

Image-based fine phasing methods for Earth observation segmented mirror telescopes

M. Hirose^{*a}, S. Sato^a, T. Anada^b, T. Tsuchikawa^b, T. Endo^b, S. Kobayashi^b, T. Mizutani^a

^aJapan Aerospace Exploration Agency (JAXA), Research and Development Directorate,
2-1-1 Sengen, Tsukuba, Ibaraki, Japan 305-0047;

^bMitsubishi Electric Corp., Information Technology R&D Center,
5-1-1 Ofuna, Kamakura, Kanagawa, Japan 247-8501

ABSTRACT

High-resolution geostationary (GEO) optical satellites for Earth observation offer advantages for the rapid and continuous monitoring of designated fields of view, attracting considerable interest across a variety of use cases ranging from economic security to disaster mitigation. We have investigated a 3.6-m-sized GEO optical imager utilizing a segmented mirror telescope system, alongside the fine phasing techniques involved in the wavefront management architecture necessary to achieve diffraction-limited imaging performance after launch. Here, we evaluated two image-based fine phasing methods: model-free fine phasing using the image sharpness function, and model-based fine phasing employing the phase diversity method. Specifically, in this research, the former approach addresses fine alignment of multiple segmented mirrors based on a time-varying dynamic scene, whereas the latter one focuses on investigating measurement and analytical conditions for realizing accurate wavefront estimation. These results showed the technical effectiveness of both approaches to achieve high-accuracy wavefront control and estimation, according to the satellite operation conditions.

Keywords: Earth observation, segmented mirror telescope, wavefront sensing and control, optimization algorithm

1. INTRODUCTION

The responsive and continuous observation capabilities of GEO optical satellites are critical for large-scale disaster monitoring and subsequent decision-making in damage mitigation. However, developing GEO optical satellites that offer a satisfactory ground sampling distance (GSD) for practical applications remains a major challenge due to the constraints of fabricating a large-scale optical imager within current launch capacities. To address this issue, we have investigated the feasibility of a 3.6-m-diameter optical telescope system designed for a GEO optical imager. This system is engineered to provide a 100 km by 100 km field of view and a nadir GSD better than 7 m in the visible wavelength [1, 2]. A key enabling technology of a 3.6-m-sized optical imager is the optical synthetic aperture telescope system with segmented mirrors. Figure 1 illustrates the conceptual image of our system, where six phased 1.3-m hexagonal segmented mirrors function as a 3.6-m diameter synthetic aperture primary mirror. Compared to an equivalent-sized, rigid monolithic mirror, a segmented mirror telescope expands the effective aperture size while significantly suppressing mass and cost increases. Conversely, on-orbit wavefront control of multiple mirror segments becomes essential to achieve diffraction-limited imaging performance. Because wavefront control requires a wide correction dynamic range, particularly during the post-launch commissioning phase, the wavefront error must be hierarchically reduced through sequential phasing stages that address different dynamic ranges and correction modes. Fine phasing represents the final stage in this wavefront management flow; it achieves the ideal imaging performance by compensating for residual piston-tip-tilt wavefront errors, *i.e.*, rigid-body misalignment modes between mirror segments, which remain a technical hurdle for operating segmented mirror telescope systems. While conventional single-aperture telescopes often employ dedicated wavefront sensors (WFS) for precise measurements, conventional WFS are inherently insensitive to inter-segment piston errors. Besides, non-common-path error occurs because a dedicated optical path needs to be prepared. An alternative approach is image-based fine phasing, where wavefront errors are estimated and controlled solely through images captured by the primary mission sensor. Image-based techniques are broadly classified into model-free and model-based approaches, both of which have potential for fine phasing in a segmented mirror telescope used for GEO Earth observation.

^{*}hirose.makoto@jaxa.jp; phone +81-70-1170-3354



Figure 1. Conceptual image of a GEO optical satellite for Earth observation with a segmented mirror telescope system.

Model-free fine phasing methods iteratively adjust corrective actuators based on image sharpness, a metric calculated from observed images that is designed to have an extremum in the absence of wavefront errors. Essentially, this approach solves a multi-variable optimization problem with hardware, where the signals to the corrective actuators serve as optimization parameters and the image sharpness acts as the cost function in the optimization. While any optimization algorithm could be applied, we selected the Stochastic Parallel Gradient Descent (SPGD) algorithm due to its structural simplicity and robust convergence property [3]. The SPGD algorithm alternately applies positive and negative wavefront perturbations by slightly moving the corrective actuators and uses the resultant deviations in image sharpness to update the actuator control signals. Thus far, fine phasing based on the SPGD algorithm has primarily been applied to single-aperture optical systems [4], leaving its feasibility in segmented mirror telescopes not fully clarified. Furthermore, most existing studies assume observations of static scenes during wavefront correction. However, in GEO remote sensing, the observed scenes can vary dynamically over time due to moving objects or changing environmental conditions within the field of view. Consequently, it is critical to evaluate the correction capabilities when operating the imager under such dynamic scene conditions.

The alternative approach is model-based fine phasing, which reconstructs the wavefront error from a dataset containing multiple images of the same scene captured with known phase diversity, combined with a priori knowledge of the system's optical parameters. For example, the James Webb Space Telescope (JWST) utilizes phase retrieval during its fine phasing stage to estimate wavefront errors from images of point-like astronomical sources [5]. Phase retrieval recovers a pixel-wise wavefront map by iteratively imposing known constraints on the pupil and image planes. A key limitation of this approach, however, is the requirement of an isolated point source. While point-source phase retrieval is highly effective for post-launch commissioning of GEO Earth observation satellites, this study investigates the feasibility of applying the phase diversity method to estimate wavefront errors directly from extended scenes [6]. The phase diversity method determines wavefront errors by identifying a set of Zernike polynomial coefficients that minimizes the cost function. Consequently, it also relies on a multi-variable optimization algorithm for wavefront reconstruction. To ensure accurate wavefront estimation using phase diversity, it is crucial to use appropriate measurement conditions, such as the image acquisition count and inter-image wavefront diversity magnitude, tailored to the specific optical parameters of our segmented mirror telescope system. Additionally, estimation accuracy depends on the selection of the underlying optimization algorithms.

In this paper, we evaluate both image-based fine phasing for future applications to our segmented mirror telescope for GEO Earth observations. First, we examine the model-free fine phasing by the SPGD algorithm that iteratively drives corrective actuators based on image sharpness. Using a laboratory imaging system equipped with a multi-aperture segmented mirror device, we demonstrated successful iterative correction of piston-tip-tilt wavefront error while viewing a time-varying dynamic scene. Next, we numerically investigated the optimal measurement conditions for the model-based phase diversity method, characterizing the impacts of image count and diversity magnitude. We also evaluated a hybrid optimization strategy combining two distinct multi-variable algorithms to achieve efficient and accurate wavefront reconstruction. Finally, we compare model-free and model-based fine phasing methods based on experimental and simulation results.

2. MODEL-FREE FINE PHASING

Principle

We first formulate the model-free fine phasing using the SPGD algorithm, followed by experimental results obtained from our laboratory tabletop imaging system. The correction accuracy of model-free fine phasing methods is primarily dictated by two influential factors: the optimization algorithm and the image sharpness function. Regarding the optimization algorithm, several candidates exist, such as hill-climbing and simulated annealing algorithms [7, 8]. However, we selected the SPGD algorithm due to its structural simplicity and robust convergence stability. For the image sharpness metric, we select a deviation-based cost function based on prior investigations defined as [9]:

$$J = \sigma[i^2(\mathbf{r})]$$

where σ denotes the standard deviation operator, and i represents the captured image. This deviation-based cost function serves as a metric for image contrast, reaching its global maximum when the wavefront error is fully compensated. Consequently, the control voltages supplied to the corrective actuators are iteratively updated to maximize this cost function. The parameter search proceeds by applying positive and negative perturbation signals to the corrective actuators, creating small perturbation wavefronts. The cost function is then evaluated at both perturbed states to determine the gradient direction that restores image sharpness. Finally, the actuator control signals are updated based on the differential cost function. Letting $\mathbf{u}$ represent the control signal vector input to the actuators, the iterative update is formulated as:

$$\begin{aligned}\Delta J &= J_+(\mathbf{u} + \Delta\mathbf{u}) - J_-(\mathbf{u} - \Delta\mathbf{u}) \\ \mathbf{u}_{n+1} &= \mathbf{u}_n + \gamma \Delta J / \Delta\mathbf{u}\end{aligned}$$

where ΔJ is the differential cost function, J_+ is the positively perturbed cost function, J_- is the negatively perturbed cost function, $\Delta\mathbf{u}$ is the perturbation signal applied to corrective actuators, and γ is a feedback gain. The parameter update continues until the J_+ and J_- are balanced, and the cost function converges to a maximum. In the context of piston-tip-tilt correction for segmented mirror telescopes, each mirror segment requires at least three independent degrees of freedom as optimization parameters.

Laboratory experiment

To demonstrate the feasibility of this model-free fine phasing under piston-tip-tilt wavefront errors, we developed a laboratory-scale tabletop imaging system equipped with a multi-mirror device. Figure 2(a) illustrates the schematic of the experimental setup [10]. The observation target consists of a liquid crystal display (LCD) that projects a dynamic image sequence. Light emitted from the LCD is collimated and directed onto a multi-mirror device comprising 37 hexagonal segments. The piston-tip-tilt states of each segment are actively controlled by adjusting the voltages applied to three MEMS actuators bonded to its rear surface. The reflected light is then focused onto a CMOS image sensor through a tube lens. The captured images are processed in real time to calculate the deviation-based cost function, which subsequently drives the control loop for all 111 MEMS actuators. The fine phasing experiments were initiated by introducing an intentional misalignment across the multi-mirror device. The initial deflection error of the MEMS actuators relative to an ideal flat reference surface was 50.0 nm rms, as shown in the deflection map of Figure 2(b). The top-left panel of Figure 2(c) presents the degraded image resulting from this initial wavefront error, which exhibits significantly lower contrast compared to a reference image acquired without loading any intentional deflection error. Following initialization, we executed the iterative closed-loop control of the actuator voltages. To evaluate the phasing performance against a non-static target, the projected frame on the LCD was advanced synchronously with the actuator control loop at a frame rate of approximately 15 Hz. A fundamental challenge of model-free phasing using dynamic scenes is that the image sharpness metric fluctuates due to both the intentional hardware perturbations and the temporal evolution of the scene itself. Figure 2(c) chronologically compares the images captured during wavefront correction against their corresponding reference images. The image sequence depicts a dynamic scene of clouds traversing the field of view over time. Concurrently with this scene evolution, the image contrast improves as the wavefront correction progresses. After 1,000 iterations, the finally corrected image is nearly indistinguishable from the reference image. The final deflection error was reduced to 20.0 nm rms, as verified by the deflection map in Figure 2(d), and the post-correction cost function values matched those of the reference scene. The present results showed the applicability of model-free fine phasing of piston-tip-tilt wavefront error in segmented mirror telescopes even for dynamic extended scenes.

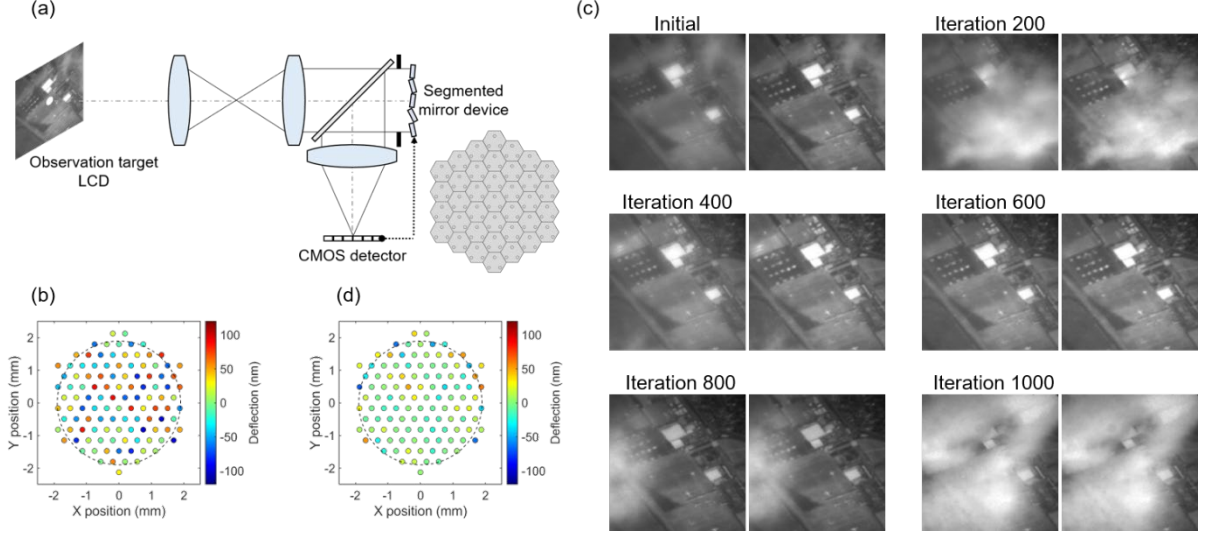


Figure 2. (a) Experimental setup. The lower right inset image is a schematic of the multi-mirror device with 37 hexagonal segments. Each segment has 3 MEMS actuators indicated in the circle in the image. (b) Deflection error map of 111 MEMS actuators before the wavefront correction. Each circle indicates the position of the MEMS actuator. (c) The image sequence is measured during the wavefront correction (left) and without the intentional wavefront error for comparison (right). The dashed circle represents the effective aperture size. (d) The same map as (c) after the wavefront correction.

3. MODEL-BASED FINE PHASING

Principle

The alternative approach to image-based fine phasing is the model-based phase diversity method, which reconstructs wavefront errors from an observed image dataset of extended scenes acquired directly by the primary mission sensor. This dataset comprises both in-focus and out-of-focus images of the same scene. In contrast to model-free fine phasing, which demands multiple hardware-in-the-loop actuator adjustments, the phase diversity method can deterministically complete fine phasing once the wavefront error is accurately estimated. This requires significantly fewer image acquisitions, though it necessitates building an accurate model of the optical system. Here, we numerically investigate the measurement and analytical conditions required for the phase diversity method within our segmented mirror telescope system. In this section, we first formulate the imaging process and wavefront estimation before presenting our simulation results. The phase diversity method assumes an incoherent, space-invariant image formation process, modeled as:

$$i_n(\mathbf{r}) = i_0(\mathbf{r}) * |h_n(\mathbf{r})|^2 + n(\mathbf{r})$$

$$h_n(\mathbf{r}) = \mathcal{F}[a(\mathbf{k}) \cdot \exp[i\{\theta(\mathbf{k}) + \varphi_n(\mathbf{k})\}]]$$

where i_n is the n th diversity image, n is observation noise, h_n is the optical response, $\mathcal{F}$ is the Fourier transform operator, a is the aperture function, θ is the wavefront error, and φ_n is the wavefront diversity. This diversity phase is most often chosen as a defocus mode due to its ease of implementation and its wavefront symmetry. The phase diversity reconstruction searches for an aberration parameter set based on the cost function E calculated as follows [11]:

$$E = \sum_{\mathbf{k}} \left[\left| \sum_{n=1}^N i_n(\mathbf{k}) h_n^*(\mathbf{k}) \right|^2 / \sum_{n=1}^N |h_n(\mathbf{k})|^2 \right] - \sum_{\mathbf{k}} \sum_{n=1}^N |i_n(\mathbf{k})|^2$$

The cost function is subject to minimization with respect to the aberration coefficient set, the detailed notation of which is presented in the next subsection.

Numerical simulations

We numerically evaluated the impact of the number of diversity images, the inter-image defocus intervals, and the choice of optimization algorithms on the wavefront estimation accuracy. Figure 3(a) illustrates our pupil model, which features six hexagonal mirror segments and an obscuration by spiders holding the secondary mirror. The wavefront error θ over the entire aperture was defined as a combination of segment-wise piston-tip-tilt wavefront error θ_l and global wavefront error θ_g as:

$$\begin{aligned}\theta(\mathbf{k}) &= \theta_l(\mathbf{k}) + \theta_g(\mathbf{k}) \\ \theta_l(\mathbf{k}) &= \sum_{n=1}^6 \sum_{m=1}^3 a_{n,m}(\mathbf{k}_n) \\ \theta_g(\mathbf{k}) &= \sum_{m=4}^6 b_m(\mathbf{k})\end{aligned}$$

where $a_{n,m}$ and b_n represent Zernike mode coefficients. The global wavefront error was given by defocus and astigmatism modes. An example of the loaded wavefront error is presented in Figure 3(a). Then, solving the optimization of $\text{argmin}_{\mathbf{a}, \mathbf{b}} E(\mathbf{a}, \mathbf{b})$, where $\mathbf{a} = (a_{1,1}, \dots, a_{6,3})$, and $\mathbf{b} = (b_4, b_5, b_6)$ parametrically reconstructs the wavefront error over the whole aperture.

Figure 3(b) shows the simulated observation of an extended scene captured through the aberrated optical system defined in Figure 3(a). The signal-to-noise ratio, defined by the ratio of the mean signal to the standard deviation of the Gaussian noise, was set to 200 throughout the simulations. The sampling factor $Q = F_\# \lambda / p$, where $F_\#$ is F-number, λ is wavelength, and p is pixel pitch, was set to unity. The under-sampling condition of $Q \sim 1$ is widely seen in Earth observation [12], but the over-sampling condition is preferable from the viewpoint of applying the phase diversity method. We need to clarify the working conditions even though the phase diversity method could still work at $Q \sim 1$, as demonstrated in previous literature [13]. Figure 3(c) illustrates an example phase diversity dataset comprising 7 diversity images generated with a 4-wave p-v defocus interval. For the initial phase of wavefront estimation, we employed a Real-Coded Genetic Algorithm (RCGA) for global optimization [14]. The fitness function in the RCGA was set to the phase diversity cost function. RCGA is known as a global optimization technique, taking advantage of its multi-point searching nature. The optimization condition in our calculation is as follows. The population of each generation was set to 50. The elite individual of each generation was propagated unmodified to the next generation, while the remaining individuals underwent sequential genetic operations of selection, crossover, and mutation. Parents were chosen via tournament selection, and offspring were generated using a unimodal normal distribution crossover (UNDX) to maintain population diversity. A mutation rate of 5% was enforced to prevent premature convergence. The optimization was run across 300 generations, with the best individual of the final generation yielding the estimated aberration coefficients.

Figure 3(d) plots the estimation accuracy as a function of the number of diversity images, keeping the defocus interval fixed at 4 waves. The curve displays a clear trend that increasing the number of diversity images reduces estimation error, and the use of five or more diversity images results in stable optimization. Conversely, Figure 3(e) plots the estimation accuracy against variations in the defocus interval size while keeping the dataset size fixed at seven images. Unlike the monotonic trend in the previous analysis, this profile exhibits a distinct minimum at a 4-wave diversity defocus. This indicates that too small defocus intervals are not sufficient for providing significant image diversity, while too large defocus causes the loss of information in the presence of observation noise. Therefore, it would be appropriate to acquire 7 diversity images with 4-wave defocus intervals. To further suppress the residual estimation error under these measurement conditions, we implemented an algorithmic handoff by applying the SPGD algorithm following the RCGA optimization. In other words, the last estimate of the RCGA served as the initial guess of the SPGD algorithm. Note that the SPGD algorithm used here for the wavefront estimation completes only on the computer, not requiring any hardware like the model-free fine phasing in the previous section. As opposed to the RCGA, the SPGD algorithm uses a single-point searching strategy, but it can finely tune the solution in the vicinity of the extremum because it refers to the cost function gradient ΔJ . The red plot in Figure 3(e) is the wavefront error after the refinement by the SPGD algorithm, and the estimation error was lowered to 1.57×10^{-3} waves rms. This simulation confirms that cascading the multi-point global search and the single-point local optimization provides a robust, efficient, and precise wavefront reconstruction for segmented mirror telescopes.

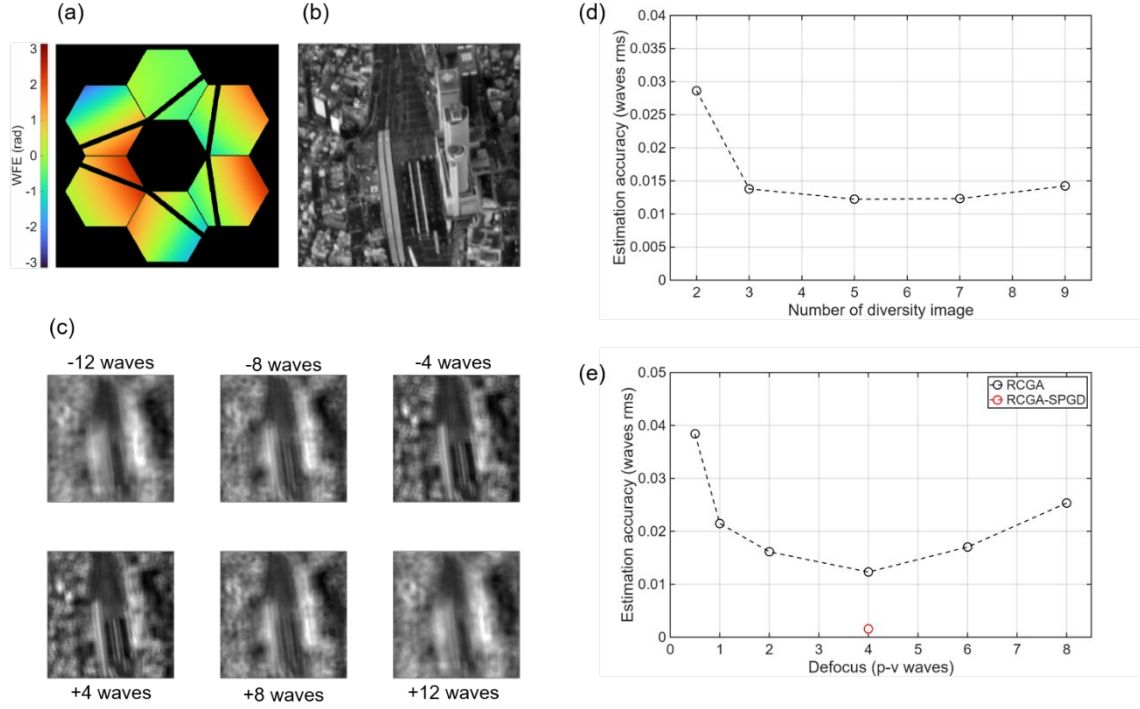


Figure 3. (a) An example of simulated wavefront error. The aperture model consists of six hexagonal mirror segments and six spider obscurations. (b) An aerial scene used in our simulation. (c) PD dataset other than the in-focus image of (b) calculated by adding four-wave defocus diversity between neighboring images. (d) Estimation error when the number of diversity images is changed. (e) Estimation error when inter-image defocus is changed. At the four-wave defocus condition, the wavefront estimation was sequentially performed using the RCGA (black) and the SPGD algorithm (red) to further improve the estimation accuracy.

4. DISCUSSION & CONCLUSION

In this study, we experimentally and numerically evaluated the feasibility of image-based fine phasing techniques in a synthetic aperture optical imager used for a GEO Earth observation satellite. GEO imagers are compatible with image-based approaches that use multiple images of the same scene because of their ability to continuously observe a designated region of interest. The model-free fine phasing using the image sharpness function and the SPGD algorithm succeeded in correcting piston-tip-tilt wavefront error in 37 mirror segments based on a time-varying dynamic scene. The model-based phase diversity method also enabled wavefront estimation from a dataset comprising seven diversity images of an extended aerial scene captured with 4-wave inter-image defocus diversity. In addition, within the wavefront estimation analysis by the phase diversity method, we showed that a combination of optimization algorithms based on multi-point global search and single-point local search, such as cascading use of the RCGA and the SPGD algorithm, is useful for better reducing the estimation error. At the same time, our evaluation elucidated drawbacks of each approach. The model-free methods require on the order of 1,000 times multiple actuator control operations when the imaging system has a large initial piston-tip-tilt error, taking a long optimization time even with implementing on-board processing. Besides, the correction accuracy depends on the scene: unless we continue observing less-changing scenes under clear meteorological conditions, high-accuracy wavefront control could suffer. More specifically, the image sharpness deviation derived from applied wavefront perturbation must exceed that caused by the image evolution itself. Otherwise, the SPGD algorithm does not reach convergence. The model-based methods, on the other hand, require far fewer corrective actuator operations than the model-free approach. A downside is that the estimation accuracy depends on the correctness of the optical model used in the analysis, including the diversity function applied to the optical system.

Another indication is that the model-based and model-free approaches can work complementarily in the phasing process. For instance, the phase diversity method is effective for initial wavefront control after launch and regular wavefront inspection during satellite operation. When observing a point light source at this phasing stage, the phase retrieval method is also available. Meanwhile, the model-free approach has an advantage in fine-tuning the small residual wavefront error left uncorrected by model-based methods due to model mismatch, including mechanical positioning errors of actuators. Note that the model-free approach can handle not only extended scenes but also point light sources by changing the image sharpness function. Also, the model-free fine phasing methods accept broadband illumination, while the phase diversity and phase retrieval methods prefer monochromatic illumination. This operation sequence will alleviate the technical difficulties in modeling accuracy on one side and reduce the hardware optimization time on the other. Another potential usage of the model-free fine phasing is found when some actuators for controlling mirrors malfunction. Even under such a situation, the model-free fine phasing methods can find optimal conditions for active actuators to maximize image quality. Although we need further elaboration on the wavefront management architecture, including fine phasing methods and operation, we believe that the image-based fine phasing approaches investigated here would help realize a high-resolution GEO optical imager.

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