

BELL-TEST SIGNIFICANCE FOR QUANTITATIVE ASSESSMENT OF SPACE-DISTRIBUTED ENTANGLEMENT FOR QUANTUM INFORMATION NETWORKS AND ENTANGLEMENT-BASED QKD

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ABSTRACT

Space-based quantum communications are expected to play a central role in overcoming the distance limitations of terrestrial quantum links and in enabling future large-scale quantum networks [3]. In this context, satellite-to-ground entanglement distribution is a promising resource for quantum services such as quantum information networks (QIN) and entanglement-based quantum key distribution (EB-QKD). Yet, beyond demonstrating entanglement distribution itself, a key challenge is to quantitatively assess the quality of the distributed resource and determine whether it is sufficient for such applications under realistic operating conditions.

In this paper, we propose a quantitative approach to assess space-distributed entanglement based on the statistical significance of Bell inequality violations under a high losses scenario, expressed in number of sigmas, together with the probability that such a significance level could have arisen by chance rather than reflecting the actual performances of the system due to the high losses regime. The objective is not only to evaluate link performance at the physical level, but also to characterize the usefulness of the generated entanglement for application-oriented quantum services. Bell-test significance is used here as a key performance indicator of entanglement quality and confidence, allowing the distributed resource to be interpreted in terms of its operational relevance for QIN or EB-QKD purposes.

To this end, we develop a system-level analysis that incorporates environmental and operational constraints affecting satellite-to-ground optical links, including optical ground station availability, meteorological conditions and turbulence contributions. These factors are integrated into a statistical model used to estimate entanglement-distribution success over repeated satellite passes and across different deployment configurations. Two representative scenarios are examined: a demonstration-oriented case based on repeated-orbit operations, and a more operationally relevant case involving a low-Earth orbit (LEO) constellation.

The resulting estimates are then mapped to the requirements and perspectives of QIN and EB-QKD, showing how Bell-test significance and success probability together provide a meaningful basis for judging service feasibility. Possible performance improvements through temporal and frequency multiplexing are also discussed. Overall, this work provides a practical characterization framework for linking the measured quality of space-distributed entanglement to its effective exploitation in quantum applications.

Keywords: Satellite-based Quantum Communications, Quantum Information Networks, Quantum Key Distribution, Optical Communications, Telecom

1. INTRODUCTION

Entanglement distribution is a key capability for quantum information networks (QIN) and entanglement-based quantum key distribution (EB-QKD). Since terrestrial quantum links are strongly limited by losses and cannot be amplified, satellite-based links are a promising solution for extending entanglement distribution to continental and global scales [1, 2].

For space missions, however, demonstrating entanglement distribution on average is not sufficient. Because satellite quantum links operate in a high-loss regime, the number of detected coincidences is limited and Bell tests are affected by finite-statistics fluctuations. As a result, the relevant question is not only whether a Bell inequality is violated on average, but whether a given realization achieves a required confidence level.

In this paper, we assess space-distributed entanglement using two complementary metrics: **Bell-test significance**, expressed in standard deviations, and **success probability**, defined as the probability of reaching a target significance threshold in a single experiment. The first metric quantifies the credibility of the observed Bell violation, while the second quantifies operational robustness.

We apply this framework to two representative satellite scenarios: a single-satellite Sun-synchronous demonstration case and a four-satellite low-Earth-orbit constellation. End-to-end simulations include orbital access, optical ground-station availability, weather, turbulence and finite observation time. The goal is to evaluate whether distributed entanglement is not only observable, but operationally useful for QIN and EB-QKD.

2. PERFORMANCE METRICS FOR HIGH-LOSS SPACE ENTANGLEMENT DISTRIBUTION

In the context of QKD, the secure key rate is very often proposed as a key performance metric. However, this metric encompasses many QKD protocol assumptions, e.g. for the sifting, error correction and for the security amplification. Therefore, the assessment of entanglement requires a more primitive figure of merit focusing only on the measurement correlations, as used by A. Aspect *et al* [4]. The objective for space-based entanglement distribution is to demonstrate with a finite observation time (dual visibilities of the satellite from the two ground stations) that entanglement has actually been distributed. The natural physical criterion for the entanglement assessment is the violation of a Bell inequality, typically in its CHSH form.

A Bell violation provides a direct operational signature of non-local correlations and is therefore a relevant indicator that the quantum link has succeeded.

However, Bell violation is not a deterministic quantity obtained from a single event. It is inferred from a finite set of coincidence measurements acquired in several polarization bases. As a consequence, the measured Bell parameter is affected by statistical uncertainty. Even if the underlying physical state is entangled, finite sampling and experimental noise can produce fluctuations in the estimated correlations. Conversely, an apparent violation may arise from statistical fluctuations if the sample is too small. For this reason, the Bell result must be expressed together with a statistical confidence metric. In this work, we use the confidence level of the Bell violation, expressed in units of standard deviation, denoted by σ . In practice, the requirement can be written as a condition of the form $S - n\sigma > 2$, where S is the measured CHSH Bell parameter and 2 is the local hidden-variable bound. The value of σ therefore quantifies how robustly the observed Bell violation excludes a local hidden-variable model. This metric captures the **scientific significance** of the result [4].

The confidence level increases with the number of detected coincidences. As more samples are accumulated, the uncertainty on the estimated correlations decreases and the Bell violation becomes statistically more reliable. In a satellite scenario, this means that the confidence level generally grows over time during one or several satellite passes. A mission can therefore be characterized by quantities such as the number of passes required to reach a target confidence threshold, for example 95% confidence or a 2σ violation.

Yet this metric alone is not sufficient for system design. The reason is that the Bell test is implemented through simultaneous measurements in four basis configurations, typically using 50:50 beam splitters in order to avoid systematic bias due to time-varying channel conditions. This architecture is important because sequential measurements of the four CHSH settings would make the Bell parameter sensitive to changes in attenuation, background noise, and pointing conditions during the pass. By measuring all settings simultaneously, one ensures that the four correlations are sampled under the same instantaneous link conditions. The price to pay is that the total set of coincidences is randomly distributed among the four basis combinations. For an infinite number of events, each configuration receives 25% of the coincidences on average (theoretical values of **Figure 1**). For a finite number of events, however, this partition fluctuates significantly.

These fluctuations are especially important when the number of coincidences is small. Two experiments performed with the same physical parameters, the same QBER (Quantum Bit Error Rate), and the same expected Bell value can lead to

different outcomes simply because the coincidence counts are not distributed in exactly the same way among the measurement settings and detector outcomes. One realization may yield a Bell violation above the required threshold, while another may fall below it. In other words, a system that is expected to achieve a 2σ violation **on average** does not guarantee that a single experimental run will actually do so.

Figure 1 shows that, for identical experimental conditions (200 coincidences and a QBER of 6%), statistical fluctuations in how coincidences are distributed among the different measurement bases can significantly affect the observed Bell inequality violation. In one realization, the experiment achieves a 2.1σ violation, whereas in another, it reaches only 1.1σ .

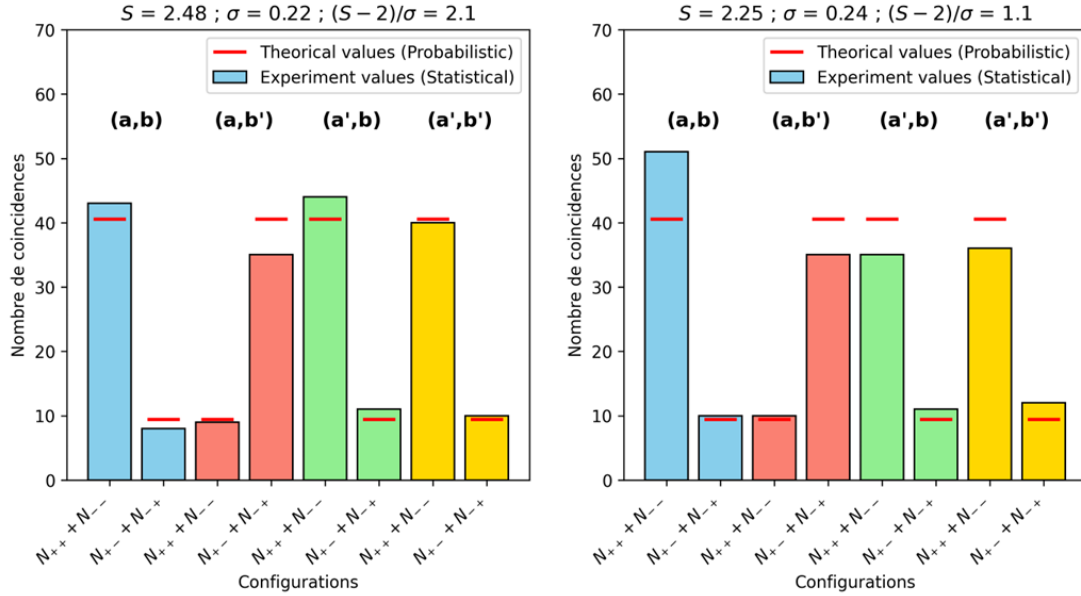


Figure 1: Statistical fluctuation of the coincidences repartition for two identical and independent experiments (200 coincidences, QBER = 6%)

This observation motivates the introduction of a second metric: the **probability of success**. While the confidence level σ characterizes the statistical significance of a Bell violation once measurement data have been acquired, the probability of success quantifies the likelihood that a single realization of the experiment will reach the required confidence threshold. It is therefore a measure of **operational robustness** rather than physical significance alone. In a mission context, this distinction is essential. A satellite pass is a finite and costly opportunity; what matters is not only the average expected performance, but also the probability that the pass will actually produce a valid Bell violation.

The two metrics are thus strongly interconnected but address different questions. The confidence level asks: *given the collected data, how strongly can a local hidden-variable explanation be rejected?* The probability of success asks: *given the system design and the expected channel conditions, what is the probability that one experimental run will produce data sufficient to meet the required confidence level?* The first metric validates the physics of the result; the second validates the engineering of the mission.

Their interplay has major consequences for system sizing. If one only considers the average expected Bell violation, the required number of coincidences may appear relatively modest. However, once the randomness of finite sampling is taken into account, the number of coincidences needed to ensure a high probability of success can become much larger. For example, **Figure 2** shows that, for a QBER of 6%, about 200 coincidences are sufficient to obtain an average Bell violation above 2σ . However, due to statistical fluctuations in the distribution of coincidences among the different measurement bases, the probability of actually observing a violation of at least 2σ in a single experimental run is only about 60%.

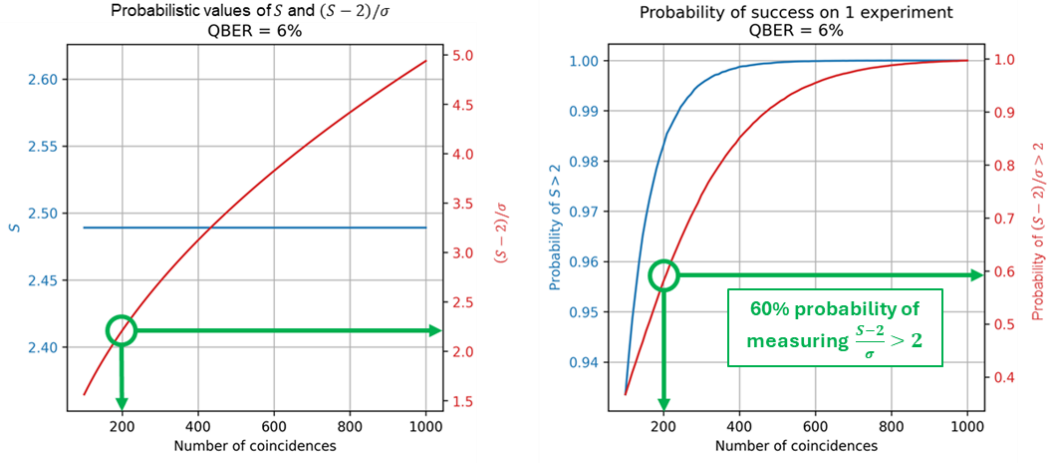


Figure 2: Success probability of one experiment

Requiring instead a 95% probability of observing a violation of a least 2σ may increase the requirement to roughly 600 coincidences. This difference illustrates that designing from average values alone leads to an underestimation of the resources actually needed to secure mission success.

The interest of the probability of success is therefore twofold. First, it converts a purely statistical Bell-violation criterion into an actionable engineering requirement. Second, it reveals the degree of over-dimensioning that is necessary in a realistic mission, where finite-duration passes, link variability and noise cannot be ignored. In practical terms, it informs decisions on source brightness, telescope size, detector performance, synchronization module performance and the number of usable satellite passes. **Without this metric, a mission could be declared feasible on paper because it reaches the target confidence level in expectation, while still having a substantial chance of failure in practice.**

For this reason, both metrics must be used together. The Bell-violation confidence level ensures that the observed result is scientifically meaningful and statistically credible. The probability of success ensures that this result is realistically achievable with a sufficiently high reliability in a single mission opportunity. Taken together, they provide a consistent framework for evaluating and dimensioning space-based entanglement distribution experiments under finite-statistics conditions.

In the following, Bell-violation confidence level is used as the physics validation metric and success probability as the operational metric for system sizing. These metrics are then applied in Section 3 to define the suitable system architecture that allow to distribute the entanglement resource reducing the potential chance of mission failure. This judicious definition prevents privileging a system that, on paper, meets the mission conditions (average expected performance) but, by ignoring the statistical fluctuations characteristic of the high-loss regime, results in mission failure during its operational execution. Moreover, these two metrics are discussed for two different mission scenarios.

3. SYSTEM ARCHITECTURE AND MISSION SCENARIOS

In this section we consider a system that enables the distribution of entanglement from space for a high-loss regime, taking into account all mission constraints (translated as Bell-violation confidence level and probability of success). In particular, the system operates in the C-band and the wavelength selected for the quantum link is 1550 nm. The preferred encoding variable is polarization. The two optical heads used are medium-sized OISL (optical inter-satellite link) units and thanks to their alt-azimuth pointing mechanism they can simultaneously point to the two optical ground stations located at two geographically distant sites in France (Toulouse and Nice). Night-time operations are privileged to limit

atmospheric stray light and reduce turbulence-related losses. The following schematic view allows to focus on the high-level architecture of the studied payload sub-system:

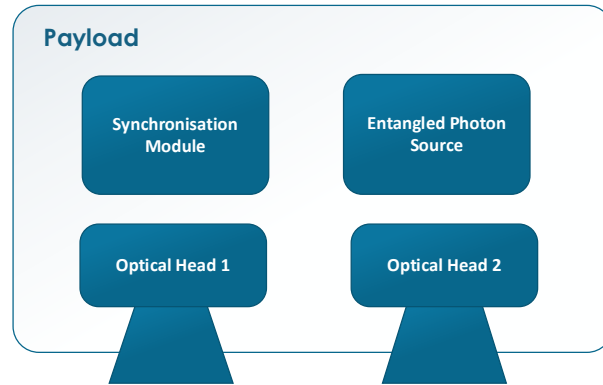


Figure 3 : High-level Payload Architecture

The payload of the space segment considered consists of two optical inter-satellite link type optical heads, an entangled Photon Source and a synchronization module. The entangled-photon source emits pairs of photons in an entangled state. Both photons are routed to the two optical heads and, thanks to them, are transmitted through the free space channel to the two ground telescopes that will receive them. A fundamental element is the payload synchronization module: its role is to ensure the most accurate possible reconstruction of the temporal timeline of the photon sequence sent to the two receivers, so that coincident events can be identified from the raw detections at the two ground sites. The precision of this module represents an essential degree of freedom that can be exploited to improve the end-to-end performance of an entanglement-distribution system even if it is commonly - but incorrectly - assumed that the size of the onboard and ground telescopes and the source rate are the only levers for improvement.

This section describes two specific mission scenarios for the space segment distributing entanglement from low Earth orbit (LEO) satellites. Both of them are based on the space segment high-level definition provided in the previous figure.

The first mission scenario is a technology-demonstration system based on a single satellite in a Sun-synchronous orbit. This configuration is chosen to provide a repeatable, predictable orbit with a defined revisit cadence, simplifying operations and maximizing joint-visibility communication windows. A Sun-synchronous orbit also tends to homogenize atmospheric conditions of the two ground sites, reducing sensitivity to local cloud variability and improving operational robustness. By adopting conventional, state-of-the-art orbit compatible with ride-share launches, the design avoids imposing unusual launcher constraints. Finally, the repetitive passes minimize uncontrolled events and enable comprehensive statistical characterization of environmental factors (e.g., turbulence, cloud cover, albedo), while the wide latitude coverage of the orbit facilitates collaboration with potential diverse ground facilities partners.

The second mission scenario, which follows the demonstration phase, is an operational system consisting of a constellation of four satellites in inclined orbits. The access to multiple satellites shifts the objective from optimizing individual passes to maximizing the globally distributed entanglement resource. The multi-satellite architecture also increases redundant access opportunities to the two optical ground stations. The constellation design and operational parameters are discussed in the following section. It is important to note that this implementation does not seek to demonstrate the maximum capacity of a LEO constellation implementation since achievable performance depends primarily on the spatial arrangement of the optical ground stations and the specific service-level requirements of the potential users.

In **Figure 4** the two mission scenarios of the space segment are shown simultaneously: the light blue orbit represents the Sun-synchronous satellite with quasi-polar inclination angle while the orange orbits portray the LEO constellation. The two white dots over France illustrate the optical ground stations located in Toulouse and Nice.

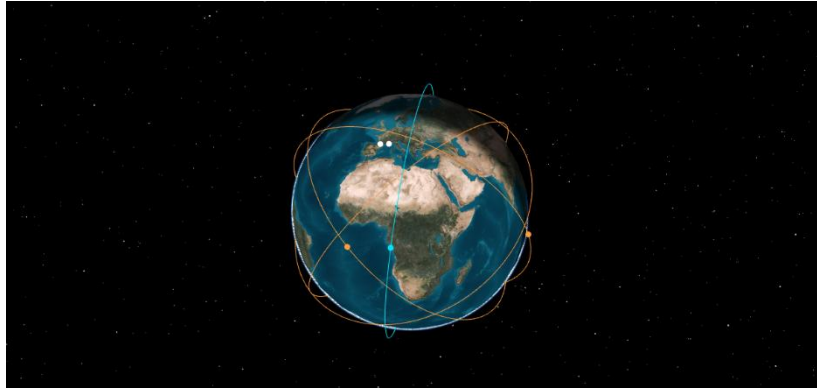


Figure 4 : 3D orbits view

The selected orbital parameters of the mission scenarios are defined in the following table:

Table 1 : Mission scenarios parameters

Mission scenario	Orbit type	Number of satellites	Altitude	Inclination	RAAN
Demonstration	Repeating-trace Sun-synchronous	Single satellite	~550 km	96 deg	Chosen to maximise dual visibility access
Operational	Inclined low Earth orbit	4 satellites	~550 km	48 deg	From 0 deg to 270 deg

The orbital altitude is chosen to minimize the distance between the emitter (the satellite) and the optical ground stations; however, a compromise must be reached to avoid lowering the altitude so much that simultaneous visibility windows of the two stations are lost and atmospheric drag excessively shortens the orbital lifetime.

For the Sun-synchronous orbit, the altitude and the right ascension of the ascending node (RAAN) are selected to guarantee repetition of the ground track over the two optical ground stations and to optimise visibility duration.

The constellation of four inclined low-Earth-orbit satellites is implemented by placing each satellite in a distinct orbital plane, offsetting their RAANs by 90° relative to one another. In this way the individual satellite accesses complement each other optimally, avoiding extended periods without any available geometric line-of-sight visibility. The inclination is chosen to maximize the duration of each satellite's visibility with respect to the geographic position of the optical ground stations.

Figure 5 depicts the orbit trajectory (light blue or orange) and the accesses over multiple days of mission (white lines when the satellite is visible from at least one ground site at an elevation angle over 30 degrees). The Sun-Synchronous satellite visibilities are repetitive and converge in two constant traces (one for the day-light pass and the second for the night pass) while LEO constellation non repetitive accesses integrated over a long period generate a large white spot for the two stations.

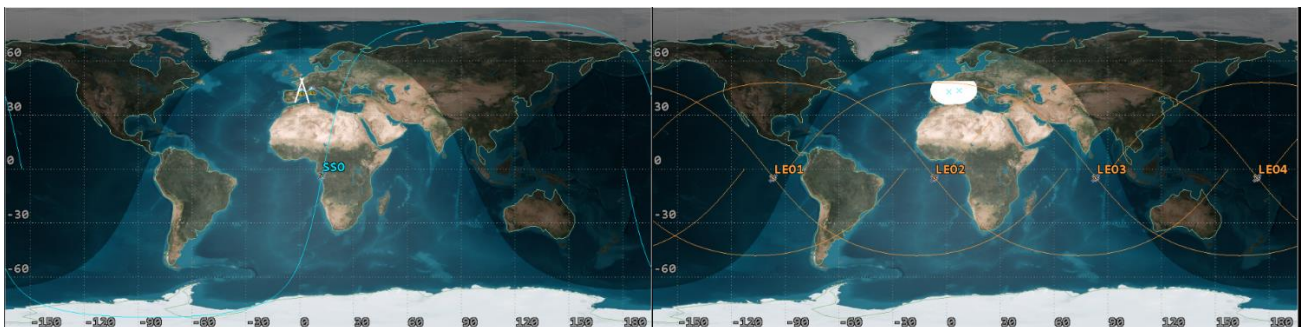


Figure 5: Ground-tracks and accesses of the two mission scenarios

The timeline of the satellite accesses (blue vertical lines) is presented in the following schedule chronogram (**Figure 6**). The LEO constellation satellites accesses allow to have no long period without availabilities of the space segment while the SSO satellite accesses exhibit the constant frequency of availabilities offered by this particular orbit.

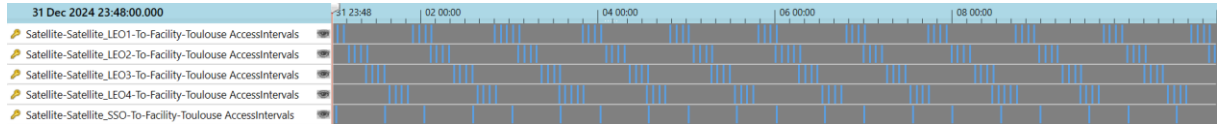


Figure 6: Accesses schedule of the two mission scenarios

The objective of the system presented for the second mission scenario is not to assess the maximum resource distribution capacity of a low Earth orbit constellation. In fact, the constellation studied in this article should be regarded as an "elementary" constellation that allows studying the impact of multiple satellites with inclined orbits, reducing the impact of cloud coverage by distributing potential visibility throughout the entire mission period. The results of this elementary constellation can undoubtedly be improved by adding more satellites in the same orbital plane or by adding additional distinct orbital planes. However, this improvement is limited by the availability of optical stations. The presented results highlight the interest of a satellite constellation; however, the constellation design is not pushed to the maximum possible performance because the latter strongly depends on the locations of the stations studied and the service level required by the end users.

4. SIMULATION ASSUMPTIONS

The main simulation parameters are summarized in Table 2:

Table 2: List of main simulation parameters

	Parameter	Value
General	Altitude	~550 km
	Minimum elevation angle qubits	30°
	E2E Intrinsic quantum error	1%
	System margin	-3 dB
Entangled Photon Source	Brightness at source output	125 Mpairs/s/nm
	Wavelength	~1550 nm
On-Board Optical Terminal	Overall Gain (including internal & pointing losses)	102 dB
Free Space Optical Link	Propagation Losses	-258 dB @30°
	Aerosol Optical Depth (AOD) losses + absorption losses	~ -0.60 dB @30° Depending on the considered ground station and elevation angle-dependent
	Cloud Coverage	Binary Cloud Mask from ICARE Database
Optical Ground Stations	OGS Aperture	Nice : large diameter > 1 m Toulouse : small diameter < 1 m
	OGS Gain (from M1 to fiber injection included)	Nice : 120 dB Toulouse : 115 dB
Quantum Bench	Quantum state analyzer efficiency (including SNSPD detectors)	-4dB
	Noise (including auxiliary signals crosstalk, albedo & dark counts)	200 cps
	Spectral filtering	1 nm
	Temporal coincidence window	200 ps

These assumptions are used in Section 5 to evaluate Bell-test significance and success probability for the considered mission scenarios.

5. RESULTS

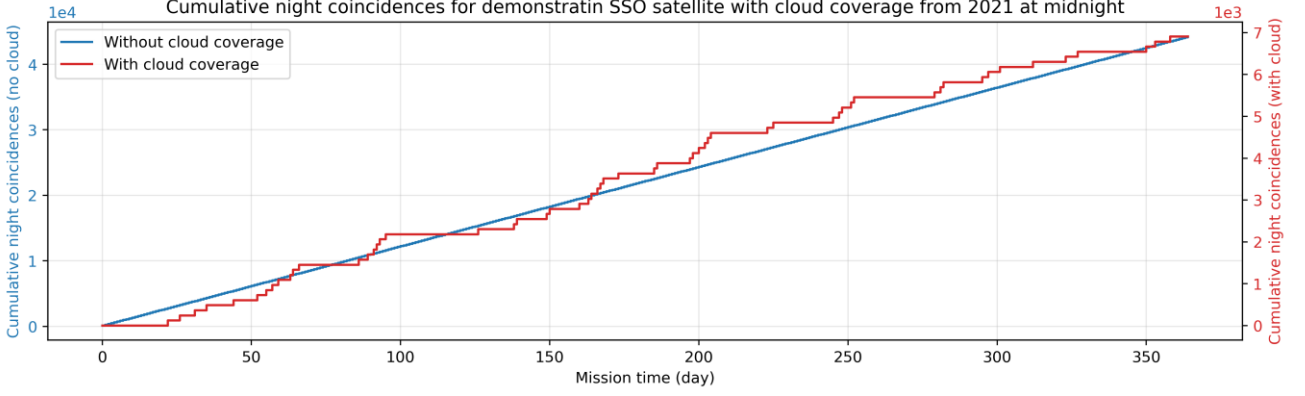


Figure 7: E2E performances for night entanglement distribution with a single non-drifting SSO satellite over a 1-year mission period ($T_0 = 01/01/2021$)

Figure 7 shows that for the single non-drifting SSO satellite, the cumulative number of coincidences grows linearly with time. This behavior is a direct consequence of the highly repetitive orbital geometry: the satellite follows essentially the same ground track every night, leading to nearly identical visibility conditions and link performances from pass to pass. As a result, each night contributes approximately the same number of coincidences, producing a regular and quasi-linear accumulation over the mission duration. Under the cloud-coverage implementation, the cumulative trend remains approximately linear but with a reduced slope, since each cloudy night suppresses only the single access scheduled for that day; this contrasts with LEO constellations (**Figure 8**), for which several accesses may occur during the same night, so a single cloudy night can simultaneously eliminate multiple transmission opportunities and may lead to a much stronger reduction in the cumulative coincidences.

For the non-drifting SSO mission scenario, the system reaches approximately 120 coincidences per access with a QBER of 5.95%, corresponding on average to a Bell-inequality violation of about 1.7σ for each pass. However, in the low-count regime, average significance alone does not fully capture the outcome of a single experimental realization, since finite-statistics fluctuations in the distribution of coincidences among the CHSH settings can noticeably affect the achieved violation. For this reason, we also consider the success probability, defined as the probability of reaching a target Bell significance on a single access.

Using this metric, the guaranteed Bell significance reduces to about 0.9σ with 80% success probability and less than 0.2σ for a 95% success probability. The chosen system (in terms of synchronization sub-system performance, end-to-end budget, end-to-end QBER, satellite passes duration, etc.) results in a slight difference between the average performance of Bell-inequality violation and the performance achieved at high success probabilities taking into account statistical fluctuations. Additionally, the above definition allows, in just a few passes, to accumulate enough

coincidences to reach robust sigma values with excellent probabilities of success. The system defined for this particular mission scenario achieves, after 5 passes, a Bell-inequality violation of about 2.1σ with a 95% probability of success.

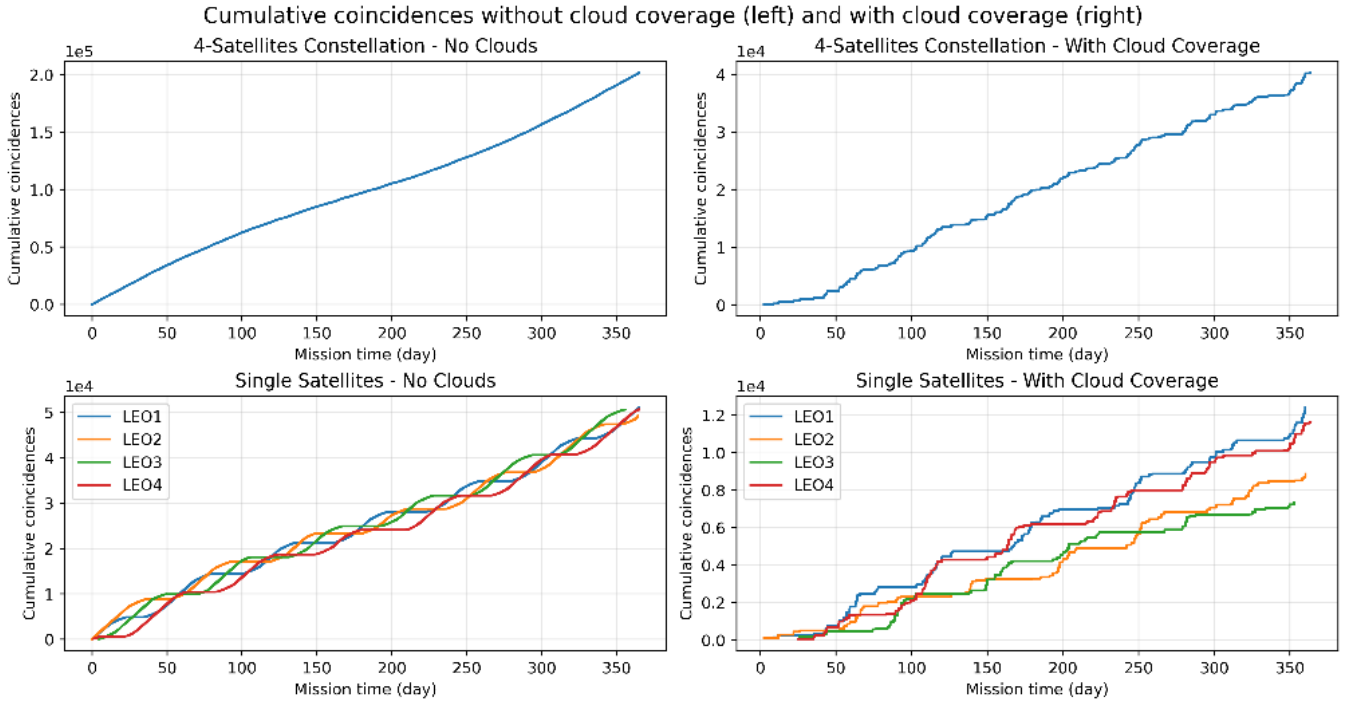


Figure 8: E2E performances for night entanglement distribution with a 4-satellites constellation over a 1-year mission period

As regard the second mission scenario, even if the total cumulative number of coincidences delivered by the 4-LEO constellation evolves approximately linearly with time thanks to the specific constellation design, this behavior does not hold at the level of an individual satellite, as highlighted in **Figure 8**. For a single satellite, the access conditions vary significantly from one pass to another because of the orbital dynamics, and extended periods of several days without visibility from the ground stations may occur. As a result, the cumulative coincidence yield of each spacecraft is highly irregular over time. The apparent linearity recovered at the constellation level arises only from the combination of the four optimally phased satellites, whose individual fluctuations compensate for each other. This contrasts with the non-drifting SSO mission scenario, for which the linear evolution is obtained intrinsically from a single satellite thanks to its highly repetitive geometry. For a demonstration mission, such an orbit is therefore preferable, since it provides a more regular and predictable accumulation of coincidences over time. However, a constellation offers a clear advantage when considering service provision to multiple users distributed over different ground stations. Because the satellites in the constellation pass over a much wider range of geographical locations, they can provide access opportunities to almost any ground station worldwide, unlike a non-drifting SSO, whose coverage remains constrained by its fixed ground track geometry. In that sense, although a non-drifting SSO is better suited for a demonstration mission requiring regularity and predictability, a constellation is more appropriate for an operational service provider, as it enables a broader and more flexible distribution of coincidences among geographically dispersed users.

6. CONCLUSION

In this paper we have discussed two essential figures of merit for the assessment of entanglement in the very high-loss regime of Space-based entanglement distribution: the well-known Bell parameter with its standard deviation (sigma) and the probability-of-success. The probability-of-success metric is essential in a realistic Space mission context: it converts a purely statistical Bell-violation criterion into an actionable engineering requirement and exposes the degree of over-dimensioning needed when finite-duration passes, link variability and noise are considered. It must be used together with the Bell-violation confidence level — the former as an operational sizing constraint, the latter as the physics validation metric — to obtain a consistent design framework for space-based entanglement distribution under finite-statistics conditions and high-losses regime.

A low-Earth-orbit entanglement-distribution system is discussed. It reduces the gap between experience results based on the average expected Bell violation and those that account for the success probability too. By accumulating coincidences over a few passes, the system can reach high- σ violations with very high success probability.

Two mission scenarios are analysed - non-drifting Sun-synchronous orbit (SSO) and a LEO satellite constellation - and quantitative results are presented. SSO orbit allows a Bell-inequality violation of about 2.1σ with a 95% probability of success after 5 successful passes. SSO orbit particularly fits for a demonstration system while LEO constellation is well adapt for an operational system to maximize total distributed resource and to address different end-users.

ACKNOWLEDGEMENTS

This work was supported by the French Government under the French space agency (CNES) supervision through QINSAT phase A study.

QINSAT phase A study was conducted in collaboration with Airbus Defence and Space, OGS Technologies and Aurea Technology.

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