IMPACT PROBABILITY ESTIMATION WITH PARTIAL BANANA MAPPING: SEARCH FOR VIRTUAL IMPACTORS. Dmitrii E. Vavilov[1,2], [1]IMCCE, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, Université Lille, Paris, 75014 France; dmitrii.vavilov@obspm.fr; [2]Institute of Applied Astronomy of Russian Academy of Sciences, 10 Kutuzova emb., St. Petersburg, 191187 Russia

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Introduction:

The probability of a collision of an asteroid with the Earth arises because of the uncertainty of the position of the asteroid in space. The better we understand the behavior of the uncertainty region the more precise we can estimate the probability of a collision. In this work I am considering only linear methods of impact probability estimation. In these methods we assume that there is a linear relation between the errors of orbital parameters at the epoch of observations and at the epoch of a possible collision. The uncertainty region is determined by the covariance matrix, which is computed via integration of asteroid's equations of motion with variational equations. Thus, this kind of methods require propagation of only the nominal orbit.

The most well-known linear method is the target plane method, which was first successfully used by Paul Chodas [1] to predict a collision of comet Shoemaker-Levy 9 with Jupiter. Vavilov & Medvedev [2] introduced a concept of curvilinear coordinate system in order to take into account that the uncertainty region is curved and stretched mostly along the nominal orbit. Later Vavilov [3] extended this approach to the Partial Banana Mapping method. Here in this work I will improve the Partial Banana Mapping method, so that for a small cost of time it will produce much more reliable results.

Partial Banana Mapping method (PBM):

The basic principle of the Partial Banana Mapping method (PBM) is that the covariance matrix in curvilinear coordinate system, as described in [2] or in orbital elements much better represent the actual shape of the uncertainty region (see Fig.1) in the two-body formalism. The region is thin and stretched mostly along the nominal asteroid's orbit. Hence, a collision can happen when the Earth comes close to the nominal asteroid's orbit. The problem of working in curvilinear coordinate system or in orbital elements is that it is difficult to represent the uncertainty region analytically in Cartesian coordinates. However, the idea of PBM is that we don't need to represent the whole region to compute impact probability. We need to estimate only the part of the region, which is close to the Earth. For this we find point B on the main axis of the curvilinear uncertainty region, and compute the covariance matrix in Cartesian coordinates as:

$$\mathbf{C}^*_{xyz\dot{x}\dot{y}\dot{z}} = \mathbf{Q}_*^{-1} \cdot \mathbf{C}_{\xi\eta M\dot{\xi}\dot{\eta}\dot{M}} \cdot \mathbf{Q}_*^{\mathrm{T}^{-1}}. \tag{1}$$

where $\mathbf{C}^*_{xyz\dot{x}\dot{y}\dot{z}}$ is the covariance matrix of the state vector in Cartesian coordinate system around point B, $\mathbf{C}_{\xi\eta M\dot{\xi}\dot{\eta}\dot{M}}$ is the covariance matrix of point A in curvilinear coordinate system, which we got before through integration of variational equations. $\mathbf{Q}$ is the matrix of partial derivatives of curvilinear coordinates and velocities over Cartesian coordinates and velocities around point B. The impact probability is computed by the target plane method, but for the found point B and $\mathbf{C}^*_{xyz\dot{x}\dot{y}\dot{z}}$ as it's covariance matrix. The final impact probability value must be multiplied by a value of the probability density function for point B, i.e. to take into account that point B is not the nominal one (in reality it can be quit far).

PBM: search for virtual impactors:

PBM method has one essential drawback that it inherits from the assumption of linear methods: the linear relation can be broken by gravitational perturbations from massive bodies, like major planets. This can distort the impact probability values. In order to suppress this drawback the following scheme is introduced. Once we found the orbital elements for point B, we can find in the original uncertainty region the orbital elements of the virtual asteroid that lead to point B:

$$\mathrm{V}^{-1}[E^{min} - E] = [x_0^{min} - x_0] \tag{2}$$

where x_0 — state vector at epoch of observations, x — state vector at time of possible collision, E — Keplerian elements at time of possible collision, E^{min} — Keplerian elements of point B, x_0^{min} — state vector at epoch of observations that should lead to point B, and matrix V is a multiplication of two partial derivative matrices:

$$\mathrm{V} = \left[\frac{\partial E}{\partial x}\right]\left[\frac{\partial x}{\partial x_0}\right].$$

Once we found the state vector x_0^{min} we propagate its orbit to the time of the possible collision and compute the impact probability to it. This virtual asteroid is much closer to the Earth than the nominal one, and also while we propagate its orbit we take into account all the perturbations. Thus

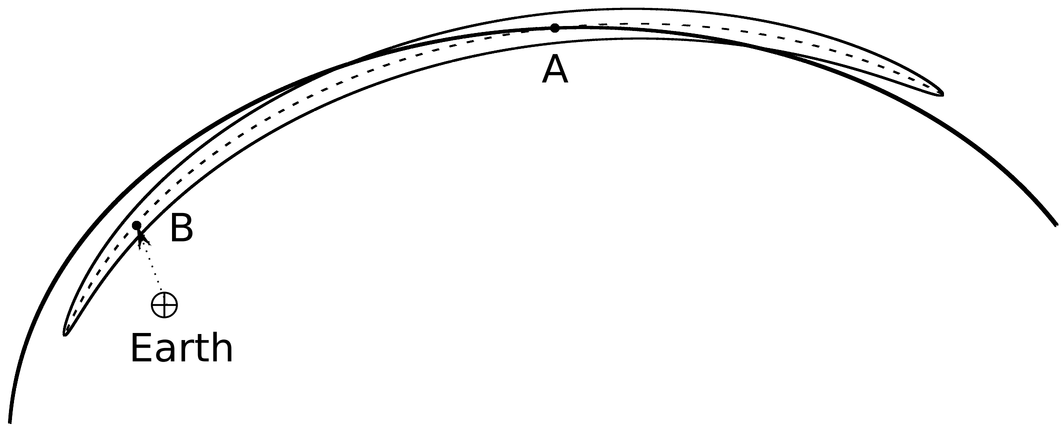

Figure 1: The scheme of the banana shaped uncertainty region of the asteroid. Point A is the nominal position of the asteroid, point B — the virtual asteroid of the main axis of the uncertainty region, which is closest to the Earth. The bold line is the nominal asteroid's orbit.

with a cost of one more orbit propagation we suppress the main drawback of linear methods.

Results:

The methods were tested on 16 asteroids. The orbits of some of the asteroids now changed, because of the addition of observations, and they are removed from the list of dangerous asteroids. However, they still can be used as modeled asteroids, which are based on real cases. For asteroid Apophis the orbit was computed by the set of observations from 2004 till 2006. The probabilities computed by the Monte-Carlo method are considered as etalon values.

As one can see from Table 1 the improved version of the method gives more reliable values of impact probabilities. They are closer to the Monte-Carlo ones. For the case of asteroid Apophis the classical partial banana mapping method could not find possible collisions, because of the very close approach in 2029 of this object with the Earth. However, for the improved algorithm, it didn't become a problem. The interesting case is for asteroid 2010 RF12. Based on our orbit and covariance matrix the Monte-Carlo method couldn't find any possible collision after propagating 564000 orbits, while the target plane method and the PBM gave large values of impact probabilities. But the algorithm of searching for virtual impactor couldn't converge and hence didn't find any collision.

Interesting observation is that if we change the possible collision time to 50 days earlier, the improved approach is still capable of finding the collision and compute probability values, however, it requires several iterations of finding x_0^{min}.

Conclusion: The Partial Banana Mapping method was significantly improved so that now it doesn't suffer from gravitational perturbations from major planets, and produces accurate results of impact probability values. The cost of the improvement is that now instead of propagating only the nominal orbit of the asteroid, it requires a couple of more propagations.

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References: [1] P. W. Chodas (1993) in *BAAS* vol. 25 1236. [2] D. E. Vavilov, et al. (2015) *MNRAS* 446(1):705 `doi`. [3] D. E. Vavilov (2020) *MNRAS* 492(3):4546 `doi.arXiv:1911.12991`.

Table 1: **Impact probabilities computed by different methods.**

Designation	$P_{MC} \pm 3\sigma_{MC}$	P_{TP}	P_{PBM}	P_{IPBM}
2006 JY26	$(5.6 \pm 1.7) \cdot 10^{-5}$	$\mathbf{1.0 \cdot 10^{-4}}$	$\mathbf{1.0 \cdot 10^{-4}}$	$6.0 \cdot 10^{-5}$
2006 QV89	$(1.8 \pm 0.1) \cdot 10^{-3}$	$\mathbf{2.0 \cdot 10^{-3}}$	$1.8 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$
2010 UK	$(3.1 \pm 0.7) \cdot 10^{-3}$	$\mathbf{1.8 \cdot 10^{-3}}$	$\mathbf{1.8 \cdot 10^{-3}}$	$2.9 \cdot 10^{-3}$
2011 AG5	$(5.3 \pm 1.3) \cdot 10^{-4}$	$4.2 \cdot 10^{-4}$	$5.7 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
2007 VK184	$(6.2 \pm 2.0) \cdot 10^{-6}$	$\mathbf{2.7 \cdot 10^{-5}}$	$\mathbf{2.6 \cdot 10^{-5}}$	$8.8 \cdot 10^{-6}$
2007 VE191	$(6.4 \pm 1.0) \cdot 10^{-4}$	$\mathbf{0.0}$	$6.8 \cdot 10^{-4}$	$7.0 \cdot 10^{-4}$
2008 JL3	$(3.0 \pm 0.4) \cdot 10^{-4}$	$\mathbf{7.5 \cdot 10^{-4}}$	$\mathbf{4.0 \cdot 10^{-4}}$	$3.3 \cdot 10^{-4}$
2014 WA	$(3.5 \pm 2.4) \cdot 10^{-7}$	$\mathbf{0.0}$	$5.4 \cdot 10^{-7}$	$5.3 \cdot 10^{-7}$
2009 JF1	$(7.4 \pm 1.2) \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	$8.0 \cdot 10^{-4}$	$7.6 \cdot 10^{-4}$
2012 MF7	$(3.1 \pm 0.8) \cdot 10^{-4}$	$\mathbf{0.0}$	$\mathbf{4.8 \cdot 10^{-4}}$	$3.2 \cdot 10^{-4}$
2008 CK70	$(6.4 \pm 1.0) \cdot 10^{-4}$	$5.8 \cdot 10^{-4}$	$6.9 \cdot 10^{-4}$	$6.5 \cdot 10^{-4}$
2005 BS1	$(1.4 \pm 0.2) \cdot 10^{-4}$	$\mathbf{0.0}$	$1.4 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$
2005 QK76	$(4.3 \pm 0.9) \cdot 10^{-5}$	$\mathbf{0.0}$	$4.1 \cdot 10^{-5}$	$4.3 \cdot 10^{-5}$
2007 KO4	$(7.3 \pm 4.0) \cdot 10^{-7}$	$\mathbf{0.0}$	$\mathbf{2.2 \cdot 10^{-6}}$	$9.7 \cdot 10^{-7}$
Apophis 2036	$(1.4 \pm 0.8) \cdot 10^{-5}$	$\mathbf{0.0}$	$\mathbf{0.0}$	$1.2 \cdot 10^{-5}$
2010 RF12	$(0.0 \pm 1.7 \cdot 10^{-6})$	$\mathbf{5.1 \cdot 10^{-2}}$	$\mathbf{4.9 \cdot 10^{-2}}$	0.0

Notes: 'Designation' is the asteroid's designation, P_{MC} is the probability computed by the Monte-Carlo method, σ_{MC} is a standard deviation of P_{MC}, P_{TP} and P_{PBM} are probabilities calculated by the target plane method and the partial banana mapping method correspondingly. P_{IPBM} is the probability computed by the improved version of PBM method.