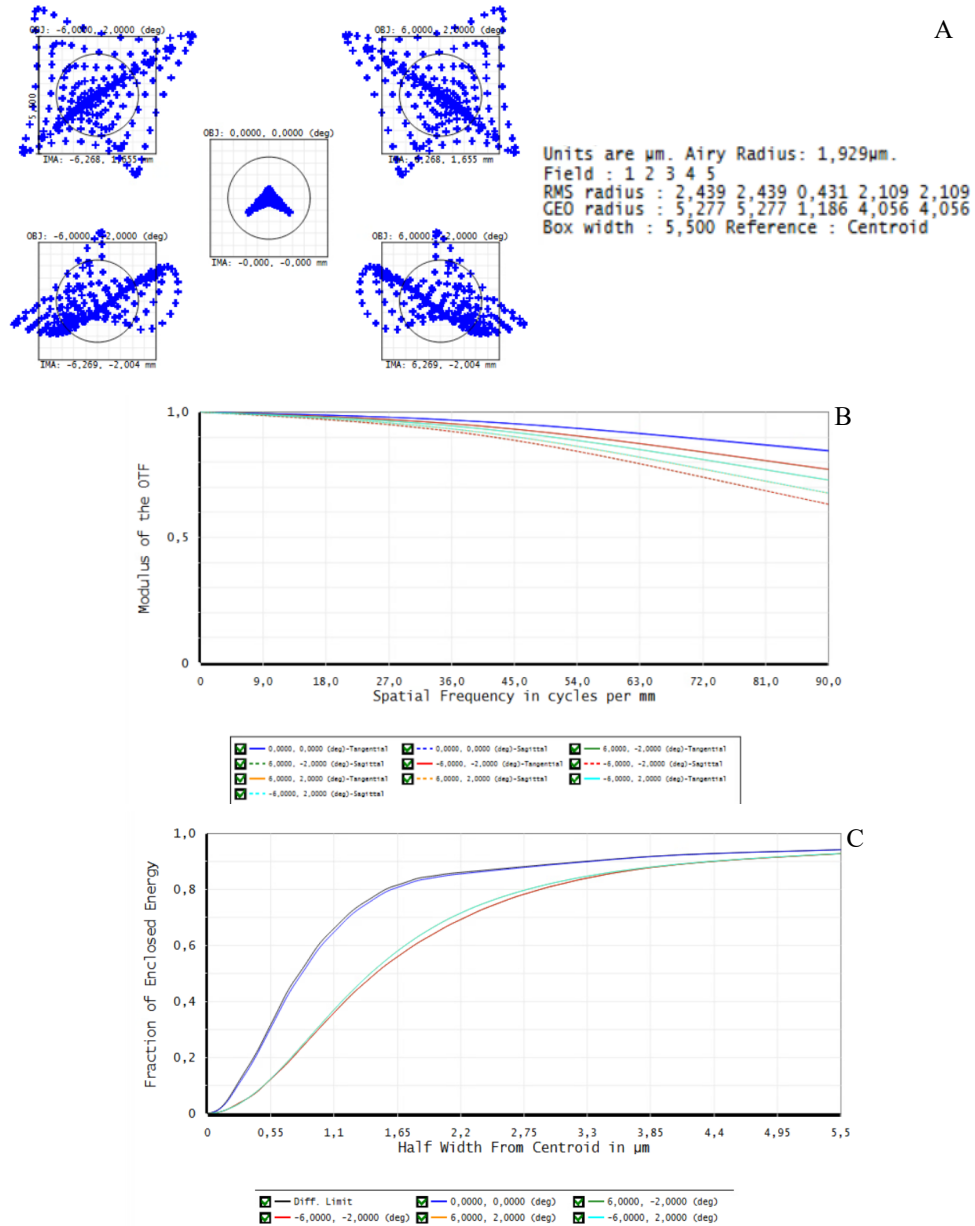


Figure 10: (A) Spot diagrams: the spots are contained within the pixel size of $5.5\ \mu\text{m}$. The black square box is $5.5\ \mu\text{m}$ in size. The Airy radius refers to $\lambda = 633\ \text{nm}$. (B) optical MTF: higher than 60% at 90 lp/mm, calculated over the 400 – 900 nm. (C) Ensquared energy: higher than 90% at $5.5\ \mu\text{m}$, evaluated from 400 to 900 nm.

4. TOLERANCE ANALYSIS

A tolerance analysis was performed to evaluate potential physical deviations from the nominal configuration that may occur during the manufacturing and alignment of the optical components, using the average RMS spot radius across the entire FOV as the evaluation criterion.

First, a sensitivity analysis was conducted to determine the impact of individual parameter variations from their nominal values on the performance criterion. Subsequently, a Monte Carlo simulation with 1000 trials evaluated the cumulative effect of all applicable tolerances by randomly combining parameter variations within their defined boundaries, thereby providing a realistic prediction of overall system performance. The back focal length and detector tip-tilt were designated as compensators to absorb accumulated deviations from the nominal configuration.

The tolerance parameters analyzed for both mirrors are summarized in Table 5 and include: (a) mirror manufacturing errors, and (b) alignment errors, specifically (i) lateral decenter along x and y, (ii) tip-tilt around x and y, (iii) vertex spacing between M1 and M2, and (iv) the clocking angle. While the clocking angle is typically neglected for conventional, rotationally symmetric surfaces, it becomes critical for freeform optics. Since their surface profiles are described by the Zernikes, which directly depend on the clocking angle θ , the rotational orientation around their z-axis must be precisely controlled during assembly.

Mirror manufacturing tolerances, particularly surface irregularities, were modelled by directly tolerancing the Zernike FRINGE polynomial coefficients via the TEXI operand [17].

Table 5: Tolerance analysis results.

Tolerances	
M1	
Radius	± 0.1 mm
Conic value	± 0.01
X decenter	± 10 μ m
Y decenter	± 10 μ m
X tilt	$\pm 0.0167^\circ$
Y tilt	$\pm 0.0167^\circ$
Z clocking	$\pm 0.0167^\circ$
Surface Irregularity	± 0.75 Fringes
M2	
Radius	± 0.1 mm
Conic value	± 0.001
X decenter	± 10 μ m
Y decenter	± 10 μ m
X tilt	$\pm 0.008^\circ$
Y tilt	$\pm 0.008^\circ$
Z clocking	$\pm 0.0167^\circ$
Surface Irregularity	± 0.75 Fringes
Vertex Spacing M1-M2	± 200 μ m
Compensators	
Back focal length	± 1 mm
Detector X tilt	$\pm 0.06^\circ$
Detector Y tilt	$\pm 0.06^\circ$

5. CONCLUSIONS

The optical design of a hyperspectral camera for Earth observation in the 400 – 900 nm spectral range was developed for a 12U CubeSat platform. By combining the push-broom acquisition technique with a LVF integrated directly on top of the detector, a full hyperspectral data cube can be reconstructed. Radiometric analysis confirmed that the system maintains a $SNR > 20$ across the entire spectral range under LEO operational conditions.

While the off-axis Schwarzschild configuration utilizing conventional optical surfaces exhibited unacceptable performance due to severe residual aberrations, the introduction of FFOs significantly improved system performance, achieving pixel limited, high quality imaging across the entire FOV. Additionally, a tolerance analysis was performed to evaluate manufacturing tolerances and alignment sensitivities, guaranteeing the physical buildability of the design. These results validate FFOs as a crucial enabling technology for high-performance, highly miniaturized space payloads.

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