Using geometrical algorithms to facilitate hand-off between SPH and N-body modelling of ejecta evolution

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Abstract

The numerical modelling of chaotic kinetic events involving asteroids can be divided into two stages. First, a high-energy event where e.g. an impactor collides with a target, leading to cratering, shattering or dispersion, which is commonly simulated using hydrodynamical codes, such as 3D smoothed-particle hydrodynamics (SPH). The next stage is represented by a lower-energy dynamical evolution, modelled with gravitational N-body codes.

We have developed an interface between SPH and N-body codes that identifies clusters of SPH particles and groups them together as fragments with physical properties based on their constituents. Our approach is unique in that it also assigns a realistic physical shape to the fragment based on the distribution of its SPH particles using a geometrical algorithm known as α-shape.

Modelling asteroid impacts

Chaotic events such as impacts are usually divided into two different regimes:

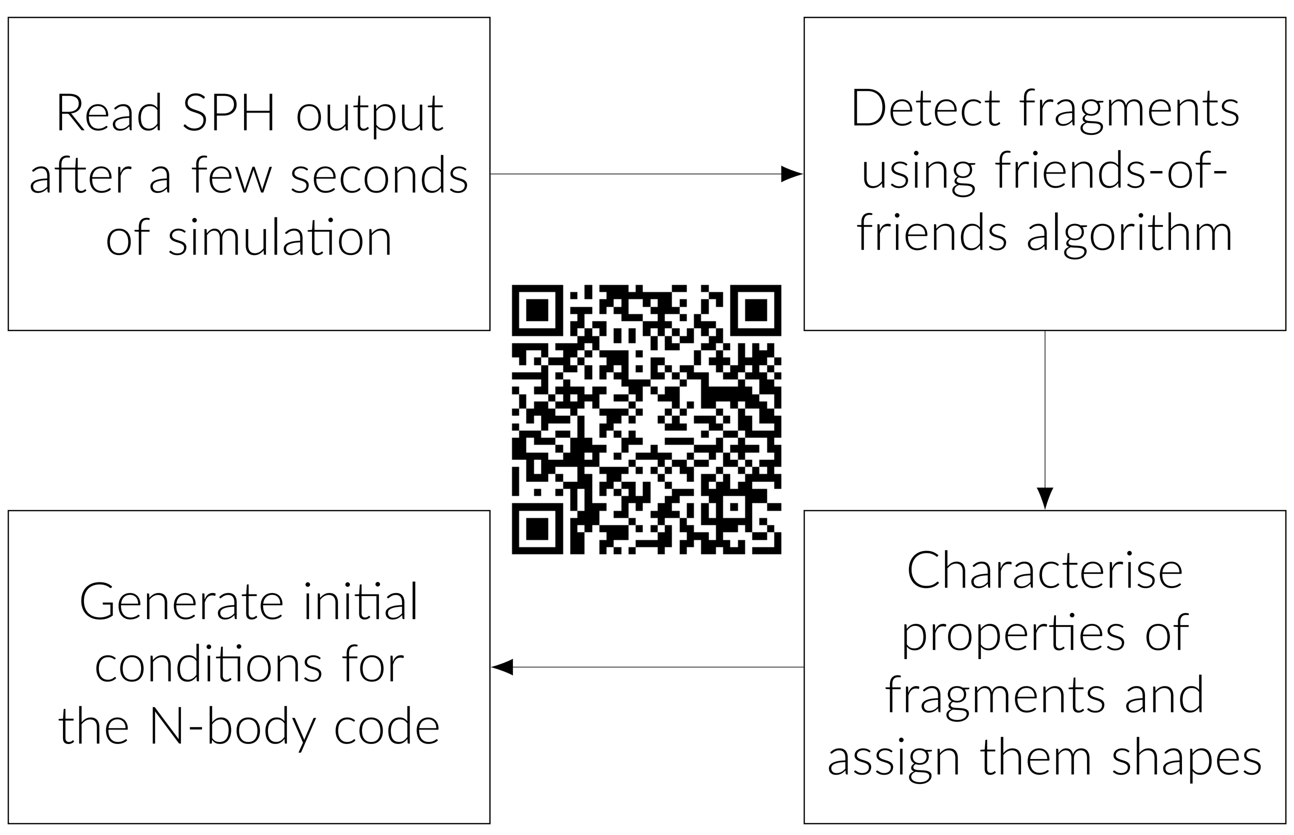
- **High-energy:** E.g. a collision followed by shock propagation and then fragmentation and cratering
- **Low-energy:** Reaccumulation of fragment particles into gravitational aggregates and dynamical evolution

Simulating the high-energy regime requires tracking of rapid changes in energy over short timescales, as well as deformation. Such dynamics are modelled using SPH. The low-energy regime is simulated using gravitational N-body.

Moving from SPH to N-body

There are three main issues that motivate the need for an interface to properly hand-off the numerical problem between the two different codes:

1. **Resolution:** SPH needs a large number of particles, on the order of $N \sim 10^6$, for realistic continuum approximations while N-body performance generally suffers for $N > 10^5$
2. **One-to-one translation:** SPH particles are essentially infinitesimal fluid elements while N-body particles directly represent matter
3. **Overlapping bodies:** SPH particles often end up overlapping in time and space. Direct translation then leads to diverging solutions for N-body



For a more detailed diagram, scan the QR code.

α shape

α-shape is a method used to determine a shape for a finite set of points, created by Edelsbrunner & Mücke (1994). It is based on Delaunay triangulation which maximises the minimum angle in all tetrahedrons formed by particles.

Imagine a ball of chocolate chip ice-cream. A set of points S is represented by chocolate pieces. We want to use a scoop to carve out as much ice-cream as possible without including any chocolate chips. Straightening the resulting round faces into triangles and straight lines, we end up with the α-shape of S. The squared radius of the ice-cream scoop is our α value and if $\alpha \to \infty$, we end up with a convex hull.

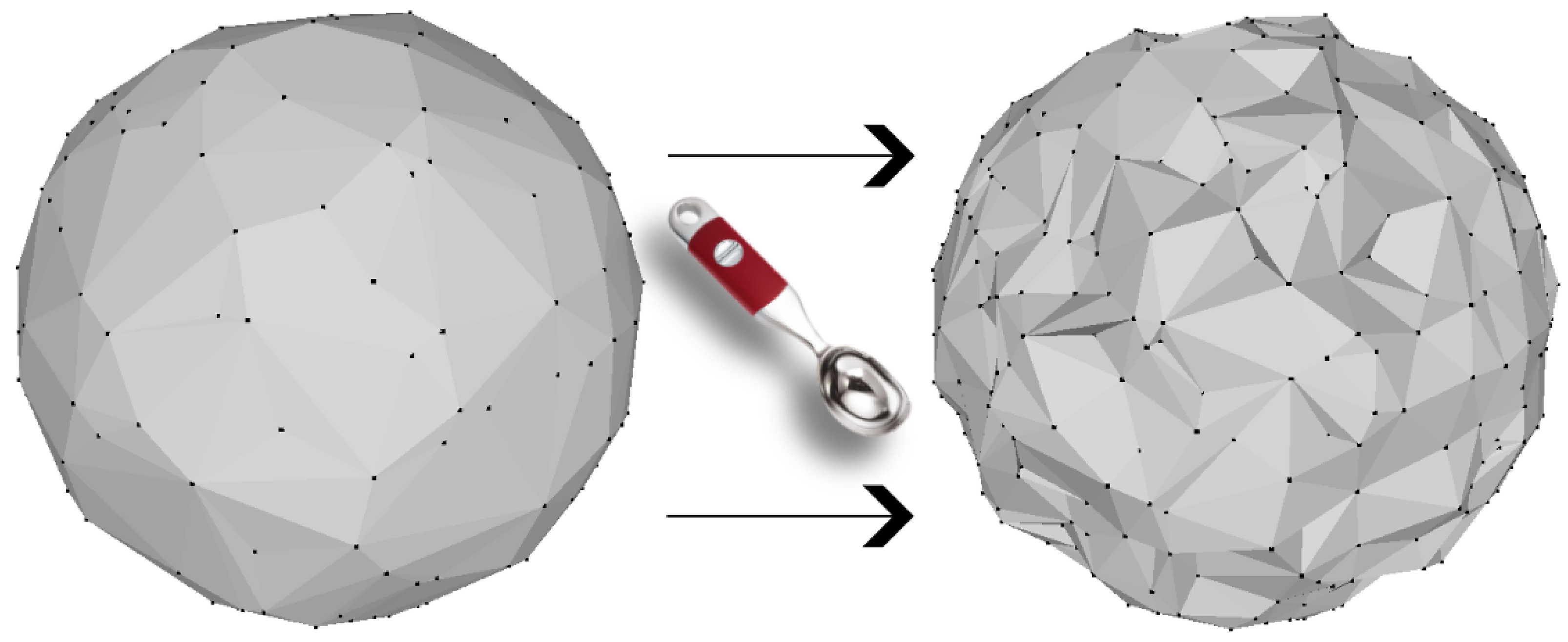


Figure 1. Applying an α-shape to a set of points, S, allows us to carve out a realistic shape. The α-shape of S on the left has $\alpha \to \infty$ making it the convex hull of S. For the right shape, α is the squared radius of the spoon.

Implementation of the interface

- `Bern SPH`: a 3D SPH code (Jutzi et al. 2008)
- `GRAINS`: a parallel GPU code that uses irregularly shaped particles (Ferrari et al. 2017; Ferrari et al. 2020)
- `CGAL`: The Computational Geometry Algorithms Library for `c++` (The CGAL Project 2023)

α-shape algorithms have been used before for hand-off between SPH and N-body by Ballouz et al. (2018), but only for the largest remnant in the system. The `CGAL::Alpha_shape_3` library allows us to generate α-shapes for all fragments with at least 10-15 SPH particle constituents.

Smaller fragments and single particles are instead represented by randomly generated convex hulls. An example of the resulting output can be seen in figure 2 where the number of particles has been reduced from 25 247 to 931.

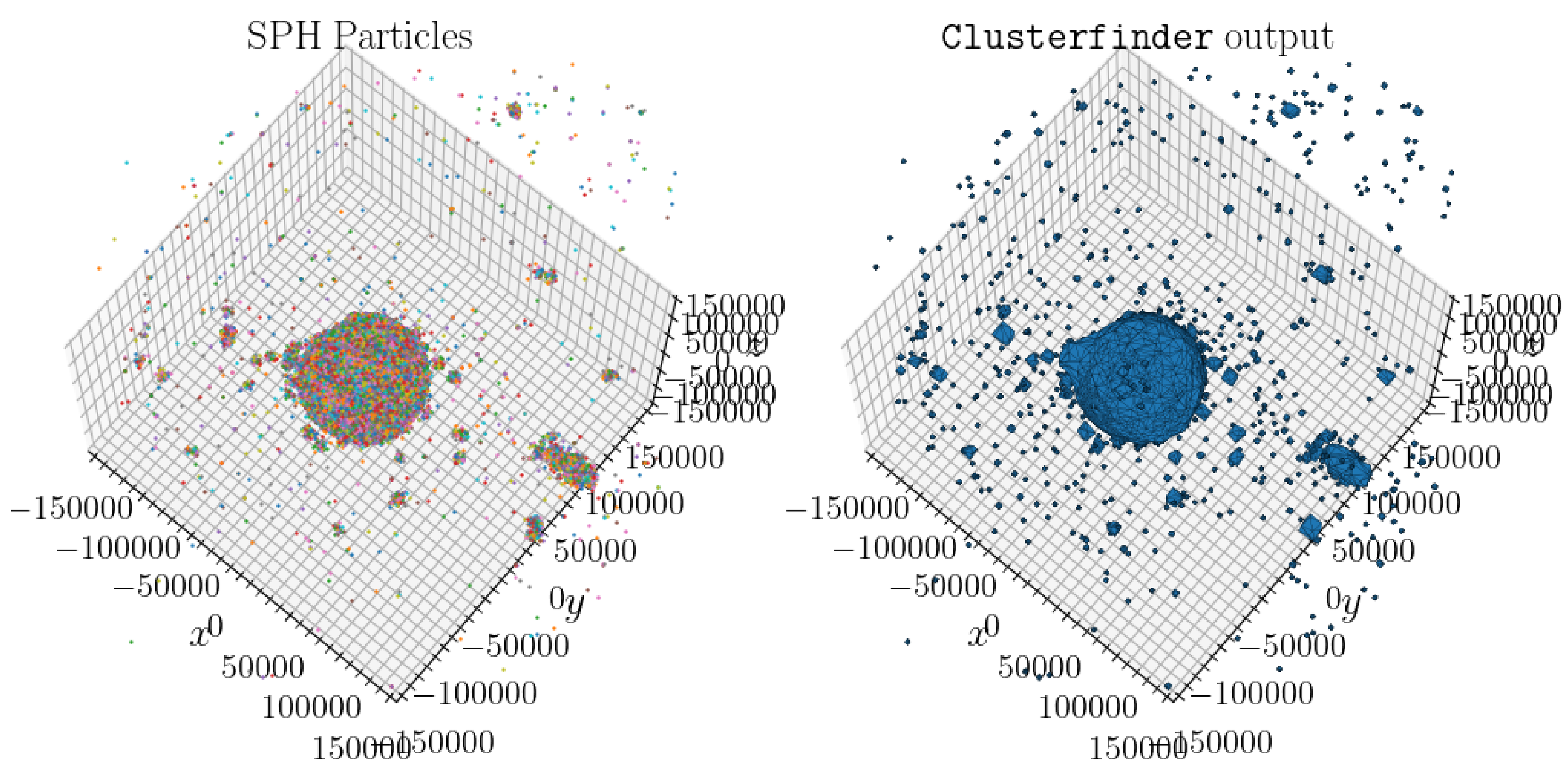


Figure 2. Hand-off based on an SPH simulation of a rapidly spinning asteroid (e.g. Didymos) which has led to deformation and shedding of mass. Our interface identifies clusters of particles and generates fragments with realistic physical properties and shapes. Credit to Zhen Xiang for the SPH simulation output.

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