

Optimal filtering for the tip-tilt Propagation Delay method at the GEO point-ahead angle

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ABSTRACT

For stable ground-to-satellite optical links, the uplink beam should be pre-compensated at the ground in order to mitigate deep signal fades at the receiver. We develop numerical simulations of the Propagation Delay method for geostationary (GEO) satellite uplink pre-compensation, to measure the tip-tilt at the point-ahead angle using a sodium laser guide star. Using a filtered combination of both the laser guide star differential tilt as seen from a monostatic aperture and the downlink tilt from the satellite, we reconstruct the point-ahead angle tilt in 80 real day and night atmospheric profiles using Monte-Carlo simulations. In this paper we investigate the filtering methods, and it is found that there is no relationship between optimal cut-off frequency values and isoplanatic angle. The cut-off frequency values are in fact sensitive to wind direction, so cannot be simply inferred from one standard integrated atmospheric parameter. Comparing to pre-compensation using the satellite downlink tilt alone, the Propagation Delay method reduces the residual tip-tilt variance for all of the tested atmospheres for a 1 m aperture using the optimal cut-off frequencies, and using median cut-off frequencies still gives an reduction 95% of the time.

1. INTRODUCTION

Ground-to-satellite free-space optical communications is a developing field, where optical wavelengths rather than radio are used to transmit data between the Earth and satellites.¹ Optical wavelengths provide multiple benefits over radio, most notably the potential for higher data rates,^{2,3} unlicensed spectrum and increased security.^{4,5} However, these shorter wavelengths are more affected by the atmospheric optical turbulence which distorts the wavefronts, leading to potential frequent and deep signal fades at the receiver. To mitigate this on the uplink (ground-to-satellite) beam, the communications laser can be pre-compensated at the ground before propagation. Depending on the satellite velocity and range, the uplink beam must be pointed at some angle ahead of the downlink, known as the point-ahead angle (PAA). The downlink beam can be used as a wavefront reference, however it is generally outside of the isoplanatic angle. A more accurate alternative is to have a laser guide star (LGS) at the PAA as the wavefront reference.

It is well known that tip-tilt (TT) cannot be directly measured using an LGS,⁶ as the measured TT is a combination of that on the upward and downward propagation path. There have been multiple methods proposed to retrieve this uplink TT,⁷⁻¹¹ and here we further investigate the Propagation Delay (PD) method proposed by Ragazzoni,¹² and develop the work by Barrett et al.¹³ We simulate the performance of the PD method in reconstructing the TT at the PAA for a geostationary (GEO) uplink, comparing this to using the satellite downlink as the TT wavefront reference. This method incorporates both the measurable LGS differential TT and the satellite downlink TT, filtering each signal before combining. In this paper we investigate the filtering method,¹⁴ in particular the cut-off frequencies and wind direction, and quantify what effect this has on the TT residual.

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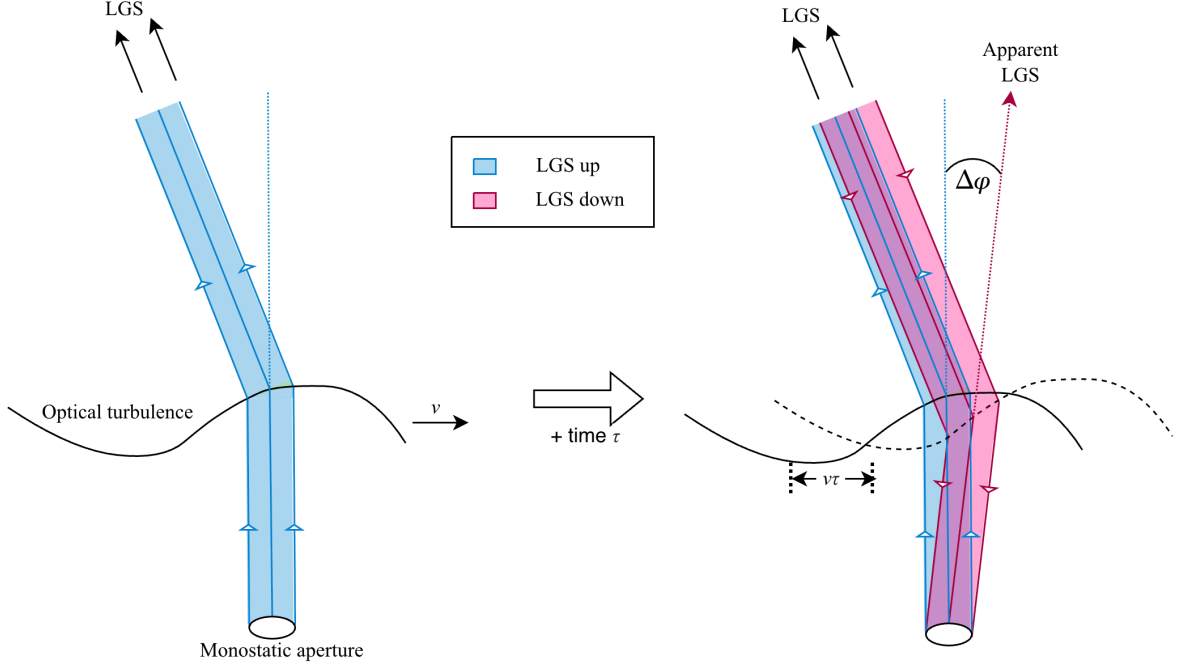


Figure 1. Concept of the Propagation Delay method. Left: upward propagation of the LGS, right: subsequent downward propagation. Using monostatic aperture, a small differential tilt $\Delta\varphi$ can be observed on the LGS as the turbulence has propagated distance $v\tau$, where v is the wind velocity of the turbulent layer and τ is the return time of the LGS to the sodium layer.

2. TILT RECONSTRUCTION AT THE PAA

2.1 Propagation Delay method

The PD method¹² is based on the ability to measure the differential tilts ($\Delta\varphi$) induced on a sodium LGS, due to the displacement of the atmospheric optical turbulence in the finite return propagation time of the light to the sodium layer, as shown in Fig. 1. It requires $\Delta\varphi$ to be measured with a monostatic configuration, in which the LGS is transmitted and received through exactly the same pupil. This return time is known as the propagation delay τ , and is ≥ 0.6 ms. The difference in the tilt θ between time t and $t + \Delta t$ is

$$\theta(t + \Delta t) - \theta(t) \approx \frac{t^*}{\tau} \sum \Delta\varphi, \quad (1)$$

where t^* is the sampling time of the detector and $\tau = \frac{2}{c}(H - \bar{h})$, where $\bar{h}$ is the average altitude of the turbulence and H is the average altitude of the LGS. The value of $\Delta\varphi$ is generally in the range 0.4-15 mas (0.002-0.07 μrad). Given an accurate starting tilt θ and frequent sampling (small t^*), this alone would be enough to reconstruct the tilt. However in a realistic system there are errors on the $\Delta\varphi$ measurements from wavefront sensor centroiding uncertainties, which when summed as in eq. 1, lead to a low frequency drift from the true TT. Various filtering steps are required to correct for this, which are explained in the section 2.4.1.

2.2 Simulation

To simulate the performance of the method for GEO satellites, we use a set up as shown in Fig. 2. The GEO satellite downlink path and the LGS are separated by PAA $\alpha_{\text{PA}} = 4''$. 80 Monte Carlo simulations were run, each using a different atmospheric turbulence profile. These profiles are taken from a dataset measured in urban Barcelona, Spain, using a 24h-SHIMM.¹⁵ They contain both day and night 4-layer profiles, each with a refractive index structure constant C_n^2 and wind speed v value, but not a wind direction. To minimise any impact from

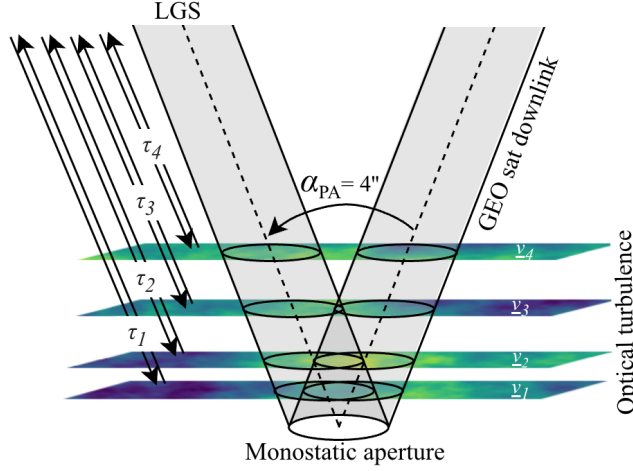


Figure 2. The simulation set up. The satellite downlink and LGS propagation paths are separated by $\alpha_{PA} = 4''$, corresponding to a GEO satellite. The LGS is transmitted and received at the same aperture. Real 4-layer atmospheric turbulence profiles from Barcelona were used, each layer with its own velocity $\underline{v}_i$. Between the upward and downward pass of the LGS to the sodium layer, the layer is displaced by $\underline{d}_i = \tau_i \underline{v}_i$, where τ_i is the return time to each layer.

a dominant wind direction, each profile has wind directions set to $[0, 270, 180, 90]^\circ$ corresponding to altitude layers $[0, 4, 12, 20]$ km, where 90° is along the direction from the LGS to the satellite. These atmospheres were selected randomly from the full dataset to represent the full range of profiles, in order to characterise the performance of the PD method in an urban environment.

The LGS uplink TT $\theta_{LGS \uparrow}$, LGS downlink TT $\theta_{LGS \downarrow}$ and the satellite downlink TT θ_{sat} are extracted from the simulation. These TT values are calculated by summing the phase in a circular aperture of diameter D from all phase screens at angles 0 and $4''$, neglecting focal anisoplanatism. The tip and tilt are then directly calculated, by projecting these phase aberrations onto the Noll normalised tip and tilt Zernike modes, giving the Z-tilt. The differential TT is given by $\Delta\varphi = \theta_{LGS \downarrow} - \theta_{LGS \uparrow}$. We are aiming to reconstruct $\theta_{LGS \uparrow}$ and so compare the results of the PD method to these values. To simulate the PD, the return time to each turbulent layer τ_i is calculated and each phase screen is propagated by $\underline{d}_i = \tau_i \underline{v}_i$ between the uplink and downlink pass of the LGS.

3 additional named turbulence profiles were also included, whose integrated parameters are listed in table 1. All are real atmospheric profiles with median / 90th percentile integrated parameters.¹⁶ TURBO90 is the 90th percentile profile from all day and night Barcelona 24h-SHIMM profiles, chosen to represent the strongest expected turbulence conditions. TURBO50 is the median profile from all day and night Barcelona 24h-SHIMM profiles, to show a median performance of the method in an urban environment. LP night is the median SCIDAR (SCintillation Detection And Ranging) profile taken in La Palma,¹⁷ compressed to 5 layers using the equivalent layers method, conserving Fried parameter, isoplanatic angle and coherence time.¹⁸ This is used to investigate how the PD method performs in weak turbulence at an astronomical site and it is the only profile not measured using the 24h-SHIMM. The atmospheric optical turbulence is simulated using the Kolmogorov infinite phase screen from the AOtools package.¹⁹

Atmosphere	Fried parameter r_0 [cm]	Isoplanatic angle θ_0 [μ rad]	Coherence time τ_0 [ms]
TURBO90	3.90	2.67	1.30
TURBO50	6.21	5.24	3.34
LP night	14.4	11.7	5.56

Table 1. Integrated parameters for the additional named atmospheric profiles. Values are quoted for $\lambda = 500$ nm and $\zeta = 0^\circ$.

N_{ph}	N_{T}	N_{samp}	N_{r}	N_{s}^2	N_{subap}
15	3 pix	3 pix	0.4	3 pix	236 / 59

Table 2. Chosen values to calculate TT error due to Shack-Hartmann wavefront sensor centroiding noise as in eq. 4, for aperture size $D = 1 / 0.5$ m. A subaperture size of $d = 0.05$ m was chosen to match the median $r_0 \approx 0.05$ m for the selected 80 $C_n^2(h)$ profiles. N_{subap} is the number of illuminated subapertures, assuming a 25% area central obscuration.

Zenith angle $\zeta = 50^\circ$ was chosen as the GEO belt is located at $\zeta \approx 48^\circ$ when observing directly south from Barcelona, and will increase if observing away from directly south. The return flux for a 70 W pulsed (shuttered 50%) laser at the ground is approximately $17.5 \text{ Mph s}^{-1} \text{ m}^{-2}$.²⁰ It was chosen to run the simulation for both $D = 1$ m and $D = 0.5$ m, to give performance results over the range of likely GEO optical ground station aperture sizes. Each simulation is 10000 iterations, with a sampling time $t^* = 1$ ms. We assume an optics efficiency of 35 %. Neglecting error sources from the AO system (e.g fitting error, temporal error) allows us to focus on the PD method's capabilities.

2.3 Noise

To simulate noise on the $\Delta\varphi$ measurements, we model the phase error resulting from shot and read noise using a Shack-Hartmann wavefront sensor.²¹ The analytical phase variance per subaperture from photon noise is

$$\sigma_{\phi, N_{\text{ph}}}^2 = \frac{\pi^2}{2 \ln 2} \frac{1}{N_{\text{ph}}} \left(\frac{N_{\text{T}}}{N_{\text{samp}}} \right)^2 \quad (2)$$

and from read-out noise is

$$\sigma_{\phi, N_{\text{r}}}^2 = \frac{\pi^2}{3} \frac{N_{\text{r}}^2}{N_{\text{ph}}^2} \frac{N_{\text{s}}^4}{N_{\text{samp}}^2}, \quad (3)$$

where N_{ph} is the number of photons in each subaperture per exposure, N_{T} is the FWHM (full width at half maximum) in pixels, N_{r} is the read noise, N_{s}^2 is the total number of pixels used in the centre of gravity calculation, and $N_{\text{samp}} = \lambda/(dp)$, where λ is the wavelength, d is the subaperture width and p is the angular pixel scale. The total TT phase variance from noise across the whole aperture is then

$$\sigma_{\phi, \text{tot}}^2 = \frac{\sigma_{\phi, N_{\text{ph}}}^2 + \sigma_{\phi, N_{\text{r}}}^2}{N_{\text{subap}}}, \quad (4)$$

where N_{subap} is the number of illuminated subapertures. To calculate this noise, we assume a system with parameters detailed in table 2, which are chosen to represent a realistic system. These parameters give $\sigma_{\phi, \text{tot}}^2(D = 1 \text{ m}) = 0.00405 \text{ rad}^2$ and $\sigma_{\phi, \text{tot}}^2(D = 0.5 \text{ m}) = 0.0162 \text{ rad}^2$. Approximating the noise ϵ to follow a normal distribution with $\epsilon \sim \mathcal{N}(0, \sigma_{\phi, \text{tot}}^2)$, random noise values are added to the $\Delta\varphi$ measurements to model the centroiding error.

2.4 Filtering

2.4.1 Method

As stated previously, summing these noisy $\Delta\varphi$ measurements causes a low frequency drift from the true tilt. To remove this, the signal is then filtered using a high-pass Butterworth filter, with cut-off frequency $f_{\text{c, high}}$. This is then combined with the low-pass filtered TT signal from the satellite downlink, with cut-off frequency $f_{\text{c, low}}$, used to provide a low frequency TT reference. Removing the high frequency TT from the satellite increases the correlation between itself and the TT at the PAA. The final TT is

$$\theta_{\text{PD}} = \mathcal{H}_{\text{HP}}\{\theta_{\Delta\varphi+\epsilon}(t)\} + \mathcal{H}_{\text{LP}}\{\theta_{\text{sat}}(t)\}. \quad (5)$$

where $\mathcal{H}_{\text{HP}}\{\theta_{\Delta\varphi+\epsilon}(t)\}$ is the high-pass filtered LGS TT and $\mathcal{H}_{\text{LP}}\{\theta_{\text{sat}}(t)\}$ is the low-pass filtered satellite TT.

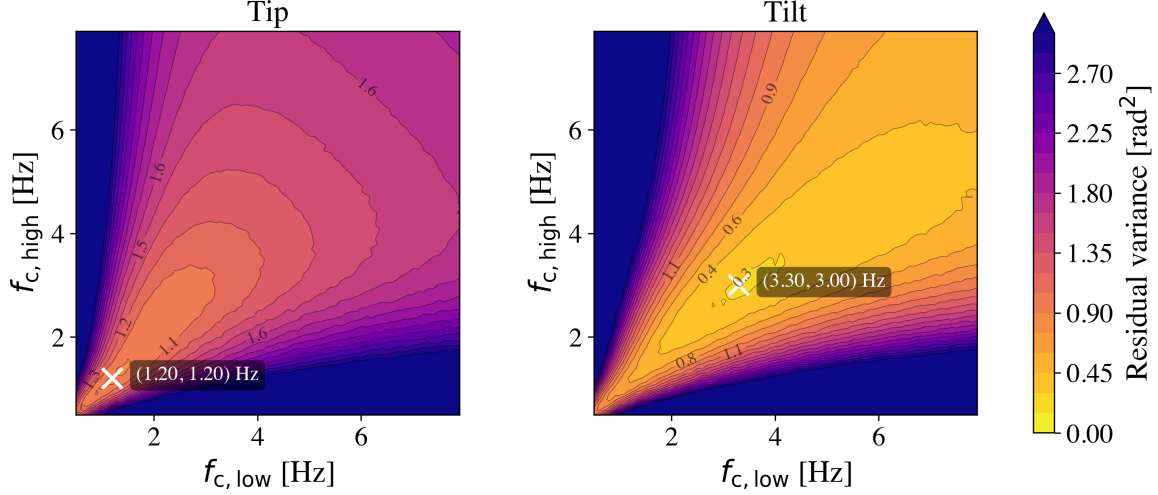


Figure 3. Residual tip (left) / tilt (right) variance for a range of $f_{c, \text{low}}$ and $f_{c, \text{high}}$ values, for $D = 1$ m and TURBO50 $C_n^2(h)$ profile. The minimum residual variance for each mode is marked with a white cross, with the corresponding optimal $f_{c, \text{low}}$ and $f_{c, \text{high}}$ values written beside it.

To quantify the performance of the PD method, we calculate the residual TT RMS at the PAA using the PD method. This is compared to the baseline method, in which the satellite downlink TT is used to pre-compensate the uplink at the PAA, which is referred to here as the ‘classical’ method. The residual TT for the PD and classical method respectively are

$$\delta\theta_{\text{PD}}(t) = \theta_{\text{PD}}(t) - \theta_{\text{LGS} \uparrow}(t), \quad (6)$$

$$\delta\theta_{\text{classical}}(t) = \theta_{\text{sat}}(t) - \theta_{\text{LGS} \uparrow}(t). \quad (7)$$

The corresponding RMS values are then denoted as

$$\sigma_{\theta, \text{PD}} = \text{RMS}(\delta\theta_{\text{PD}}(t)) \quad (8)$$

and

$$\sigma_{\theta, \text{classical}} = \text{RMS}(\delta\theta_{\text{classical}}(t)). \quad (9)$$

2.4.2 Optimal cut-off frequencies

To find the optimal high-pass $f_{c, \text{high}}$ and low-pass $f_{c, \text{low}}$ cut-off frequencies, combinations of values between $0.5 \leq f_c [\text{Hz}] \leq 10.0$ were ran and the residual tip/tilt variance ($\sigma_{\theta, \text{PD}}^2$) calculated. Each combination was repeated 10 times with new noise generated each time, and the mean residual variance was calculated. The combination with the minimum mean residual tip/tilt variance was selected as the optimal $f_{c, \text{high}}$ and $f_{c, \text{low}}$ values for that atmosphere, aperture size and mode combination.

Figure 3 shows the residual tip (left) and tilt (right) for a range of cut-off frequency values for the atmospheric profile TURBO50, $D = 1$ m. The optimal values for each are marked with the white cross. Larger values of $f_{c, \text{high}}$ and $f_{c, \text{low}}$ mean that more of the reconstructed LGS signal is removed by the filtering, and more of the satellite signal is used. Using the satellite downlink alone, there is a smaller residual variance between the apertures with the tilt mode (longitudinal) compared to the tip mode (transverse).²² Hence, the tilt $f_{c, \text{high}}$ and $f_{c, \text{low}}$ values are larger than those for tip to use more of the satellite signal in θ_{PD} .

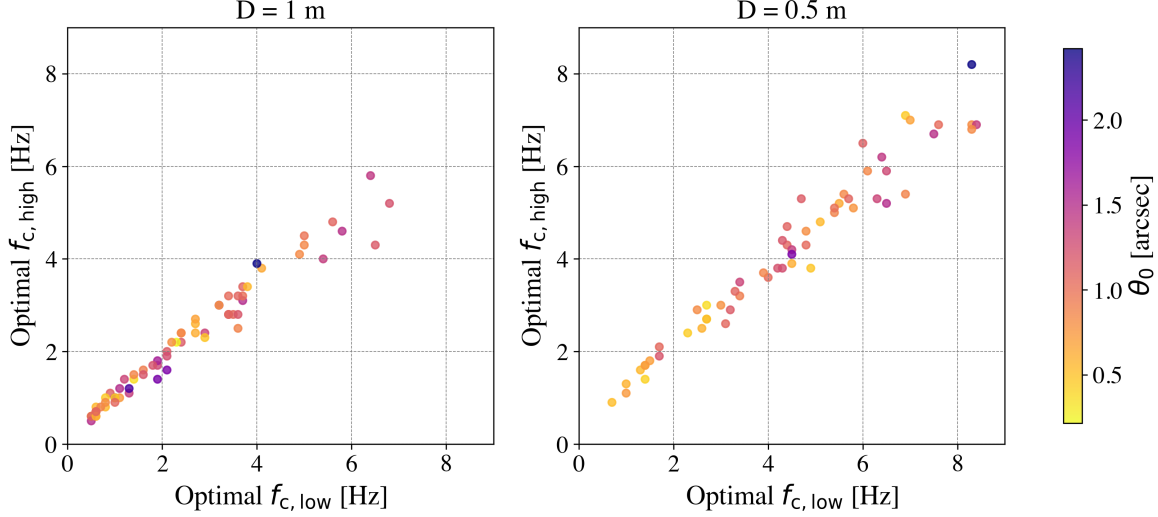


Figure 4. Optimal values for $f_{c, \text{low}}$ and $f_{c, \text{high}}$ for tip for all 80 atmospheres. The colour of the point corresponds to the isoplanatic angle θ_0 of that atmosphere. Most atmospheres have $f_{c, \text{low}} \approx f_{c, \text{high}}$. A larger value of $f_{c, \text{low}}$ and $f_{c, \text{high}}$ corresponds to removing more of the LGS reconstructed TT and keeping more of the satellite downlink TT.

Figure 4 shows these optimal $f_{c, \text{high}}$ and $f_{c, \text{low}}$ values for all 80 atmospheres, with the colour of the point corresponding to the isoplanatic angle θ_0 of that atmosphere. Most points lie around $f_{c, \text{high}} = f_{c, \text{low}}$. There does not appear to be any correlation between the optimal $f_{c, \text{low}}$ and $f_{c, \text{high}}$ values and θ_0 . $D = 0.5$ m has a larger mean value of $f_{c, \text{low}}$ and $f_{c, \text{high}}$ compared with $D = 1$ m, which may be because the noise on the $\Delta\varphi$ values is larger, therefore more of the LGS signal must be removed with the filter.

3. RESULTS

3.1 Optimal results

A 10 s simulation for all 80 atmospheres and 2 aperture sizes was run as described in section 2.2 and the TT at the PAA was reconstructed using the described filtering process, using the optimal $f_{c, \text{low}}$ and $f_{c, \text{high}}$ values for the given atmosphere, aperture size and mode. The resulting TT RMS error $\sigma_{\theta, \text{PD}}$ is plotted in purple in Fig. 5, with a separate plot for each aperture size. The TT RMS error for the classical case, $\sigma_{\theta, \text{classical}}$, in which the satellite downlink is used to pre-compensate at the PAA, is shown in blue. To verify the simulation, the classical results were also calculated analytically,²³ and those point are shown in black. The named atmospheres are plotted with different shaped markers, as shown in the figure legend. Below the reduction is plotted, which is calculated as $\text{reduction} = (1 - \sigma_{\theta, \text{PD}} / \sigma_{\theta, \text{classical}}) * 100$. The PD method gives a larger improvement at smaller isoplanatic angles, as this is when the classical method is least effective due to the limited TT correlation between the propagation paths. For both apertures sizes, the PD method gives little to no improvement over classical at large θ_0 , as θ_0 is close to α_{PA} and so classical pre-compensation is sufficient. For some atmospheres with $D = 0.5$ m, classical outperforms PD as seen by points in the shaded grey region. This may be due to the noise on the LGS $\Delta\varphi$ measurements, as the noise is larger for the smaller aperture. The cut-off frequencies were searched up to 10.0 Hz, but if given a larger upper limit, these atmospheres would tend towards larger values of $f_{c, \text{low}}$ and $f_{c, \text{high}}$, to remove more of the LGS TT. For $D = 1$ m, PD gives a reduction in all 80 atmospheres, and in 91% for $D = 0.5$ m.

3.2 Effect of cut-off frequency

In practice it may be challenging to find the optimal cut-off frequencies for each atmosphere, especially given that there is little relationship between the optimal values of $f_{c, \text{low}}$, $f_{c, \text{high}}$ and θ_0 , as was shown in Fig. 4. Here we find the median values of $f_{c, \text{low}}$ and $f_{c, \text{high}}$ for each aperture size and mode, and apply these values for filtering every atmosphere. The resulting reductions in TT RMS error compared to the classical case are

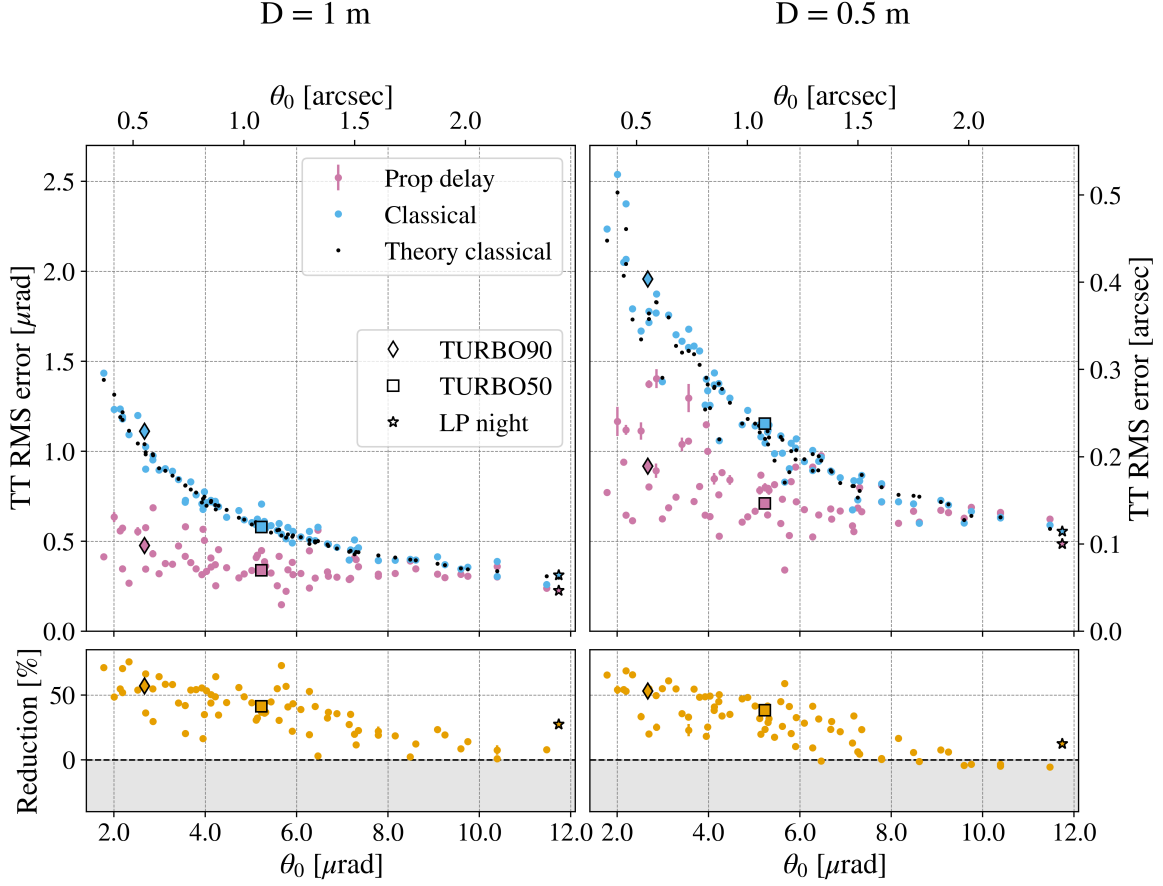


Figure 5. Performance of the PD method in comparison to classical pre-compensation for 80 different atmospheric turbulence profiles. Named representative profiles are shown with different markers. The lower panels show the reduction of the TT RMS error, where reduction = $(1 - \text{prop delay} / \text{classical}) * 100$. Shaded grey region indicates where classical performs better than the PD method. Isoplanatic angles θ_0 are quoted for $\lambda = 500\text{ nm}$ and $\zeta = 0^\circ$.

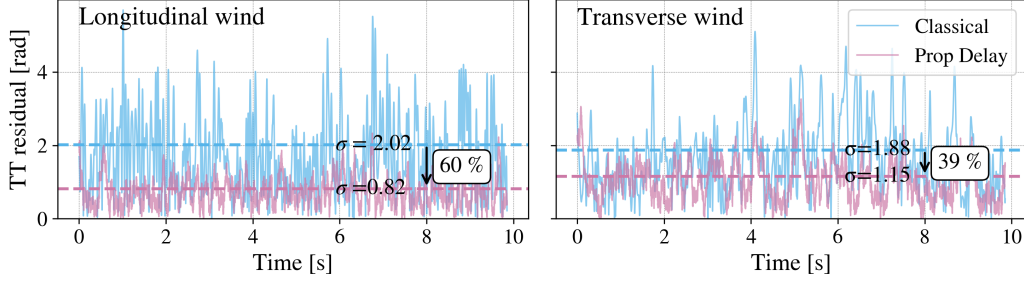


Figure 6. TT residual plots for TURBO50, $D = 1$ m, for all wind in longitudinal (top) or transverse (bottom) direction. Longitudinal is along the axes between the satellite and PAA, and transverse is perpendicular to this. The 4-layer profile was given alternating wind directions. For example, for longitudinal, ground layer is in direction from the satellite to the PAA, next layer is the opposite direction, etc. TT residual = $((\text{tip residual})^2 + (\text{tilt residual})^2)^{0.5}$, therefore while tip residual and tilt residual can be less than 0, TT residual is always positive. This plot demonstrates the impact of the wind directions on the performance of the PD method in comparison to classical precompensation, with a reduction in the TT RMS = 60 % for longitudinal wind and TT RMS = 39 % for transverse wind, for the same $C_n^2(h)$ profile.

calculated, and the mean decrease in the reduction using the median cut-offs rather than optimal is -7.1%. PD goes from giving a reduction 100% to 95% of the time for $D = 1$ m, and from 91% to 88% for $D = 0.5$ m. This is minimal and suggests that it may be suitable in practice to use only the median $f_{c, \text{low}}$ and $f_{c, \text{high}}$ values.

3.3 Effect of wind direction

To investigate how sensitive the PD method is to wind direction, each layer in TURBO50 was set to the longitudinal direction between the PAA and satellite, $[270, 90, 270, 90]^\circ$, or transverse direction $[0, 180, 0, 180]^\circ$. The time series of the results is shown in Fig. 6, with the classical TT residual $\delta\theta_{\text{classical}}(t)$ plotted in blue, and the PD TT residual $\delta\theta_{\text{PD}}(t)$ in purple. The RMS of each is shown on the dashed lines, with the reduction in RMS between them. For the same $C_n^2(h)$ and wind speed profiles, the PD gives a reduction of 60 % for longitudinal wind compared with 39 % for transverse wind. When all wind is in the longitudinal direction, there is a larger correlation between the low-frequency TT in the PAA and satellite directions, as each aperture is seeing the same TT with some time offset. These results suggest that there is no relationship between the residual TT variance reduction and the optimal cut-off frequency values with any standard integrated parameter of the atmosphere, such as θ_0 or coherence time τ , as these values are sensitive also to the direction of the wind.

4. CONCLUSION

We simulated the Propagation Delay method in the application of measuring tip-tilt at the point-ahead angle for GEO uplink pre-compensation. Monte Carlo simulations were run for 80 different atmospheric profiles measurement in urban Barcelona, Spain, to give a wide range of atmospheric turbulence conditions. We describe the filtering process, using both the differential laser guide star tilt and the satellite downlink tilt combined to reconstruct the point-ahead angle tilt, and investigate the optimal cut-off frequencies. We found that there was no relationship to isoplanatic angle, and that the optimal values and performance of the method are highly sensitive to wind direction. It may be sufficient to use only median cut-off frequencies, as we show that for a 1 m aperture, using the median values still gives an improvement on pre-compensation with the satellite downlink tilt 95% of the time, compared with 100% with optimal values. Experimental results are sought and required to confirm these simulations.

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